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NPTEL ONLINE COURSE

**Structural Dynamics
Week 11: Module 01**

Dynamics of Non Structural Elements

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Welcome to structural dynamics class. So in this class we will study about dynamics of non structural elements. So this non structural elements are of two types one is force based non structural elements and second one is deformation based non structural elements. So a simple example like this if you take a bottle okay. So this is almost a rigid body kind of thing and it behaves like if I do the shaking, so it topple or it may slide or it may rock.


So these are rigid blocks, so this is one type of non structural elements other type of non structural element is long say pipelines which are there inside the building. So these two are non structural elements, so in this class we are going to study non structural elements which are rigid in nature, so for example say furniture in the building say water tanks on the roof top of building and say electronic equipments which are placed on table, the tables also all these come under rigid non structural elements.

So they do not have internal deformation, but they may slide, they may rock, they may topple. So during earthquakes even if the structure is the building is safe there are lot of injuries leading to death sometimes mainly due to non structural elements.

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Outline

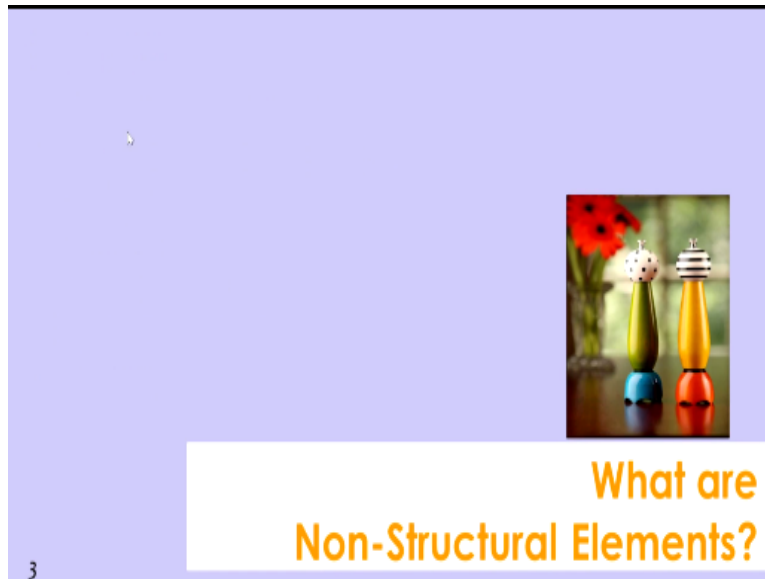
- **Nonstructural Elements**
- **Description and Conditions for**
 - *Sliding*
 - *Rocking*
 - *Toppling of NSE*
- **Equation of motion**
- **Summary**



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Non structural elements we will discuss.

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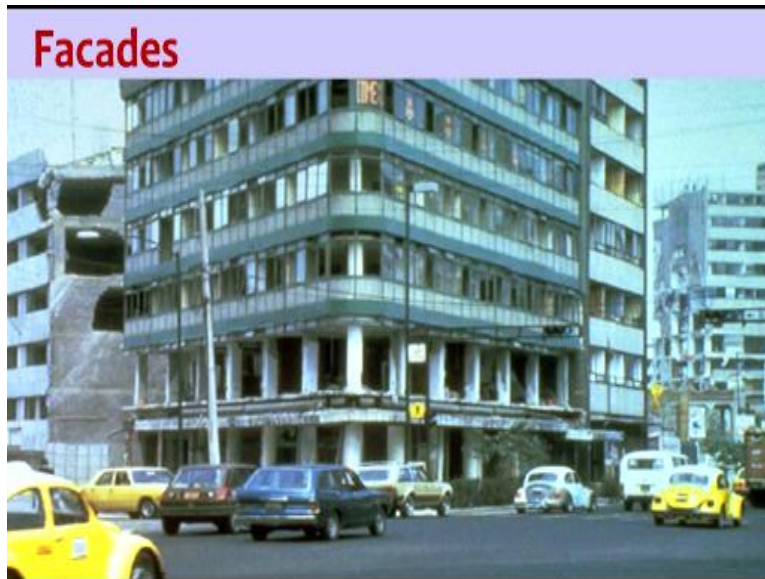
So what are non structural elements.

