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Structural Dynamics
Week 10: Module 03

Theory of Seismometer

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Welcome to structural dynamics class. In this class we will discuss the theory of seismometer. So how we use the principles of dynamics in designing an equipment which measures earthquake record or earthquake ground motion.

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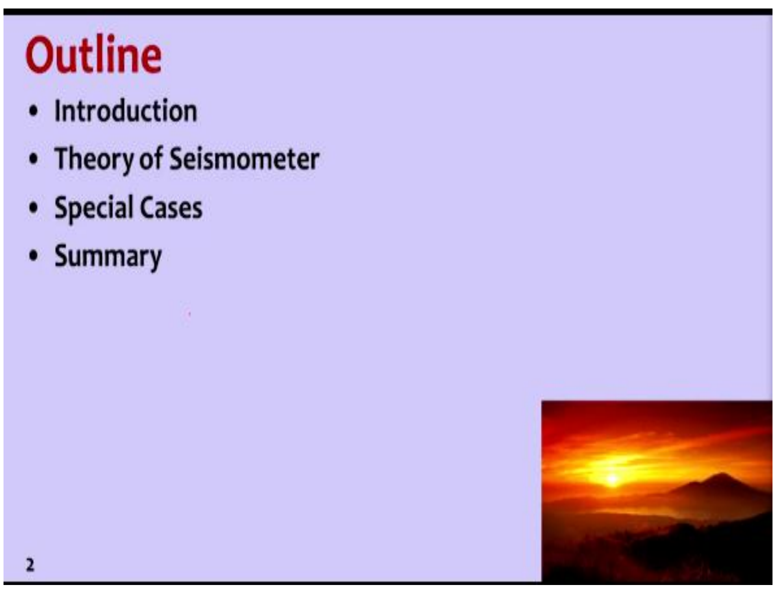

So before going into the theory let us watch some earthquake ground motion records and equipments how they are recording this one.

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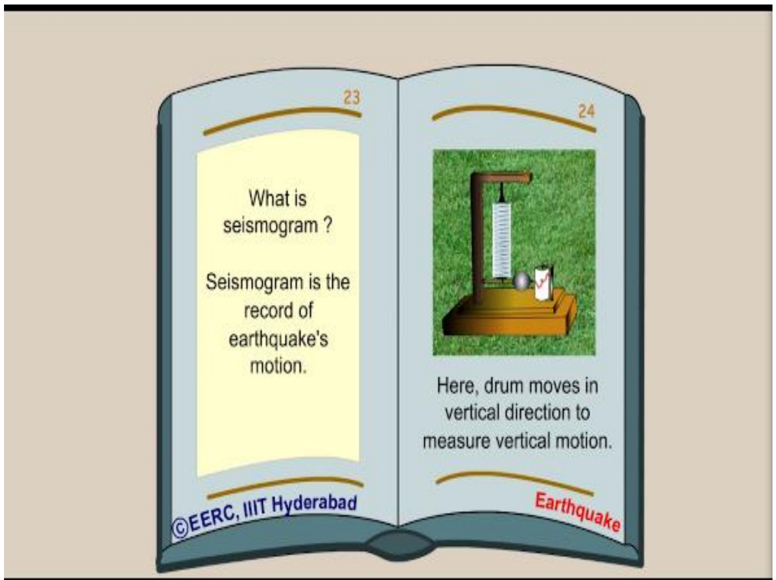

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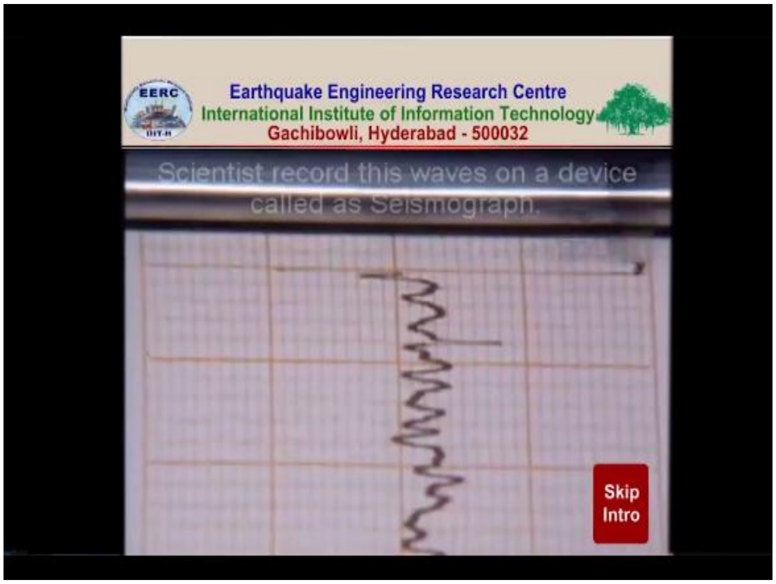

