Mechanics of Material

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Constitutive relation, strain energy and potential
Lecture – 37
Compatibility condition

We saw that there are 4 concepts in mechanics and 4 equations that connects them right that is what we have been looking at. So, we have seen all the 4 concepts we have seen what forces.

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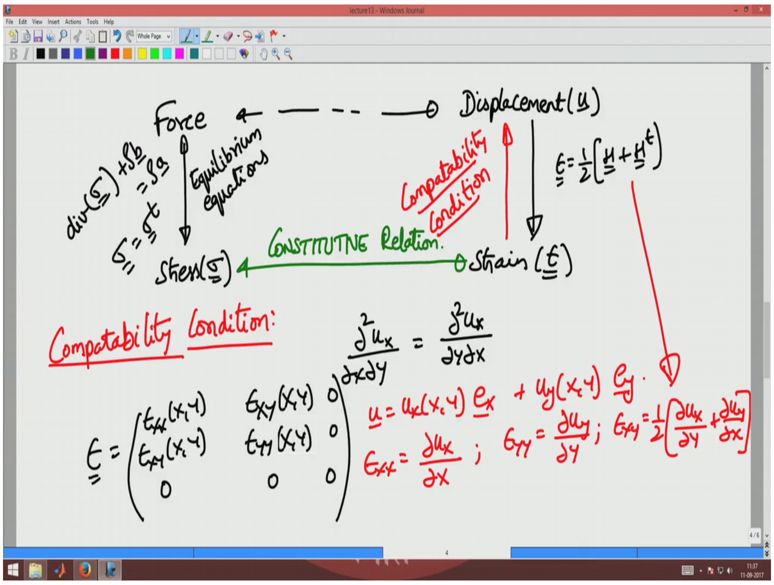

We have seen what Displacement is and then we have seen now what Strain is and we have seen what Stress is right. So, we have seen the equation that connects the displacement to the strain say strain and displacement relation to be 1 half H plus H transpose and then we have seen the equation that connects the stress and the force which is equilibrium equations which is divergence of sigma plus rho b equal to rho a and sigma equal to sigma transpose as seen these equations.

Now, what we are going to see is the equation that connects the strain into the displacement that is the compatibility condition this is what we are going to look at now what did we say about a displacement field we said that displacement field would be in such a way that if I have a surface the surface will move like this, but there would not be opening up of surfaces or their would not be inter penetration of surfaces right that is a material particle will be mapped to a material particle in the current configuration there will would not be a case or in a material particle becomes 2 different material particles or there would not be a case having 2 different material particles will become fused to a single material particle.

So, that is what we said about the displacement field if that were to be a case then what you know is the order of differentiation of the displacement field will not matter, mathematically what this means is if you have dou square u x by dou x dou y this will be same as dou square u x dou y dou x. That is the order in which you differentiate a component of the displacement field will be immaterial because it is a smooth continuous displacement field. So, what happens then is now I have 6 components of strain that I can prescribe I ask 6 components of strain that I can prescribe from which I had to find only 3 components or a displacement field, there is a disparity in the number of unknowns. So, what will happens there will be a restriction on how I can prescribe the component of the strain because this prescription of the component of the strain should be consistent.

So, that I find a smooth continuous displacement field from the prescribed strain components to give an idea let us look at the plane strain case let assume the strain is given by Epsilon x x function of x comma y Epsilon xy some function of x comma y 0 Epsilon xy some function of x comma y Epsilon yy some function of x comma y 0 0 0 0 say I prescribe this Epsilon x x Epsilon xy and Epsilon yy.

Now from this prescription I have to be able to find the component or displacement dou ux and duo uy. So, for this I allow the displacement field as u to be ux function of x comma y ex plus uy function of x comma y ey. So, that Epsilon x x becomes dou u x by dou x Epsilon y y becomes dou u y by dou y and Epsilon xy becomes 1 half dou x by dou y plus dou uy by dou x, this comes from the fact that I use this in near to compute the strain components in terms of a displacement field.

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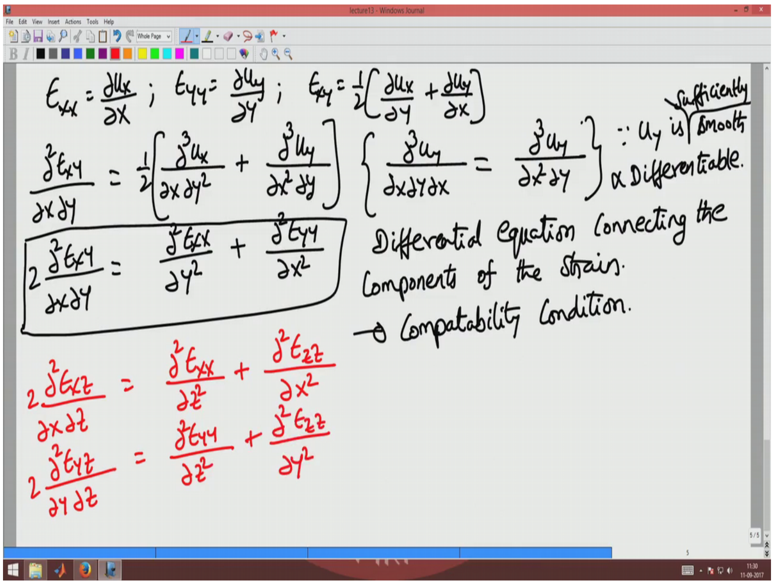