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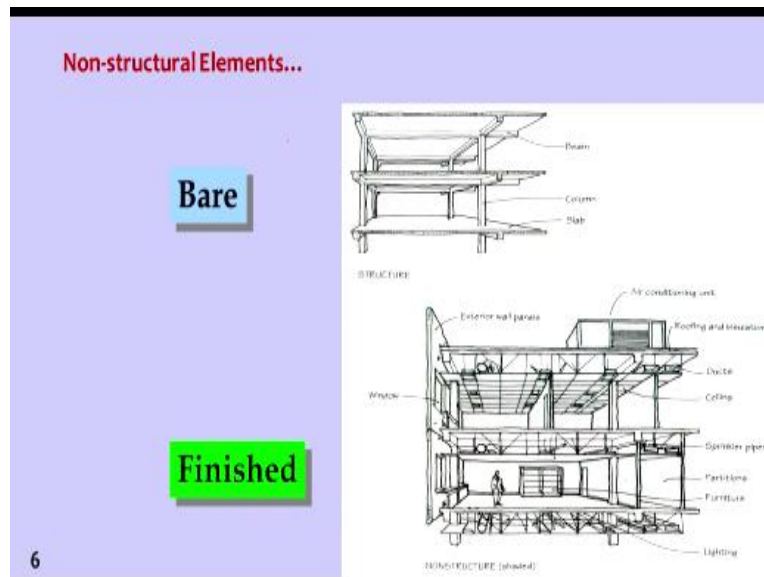
So we can see this one say a glass façade okay so that is a non structural element as you can see in this slide.

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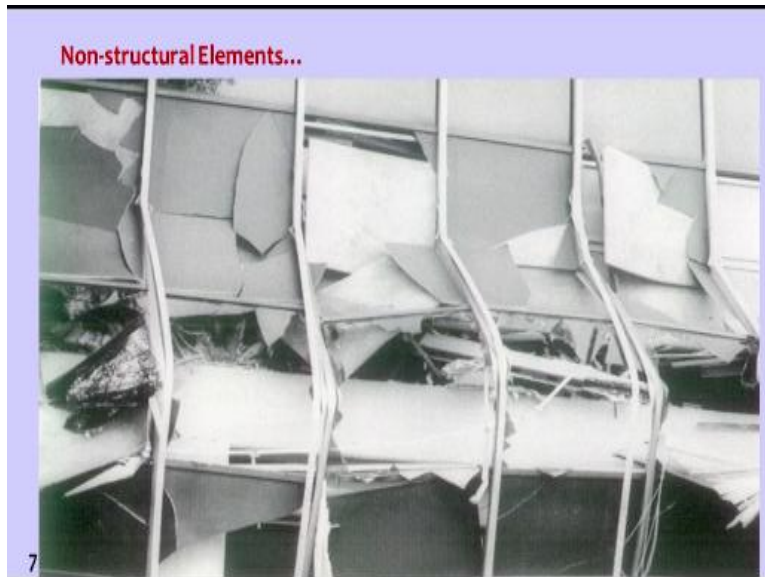
And then you can see heavy facades, so these days especially in urban areas so glass panels has 3m/3m glass panels or installed at the top so maybe 30m height, 40m height. So we need to understand the dynamics of this non structural elements.

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This is you can see in this one this is a bare film and then the finished one will have so many non structural elements. So like say duped the ceilings, sprinklers, partitions, furniture, lighting arrangement, air-conditioning units so many things are there.

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As you can see this one so during earthquakes because of deformation and also because of forces so they got damaged.

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Non-structural Elements...

- Infringements understood



IMPOSED DEFORMATION

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The diagram shows a yellow structural frame on a black background. The frame consists of two vertical columns and a horizontal beam. The left column is fixed to a base, indicated by hatching. The right column is also fixed to a base. The horizontal beam is supported by the two columns. An arrow points to the right from the top of the right column, indicating a horizontal displacement. The frame is shown in a deformed state, with the columns and beam curved. The text 'IMPOSED DEFORMATION' is written in yellow below the diagram. The number '8' is in the bottom left corner of the slide.

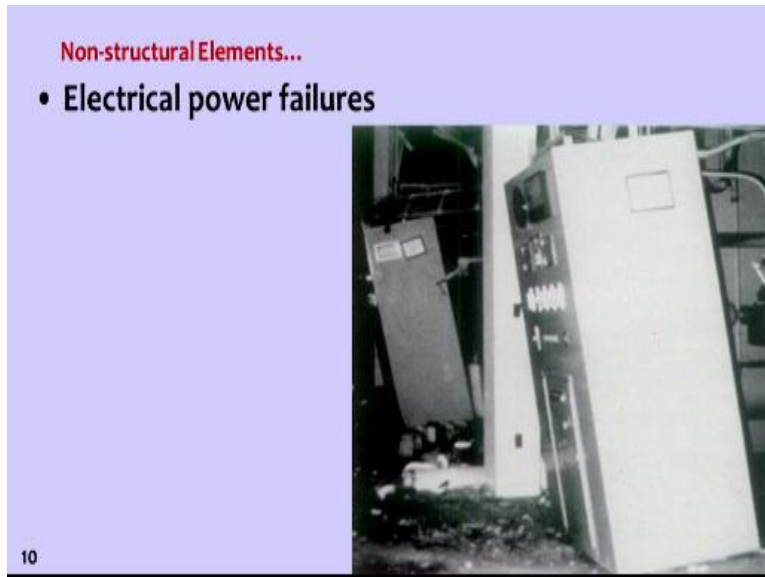
This is imposed deformation.