Female Speaker: Scientist record this waves on a device called as seismograph. The zig-zag lines show the strength of the various seismic waves.

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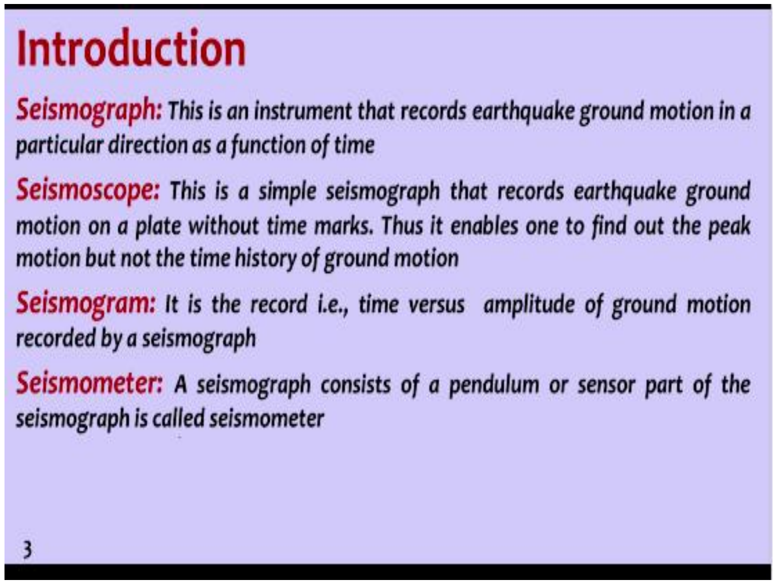

So let us discuss the various terms which are used in this measurement. So first one is called seismograph. So seismograph is an instrument that records earthquake ground motion in a particular direction as a function of time. So that is called seismograph and then seismoscope, this is a simple seismograph that records earthquake ground motion on a plate without time marks. So thus it enables one to find out the peak motion but not the time history of the ground motion.

And third one is seismogram, seismogram is a record so it is a record that is time versus amplitude of ground motion recorded by seismograph. And the fourth one is seismometer, a seismograph consisting of a pendulum or sensor part of the seismograph that is called seismometer these are the terms.

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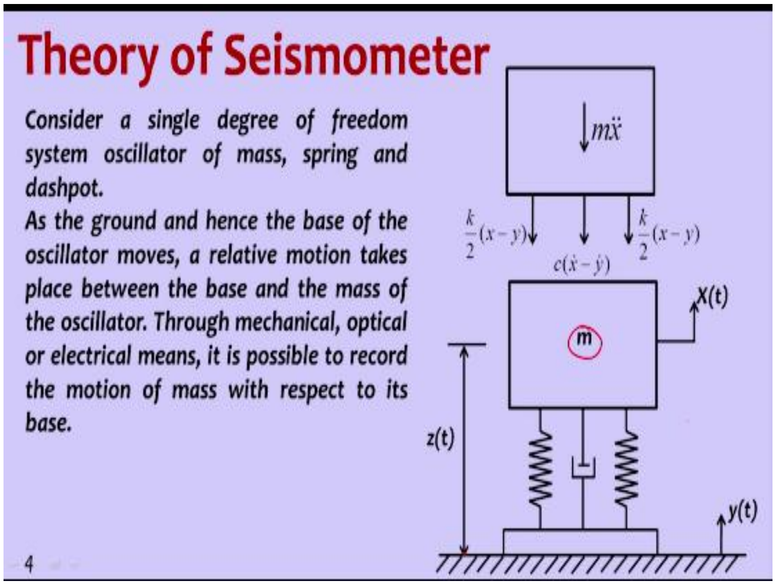

Now what is the principle of this one. So let us consider a single degree of freedom system oscillator which has mass, spring and dashpot so something like this. So we use the basic structural dynamics principles of single degree of freedom system and try to device an instrument there. So this is a free body diagram of that, so if we give motion X so it is a free body diagram and the relative displacement between the ground and the center of mass is given as Z.

