Mechanics of Material
Dr. U. Saravanan
Department of Civil Engineering
Indian Institute of Technology, Madras

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Lecture – 101
Pressure vessel and failure theory

Welcome to the last lecture in Mechanics of Materials. In this lecture we will solve 2 boundary value problems; find the stresses in the body, and utilize the failure theory that we have learned to ensure that the applied load is less than a particular value so that the body does not fail, ok.

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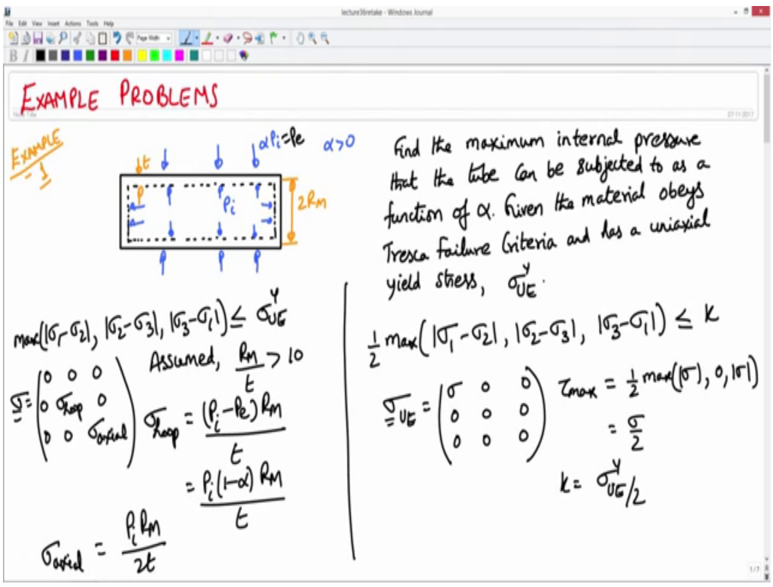

First problem that we are going to look at is a thin walled pressure vessel pressurize from inside subjected to an external pressure P e which is alpha times the internal pressure P i. This pressure vessel has a diameter of 2 times R m, or R m is the radius of this vessel. And t is the thickness of the vessel, ok.

Now, we want to find; what is the maximum pressure internal pressure that can be applied P i, such that this body does not fail by yielding ok. So, we assume that and the material obeys tresca failure criteria, and has a uniaxial yield stress value of sigma y UE ok. So now, you want to find out what will be the pressure internal pressure that can be applied as a function of alpha so that this body does not fail, ok. Since we know from by tresca failure criteria what we mean is the maximum of the absolute value of the principal stress is sigma 1 minus sigma 2, sigma 2 minus sigma 3 sigma 3 minus sigma 1, one half of this that is the tau max the maximum shear stress in the body should be lesser than or equal to kappa ok.

And we know that for uniaxial state of stress sigma uniaxial is given by can be written in this form 0 0 0 0 0 0 0 0, now tau max or the maximum shear stress in this for the state of stress uniaxial state of stress is one half maximum of absolute value of sigma 0 absolute value of sigma which will be sigma by 2 ok. And you know that the maximum stress that it can take in a uniaxial is given by sigma y by sigma y UE and hence kappa has to be sigma y UE by 2 ok. Because that is a stress at which the uniaxial state of stress begins to yield ok.

So, kappa has to be this so now, putting all this together we know that you have found the kappa the material parameter in the failure theory tresca failure criteria. So, you have now, you have to limit maximum of sigma 1 minus sigma 2, absolute value of that sigma 2 minus sigma 3, absolute value of sigma 3 minus sigma 1 should be lesser than or equal to sigma y UE. Substituting for kappa here are sigma y UE by 2 ok.

Now, let us go over to the boundary problem that we are interested in. And a thin walled pressure vessel, you know that the state of stress can be written as you assume there is no radial stress and there is only hoop and axial stresses 0 0 0 sigma axial ok. Now what I have done is I assumed R m by t is less is greater than is greater than 10 ok. So, I can assume the pressure vessel to be a thin walled vessel so that the hoops stress as you saw in the previous lectures is given by P i minus P e into R m by t, which in our case will be P i into 1 minus alpha R m by t ok.

Similarly, the axial stress is given by P i R m by 2 t, now assuming the external pressure it is not contributing to the axial stress ok. We gave an argument why it will would not when we saw thin walled pressure vessels so, this is given by P i R m by 2 t ok. So now, for this sigma 1 would be 0 sigma 2 would be P i R m by t into 1 minus alpha and sigma 3 the third principle stress would be P i R m by 2 t.