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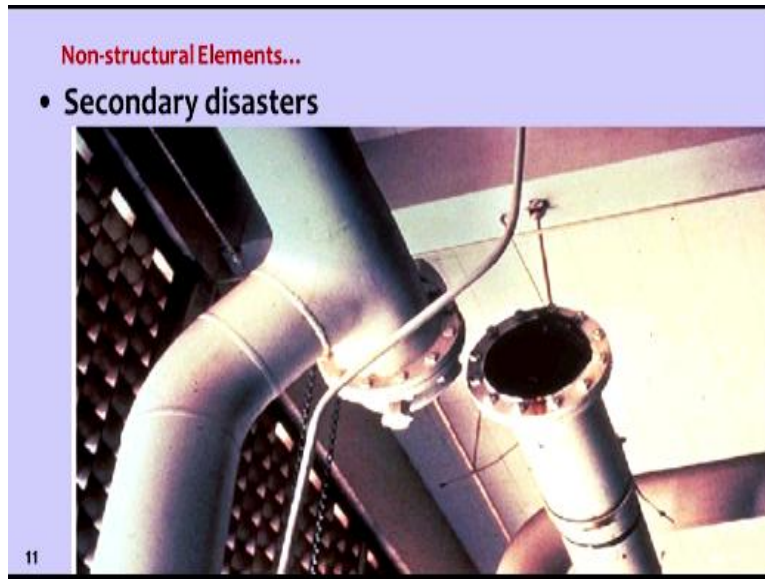
And you can see deformation of the rack.

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And also you can see like equipment.

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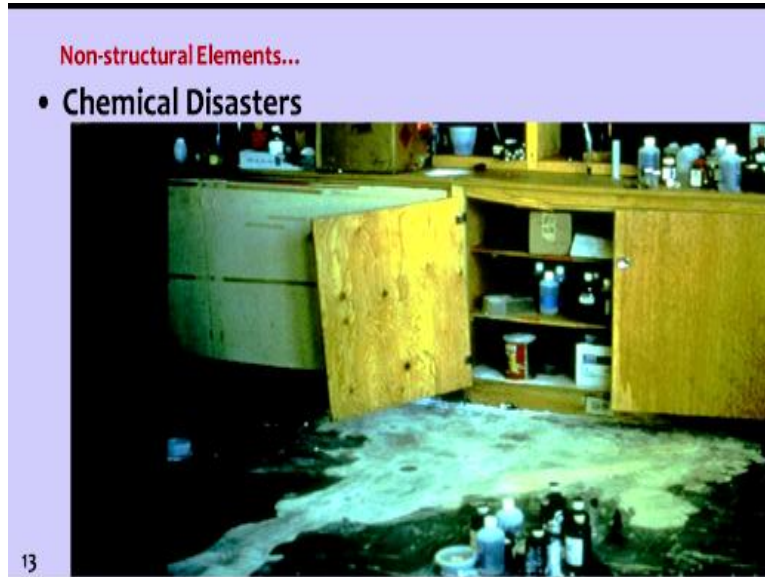
So this is secondary disaster this is also non structural element.

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So but mainly we will study non structural elements which are rigid in nature, so how dynamics of that will play.

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Non-structural Elements...


- **Overhead safety**



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Non-structural Elements...

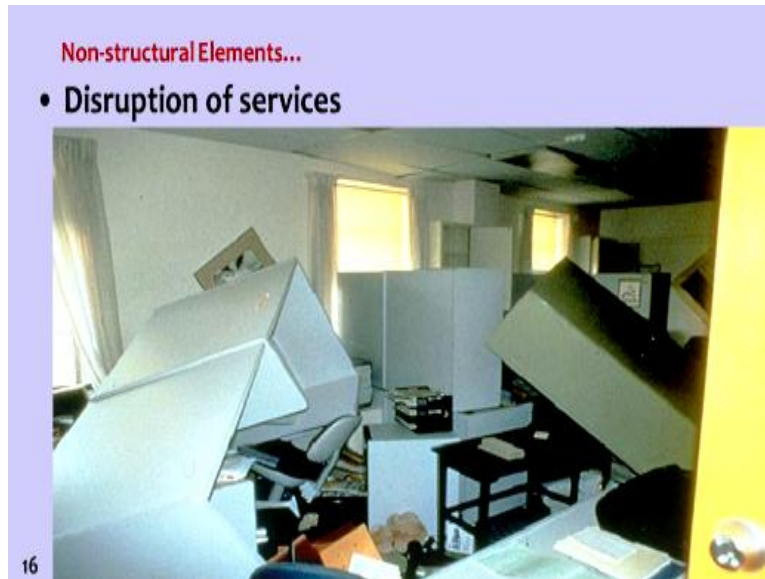
- **Indirect losses**



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In direct losses.

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Due to this and this furniture behavior.