So as the ground hence the base of the oscillator moves, a relative motion takes place between the base and the mass of the oscillator. So base and the mass of the oscillator. Through mechanical, optical or electrical means, it is possible to record the motion of mass with respect to its base. So that means what, we cannot independently record the motion of the mass, but we can record the relative displacement between base and the mass.

So let Y be the ground displacement which we do not know we do not know how to measure it right now. And let X be the displacement of the mass and Z is the relative displacement between these two.

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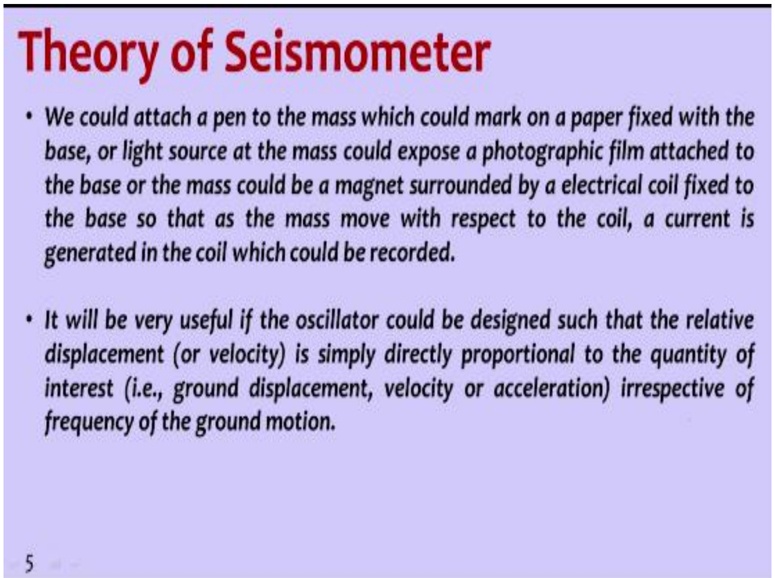

So if you draw the free body diagram, so what we get is.

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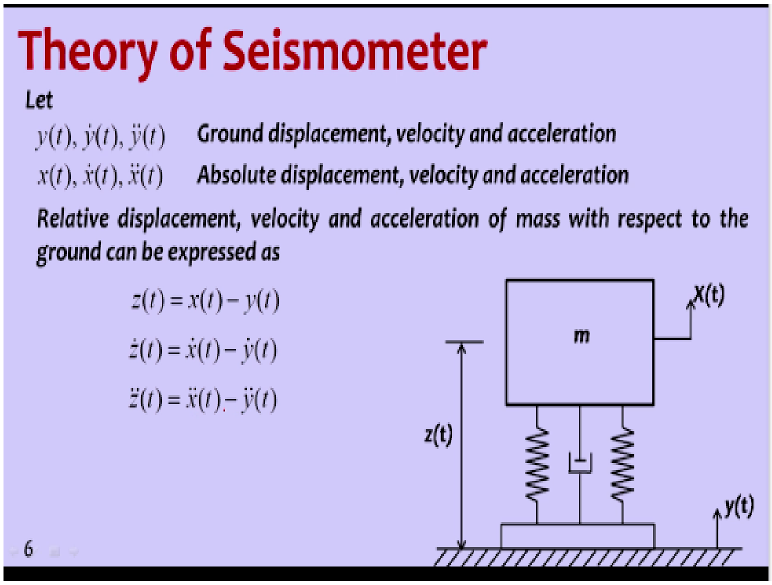

Let y y. and y.. are ground displacement, ground velocity and ground acceleration respectively, x x. and x.. are absolute displacement of the mass, absolute velocity and absolute acceleration of the mass itself. So what we can do is we can write, we cannot measure X directly, we cannot measure Y directly, but what we can measure is the relative displacement between these two.