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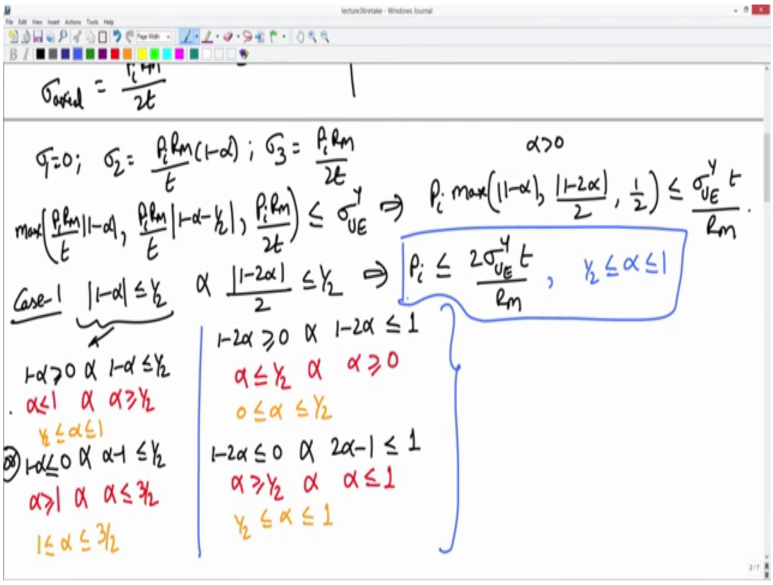

Substituting these values into the star equation; what we get is, we get maximum of P i R m by t absolute value of 1 minus alpha comma P i R m by t absolute value of 1 minus alpha minus 1 by 2 comma P i R m by 2 t, must be lesser than or equal to sigma y UE.

What this tells us is, this tells us that P i into maximum value of absolute value of 1 minus alpha 1 minus 2 alpha by 2 comma 1 by 2 must be less than or equal to sigma y UE t by R m ok. Now the task is to find out what is this maximum value of this thing as a function of alpha. You know that is given that alpha is greater than 0 because I am fixing up the direction of the external pressure was acting readily inward ok. So, the alpha is greater than 0 that information is known to us.

Now, we have to find the maximum value of this subject to the constraint that alpha is greater than 0 which is given to us, because we are fixed the direction of the externally acting pressure as being readily inward and hence this alpha has to be greater than 0 ok. So, basically now what we are interested is we are interested in finding the maximum value of this subjected to the constrained that alpha is greater than 0 ok. So, there will be 3 cases that we have to consider. Let us do it case by case 1; we want 1 minus absolute value of 1 minus alpha to be lesser than or equal to half and absolute value of 1 minus 2 alpha by 2 also must be lesser than half, in which case the maximum value would be in which case a maximum value would be half and hence P i must be lesser than or equal to 2 times sigma y UE t by R m. This implies if this if these 2 holds you know that this one is lesser than half this quantity is lesser than half this quantity is lesser than half. So, the maximum value of these 3 quantities will be half ok.

So, if the maximum of these 3 quantities is half it is P i by 2 must be less than or equal to this quantity, which means P i must be lesser than this quantity. Now we have to find out what restriction? This places on this condition and condition places on alpha ok. So, to do that what we will do is, first we will consider this term, this term, what I want to say I want 1 minus alpha it should be greater than 0. So, 1 minus alpha is greater than 0, then 1 minus alpha has to be and 1 minus alpha must be lesser than half ok.

If 1 minus alpha were to be lesser than 0, then the absolute value of that would be alpha minus 1 ok, the next condition is or 1 alpha can be lesser than 0, in which case the absolute value of that would be alpha minus 1, which must be lesser than half ok. Now what does this condition tell me this condition tells me that alpha must be lesser than or equal to 1 and alpha must be greater than or equal to half right ok. So, combining these 2 conditions alpha must be lesser than 1 and alpha must be greater than half, means alpha must lie between half and 1. That is what this tells us ok.

Let us look at what is second condition tells us the second condition tells us that, alpha must be greater than or equal to 1, and alpha must be lesser than or equal to 3 by 2 ok. This tells us that alpha must vary between one less than alpha less than 3 by 2 ok. So, to satisfy this condition, alpha must lie either between half and 1 or one and 3 by 2 ok. Now let us explore what happens to this second condition, the second condition will tell us that the again there will be 2 cases; one is 1 minus 2 alpha is greater than or equal to 0 ok, and in which case 1 minus 2 alpha is positive. So, absolute value of that will be 1 minus 2 alpha, this should be lesser than or equal to 1.

