

Mechanics of Material
Dr. U. Saravanan
Department of Civil Engineering
Indian Institute of Technology, Madras

Buckling of columns

Lecture – 102

Determination of maximum load carrying capacity of a simple truss

(Refer Slide Time: 00:16)

Example
 $\cos \theta = 3/5$
 $\sin \theta = 4/5$

Find the maximum load, P , that can be applied, assuming that the plane truss fails when its members yield or buckle in the plane. Given the members of the truss has a circular cross section of diameter 15mm & made of a material with Young's Modulus, $E = 200 \text{ GPa}$ & uniaxial yield stress of 250 MPa. Also it is required that the displacement of point C should be less than 0.15 mm.

Joint B

$$\sum F_x = 0 \quad -F_{BA} \cos \theta + F_{BC} \cos \theta + P/2 = 0$$

$$\sum F_y = 0 \quad -F_{BA} \sin \theta + F_{BC} \sin \theta + P = 0$$

$$F_{BA} = -\frac{5P}{24} \quad \& \quad F_{BC} = \frac{-25P}{24}$$

Now, we will move on to the second example, in the second example we are interested in solving a truss problem what we want is we want to descend a truss. So, that it will take the load p , there is rv the members of the truss are fixed, you want to find what is a maximum load P that can be applied. So, that the members zone fail and as a restriction on displacement of this point C. The restriction is I want to C to B displaced by less than 0.15 mm ok.

I want to C to B displaced by less than 0.15 mm and I want to find what is the maximum load it can be applied so, that this serviceability condition on displacement of C as for as a strength requirements of the members AB BC and AC are met.

Now, we will solve for the member forces first, find which of these member forces is in tension which are these members are in compression then we have to limit the compressive stress so that it does not exceed the all the critical load for this member and

then it should also ensure that the material strength is not exceeded in compression and then in tension also the material strength should not exceed for the given load.

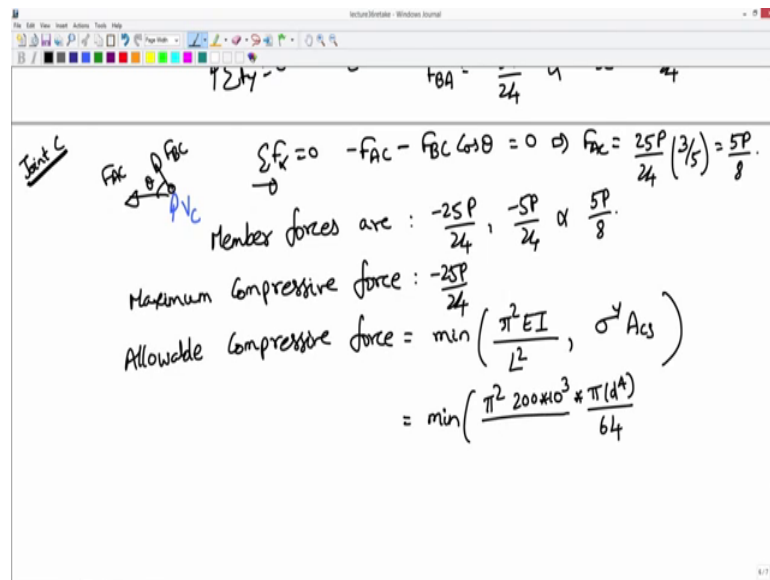
So, let us go about systematically solving this problem first task is to find the member forces. So, I will use method of joints to find the member forces, first I will look at joint B I am looking at joint B, where in there are two members coming either joint I assume the tensile force acting on this members as positive. So, this will be F_{BA} and this will be F_{BC} and I have a load P acting vertically downward $P/2$ acting like that let me assume a coordinate system to be x and y like that orient will like that.

Now, I want write the equilibrium of forces, I want write the horizontal equilibrium of forces $\sum F_x = 0$ assuming the force acting in this direction as positive. So, this angle would be θ from the figure and this angle also would be θ . So, horizontal force for member AB would be acting opposite to the positive direction of x assume positive direction of the force. So, it will be $-F_{BA} \cos \theta + F_{BC} \cos \theta + P/2 = 0$ this is the horizontal force equilibrium statement.

