Mechanics of Material
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Constitutive relation, strain energy and potential
Lecture – 36
Transformation of strain components/ Strain Rosette

Welcome to 13th lecture in mechanics of materials the last lecture we saw how to compute the change in length by original length for line element original longer direction.

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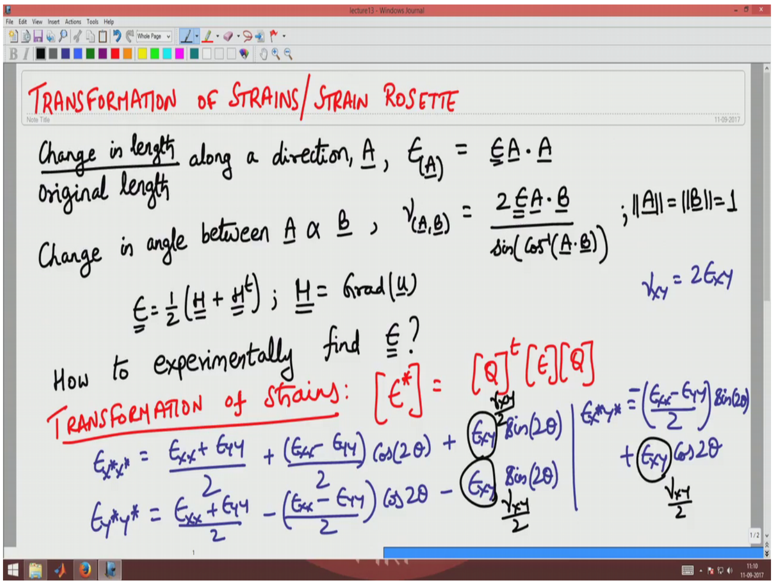

We saw that the expression for finding the change in length along a direction A is given by this expression where Epsilon is E linear A is strain or this one half of credent of U plus credent of U transpose. So, if you take the strain tensor or linear strain tensor as 1 half of the gradient of U plus gradient of U transpose that tensor multiplied by the direction along which you want find the change in length by original length and dotted with the same direction to give you the change in length by original length. Similarly the change in angle between 2 line elements A and B is given by this expression in here which is 2 times Epsilon A dotted with B by sin of the angle between A and B which is cos inverse of A dotted with B where A and B are unit vectors.

Now, will address question on how to experimentally find the strain tensor now we cannot measure changes in the angle directly using a instrument and hence you have to in for the change in angle also by measure changes in length along 3 different directions, for this you have to use for what is called as a transformation or strain concept or the more circle for strains to understand how to compute the strain especially strain components Epsilon x y because you cannot measure a changes in angle directly.

Towards that will look at Transformation of Strains first strains in the last class we saw that if you are write strain as 1 half gradient of U plus gradient of U transpose this is second order tensor and we show that how in the previous lectures how a second order tensor will transform when you looked at the stress tensor, since strain is also second order tensor will transform similarly.

If I write strain in one coordinate system and want to compute this strain tensor in the star coordinate system this will be what we show before as Q transpose Epsilon Q, where Q I J we define it as E J star dotted with T I. So, basically from that we saw the strain also will transform similar manner like the stress transform. So, whatever equation we have for the plane stress transformations will all for plane strain transformation also.

In particular these expression that we got for sigma x star, x star will whole for Epsilon x star x star also plus Epsilon y y by 2, plus Epsilon x x minus Epsilon y y by 2 cos 2 theta plus Epsilon x y sin of 2 theta and Epsilon y star y star would be Epsilon x x plus Epsilon y y by 2 minus x x minus Epsilon y y by 2 cos of 2 theta minus Epsilon x y sin of 2 theta and Epsilon x star y star would be given by Epsilon x x minus Epsilon y y by 2 it is a negative sign sin of 2 theta plus Epsilon x y cos of 2 theta. So, these are the more circle equation that we obtain for the stress transformation the same equations holds here.

However, if what to replace Epsilon x y by change in angle between x and y which is related to Epsilon x y as gamma x y was 2 I Epsilon x y like what we show in the last class then what happens is here it will become gamma x y by 2 and here this will be gamma x y by 2 and here also it will be gamma x y by 2. In that case we saw that gamma x y if you write in strain in terms of gamma x y no longer second order tensor and then the transformation rules defer by a 2 there. So, this why it is important to treat the same also second order tensor so that you use same transformation like in the most circle for stresses you can use this same expression for the more circle of the strains.

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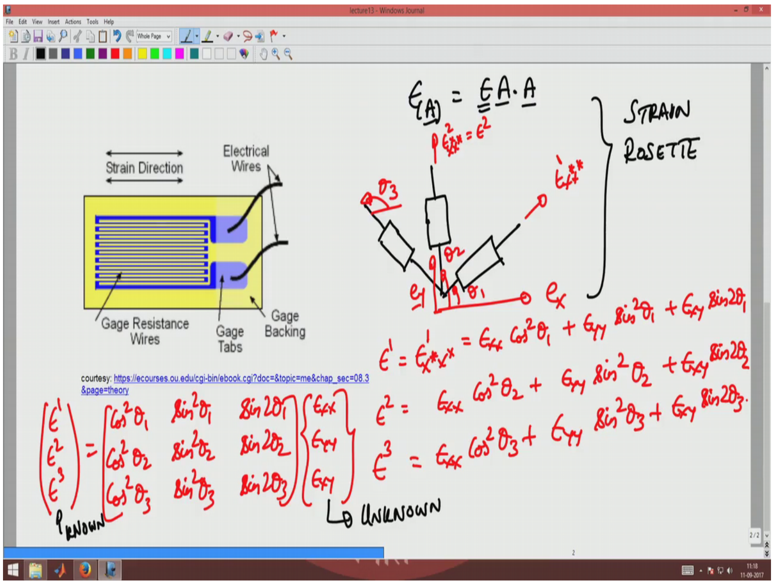

Let see how electrical strain gage functions this is a schematic of electrical strain gage what it measure is the change in resistance you know that the resistance is proportional to the length of the member. So, the length change is the resistance will change and hence it will cause that change in resistance is what you measure as change in the balance in the Wheatstone shown bridge network.

