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Structural Dynamics
Week 12: Module 01

Classical and Non-classical Damping

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Welcome to structural dynamics class. So in this class we will study classical and non-classical damping. So what is actually damping, so let me show you an example say assume that this is a pendulum and I am giving say initial displacement and it is oscillating. So after few cycles it is coming to rest. So how it is coming to rest, so it is coming to rest because the frequency that the vibration energy is dissipated.

So where it is dissipated, it is dissipated in two forms, one is friction between this point, this edge and my finger that is friction in the form of heat it is getting dissipated and then second one is their resistance. So in this two forms the entire energy what I am supplying for oscillation is getting dissipated. So this is damping, so in a way damping removes the vibrating energy and in each cycle it removes. Now let us discuss what is classical and non-classical damping.

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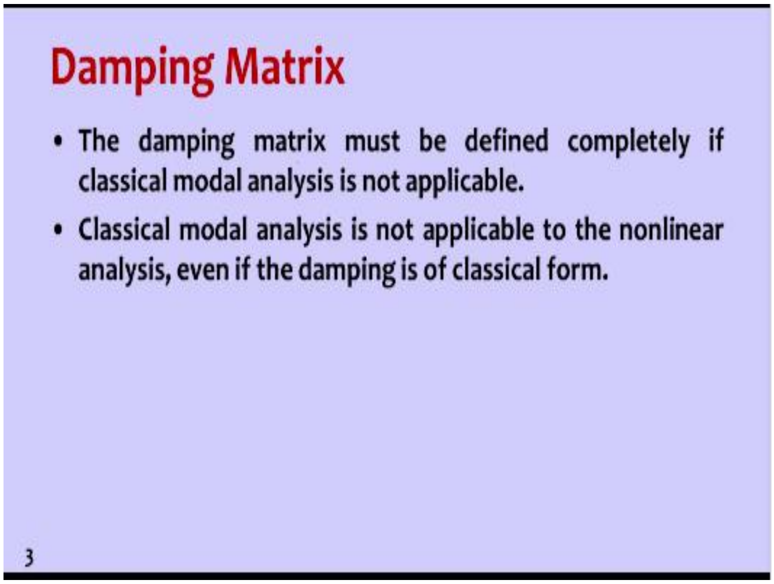

Now damping matrix, so if we are talking about multi-degree of freedom systems we need to define damping matrix. So in single degree of freedom system we define damping as a term. So generally damping is a nonlinear phenomenon. So because it depends on the displacement and velocity time history. So if more velocity damping is proportional to that, so in earlier cases what we have done was we have made damping term as constant in linear elastic analysis.

Now when it comes to non-linear analysis it is not anymore linear, so damping term also is nonlinear it depends on the history that is U and U. that is displacement time history and velocity time history. Now this damping matrix how do we define, damping matrix must be defined completely if classical modal analysis is not applicable, so that means what we cannot fix this damping value, so damping value varies.

Now classical modal analysis is not applicable to nonlinear analysis even if damping is in classical form. So in these two cases when non-classical damping is there so there we have to define damping matrix and when it is nonlinear analysis even if damping is classical we have to define them.

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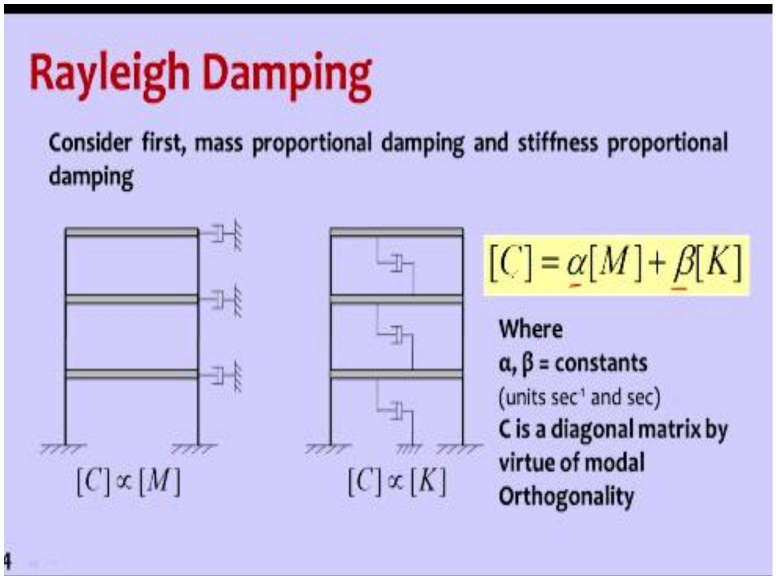

So how like we will define, so Rayleigh has given a way to define this damping matrix, so consider first mass proportional damping and also stiffness proportional damping. So if you take this structure this in this picture you can see this one, so when structure is oscillating or structure is vibrating, so damping result in the degree of freedom or in that floor is proportional to the motion of that floor.