So displacement X-Y is a relative displacement x. – y. is a velocity relative and then x.. – y.. this we can measure.

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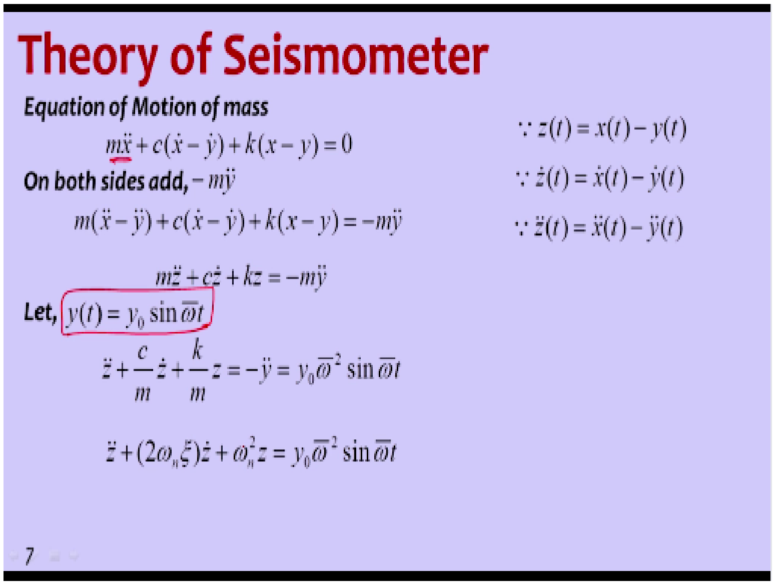

Now if we draw a free body diagram so what we get is say mx.. + c(x. – y) = k(x-y) = 0 so mass is moving by x.. only so inertia force is related to the absolute motion of the mass which is x.. absolute acceleration of the mass and then damping and stiffness are relative terms so that means what as long as there is extension or compression in the springs then only force in the springs will appear otherwise that is it is 0.

So that is why m_h.. + c(x. – y) + k(x – y) = 0, so if we introduce a new term which is –my.. in the equation so we will get this and on the left side and the right side so if we call this x-y as z, x. – y. as z. and x.. – y.. as z.. so we can rewrite the equation like this so this is governing differential equation for single degree of freedom system under earthquake excitation this is similar to that.

Now comes to the solution of it, so how do we solve this one, we will approximately will assume that y(t) = y0 is the amplitude of ground displacement Sin ω dash t so this ω dash is forcing frequency that is a ground frequency or frequency of the ground motion, so if we divide this above equation by mass we get z.. + c/ mz. + k/m z = -y.. so that is if you double differentiate it y0 ω^2 Sin ω t. So minus, minus will get cancelled so we can write the terms like c/m is 2ζ ω n and k/m is ω n^2 so this from the first principles of structure.

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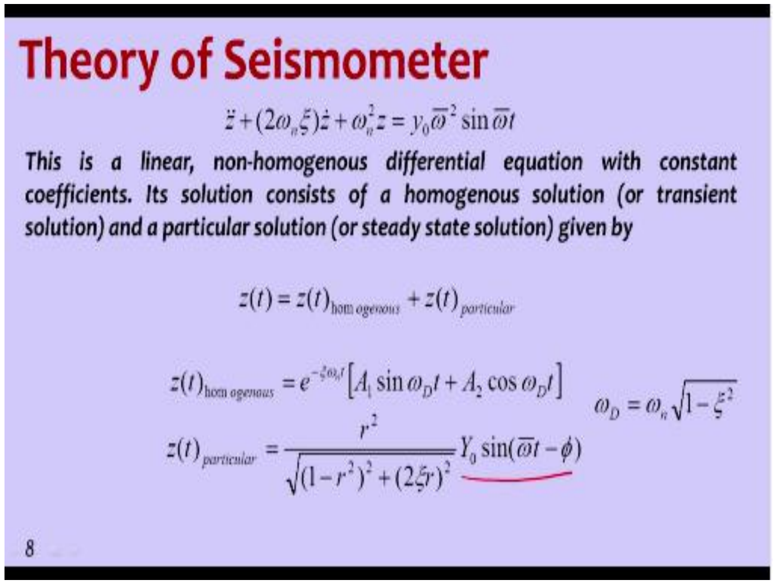

We already know that, so the above is the linear non homogenous differential equation with constant co-efficient again we know this one, so it is a solution consists of a original solution or transient part and a particular solution which is steady state part so this zt homogenous part will be there and z t particular integral part will be there both will be there, so but we know that this homogenous part will die out because of the presence of damping.

