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Structural Dynamics

Week 12: Module 04

Tuned Mass Damper

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Welcome to structural dynamics class. So in this class we will study about tuned mass damper. So what is tuned mass damper, so tuned mass damper is controlling the vibration of a structure by using mass. So the tuning is done according to the frequencies, so there is a structure and this structure the primary structure so this has severe vibrations because of some given input. So when vibration amplitude is more so there will be safety issues to the occupancy and comfort issues to the occupance.

So what we need to do is we need to control the vibration, so tuned mass damper is tuning the frequency of the secondary structure which we are going to install either on the top or anywhere in this structure. And the frequency of that structure matching with the frequency of the primary structure and using these two we will derive the damping which needs to be present in the secondary structure.

Because the damping already present in the primary structure is insufficient to control the vibration. So this is the main concept of tuned mass damper.

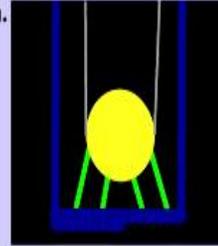
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Introduction

- A tuned mass damper (TMD) is a device consisting of a mass, a spring, and a damper that is attached to a structure in order to reduce the dynamic response of the structure.
- The frequency of the damper is tuned to a particular structural frequency, so that when that frequency is excited, the damper will resonate out of phase with the structural motion.



Location of Taipei 101's largest tuned mass damper.

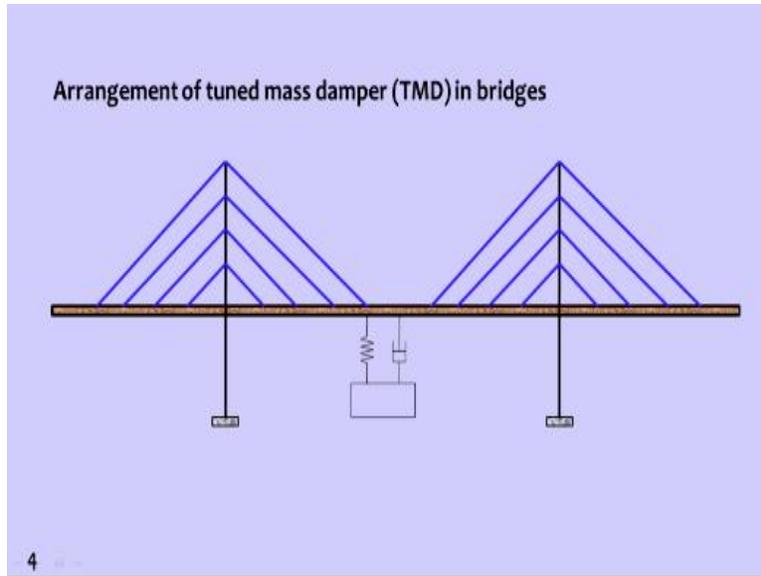


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So the tuned mass damper are popularly called as TMD is a device consisting of a mass, a spring and a damper that is attached to a structure in order to reduce the dynamic response of the structure. So the frequency of the damper is tuned to a particular structural frequency so that when that frequency is excited, the damper will resonate out of phase with the structural motion. So let us look at this structure this is thalpy 101 building.

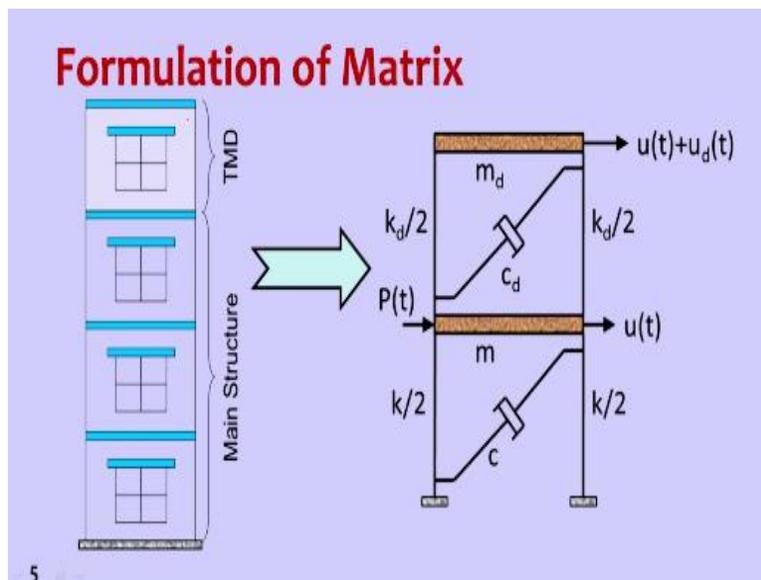
And on top tuned mass damper is installed. So if you look at closely it looks like this, so this is a mass system hanging from this location so in the form of pendulum so this makes the severe vibration out of phase so that the total vibration or the maximum displacement will reduce. So as you can see this is the motion animation of it you can clearly see that.