Similarly, vertical force equilibrium I am assuming the upward acting forces positive equal to 0 here all the forces are acting the negative y direction. So, all will be negative. So, you will have $-F_{BA} \sin \theta + F_{BC} \sin \theta + P = 0$ ok. For this $\cos \theta$ is for this $\cos \theta$ would be $3/5$ and $\sin \theta$ would be $4/5$ or 0.8 is this length of the hypotenuse this length would be 5 from pythagoras theorem you can get that. So, that will be 0.6 and hence $\cos \theta$ is $3/5$ and $\sin \theta$ is $4/5$ ok.

So, substituting that in here I am solving for F_{AB} and F_{BC} what will get is we will get $F_{BA} = 5P/24$ and F_{BC} would be $-25P/24$ what does in it is an indicate? It indicates that member AB and BC or in compression. So, that is what it indicates ok.

(Refer Slide Time: 04:56)



Joint C

$\sum F_x = 0 \quad -F_{AC} - F_{BC} \cos \theta = 0 \Rightarrow F_{AC} = \frac{25P}{24} \left(\frac{3}{5}\right) = \frac{5P}{8}$

Member forces are: $-\frac{25P}{24}, -\frac{5P}{24}, \frac{5P}{8}$

Maximum Compressive force: $-\frac{25P}{24}$

Allowable Compressive force = $\min\left(\frac{\pi^2 EI}{L^2}, \sigma_y A_{cs}\right)$

$= \min\left(\frac{\pi^2 \cdot 200 \cdot 10^3 \cdot \pi(d^4)}{64}, \sigma_y A_{cs}\right)$

Similarly, let me solve for the member force AC by looking at joint C I am going to look at joint C now ok. In joint C I have F BC again I am assume tensile force are positive, F AC I will have a reaction force VB there or VC there I will have reaction force VC there ok.

Now, I will write the horizontal force equilibrium first F_x equal to 0 right in the horizontal force equilibrium this angle is theta. So, what I have is, I have F AC with a negative sign minus F BC cos theta must be equal to 0 ok. This implies F AC is 25 P by 24 to cos theta is 3 by 5. So, that will be 5 P by 8 and then I can find VC I writing the vertical force equilibrium, which is the unnecessary in this problem. So, I will would not do that.

So, what is the maximum what are the member forces? Member forces are minus 25 P by 24 minus 5 P by a 24 and 5 P by 8. So, what is the maximum compressive stress? Compressive force maximum compressive force is minus 25 P by 24 in member BC ok.

Now, what is the allowable compressive force? Allowable compressive force would be minimum off we are all a buckling load it is a pin pinned truss a pin pinned structure. So, it will be pi square E I divided by L square and the material strength, which is sigma y the yield stress value in uniaxial extension uniaxial this thing multiplied by area of the cross section ok.

For our case it is given as in our case is given as a circular section it is a circular section of diameter 15 mm made of material with Young's modulus 200 and uniaxial yield stress of 250 MPA 250 MPA ok. So, this will be minimum of pi square 200 into 10 power 3 MPA for e into pi into d power 4 divided by 64 for I divided by L square L square for member for member BC we found that its of length 0.5 meters we found that the member BCs of length same as member AB which 0.5 meters ok.

So, that will be I have to use in millimeters. So, that will be 500 whole cube whole square comma sigma y is 250 into pi into d square by 4 I want to find the minimum of these two, minimum of these two.

(Refer Slide Time: 09:16)

The image shows a handwritten derivation on a whiteboard. The top section calculates the maximum and allowable compressive forces. The maximum compressive force is given as $-\frac{25P}{24}$. The allowable compressive force is the minimum of two values: $\frac{\pi^2 EI}{L^2}$ and $\sigma_y A_{cs}$. The first term is calculated as $\frac{\pi^2 \cdot 200 \cdot 10^3 \cdot \pi (d^4)}{(500)^2 \cdot 64}$, which simplifies to 19.6 kN. The second term is $\frac{250 \pi d^2}{4}$, which simplifies to 44.1 kN. The minimum of 19.6 and 44.1 is 19.6 kN. The final constraint is $\frac{25P}{24} \leq 19.6 \text{ kN} \Rightarrow P \leq 18.8 \text{ kN}$. The bottom section calculates the maximum and allowable tensile forces. The maximum tensile force is $\frac{5P}{8}$ and the allowable tensile force is $\sigma_y A_{cs} = 44.1 \text{ kN}$.

