Mechanics of Material
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Lecture -100
Buckling of columns
Secant formula

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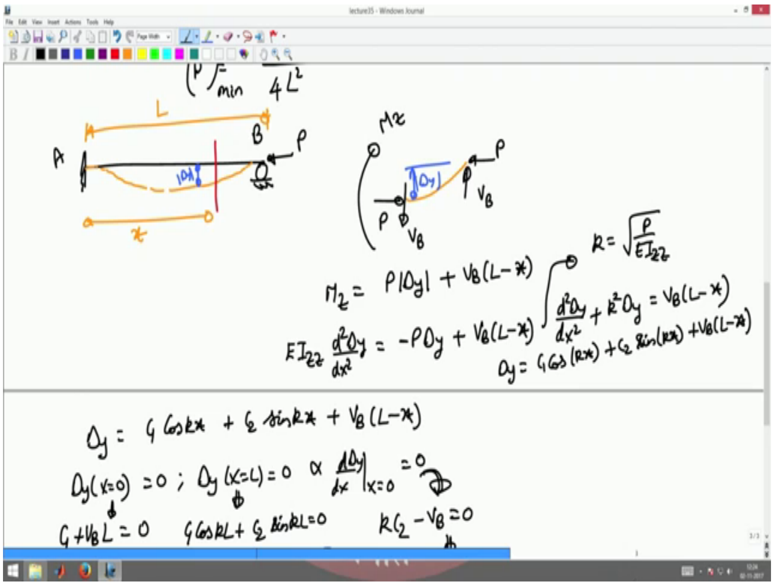

So, essentially you will find that the statement that, if there are no vertical forces acting on the beam there would not be any vertical reaction first (Refer Time: 00:24) is not always true. It depends upon whether the beam is statically determinate or indeterminate the indeterminate system there can be a vertical reaction first (Refer Time: 00:34) even though there is no vertical forces acting on a beam ok. So, that is what I want to emphasize here.

Similarly, you can work out the condition for different end conditions say is a fixed beam or fixed guided support and things like that, but in all these cases you are estimating a P critical load as a single load which occurs suddenly and the deformation was such that either the deformation is 0 or it shows a large deformation at a particular value of stress. As I demonstrated in the last lecture through a scale the deformation is continuously bending along the lateral direction when I apply a load. So, this does not seem to reflect the reality.

So, let us now analyze a case where it reflects a reality the reality is the column is not ideal. What I mean by that is the column is not prismatic or straight or the load is not applied to the center of the cross section exactly. You know that in experiment is not possible to always apply a load exactly through a particular point. If you apply a load through a particular point is a point load and it has to a some distribution some length to distribute itself to a uniformly distributed load. So, that is also not advisable.

So, in a distributed load over an area it is not possible for you to ensure that the distribution is uniform over that entire area ok. So, there will be minor eccentricities in the load ok. Now, let us analyze what happens when there is a load up it slightly eccentric to the cg of the cross section.

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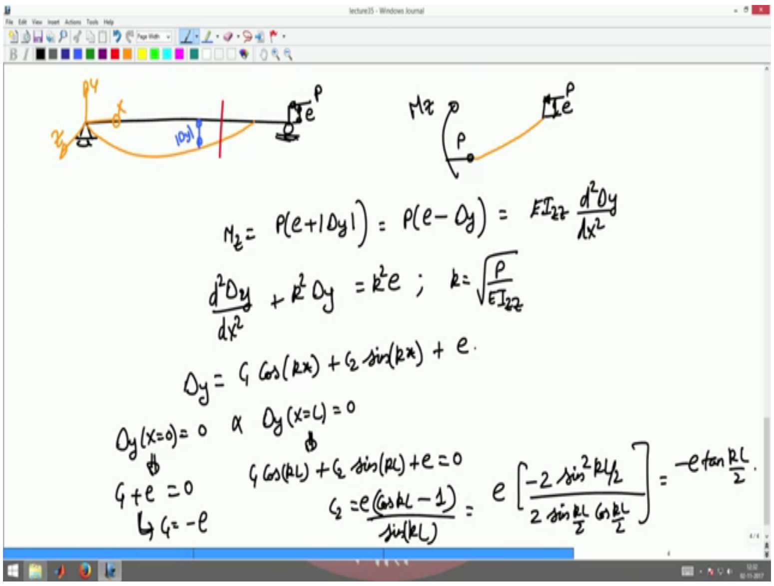

I will go back to a simply supported beam, simply supported column width and eccentrically applied load P I add this eccentricity e coming in there. E might be a small value, but it has an eccentricity e.