Now let us take forward these equations I add Epsilon xx as dou u x by dou x I add Epsilon yy as dou u y by dou y I add Epsilon xy as 1 half dou u x by dou y plus dou y by dou x.

Now let us compute dou square epsilon xy by dou x dou y what is this going to be this is going to be 1 half dou cube u x by dou x dou y square plus dou q u y dou x square dou y. Here again I use the fact that dou x dou y dou x are used dou cube uy by dou x dou y dou x can be written as dou cube uy by dou x square dou y since u of y is smooth and differentiable is sufficiently smooth and differentiable because I should have second derivatives third derivatives of u of y possible sufficiently smooth and differentiable.

Now what is this what is 2 times dou square Epsilon xy by dou x dou y this is nothing, but dou square Epsilon xx by dou y square plus this dou square Epsilon yy by dou x square. So, basically what do you obtain is a differential equation connecting the components of this strain differential equation connecting the components of the strain, this is called as compatibility condition.

If the displacement field is sufficiently smooth and differentiable there will exist some relationship between the components of the strain only prescriptions of strain and satisfy those relationships or admissible strain fields. So, that is why the computed condition tells us this is if I assume the strain is plane in terms of x and y directions, similarly I can write the condition for the strain being plane in terms of the other 2 planes.

If the plane is plane in the xz plane I will Epsilon xz loaded by dou x dou z to be given by dou square Epsilon xx by dou z square, but I am writing this way analogy which you can show also will all true in the real case. So, on z z by dou x square and similarly I have dou square Epsilon y z by dou y dou z to be equal to dou square Epsilon yy by dou z square plus dou square Epsilon z z by dou y square. So, this is the strain is plane in the x and z coordinate or the strain is plane in the y and z coordinate.

Apart from this the general set of comparability conditions in 3 dimensions are own derived this

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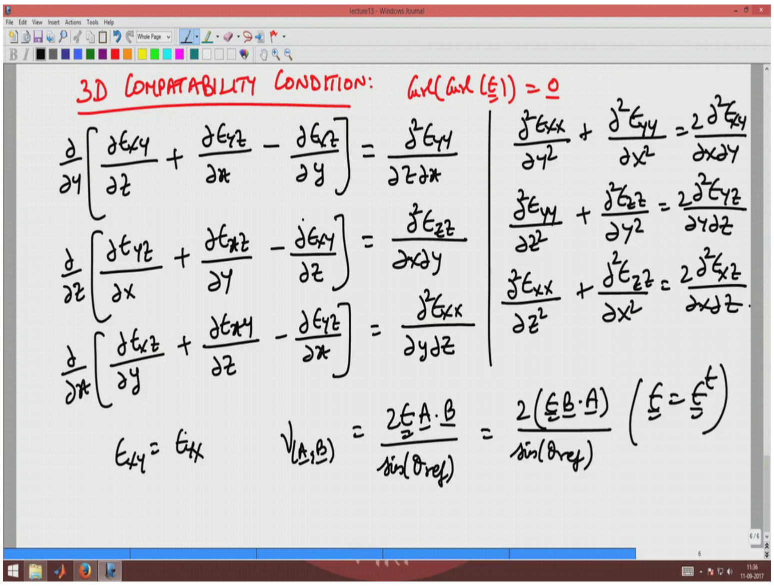

Condition is in general represented as curl of curl of Epsilon being equal to 0 only 6 of them would be independent those 6 are the 3 that are listed here is one set of equation the other equations are what I will give now dou z and a 3 equation that we add before were, these are the comparability conditions. So, these 6 equations are the compatibility conditions.

Now since strain is a symmetric tensor you can write Epsilon xy is equal to Epsilon y x. So, I would not bother about being precise about writing xy yz or xz. So, either you can write it with a symmetric tensor it is for the same reason that when you look at the angle change A dotted with B between A and B this will be 2 times Epsilon A dotted with B divided by sin of theta ref from the definition of transpose this will be same as 2 times Epsilon B dotted with A divided by sin of theta ref. So, it does not matter with what vector you prefix the Epsilon it does not matter on what vector the linear chain acts this will be same because Epsilon is equal to Epsilon transpose by the way we are defined Epsilon as. So, we have seen what a compatibility condition now is the only equation that is remaining is going back to this chart we have to connect the stress and the strain the equation that connects the stress and strain is called as the Constitutive Relation.