So we can call that as damping is proportional to mass. So and the second one is damping is also related to the inter particle friction, so that gets emitted in the form of heat. So it is also proportional to stiffness. So then combined form we can write, so damping is equal to α because if we have to remove the proportionality we have to introduce some constant, so damping is equal to α times mass matrix plus β times k matrix that is stiffness matrix.

So now we need to find out what are this α and β values. So α and β are constants so as we can see if you match the units so this α units must be per second and β units must be in the form of second so that it matches with the damping. So C matrix can bam de diagonal by virtue of its model orthogonality if you use model orthogonality principle and apply that principle so we can make that has diagonal matrix so C matrix can be made as diagonal matrix.

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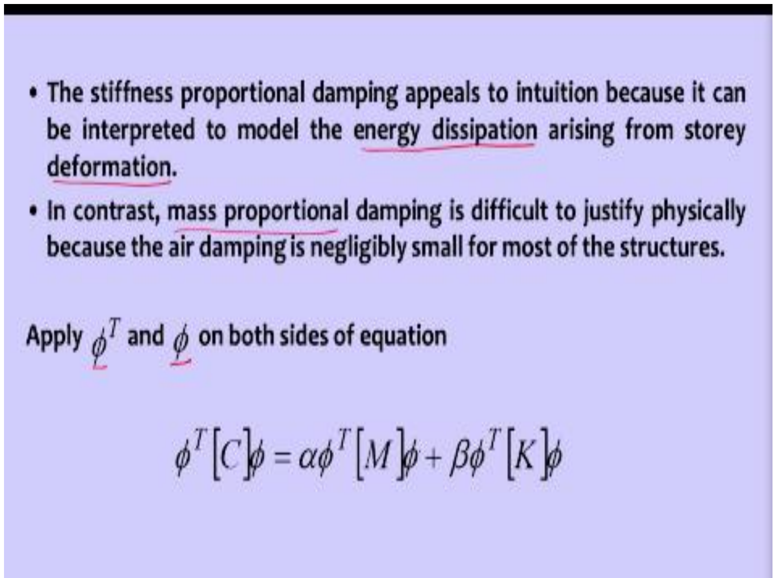

So now if the stiffness proportional damping appeals to intuition why, because it can be interpreted to model energy dissipation arising from the story deformations and in contracts mass proportional damping is difficult to justify physically because as I told you it is related to air friction quickly so it is mostly negligible, so now when we have say model matrix or model vector so if we apply model matrix as Ø transpose and multiply by Ø on both sides of the equation.

We will get the diagonal matrix because decouple the equations and solve simultaneous equations so C = α times m + β times k so we are pre-multiplying and post multiplying with Ø transpose and Ø respectively all these three terms.

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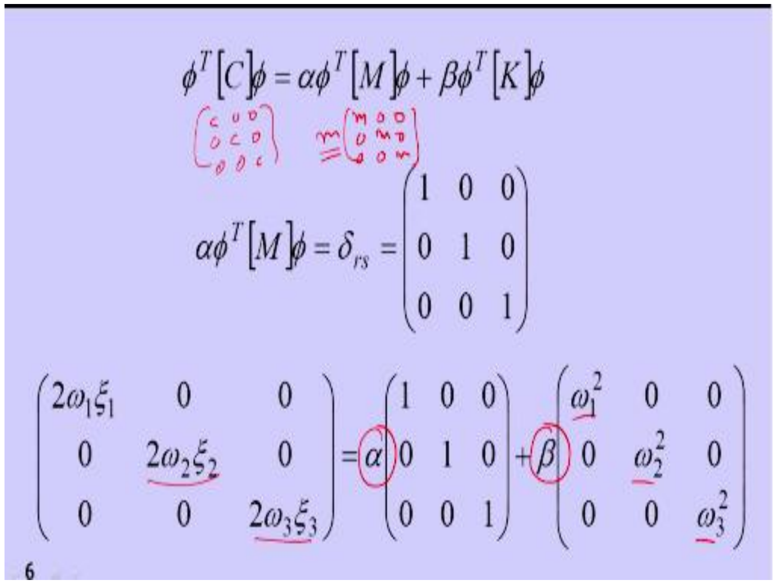

So if we do that what we get is α times Ø transpose n Ø is a diagonal matrix that is 111 so these are masses and then if we take this 2 ω 1, ζ 1 00 and a second root 2 ω 2 ζ 2 third term 2 ω 3 ζ 3 so this is coming from something like this so mass matrix has m00, 0m0, 00m now if you take m as common outside so in this one ion C 00, 0c0, 00c something like this.

So n/c term so if I divide entire all these terms by m so C/M is 2 ω 1 ζ 1 2 ω 2 ζ 2, 2 ω 3 ζ 3 so that is a culture, so how this has come please refer to single degrees of freedom system lecture and here mass by mass will be 111 diagonal term and k/m will become say ω 1^2 ω 2^2 ω 3^2 corresponding natural frequencies of mode shapes so now the thing is how to get this α value and β value.