Now, I go through the same procedure, the column deforms into some shape like this, I cut it here I draw the free body diagram, draw the free body diagram there is a load P acting at a eccentricity of e from the neutral axis neutral axis of the beam I have P here. Now the moment this P in again I assume that the vertical deflection is absolute value of delta y ok.

Now, then I have a moment MZ coming in here which will be a clockwise moment, the counter the anticlockwise moment produced by the axial force ok. So, that will be now M Z moment is given by P into e plus absolute value of delta y P into absolute value of delta y now delta y is negative downward. So, this will be P into e minus delta y because delta y is negative in the downward deflection I am assuming still my coordinate system to be x y and z here.

Now, this should be equal to from the beam bending problem e times I zz d square delta y by dx square ok. Now, solving this differential equation as before, I can write that as d square delta y by dx square plus k square delta y equal to k square e where k is square root of P by E times I zz ok.

Now, the solution for this equation is delta y is C 1 cos kx plus C 2 sin kx plus e ok. Now the boundary conditions are delta y at x equal to 0 has to be 0 and delta y at x equal to L has to be 0 ok. This tells me that C 1 plus e has to be 0 and this tells me that C 1 cos kL plus C 2 sin kL plus e has to be equal to 0 ok. From here I get C 1 to be minus e then I get C 2 to be cos kL minus 1 into e divided by sin kL ok.

Now, I am found C 1 now I can simplify C 2 further as e times cos kL minus 1 can be written as minus two times sin square kL by 2 and this is 2 times sin kL by 2 into cos kL by 2 and this becomes minus e times tan kL by 2 this becomes two times tan kL by 2 I am found C 1 and C 2 delta y.

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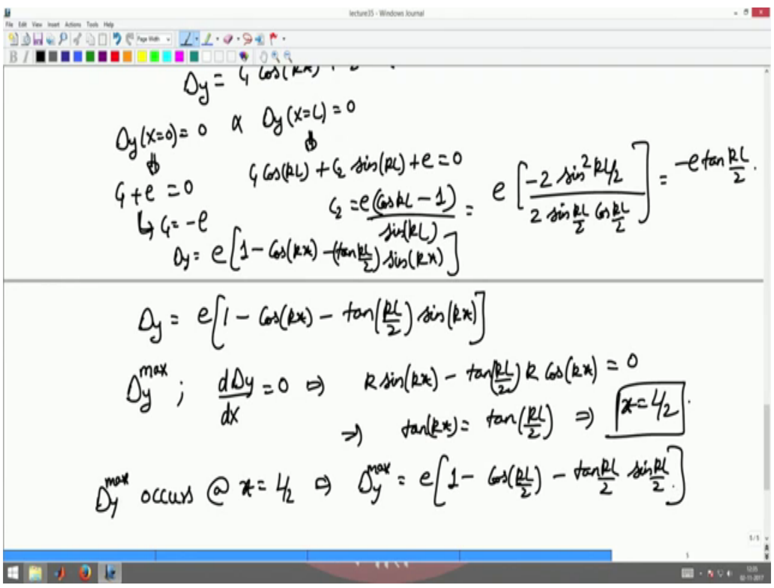

Now, substituting for this becomes e times 1 minus cos kx minus tan kL by 2 into into sin kx ok.

Now, let us proceed further we have found delta y to be e times 1 minus cos kx minus tan kL by 2 into sin kx here we did not find anywhere we were able to apply the boundary conditions without requiring P to take a particular critical value, that is the first observation ok.