So we will not consider that so then particular integral part will be z particular r^2 under root $(1-r^2)^2 + 2\zeta\ r^2$ x y so y0 sin ω t – Ø so if there is any confusion in this equation we may refer to single degree of freedom system forced vibration damped forced vibration class.

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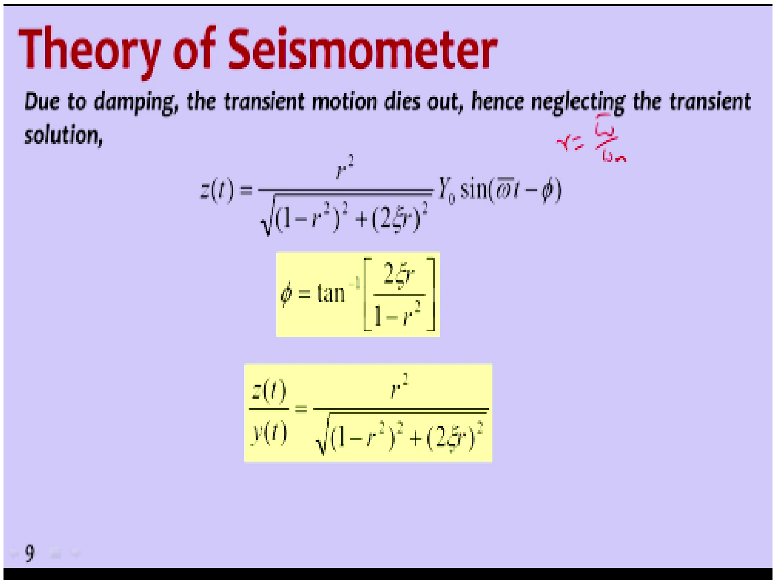

Then due to damping the transient motion dies out hence neglecting the transient solutions we have particular integral solution this, so here phase angle is Ø = tan $^{-1}$` 2 ζr / 1- r^2 r is a frequency ration here in this case r is ω bar by ω n, ω n is a natural frequency which is forcing from under root k/m and ω bar is a earthquake forcing frequency, so we can write it like this amplitudes of both the vibration if 0, z/zt maximum value = r^2 / 1- r^2 $)^2$ + 2ζ r^2 .

So let us try to understand some special cases in this equation, if you look at this equation there are some special cases in this equation.

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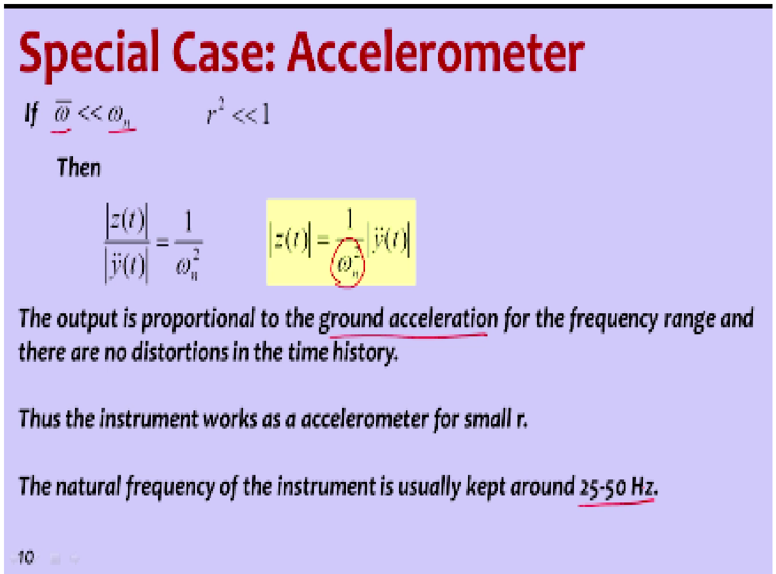

So this special cases come from the ranging of r value, so r means frequency ratio so if forcing frequency is very less than natural frequency so that means what it is almost in the state of rest, so that means r value is much, much less than 1 so that means it is equivalent to 0. So then, $z/\ddot{z}=1/\omega_n^2$ so it is something like this you can see z if we measure z that means it is equivalent to measuring acceleration if you multiply that z value with ω_n^2 so the output is proportional to ground acceleration.