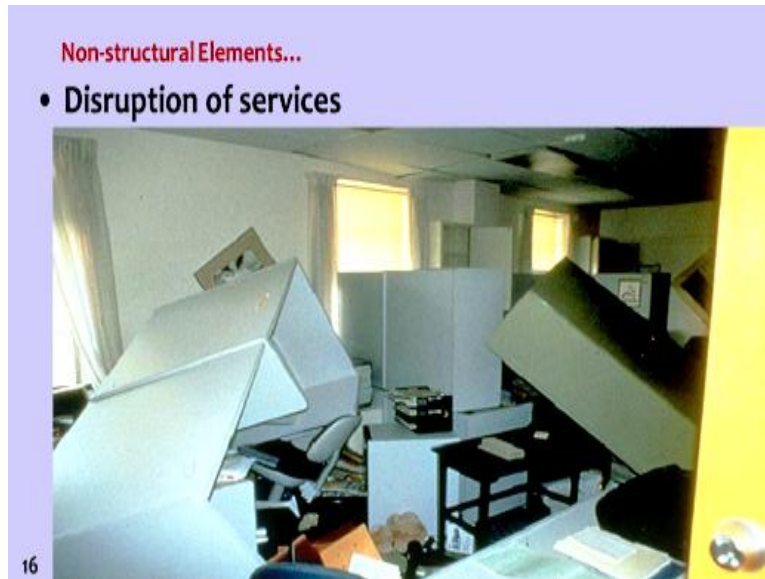
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Non-structural Elements...

- **May be acceptable**

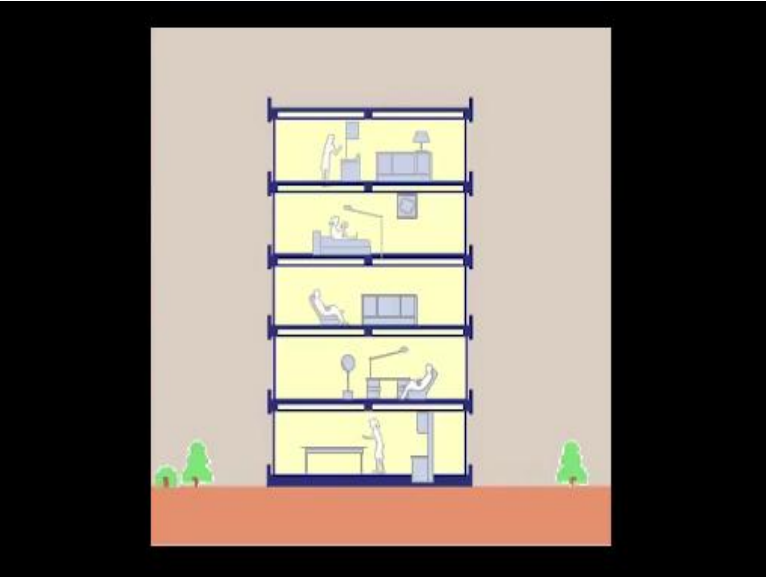


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So let me show you a small video clip which is describing this furniture behavior.


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Non-structural Elements...

- **May be acceptable**



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A long boundary walls also act as a non structural element or rigid block dynamics and you can see this on the view.

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Another example.

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Classification

- **Three sets**
 - *Building Façades*
 - e.g., Glass walling on perimeter
 - *Building Contents*
 - e.g., Furniture
 - *Building Services*
 - e.g., Electrical wires, water pipes



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So there are three sets of these non structural elements one is building facades, so for example glass volume on the perimeter. And then building contents that is example furniture and say equipments, and then building services, electrical wires, water pipelines. So we are mainly focusing on this contents of the building in this class.

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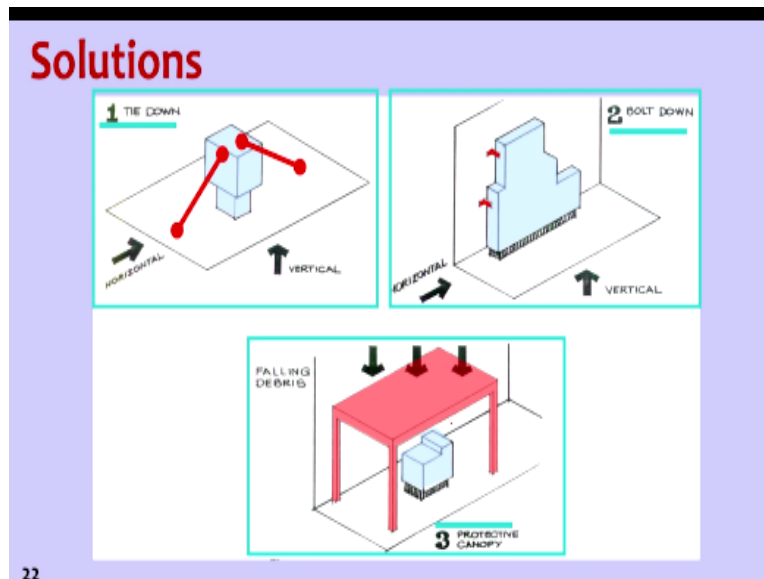
Types

- **Two**
 - Those that are affected by force
 - **Stiff and massive**
 - e.g., Diesel generator
 - Those that are affected by displacement
 - **Flexible and light**
 - e.g., Water pipes
 - Those that are affected by BOTH
 - **Flexible and massive**
 - e.g., Massive hoardings and billboards

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So they are of the type though that are affected by force. So they are stiff and massive, so for example, it is diesel generator installed inside the building and those that are affected by displacement, so they are flexible and tight. So for example water pipelines. And those that are affected by both flexible as well as massive, massive hoardings, billboards like that.