The 2 gets cancelled ok, now what will this tell us, this tell us that alpha must be greater than or equal to no, alpha must be lesser than or equal to half, and it tells us that alpha must be greater than or equal to 0, right ok. So, combining these 2 you get the recommend that this should lie between alpha 0 and half ok. Now the second recommend here is 1 minus 2 alpha is less than or equal to 0, and no it is here the absolute value of that will be 2 alpha minus 1, and this should be lesser than or equal to 1. Same argument 2 2 gets cancelled here.

So, I have this inequality in here ok. In which case alpha must be greater than or equal to half and alpha must be less than or equal to 1, which will tell me that alpha must lie in the range one half less than or equal to alpha less than or equal to 1 ok. Now combining these all these 2 inequalities it is and condition there alpha must lie between half and 1 or one and 3 by 2. In this case it should lie between 0 and half or half and 1 ok. The common overlapping domain between these 2 cases is half and 1 ok. So, you get the condition that this condition holds when alpha lies between half and 1 ok. So, you got that this condition is between alpha lying between half and 1 ok.

Now, let us move further, you have to consider the next case.

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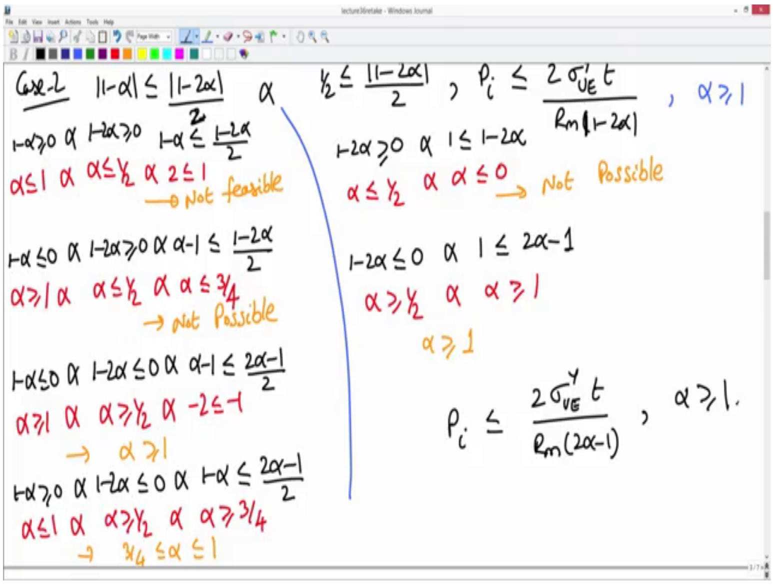

Case 2 would be when I want, now I have ensure that half is now the greatest of the quantities that I have looked at next, I will ensure that 1 minus 2 alpha by 2 is greater than any of the other 2 ok. So, in which case what will be the condition would be, absolute value of 1 minus alpha must be lesser than or equal to absolute value of 1 minus 2 alpha by 2 by 2, and one half must be lesser than or equal to absolute value of 1 minus 2 alpha by 2 right ok.

So, reverse of that condition. So, again if this were to be true then P i must be this were to be true then P i should be lesser than or equal to 2 times sigma y UE t divided by R m into absolute value of 1 minus 2 alpha ok. You have to resolve what that absolute value is when we give the restriction on alpha ok. Like before you have to look at different cases, but now there will be 4 cases for this inequality will be 4 cases for this inequality and 2 cases for this inequality, because there is a absolute chain on either side of the inequality ok.

So, first we will look at the second one, looking at the second one, I want 1 minus 2 alpha to be greater than or equal to 0 ok, and in which case I want one to be less than or equal to 1 minus 2 alpha ok. What does this tell us? This tell us that alpha must be less than or equal to 1 half, and I have alpha must be lesser than or equal to 0. This is not possible so, this is a not possible case, this is not possible ok. Now let us look at the second case. Second case would be 1 minus 2 alpha is less than or equal to 0, and one should be lesser than 2 alpha minus 1 ok. Since this is less than 0 the absolute value of that will be positive and is 2 alpha minus 1 ok. That is what I have used this will tell us that alpha must be greater than or equal to half and alpha must be greater than or equal to 1 ok.

So, combining these 2 I get the condition that alpha must be greater than or equal to 1 ok. Now let us go back and look at what this restriction tells us ok. First case is 1 minus alpha is greater than or equal to 0, and 1 minus 2 alpha is greater than or equal to 0, in which case I will have 1 minus alpha must be lesser than or equal to 1 minus 2 alpha by 2 ok, both are positive. So, I have indicated their positive values here and remove the absolute sign. Now what will this tell us this tells us that alpha must be less than or equal to 1, and alpha must be less than or equal to half and 2 is less than or equal to 1 is what condition will get, it is not true, hence this is a not possible scenario.