Which is nothing, but if I evaluated it will be minimum of 19.6 and 44.1, which is 19.6 kilo Newton's this is in kilo Newton's and in once its kilo Newton's.

So, what should I do? I will limit this 25 P by 24 to be 19.6 kilo Newton's, which will imply that P should be less than or equal to that. So, P should be lesser than or equal to 18.8 kilo Newton 18.8 kilo Newton's ok.

Now, I know the maximum tensile forces, forces in member AC which is 5 P by 8 and allowable tensile force would be sigma y to area of cross section which will be 44.1 kilo Newton's this will imply that P should be less than or equal to 8 into 44.1 divided by 5

which will be 70.6 kilo Newton's. Clearly this is not the governing case 18.8 kilo Newton's as the governing case.

(Refer Slide Time: 11:03)

Maximum tensile force - $\frac{5P}{8}$
 Allowable tensile force - $\sigma_{ACs} = 44.1 \text{ kN}$
 $\Rightarrow P \leq \frac{8 \times 44.1}{5} = 70.6 \text{ kN}$

$\delta_{AC} = \left(\frac{F_{AC}}{A_{AC}}\right) \frac{L_{AC}}{E_{AC}} \leq 0.15 \text{ mm.}$
 $\frac{5P}{8 \left(\frac{\pi(15^2)}{4}\right)} \frac{600}{200 \times 10^3} \leq 0.15 \text{ mm}$
 $\Rightarrow P \leq 14.1 \text{ kN}$

$P \leq 14.1 \text{ kN}$ to satisfy Serviceability
 $P \leq 18.8 \text{ kN}$
 \rightarrow To satisfy Strength
 $P_{\text{Allowable}} = 14.1 \text{ kN.}$

But you are one more restriction that, the deflection at c must be less than 0.15. Now, I have to find the deflection of C to find the deflection of this point, this member is horizontal member. So, what are this member along which will be the displacement of the point C. So, you are interested in finding the elongation of member AC.

So, what is the elongation of member AC given a force, we saw that in the previous lecture when we did axial members. So, delta AC would be the stress in the member AC divided by area of AC into divided by the Young's modulus of AC would give me the strain in that member into the length of AC will give me how much is the member as elongated. So, this has to be less than or equal to 0.15 mm ok.

I know F AC is 5 P by 8 I know area of AC is pi 15 square by 4 length of AC is 600 mm because of the 0.3 plus pi 3 the 600 mm divided by Young's modulus this 200 into 10 power 3 MPA I am using all units in millimeter. So, I have to use MPA for the Young's modulus this has to be lesser than or equal to 0.15 mm.

So, this will tell us that this will give us that P has to be lesser than or equal to 14.1 kilo Newton's ok. So, give us P has to be lesser than 14.1 kilo Newton's. So, combining all the three cases what we are P must be less than or equal to 14.1 to satisfy serviceability

or the recommend that it displacement C has to be lesser than of particular value and it has to be less than or equal to 18.8 kilo Newton's to satisfy strength recommends ok.

Hence the allowable value of P allowable, which is what we are interested in finding is 14.1 kilo Newton's ok. So, this structure will failed by serviceability requirements, if it is going to fail rather than by strength requirements this is the P allowable.

So, what we have seen in this lecture is two problems one three dimensional stress analysis problem and one a simple one dimensional stress analysis problem, and we saw how to find what is the maximum load that can be applied and maximum stress that can be applied on the structure to satisfy strength requirements as well as serviceability requirements as well as the stability requirements ok. So, I hope this gives you a overall feel of what you will be doing in your designing courses in a future semesters that will be taking ok.

To summarize in this course what we have seen is four concepts force displacement stress and strain and we have seen four concepts, which relates this four equation at relates this four concepts basically your stress strain relationship you are called constitute relation, your force stress relationship, which is given by the equilibrium equations your strain displacement relationship and the comparability condition.

And then you solved some simple boundary value problems like an axial member a beam bending problem, inflection of a pressure vessel and torsion problem to estimate this stresses and then we saw failure theories basically we saw Tresca criteria, von Mises criteria for ductile failure or pressure, hydrostatic pressure, independent failure and we saw Rankine criteria and Mohr-Columb criteria for hydrostatic pressure dependent or brittle failures and we saw how to limit the maximum stresses or the maximum displacements to ensure the safety of the structure.

Thank you.