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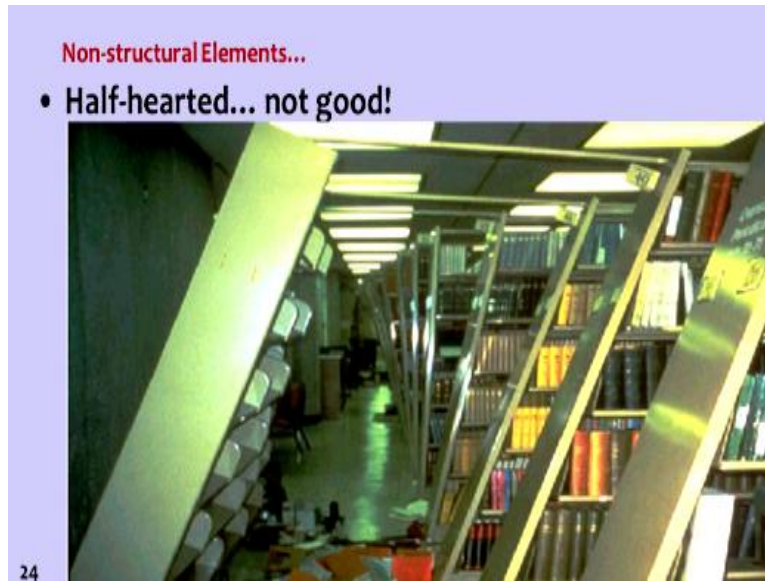
The solution to this one is liquid tied down, so tie down here you can see we are tying it down horizontally, vertically we are tying it down. The second solution is bolt down, and third solution is falling debris so you put some safety over the equipment. So this is protective canopy, so these three solutions are there, but for executing the solutions we need to do some calculations. So what will be the strength of this protections.

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Okay, so you can see this one.

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Okay.

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Seismic Design force

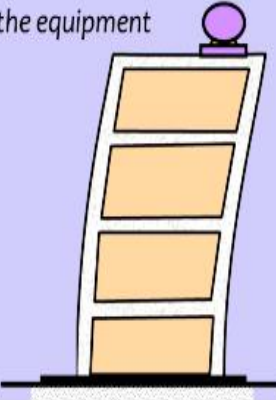
- Equipment
 - Stiff and massive
 - Need proper bolting
 - Overturning problems



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Challenge

- **Tightening the bolts very hard**
 - *Shaking of the floor transferred to the equipment*
 - *Equipment can get damaged*
 - **Manufacturers need to PRE-QUALIFY equipment**
 - *Test their safety under different ground motions*
 - *Shake table test on prototypes*



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Tightening of bolts very hard so shaking of load transformed to equipment from base 2 on the roof. So we will solve an example problem and understand this one. So tightening of bolts very hard so shaking of the floor transfer to the equipment, so equipment can get damaged so that is another issue we need to address.

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Challenge

- **Not anchoring at all**
 - *Shaking of the floor can cause sliding of the equipment*
 - **Can get damaged due to effects of sliding**
 - *Fall off the floor*



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And not anchoring at all, so then what the, it may cause sliding failure of the equipment.

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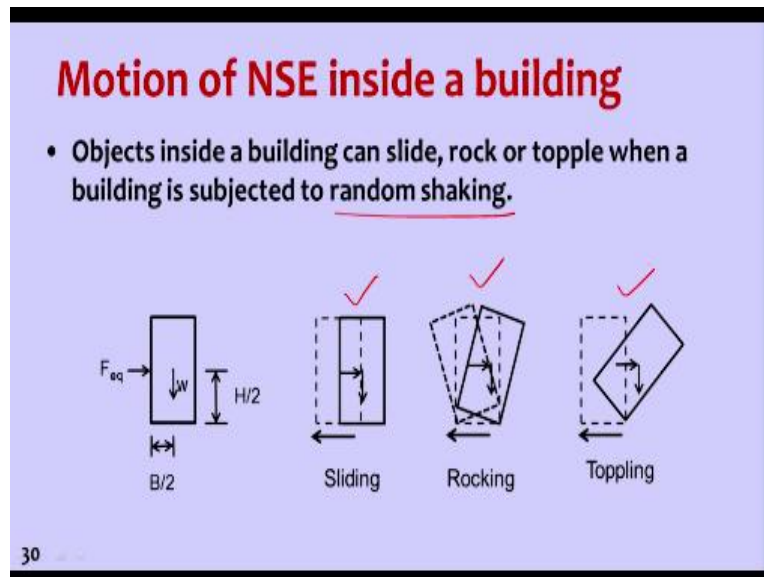
Non Structural Elements

- The Non-structural elements of a building are the ones which are not a part of any load resisting system of the building.
- Hence these are often neglected from the design point of view.
- But the past earthquakes clearly point out the poor performance of several building due to the inadequacy of design provisions for the NSE.