This is a not feasible scenario ok. Next let us look at the next condition where 1 minus alpha is less than or equal to 0, and 1 minus 2 alpha is greater than or equal to 0, in which case absolute value of 1 minus alpha would be positive for alpha minus 1 ok, and this should be less than or equal to 1 minus 2 alpha this is again positive because 1 minus alpha is positive divided by 2. Now what is the condition you get? You get alpha must be greater than or equal to 1 ok, and alpha must be less than or equal to half, and alpha must be less than or equal to 3 by 4 ok.

Now, combining all this, what we will get is we will get it as alpha must be greater than 1 and less than 3 by 4, which is a not possible ok. So, there is no it is not possible that alpha can be greater than 1 and as well as a less than 3 by 4. This is a not possible solution possible to find such an alpha ok. Next let us consider the next case, which will be 1 minus alpha is less than 0, and 1 minus 2 alpha is less than 0, and since 1 minus alpha is less than 0 the possible will be alpha minus 1 this should be lesser than 1 minus 2 alpha is less than 0. So, possible will be 2 alpha minus 1 by 2 ok.

So, what will this tell us? This tells us that alpha must be greater than or equal to 1, and alpha must be greater than or equal to half, and minus 2 is less than or equal to minus 1 which is true, always. This implies that alpha must be greater than or equal to 1 ok. The final case that we have to consider is 1 minus alpha is greater than or equal to 0 and 1 minus 2 alpha is less than or equal to 0. And since 1 minus alpha is greater than 0 it will be 1 minus alpha should be lesser than 1 minus 2 alpha is less than 0 the positive value will be 2 alpha minus 1 by 2 ok.

Now, for this case, you will find that alpha must be less than or equal to 1 and alpha must be greater than or equal to half, and be here and alpha must be greater than or equal to 3 by 4 ok. So, here alpha must be lesser than half, greater than lesser than 1 and greater than 3 by 4 which means alpha can lie between 3 by 4 less than or equal to alpha less than or equal to 1 ok. Now the common domain you have to satisfy these 2 restrictions along with this restriction on you have to satisfy these 2 restrictions, along with the restrictions on this side.

So, the common domain here is alpha greater than 1 ok. So, this will become possible if alpha is this is if alpha is greater than or equal to 1 ok. If alpha is greater than or equal to 1, what happens? 1 minus 2 alpha will be lesser than 1. So, the correct way to write this condition would be P i must be less than or equal to 2 sigma y UE t divided by R m into 2 alpha minus 1. I removed the absolute sign now ok. Because I know that when alpha is greater than 1, this will be negative and hence I have to absolute value will be this for alpha greater than or equal to 1 ok.

Now, the next case that we have to consider is the following. Now we are assume that this is greater than in one case, and this is greater in the other case, you have to assume that this is greater than in third case ok. So, if I assume that that is greater in the third case what I will get is the case 3, which will be similar to case 2 except that 1 minus 2 alpha by 2 must be lesser than or equal to absolute value of 1 minus alpha.

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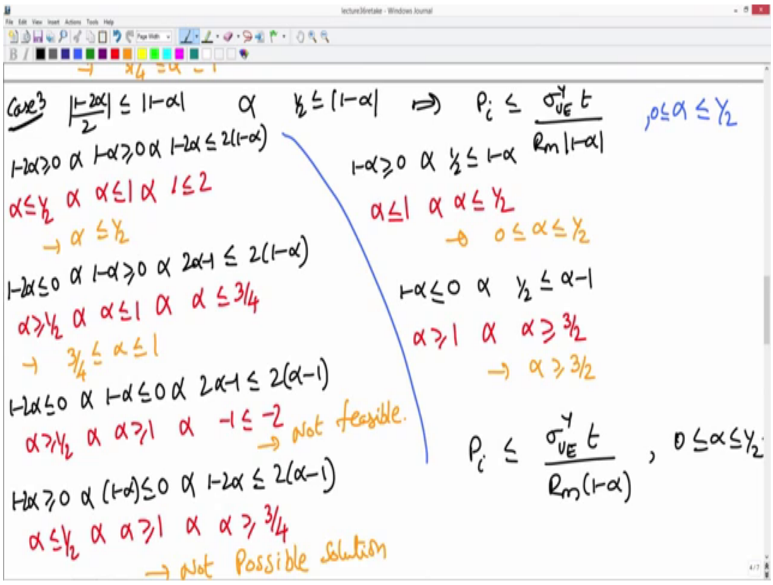

And I will have one half should be less than or equal to absolute value of 1 minus alpha ok.