So what we are measuring actually we are measuring with instrument z value only that relative displacement between mass and the ground, so with that mass and the ground discrete displacement if I multiply with ω_n^2 I am getting acceleration value. So the output is proportional to ground acceleration for the frequency range that are, there are the n there are no distortions in the time history, so thus the instrument works as accelerometer for small values of r. So the natural frequency of the instrument is usually kept in between 25 to 50 Hz to measure acceleration so accelerometer will have this range.

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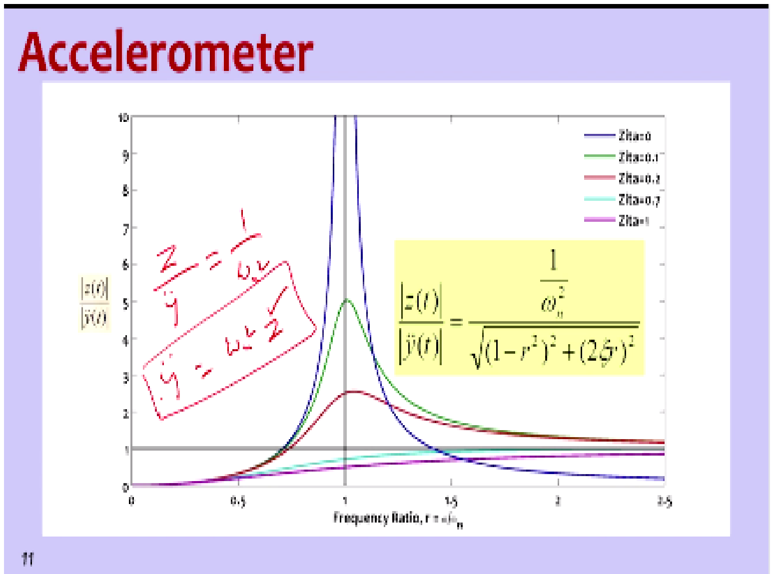

So if we plot that z curve okay, so by double differentiating y so what we get is so $1/\omega_n^2 y$ so $\ddot{y}=1/\omega_n^2 y$ so then we will back substitute that $z/\ddot{y}=1/\omega_n^2\sqrt{(1-r^2)^2+(2\xi r)^2}$ so now in this if we substitute r as 0 so denominator term this is disappear 0 and this will become 1 so $z/z\ddot{y}=1/\omega_n^2$ so that is how if we measure acceleration or measure z we are getting ground acceleration from this one, so it is something like $z/\ddot{y}=1/\omega_n^2$ so $\ddot{y}=\omega_n^2 z$ so we are measuring z, so multiplying with ω_n^2 is giving us ground acceleration so this n this called accelerometer.

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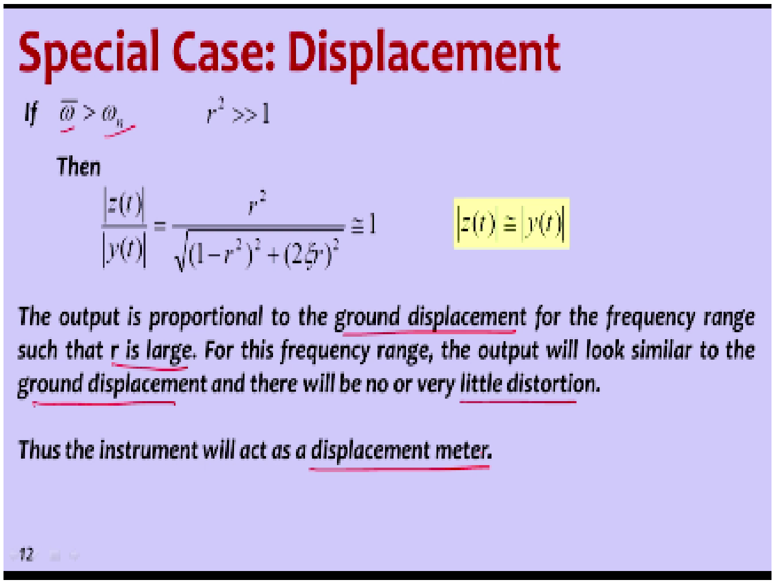