The next observation is you want to find delta y max to find delta y max I have to write t delta y by dx has to be equal to 0. So, this will imply that k times sin kx minus tan kL by 2 into k times cos kx must be equal to 0 ok. So, this will imply that tan k x must be equal to tan kL by 2, this implies that x must be equal to L by 2. So, d delta y by dx equal to 0 gives me the location of the delta y max. So, delta y max occurs at x equal to L by 2 and this is delta y max.

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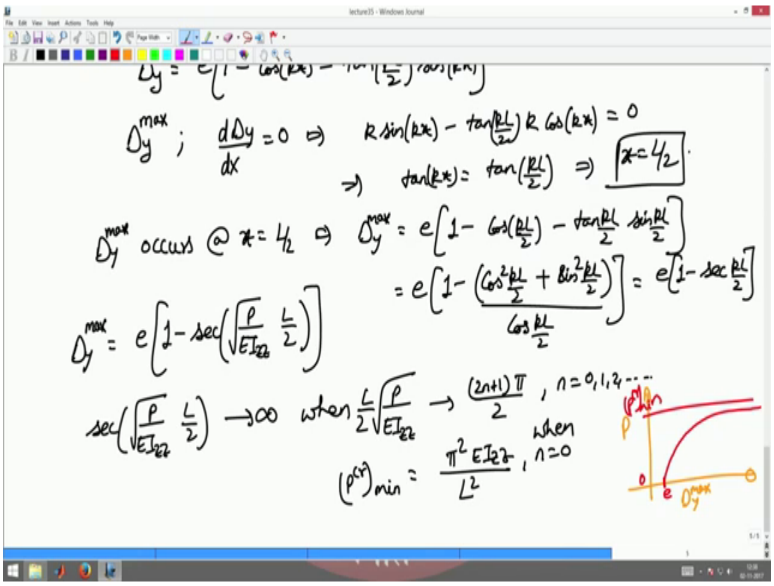

Then this e times 1 minus cos k L by 2 minus tan kL by 2 to sin kL by 2. This I can simplify further as e times 1 minus cos square k L by 2 plus sin square kL by 2 divided by cos kL by 2 this gives me the C times 1 minus secant kL by 2 because cos square theta plus sin square theta is 1 ok. So, I got delta y max is c times e e times 1 minus cos kL by 2.

Here again now I am interested in plotting how does this function delta y max vary with respect to P ok. Now, I have delta y max to be given by eccentricity times 1 minus secant square root of P by E I zz into L by 2.

Now, we know that secant is a function which varies from 1 to infinity. So, secant tends to infinity when secant square root of P by E times I zz 2 L by 2 tends to infinity when square root of P by E I zz into L by 2 tends to 2 n plus 1 pi by 2 n equal to 0 1 2 and so on ok.

Now, the P critical minimum then is P critical minimum then is pi square E I z z by L square corresponding to n equal to 0 when n is 0 ok. Now for this P critical load is same as what we found from the idle column buckling or the Euler bucking load ok.

So, if I were to plot delta y max, if I were to plot sorry delta y by ax as a function of P then what will happen is, as P tends to P critical as P tends to P critical, this is P critical minimum this delta y max will vary from and P 0 it is e and P is 0 and this is 0 this is e and it will go asymptotically to that P critical value ok.

So, this is like what the this is like what a scale did; when I applied a load for a small load there was some eccentricity and eccentricity grew or the lateral deflection grew as the load was applied more and more. So, this how it behaves for delta y max is a P critical.

Now, let us find what is the maximum stress that this section can or this metal cannot see this body cannot see ok.