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$$2\omega_i\xi_i = \alpha + \beta\omega_i^2$$

$$\xi_i = \frac{\alpha}{2\omega_i} + \beta\frac{\omega_i}{2}$$

$$\xi_i = \frac{1}{2}\left(\frac{\alpha}{\omega_i} + \beta\omega_i\right)$$

$$\begin{pmatrix}\xi_i \\ \xi_j\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1/\omega_i & \omega_i \\ 1/\omega_j & \omega_j\end{pmatrix}\begin{pmatrix}\alpha \\ \beta\end{pmatrix}$$

Now let us look at the equation so if we expand this equation earlier equation so what we get is 2 ζ ω 1 = α + β ω 1^2 so the same thing I am writing 2 ζ I ω I = α + β ω i^2 so this is corresponding to that mode shape or that idealization single degree of freedom system, so in mode analysis when we have mass matrix ,when I have k matrix and when we have c matrix so in mode analysis what we do is.

N degree of freedom system so if it is 3 degree of freedom system we are converting that into 3 single degree of freedom system something like this, so this I is corresponding to this so ω I so ω 1 ω 2 ω 3 like that so now from this we can get ζ I = α i/ 2 ω I + β x ω I / 2 so ζ I is so 1/1/2 so α / ω I + β into ω I so this is a ζ equations.

So damping equation so what is a damping present in say each mode we can calculate from this one so for that so what we do is ζ I, ζ j so if I replace j with i. So what I get is corresponding values so I get alpha and beta as matrix so in this one so what I need to do is like I estimate the damping values and then find out alpha and beta doing inverse of this one so that how.

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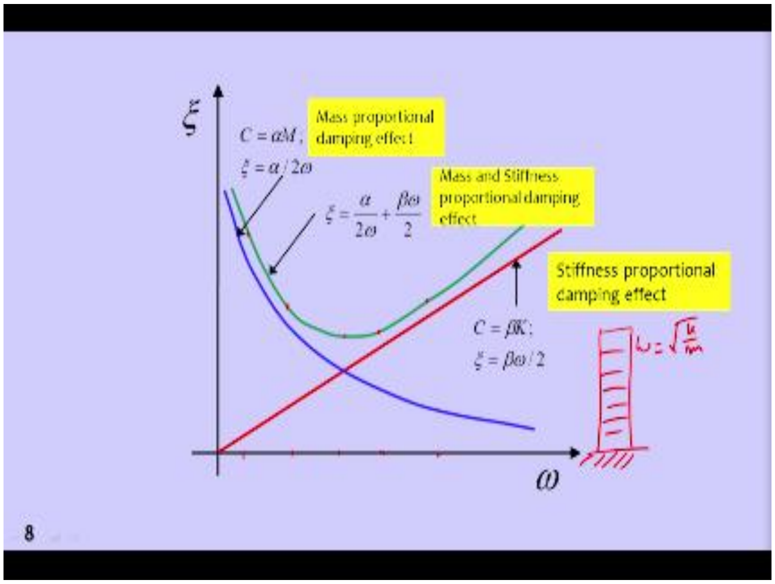

We find alpha and beta value so when alpha and beta values are there we can find the damping in any mode shape so c =beta times k so that means what beta is beta Ω by 2 so stiffness proportional damping effects so it is linear and then when it comes to mass proportional damping value damping effect so c =alpha times m so in that case beta = alpha /2 Ω so it is mass proportional damping effect so in case of rallies damping both are combined so that means what so what we are getting is you can see this mass.

And stiffness proportional damping so, so in the horizontal axis you can see Ω so now what we have is as the natural frequency of the structure or say mode shape is different in first mode shape it can be here so this damping value we use second mode shape we use this damping value third mode shape mode we will use like this different, different this okay this is one thing and then there is a another use of this one so another use of this one this means say if I have taken a tall building.

So I have found out say Ω of that tall building = under root k/m in this form now because of the vibration so or the earthquake effect so continuously see this natural frequency of the structure is reducing or natural period is elongating so now what happens here is the natural frequency is reducing and natural period is increasing so damage increases natural period so that means what reduces frequency so what happens is in first case my damping is here then if this frequency is reducing so then I'm getting reduced damping in some cases.

And in other cases my natural frequency is here and because of damaged it is reducing so then my damping increases so that means what were we are in this curve we need to be very careful in Appling damping so because ξ has effect of reducing a vibration amplitudes so this vibration amplitudes maximum amplitude if we are calculating in a wrong manner that leads to either over evaluation of the structural response or under evolution of the structure response.

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In this class we have studied so how to calculate say non classical damping matrix so the two forms are mass proportional damping as well as stiffness damping so we have developed equation in this class that is c=alpha times m – beta time k so by using different mode shapes we have evaluated this alpha value and beta value and then we have developed drawn the curve like how the damping values change with the natural frequency or the natural period of the structure.

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