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So in long span bridges also it is done so this is a exaggerated cartoon so showing how the dampers are installed.

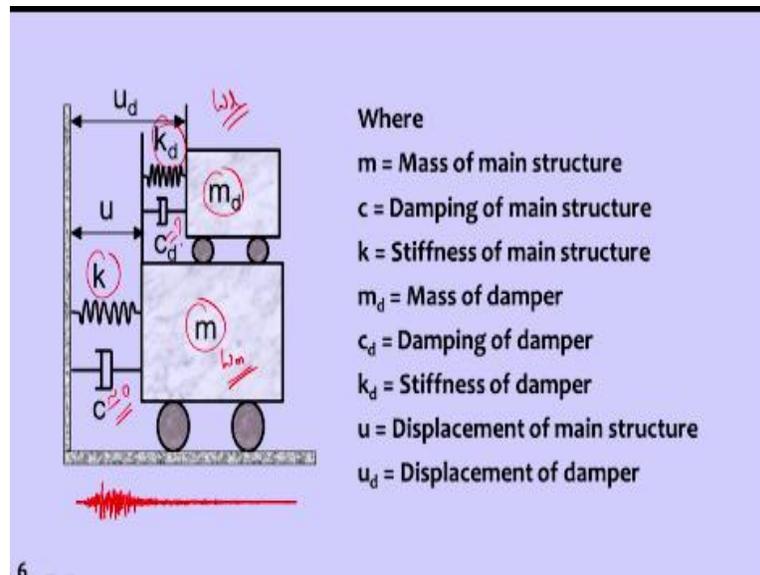
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Now let us look at formulation part in this formulation you can see this is a primary structure on which a secondary structure is constructed for explanation of concept I have kept one additional floor but it is not needed it can be a very, very small component which is having some

percentage of weight of the building. So the weight can be 5% to 10% the weight of the building. So this is called main structure and this is tuned mass damper. And as you can see this tuned mass damper acts like this so this is the conceptual explanation.

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So this is primary structure, secondary structure. So this secondary structures weight or mass can be some 5% to 10% okay in some cases it is even less. So in this one primary structure is stiffness, primary structure is damping so this is approximately 0 it is not present or very less value so this value we have to find out and mass and stiffness together will offer ω_d that is frequency damped natural frequency of the system.

And these two offer natural frequency of the primary system. So tuning is done between primary structure frequency and the secondary structure frequency so mass of main structure and see damping of main structure, k is a stiffness of main structure M_d is mass of damper, C_d is damping of damper, K_d is stiffness of damper and U is the displacement. And U_d is the displacement of the damper.

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When the support excitation is harmonic and its displacement amplitude is fixed and independent of input frequencies, i.e. $u = H e^{i\omega t}$, the steady-state response of the system can be solved from

$$\begin{bmatrix} \omega^2 + \gamma\omega_d^2 - \omega_a^2 + i2\omega_a(\xi\omega + \gamma\xi_d\omega_d) & -\gamma\omega_d^2 - i2\gamma\xi_d\omega_d\omega_a \\ -\gamma\omega_d^2 - i2\gamma\xi_d\omega_d\omega_a & \gamma\omega_d^2 - \gamma\omega_a^2 + i2\gamma\xi_d\omega_d\omega_a \end{bmatrix} \begin{Bmatrix} u \\ u_d \end{Bmatrix} = \omega_a^2 H e^{i\omega t} \begin{Bmatrix} 1 \\ \gamma \end{Bmatrix}$$

Where

$$\omega = \sqrt{\frac{k}{m}}; \quad \omega_d = \sqrt{\frac{k_d}{m_d}} \quad \xi = \frac{c}{2m\omega}; \quad \xi_d = \frac{c_d}{2m_d\omega_d} \quad \gamma = \frac{m_d}{m}$$

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Now if we formulate the two degree of freedom system mass matrix so this is actually this m is n degree of freedom system already this is n degree of freedom system it is there and the additional damper system is adding one more degree of freedom system, so $m_0, 0, m_d$ u.. and u..d $C + C_d - C_d - C_d$ Cd u. velocity of primary system velocity of damper $k + k_d - k_d - k_d$ and k_d u and u_d u is a displacement of primary system and then u_d is the displacement of damper.