In which case my this will imply that the maximum value is 1 minus alpha. So, should be lesser than or equal to sigma y UE t by R m the absolute value of 1 minus alpha ok. Now I have to go through the same procedure again, what I did for case 2 to show that this will hold when alpha is less than half, to show that this will hold for alpha less than or equal to half between 0 and half ok, let me do that. So, first let us look at this inequality. So now, what I will have I should have 1 minus alpha is greater than 0, and one half should be less than or equal to 1 minus alpha, will tell me that alpha must be less than 1 and alpha must be lesser than or equal to half ok.

This implies that alpha must lie between 0 and half 0 because alpha is bounded to be greater than 0 ok, from our general restriction. The second restriction here is if 1 minus alpha is less than 0 less than or equal to 0, and if this less than 0, I should have one half must be less than or equal to alpha minus 1 ok, in which case what I will get is I will get alpha to be greater than or equal to 1, and alpha must be greater than or equal to 3 by 2 ok. This will imply that alpha must be greater than or equal to 3 by 2 ok, these are 2 possible solutions here.

Now, coming on to the other side, just like before I will have that may assume 1 minus 2 alpha is greater than or equal to 0, and 1 minus alpha is greater than or equal to 0, and 1 minus 2 alpha is positive should be lesser than 2 times 1 minus alpha which is also positive ok, now what we will get is we will get that alpha must be less than or equal to half, and alpha must be less than or equal to 1. And 1 is less than or equal to 2; which is always true ok. So, the general condition here is alpha this implies alpha must be less than or equal to half ok.

Now, the second condition is, and alpha greater than 0 which is implied here ok. So, the second condition would be 1 minus 2 alpha is less than or equal to 0, and 1 minus alpha is greater than or equal to 0. And since 1 minus 2 alpha is less than 0 I will have 2 alpha minus 1, must be less than or equal to 2 times 1 minus alpha ok. What this will tell us is, this will tell us alpha must be greater than or equal to half and alpha must be less than or equal to 1, and alpha must be less than or equal to 3 by 4 ok.

So, here the common ground is here the common ground is, this will imply that alpha must be between 1 and 3 by 4, 3 by 4 less than or equal to alpha less than or equal to 1 ok. The next condition is 1 minus 2 alpha less than or equal to 0, and 1 minus alpha is less than or equal to 0, and 2 alpha minus 1 because 1 minus 3 alpha is less than 0 must be less than or equal to 2 times alpha minus 1; again because, 1 minus alpha is less than 0 ok. Now what will this tell us? Will tell us alpha must be greater than equal to half, and alpha must be greater than or equal to 1 and minus 1 is less than minus 2, which is not true ok, this is not feasible. This is not feasible option ok.

And now the final condition is our final condition is 1 minus 2 alpha must be greater than or equal to 0, I did 2 less than 0s and one greater than 0, here I have 2 greater than 0s and one less than 0 for 1 minus alpha. So, it will be 1 minus alpha must be less than or equal to 0, and 1 minus alpha is greater than 0. So, 1 minus 2 alpha must be less than or equal to 2 times 1 minus alpha is less than 0. So, I will have it as alpha minus 1 the absolute value to be positive ok.

What will this tell us? This tells us that alpha must be less than or equal to half, and alpha must be greater than or equal to 1, and alpha must be greater than or equal to 3 by 4 ok. Now this is again not possible, because alpha cannot be greater than 1 and less than half for the same time, this is a not possible solution ok; so again combining these domains. In this domain, you need you need alpha to be less than half and alpha must be less than half even here alpha must be greater than 3 by 2 which is not possible in this domain.

So, the common domain is alpha must be less than half ok. So, this will be a domain for this 0 less than alpha less than half will be a domain for this thing, in which case 1 minus alpha is positive. And the hence the right way to write this inequality is P i must be less than or equal to 1 minus alpha removing the absolute value when 0 less than alpha less than equal to half ok. So, this is for 0 to half if you have that this is the inequality 0 to half you have this restricting the value of Pi, the value is greater than 1 you have this in equality restricting the value of Pi. And if alpha lies between 0 and 1, you have this inequality restricting the value of P i ok.

So now, all the domain 0 to infinity you have some restriction on P i ok. So, that completes this problem. So, you find a P i corresponding to what value of alpha you have.