Similarly, we measure displacement how do we measure displacement when forcing frequency is much, much larger than natural frequency of the system so r^2 is much, much greater than 1, so when r^2 is much, much greater than 1 so the ratio is same $r/(1-r^2)^2$ so what it will remain is so this value is approximately equal to 1, so if you look at if substitute some value here say 100 r value has 100 and then check we will get approximately numerator and denominator same value damping will not have any effect there so this z/y=1.

So that means what, for higher frequency exhaustion frequencies whatever displacement we are measuring that is equal to relative displacement we are measuring that is equal to ground displacement. So the output is proportional to ground displacement for the frequency range such that r is large, so for this frequency range the output will look similar to ground displacement and there will be no or very little distortion so the instrument is called displacement meter.

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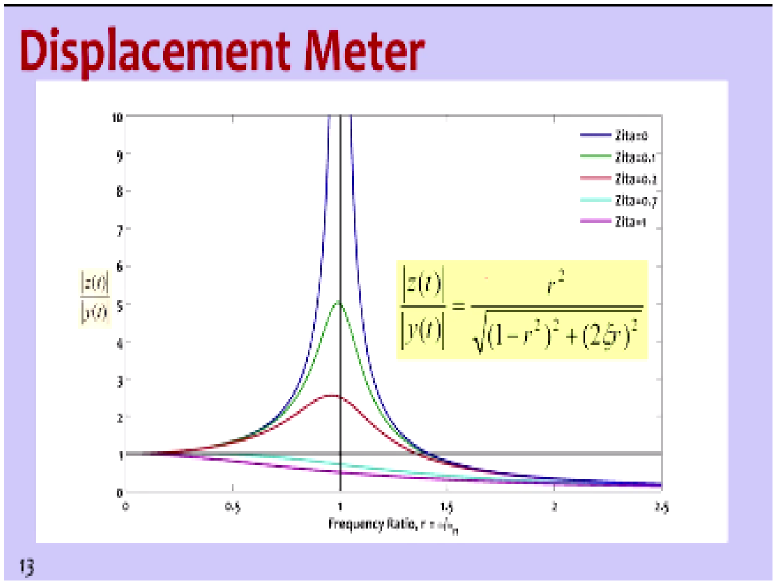

So this a displacement meter z/y=1 so z=y

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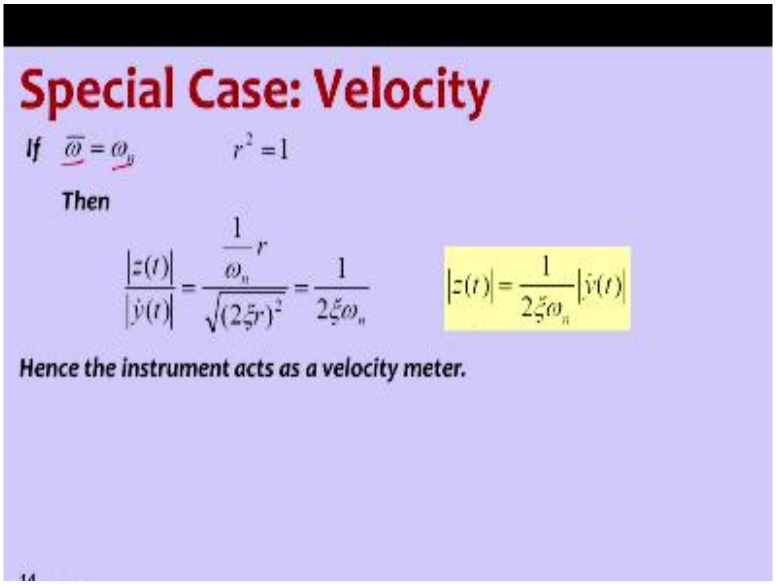

And third one is velocity meter velocity meter like when frequencies are same forcing frequency and natural frequency are the same then R square = 1 R =1 so what will happen is the term will undergo 1-0ξ square ξr will remain the 1/Ωn into r so we will be left with z =1/2ξΩ into y dot. So then we get y dot = from this.