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So this non structural elements of building are ones which are not part of any load resisting system of the building. And hence these are often neglected in from the design point of view. So but past earthquakes clearly point out the poor performance of several buildings due to the inadequacy of design provisions for the non structural elements.

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So motion of non structural elements inside the building so objects inside the building can slide, can rock or topple when a building is subjected to random shaking. So the first example is say this is a non structural element so whose weight is given by w and earthquake force is acting at a center of mass and a centre of mass is exactly at $H/2$ and from horizontal direction $B/2$ and $H/2$.

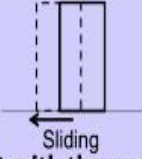
So if it is sliding means what so when the motion is given then it slides in horizontal direction then it can also rock depending on certain conditions which will discuss and then third one is topple, so it can slide it can rock or it can topple so all these three things we need to avoid.

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Sliding and Rocking

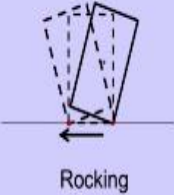
Sliding

- Base of the NSE is completely in contact with the surface on which it is rested
- But is horizontally translating on that surface with friction



Rocking

- NSE is not sliding
- But is locked at toe and lifts from its head



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Sliding and rocking this is a combined thing so the base of the non structural elements is completely in contact with the surface on which it is restrict this is because of the friction but is horizontally translating on the surface with friction so it is horizontally translating and then rocking so non structural element is not sliding but it is locked at the toe and lifts his head something like this, okay.

So if you move it then it is locked at the top and lifting from the heel or you look it from other direction it is locked at the toe here and lifting from the heel.

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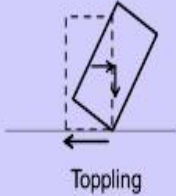
Toppling

Toppling

- NSE is rocking, losing its balance
- Finally ends up sideways on surface

These conditions depend on the

- Geometry of the object
- Ground shaking
- Frictional Resistance



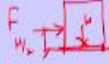
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Then third one is toppling, so toppling is non structural element is rocking losing its balance finally ends up sideways on the surface so when this condition comes we will discuss, so these conditions depend on geometric of te object, ground shaking and then frictional resistance.

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Governing conditions – Only sliding

- Push is more than frictional resistance
- Restoring moment is more than overturning moment


$$F_{eq} > \mu W$$
$$ma_{eq} > \mu mg$$
$$a_{eq} > \mu g$$
$$F_{eq}(H/2) < W(B/2)$$
$$ma_{eq}(H/2) < mg(B/2)$$
$$a_{eq} < g B/H$$
$$a_{eq} < \mu g$$

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So governing conditions for only sliding so when push is more than frictional resistance and second condition restoring moment is more than overturning moment so let me discuss this one so as we know weight here on the base weight is acting and we want to push so this push that is force of earthquake as to overcome weight times frictional resistance so that means what force = earthquake force = mass of the object x earthquake acceleration.

Should be greater than frictional resistance weight means again mass into gravity mass of the object so this mass of the object can be removed so if acceleration is greater than frictional resistance times gravity it will slide so that is how this equation is this equation is governing the sliding only sliding condition, so that means what $a > \mu g$ it is interesting to note here that geometry is not in the picture.

So then the second one is over turning moment and restoring moment so over turning moment if I look at it so from this is $F_{eq} H/2$ so moment so if I take at this point so this force F_{eq} multiply by lever arm that is $H/2$ = w so this in the clockwise direction force x $H/2$ is clockwise direction w is weight, weight x $B/2$ is an anti clockwise direction so this if it has to overturn moment due to applied force should be greater than this one but for not over Turning

only sliding condition this has to be less so $F \times H/2$ should be less than $w \times B/2$, so F to be known $m \times a^3$ earthquake acceleration multiply by mass.

Multiply by $H/2$ should be less than $m \times g \times B/2$ if we this type of m then what we get is acceleration should be less than g multiply by aspect ratio so B/H is aspect ratio, so acceleration less than $g \times B/H$ is the condition, so two condition missed two conditions are there for only sliding case so acceleration should be greater than frictional resistance times gravity and again acceleration should be less than gravity times aspect ratios. So then sliding only phase will take place that means object will only slide.