So, basically now this is an example of real electrical strain gauge.

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So what you see there is essentially this small thing which is there it is roughly 5 mm in gauge length this length is essentially 5 mm this red dimension is essentially 5 mm which is gauge length of this electrical strain gauge.

Now, what you measure is the change in length along this direction that is the strain direction that is where the windings of the metallic wires are there, that is where the resistance wires or the gauge resistance wires are there and you measure how much change in length thus this gage resistance wires up. So, what you measure is this quantity Epsilon A that is Epsilon A dotted with A this what you measure. Now I will measure this along 3 different direction you can get the 3 component of the strain tensor that is what is do is you measure you put 3 of this together you measure 3 and if I take e x and e y as my coordinate system and say this my x angle theta 1 with the x, this makes an angle theta 2 the x and this forms an angle theta 3 with respect to the x axis, then what I want to do is find this with respect to plus 3 orientation.

In other words if I had to find Epsilon x star x star 1 which is for this theta 1 direction it will be Epsilon x x cos square theta 1 plus Epsilon y y sin square theta 1 plus Epsilon x y sin 2 theta 1, what I have done is the previous expressions I have combined again the previous expression I have combined 1 plus cos 2 theta by 2 as cos square theta 1 have combined 1 minus cos 2 theta by 2 as sin square theta 1 that is what I have done here because that will yield me a equation which is convenient to handle. So, this is Epsilon x x star 1 which is nothing, but the strain measured along this is Epsilon x x star 1. Then similarly let us say this Epsilon 1 Epsilon 2 is a strain measure along this direction is Epsilon x x star 2 is an Epsilon 2 that will be Epsilon x x cos square theta 2 plus Epsilon y y sin square theta 2 plus Epsilon x y sin 2 theta 2. Similarly for a third direction will have Epsilon 3 as Epsilon x x cos square theta 3 plus Epsilon y y sin square theta 3 plus Epsilon x y sin 2 theta 3.

Now, this is 3 equations in terms of 3 unknowns the Epsilon x x y y and Epsilon x y which you can solve to get Epsilon 2 Epsilon 3 is equal to I have cos square theta 1 sin square theta 1 sin 2 theta 1 cos square theta 2 sin square theta 2 sin 2 theta 2 cos square theta 3 sin square theta 3 sin 2 theta 3 times Epsilon x x Epsilon y y Epsilon x y here I do not know this is known, this is unknown, I inverse this matrix and I can find the unknowns in terms of this know vector, this how you measure Epsilon x y. There are different Strain Rosette this is called as strain rosette there are different kind of Strain Rosettes.

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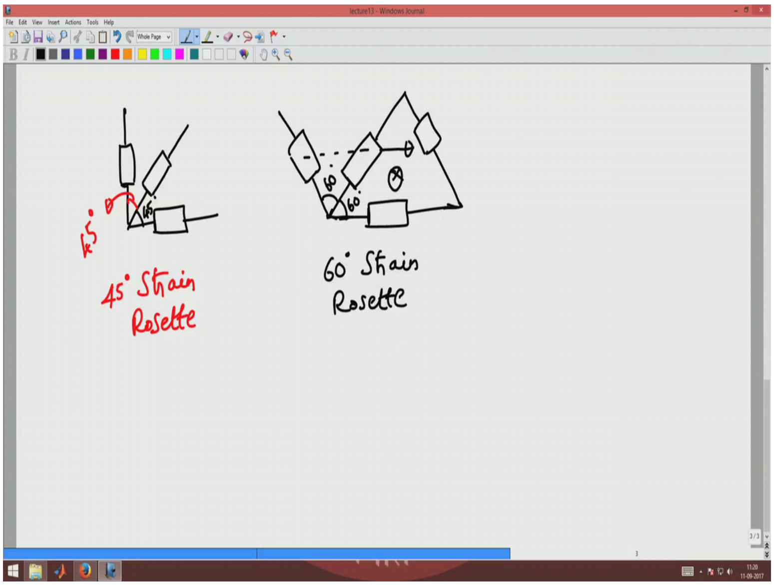

One which makes an angle 45 degrees where this angle is 45 degrees and this angle is also 45 degrees this is called as 45 degree Strain Rosette.

Similarly, there is a 60 angle strain rosette where in now the angle between the strain gauge is 60 degrees now this is 60 degrees and this also 60 degrees this is called as a 60 degrees Strain Rosette, typically what will happen is in real life this branch, this branch would be drawn here and it will be like a equilateral triangle this branch should be translated here and it will be like an equilateral triangle.

So, that is how a really rosette will look like and this rosette is supposed you measure this strain and the C J of this triangle equilateral triangle. So, the strain is at this point the measurement point is at point. So, what you do is you measure changes in length and from the changes and length you measure a change is an angle and if you are going to follow this procedure to compute a strain you will use what is call strain rosette to compute the in plane strain components.