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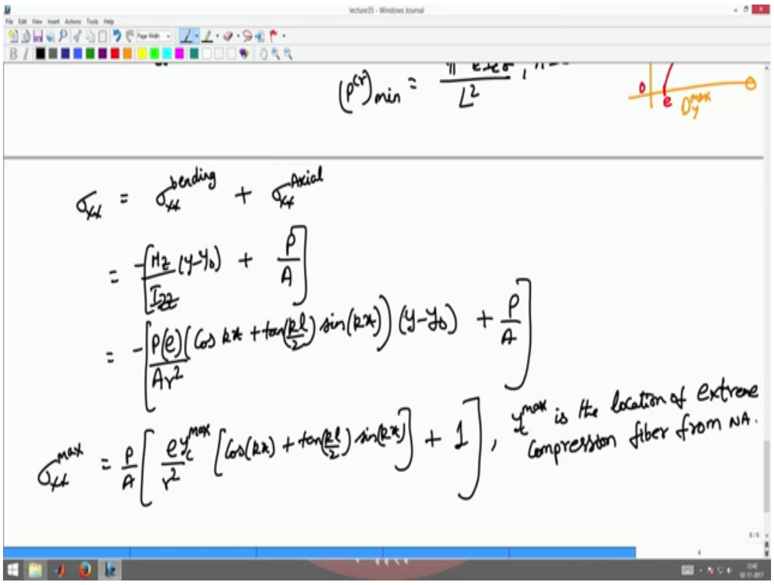

So, we found delta y max, we found delta y I have to find the stress now the stress is given by sigma xx is due to bending plus sigma xx due to the axial force that is applied bending stress is given by minus Mz by Izz into y minus y naught from the bending equation and this is given by P by area of cross section A.

Now, I know Mz from a previous expression I know Mz from our expression here, that is given by P minus delta y e minus delta y P into e minus delta y ok. So, and I found delta y to be this then I found delta y to be this. So, e minus delta y would be. So, I will get this as minus P into e into cos kx plus tan kl by 2 into sin kx to sin kx into y minus y naught by A into r square where r is a radius of gyration as before plus P by A. Here also I should have add the bracket there because axial stress is negative into P by A ok.

So, now this I can write it as P by A into e by r square into I want sigma xx max, the maximum stress will occur at the extreme fiber. So, I will have y extreme fiber in compression y max in compression in to cos kx plus tan kl by 2 into sin kx plus 1 where yc max is the location of compression fiber location of extreme compression fiber from neutral axis.

Now, I know that the maximum deflection occurs at L by two. So, if I were to maximize this.

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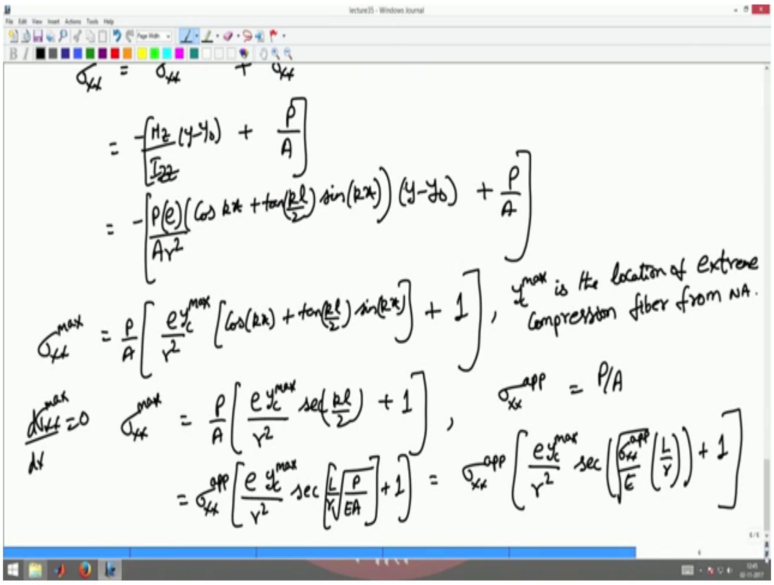

If I were to find the sigma axis max by dx equal to 0 to find the maximum location along the z direction plug in be L by 2 and ends sigma xx max both in x and y would be P by A e yc max by r square into same expression as before this will be secant kl by 2 kl by 2 plus one.