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Governing conditions - Only Rocking

- Frictional resistance is more than push
- Overturning moment is more than restoring moment

| | |
|--------------------|--------------------------|
| $F_{eq} < \mu W$ | $F_{eq}(H/2) > w(B/2)$ |
| $ma_{eq} < \mu mg$ | $ma_{eq}(H/2) > mg(B/2)$ |
| $a_{eq} < \mu g$ | $a_{eq} > gB/H$ |

The second case only rocking in only rocking frictional resistance is more than push so that means what we cannot push but at the same time over turning moment is more than restoring moment so this is a similar case but slight change in the condition here the earthquakes acceleration is peak acceleration so earthquake acceleration should be less than friction time as gravity here earthquake acceleration should be gravity times aspect ratio should be greater than that so these are two conditions for only rocking, earlier case was only sliding second case is only rocking.

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Governing conditions -Only Toppling

- **Overtuning moment is more than restoring moment**
 - Also Frictional resistance is more than push
$$a_{eq} < gB/H$$
- **The second equation can be derived from simplifying the non linear governing differential equation for toppling.**

$$a_{eq} > \alpha g \left[1 + \frac{1}{6} \left(\frac{\omega}{p} \right) \right]$$

Derivation of this condition will be described in the following sections

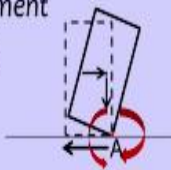
And the third one is toppling in toppling over turning moment is more than restoring moment also frictional resistance is more than push, so in that case A acceleration peak acceleration is less than gravity times aspect ratio B/H the second equation can be derive from simplifying the non linear governing differential equation for toppling, so what is that let us look at it, so thi9s is non linear governing.

Solution of non linear governing differential equation for toppling condition so if acceleration is more that α times g multiplied by $1 + 1/6 \times \omega/p$ so now what is this p and ω we will discuss now.

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Toppling – Equation of motion

- **As long as CG remains within the width (B),**
 - inertia force causes overturning moment (about point A)
 - gravity force causes restoring moment
- **Once CG crosses B,**
 - Both cause overturning moment
 - Resulting in toppling of NSE



So ω is a force free frequency so as long as CG center of gravity remains within the width inertia forces causes over turning moment about point A so this is point around which over turning is taking place, gravity force causes restoring moment so this you all know once CG crosses B both cause over turning moment resulting in toppling of non structural element so it is something like this.

Okay so this is a overturning movement and this is a restoring movement so if we take from this point moments so if the cg crosses this vertical line then both will act in the same direction so currently what is happening is applied force is acting in clock wise direction and then restoring movement is acting in the anti clock direction so that is why there is a balance but if restoring movement that cg of this one crosses vertical line then applied force and restoring movement.

Both act in the same direction so there by non structural element will topple so it's something like this you can see this one this is case one this case two in case one movement due to applied force is acting in clockwise direction the restoring movement is the anti clock wise direction here once the cg has crossed the line vertical line then applied movement is applied force

creating movement in clock wise direction and also self wait creating movement in clock wise direction so this a condition for toppling.

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Toppling-Equation of motion

• Equation of motion can be written as

$$I\ddot{\theta} + mgR \sin(\alpha - \theta) = -ma_{eg}R \cos(\alpha - \theta) \quad \theta > 0$$

$$I\ddot{\theta} + mgR \sin(-\alpha - \theta) = -ma_{eg}R \cos(-\alpha - \theta) \quad \theta < 0$$

• Assuming the earthquake excitation to be sinusoidal and the equations are linearized into

$$\ddot{\theta} - p^2\theta = -\frac{a_{eg}}{g} p^2 \sin(\omega t + \psi) - p^2\alpha \quad p = \sqrt{\frac{3g}{4R}}$$

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Now let's look at the equation for toppling so we can derive that covering derived equation in the picture the free body diagram so I is a moment of inertia okay θ double dot this rotation moment of inertia θ double dot + mg r sin alpha - θ so alpha is the angle with which non structural element is getting lifted this is alpha and θ is this is θ . So as you can see sin alpha - θ is = restoring that is m earthquake acceleration multiplied by r cos alpha - θ alpha - θ so when alpha is increasing so alpha you can consider this as the degree of freedom when alpha is increasing if it is = θ then toppling condition takes place.

So θ is greater than 0 and we have another one θ_1 is less than 0 so this is from this location θ is better than 0 this location on other side it is θ less than 0 we use this equation so two, two equations we use so assuming the earthquake excitation to be sinusoidal and the equations are linearized into so what we will get is a solution so we need solution for θ .

So this is a non linear equation solve this by doing the linearization that p is frequency ratio that is $3g/4r$ p is a frequency parameter so were r is from center of mass or center of gravity up to the diagonal length is r.

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Topping-Equation of motion

- On solving we get,

$$\theta(t) = A_1 \sinh(pt) + A_2 \cosh(pt) + \alpha + \frac{1}{1 + \frac{\omega^2}{p^2}} \frac{a_{eq}}{g} \sin(\omega t + \psi)$$

- where

$$A_1 = \frac{\theta_0}{p} - \frac{\omega/p}{1 + \frac{\omega^2}{p^2}} \frac{a_{eq}}{g} \cos(\psi)$$

$$A_2 = \theta_0 - \alpha - \frac{1}{1 + \frac{\omega^2}{p^2}} \frac{a_{eq}}{g} \sin(\psi)$$

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On solving we get this θ every complex equation.