So $m_0 g$.. is a force acting due to earthquake ground motion $M_d u_g$.. is force acting on the damper due to earth quake ground motion so the total equation of motion is $M \ddot{u} + C \dot{u} + k u = -m \ddot{u}_g$ so where m is a mass matrix compound mass matrix, C is damping matrix compound damping matrix k is compound stiffness matrix u .. is acceleration vector velocity vector and displacement vector is u.

When the support excitation is harmonic its displacement amplitude is fixed and independent of input frequencies that is $u = H e^{i\omega t}$ so this is assumed displacement profile this is in steady state response if we take the displacement response that is $k \times u$ vector is equal to the response vector so as you can see ω is a natural frequency $\gamma \omega_d$ $D \omega_d$ A_i $2 \omega_d A$ all this complex notations where ω is natural frequency of the primary system.

Omega D is K_d / M_d so natural frequency of the damper and zeta value is a damping $C/2M$ omega and zeta d is this is zeta present in the primary structure this is damping present in the damper and gamma is a mass ratio so only on m so mass the damper by damper so now you know all these values if we plug in this.

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The ratio of the displacement amplitude of the main mass to the input amplitude can be solved as

$$\frac{|u|}{H} = \frac{g^2 \sqrt{[f^2(1+\gamma) - g^2]^2 + 4g^2 f^2 \zeta_d^2 (1+\gamma)^2}}{\sqrt{[f^2 g^2 - (g^2 - 1)(g^2 - f^2) + 4\zeta \zeta_d f g^2]^2 + 4g^2 [\zeta_d f (g^2 + \gamma g^2 - 1) + \zeta (g^2 - f^2)]^2}}$$

Where

$$f = \frac{\omega_d}{\omega}; \quad g = \frac{\omega_a}{\omega}$$

We will get the ratio of displacement amplitude of the main mass to the input amplitude can be solved so this is displacement of the mean mass and this is input amplitude so A is complex where frequency ration is omega D. omega and G is omega A. omega.

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Optimum parameters for undamped systems

Harmonic main mass excitation	Harmonic base excitation
$f_{opt} = \frac{1}{1+\gamma}$	$f_{opt} = \left(\frac{1}{1+\gamma} \right) \left(\sqrt{\frac{2-\gamma}{2}} \right)$
$\xi_{dopt} = \sqrt{\frac{3\gamma}{8(1+\gamma)}}$	$\xi_{dopt} = \left(\sqrt{\frac{3\gamma}{8(1+\gamma)}} \right) \left(\frac{2}{2-\gamma} \right)$

So optimum parameters for un damped system are harmonic main mass excitation so f optimum there is $1/1 + \gamma$ so γ is a mass ratio so zeta damping so damping of the damper optimum is $3 \gamma/ 8 \times 1+ \gamma$ under root, harmonic base excitation so f optimum and zeta optimum values are given.

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Example Problem

A single storey single bay structure with plan dimensions $7\text{ m} \times 7\text{ m}$ is taken and a tuned mass damper is designed for no damping condition. The properties of structural elements are given below:

So for optimum parameters for damped systems there if optimum if given and zeta optimum is given like this so let us solve one example problem using this complex notations so a single story single based structure with one dimension $7\text{m}/7\text{m}$ I had taken a tune mass damper is designed for low damping condition so that means what, then primary structure has no damping present in it so we want to damp the reduce the vibration energy or the vibration amplitude by using tuning of the mass.

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Example Problem

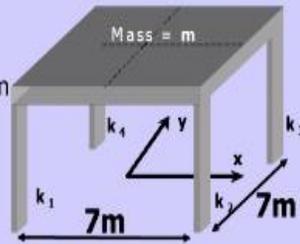
A single storey single bay structure with plan dimensions $7\text{ m} \times 7\text{ m}$ is taken and a tuned mass damper is designed for no damping condition. The properties of structural elements are given below:

Cross section of beam: $0.23 \times 0.23\text{ m}$

Cross section of column: $0.45 \times 0.45\text{ m}$

Thickness of slab: 0.15 m

Grade of concrete: M_{25}



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Cross section of beams columns thickness of slab concrete grade is given so it is the structure figure of the structure and then let us calculate.