Now, this k also contains p. So, I want to write everything in terms of let me define sigma xx apparent as P by A the axial stress that is applied. So, this will be sigma xx apparent in to e yc max by r square into secant sigma xx secant square root of P by EA 2 L by r where r is the radius of gyration and a is the area of the cross section plus identity this will can be written as sigma xx apparent into e yc max by r square into secant sigma xx apparent by e square root into L by r to L by r plus 1.

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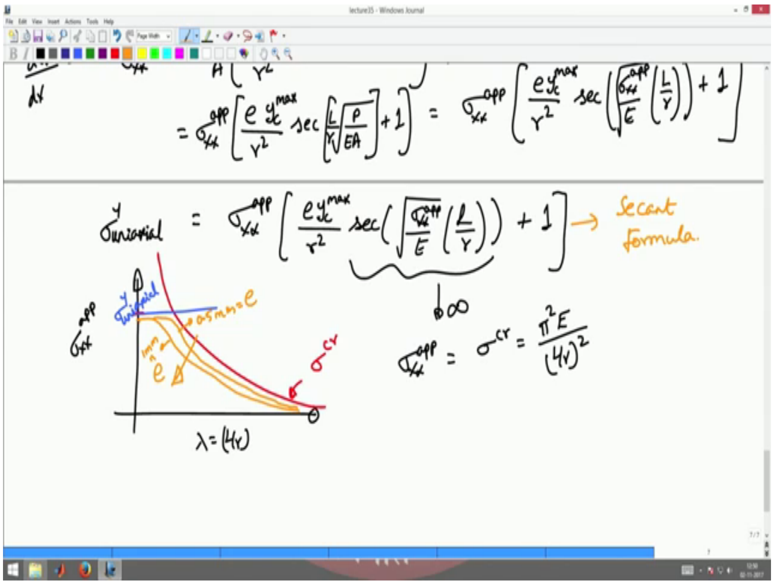

Now, so, I have this to be limited to sigma y uniaxial since the state of stress is uniaxial and if the yield stress is sigma y uniaxial this will be limited to sigma y uniaxial , this will be sigma xx apparent into e yc max divided by r square into secant square root of sigma xx apparent by E into L by r plus 1 ok.

Now, I am interested in plotting sigma xx apparent versus the slenderness ratio lambda is L by r. Now you find that this secant term will tend to infinity as sigma apparent tends to P sigma critical or a buckling load you find that this term tends to infinity and sigma xx apparent is equal to sigma critical that we found before, which will be nothing, but E times pi square E times L by r the whole square ok.

So, if the sigma critical curve are to be like this as a function of slenderness ratio, this is sigma critical curve and this is sigma y if this is sigma y uniaxial line when the load is in the slenderness ratio is close to 0, slenderness ratio is close to 0 secant will be close to one then this curves will look something like this. Do something like that where this is with increasing eccentricity e for a given section e increases along this direction section we have the cross section.

So, if the eccentricity of this is say one mm the eccentricity of this might be 0.5 mm equal to e ok. So, as a simplicity increases the curve will tend down the critical load ok. So, this wow the apparent stress will vary as a function of slenderness ratio.

So, this reflects a realistic behavior of the column wherein you have the lateral deflection also depending upon the load and you have the stress going to the yield stress value of the uniaxial yields stress value, depending upon was the eccentricity and was the slenderness ratio is and by the way this is called as the secant formula.

Now, there are other variations by which you can generate realistic behavior, the other possibility is since of applying an eccentric load it can be just like what we did for the equilibrium in your disturbance.