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Condition for toppling

- Now let $z = \alpha - \theta$
 $\ddot{z} = -\ddot{\theta}$
- Equation becomes $-I_o \ddot{z} + mgR \sin(z) = -m \ddot{u}_g R \cos(z)$
- For small angles $-I_o \ddot{z} + mgR \sin(z) = -m \ddot{u}_g R$ ✓

- On solving we get
$$\hat{z}(T) = \frac{-mR}{\left(\frac{2\pi}{T}\right)^2 I_o + mgR} \hat{u}_g$$

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Then conditions for toppling = $\alpha - \theta$ \ddot{z} is $-\ddot{\theta}$ then finally equation is becomes for small angles we use this equation complex equation and solving we get \hat{z} this is frequency domain.

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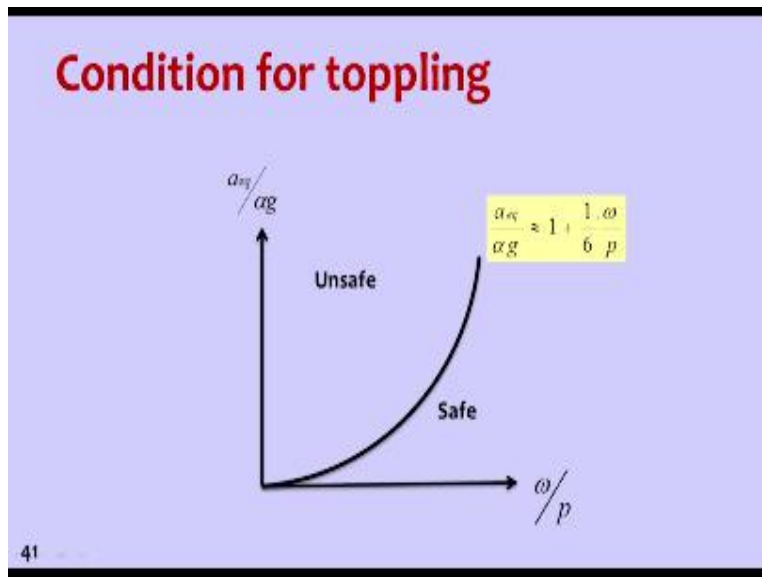
- For a rigid block $I_o = \frac{4}{3}mR^2$
- Hence $\ddot{z}(T) = \frac{-1}{\left(\frac{2\pi}{T}\right)^2 \frac{4}{3}R + g} \dot{u}_g$

$$a_{eq} > g \left[1 + \frac{\pi}{3T} \sqrt{\frac{\sqrt{B^2 + H^2}}{1.5g}} \tan^{-1}\left(\frac{B}{H}\right) \right]$$
$$a_{eq} > \alpha g \left[1 + \frac{1}{6} \left(\frac{\phi}{p} \right) \right]$$

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So with this we get the condition acceleration peak acceleration if it is greater than g times all these factor so that is the condition for toppling so this is a simplified forms so.

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So if you draw the graph between say frequency parameter and forcing frequency and normalized acceleration value so this is a condition for safe and unsafe reason so if like if acceleration ratio is crossing this line it is unsafe for the rigid block and if it is below this one it is safe so this the necessary conditions for designing the safety of rigid blocks.

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| Governing conditions | | |
|-----------------------------|------------------|--|
| Sliding | Rocking | Toppling |
| $a_{eq} > \mu g$ | $a_{eq} < \mu g$ | $a_{eq} < gB/H$ |
| $a_{eq} < gB/H$ | $a_{eq} > gB/H$ | $a_{eq} > \alpha g \left[1 + \frac{1}{6} \left(\frac{\omega}{p} \right) \right]$ |

Now for governing equations for governing conditions for sliding rocking and toppling so for us to design if acceleration is greater than friction times the gravity and acceleration is greater than less than gravity aspect ratio when this two conditions are met then sliding takes place when acceleration is less than new time gravity and acceleration is greater than the g times aspect ratio rocking conditions takes place.

Rocking takes place and toppling is this condition and acceleration is less than g times aspects as aspect ratio as well as acceleration is greater than the alpha g $1 + 1/6\Omega/p$ so these are the governing conditions in summary.

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What we have discussed in this class of dynamics of non structural elements so there are many non structural elements which are there in the building so earlier non structural elements cost percentage was around 10 15 % but these days non structural element is has high as 80% of the building cost but the issue is the non structural element are not taken into consideration while doing the design so that's why even if the main structure is safe sometimes there are seiver injuries leading to death.

Just because of the non structural elements behavior dynamic behavior so what we have studied was in this class non structural elements which are rigid in nature how they behave so either slide they rock or they topple so what are the necessary conditions do sliding rocking and toppling we have discussed in this class so in the next class we will solve one numerical problem of designing the safety of non structural elements.

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