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Stiffness of columns:

Modulus of Elasticity = $5000\sqrt{f_{ck}} = 5000\sqrt{25} = 25000 \text{ N/mm}^2$

Along x direction:

Moment of inertia of column section: $450 \times 450^3 / 12 = 34.17 \times 10^8 \text{ mm}^4$

Lateral stiffness: $4x \frac{12EI}{l^3} = 4x \frac{12 \times 25000 \times 34.17 \times 10^8}{3000^3} = 151.875 \times 10^3 \text{ kN/m}$

Along y direction:

Moment of inertia of column section: $450 \times 450^3 / 12 = 34.17 \times 10^8 \text{ mm}^4$

Lateral stiffness: $4x \frac{12EI}{l^3} = 4x \frac{12 \times 25000 \times 34.17 \times 10^8}{3000^3} = 151.875 \times 10^3 \text{ kN/m}$

And then let us calculate modulus velocity that is known as given in I s 456 5000 under root of f_{ck} so we know this value and then the movement of inertia in x direction and along y direction so we get in both symmetric in both directions so I_x same in both directions and then calculation of the mass so we take.

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Mass calculation:

$$\text{Weight of slab} = 7 \times 7 \times 0.15 \times 25 = 184 \text{ kN}$$

$$\text{Weight of columns} = 4 \times 0.45 \times 0.45 \times 25 \times 1.5 = 31 \text{ kN}$$

$$\text{Weight of beams} = 4 \times 7 \times 0.35 \times 0.23 \times 25 = 56 \text{ kN}$$

$$\text{Total mass} = 184 + 31 + 56 = 271 \text{ kN} = \underline{27620 \text{ kg}}$$

Natural frequency:

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{151.875 \times 10^6}{27620}} = \underline{74.15 \text{ rad/sec}} \quad T_n = \frac{2\pi}{\omega_n} = \frac{2\pi}{74.15} = \underline{0.08 \text{ s}}$$

Weight of the slab weight of the column weight of beam weight of total mass so 27620 kg and then natural frequency of primary system so 74.15 radian per second so that offers 0.8 seconds then assume mass ratio as 0.03 so that means 3% mass is taken so mass of the secondary structure damping is 828 kg the stiffness of the damping is 0.03% is

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Assume mass ratio is 0.03

Mass of damper = $0.03 \times \text{mass of structure} = 828 \text{ kg}$

Stiffness of damper = $0.03 \times \text{stiffness of structure} = 4556250 \text{ N/m}$

$$f_{opt} = \frac{1}{1+0.03} = 0.9708$$
$$\zeta_{dopt} = \sqrt{\frac{3 \times 0.03}{8(1+0.03)}} = 0.1045$$
$$k_{opt} = \gamma k f_{opt}^2 = 0.03 \times 151.875 \times 10^6 \times 0.9782^2 = 4.36 \times 10^6 \text{ N/m}$$
$$c_{opt} = 2m\omega \zeta_{dopt} \gamma = 2 \times 27620 \times 74.15 \times 0.104 \times 0.03 = 1.278 \times 10^4 \text{ N/m/s}$$

Taken this one just we need to tune it now because we need to match the frequency that's why same ratio used in mass as well as in stiffness so f_{opt} is $1/(1+\gamma)$ that is mass ratio so 0.978 and ζ_{dopt} that is damping damper optimum value is taken as 0.1 so that will become 10% of damping so stiffness optimum damping optimum if we calculate so we get this values and then so these values when we adopt and apply in that compound mass matrix stiffness matrix.

We get the final response as the reduced response so damping in the secondary system is helping in the primary system in two ways number one it is changing the façade of the primary system that means frequency because the frequency of the compound system is different from the frequency of the primary system if the frequency of the primary system and exact frequency matches then it will go into resonating condition and amplitude will become high so because the presence of the secondary system.

So frequency is changed so that your frequency ratio is compared to compound structure and exact frequency is now either lesser than the original frequency or more than the frequency ratio in both the cases the amplitude reduces severe so in summary so we have studied how to design a tuned mass damper in this class.

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