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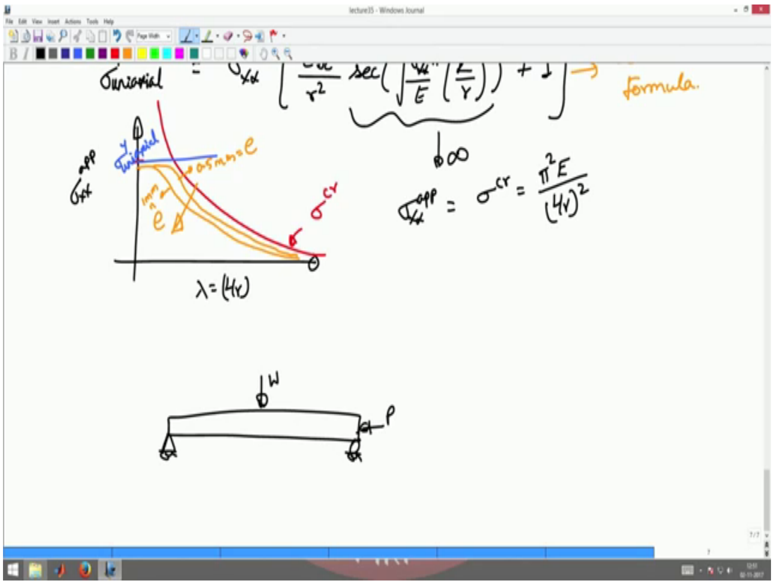

Wherein you are simply supported beam whereas, simply supported beam wherein you can say I applied a lateral load w like this ok. Then also you can redo the analysis get the critical load for this beam to be the same as pi square e by lambda square and you can show that all that what we did now also volts I will would not be doing that in this course.

The other point notice once you an eccentricity, the direction in which the column bends would be the direction with the eccentricity of the load is ok.

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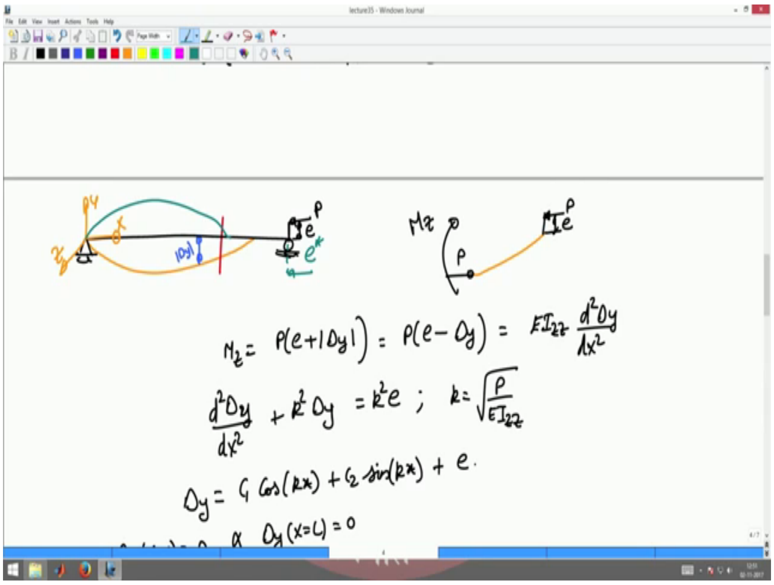

So, if I add a load acting with which is a eccentrically displace upwards, then this column will deform only down. On the other hand if I add this load eccentric downwards this column would deform something like that ok.

So, the eccentricity governs the direction of the displacement of the column and its no longer a random event ok, but whether it deforms in along y axis or whether it deforms along the z axis, difference again upon the moment of inertia along y or a z whichever is the least it will deform perpendicular to the least moment of inertia direction ok.

So, with this we conclude our discussions on stability induced failure, we saw how a ideal column will be given how to get the critical load for an ideal column and then we saw case wherein we add a real column behavior exhibited by a simply supported column subject to an axial load ok. How we could capture all the realistic behavior by assuming the load is acting somewhat eccentrically to the center of the e cross section ok.

Now, the next remaining lecture, we will see an example problem or an we will work out an example problem involving the failure theories as well as the stability induced failure will work out an truss problem or an we will analyze what is a maximum load as a truss can take, incorporating both the failure theories and this stability condition and then we will work out an example problem involving failure theories and the inflation of a analyze cylinder problem ok.

Thank you.