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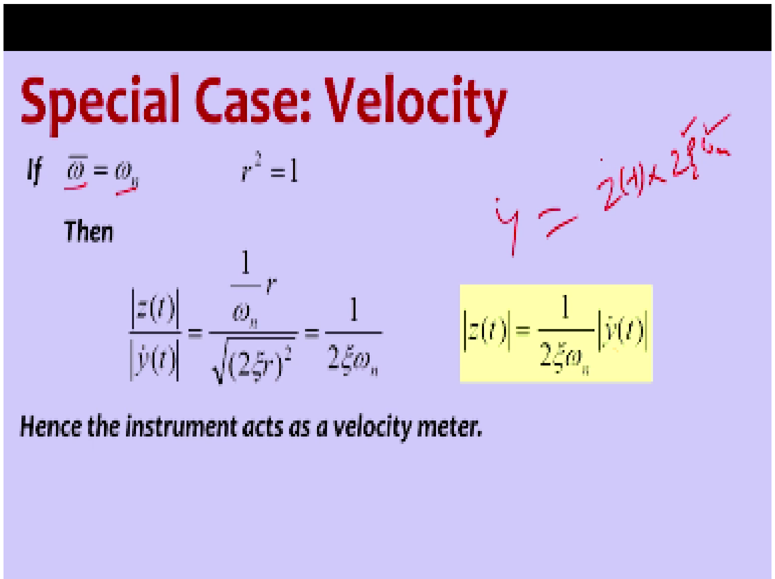

We get y dot = z multiplied by as you can see this equation okay so y dot =z multiplied by 2ξΩn so we know ξ value we know Ωn value anyway measuring z value so we get velocity.

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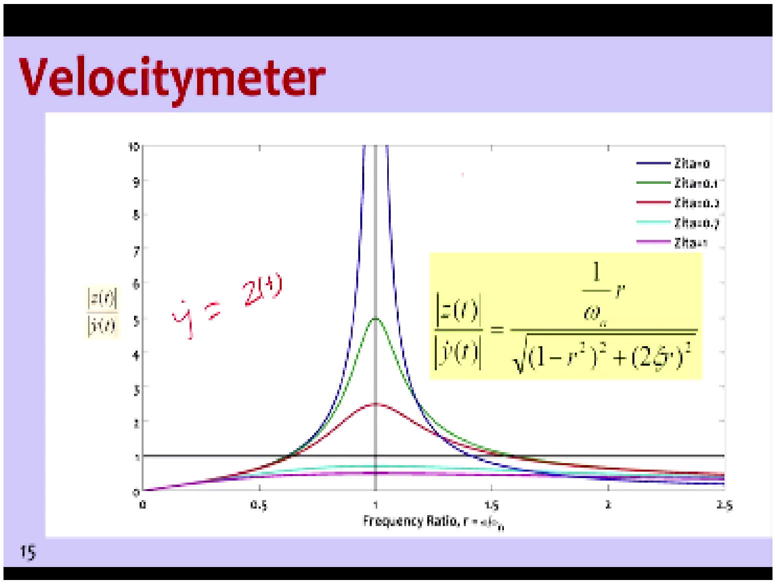

So hence this is called as velocity meter.

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So in summary by using a principles of single degree of freedom system we are understanding how to measure earthquake displacement velocity and acceleration so in a way earth quake acceleration is measured when r value is very low o that means forcing frequency is much, much less than natural frequency of the system so this is acceleration acclerometer and on the other side if the forcing frequency is very, very high compared to natural frequency it is displacement so the so for acceleration acclerometer stiffness should be very high stiffness of the structure is very high and the mass is low less mass high stiffness and on the other side displacement meter more mass less stiffness.

So these two are this next extreme cases as acclerometer and displacement velocity frequency forcing frequency natural frequency if they match that means at a resulting condition so we get a social relationship between ground displacement that is relative displacement and ground velocity so we have studied these three cases of accelrometer displacement meter and velocity meter in the seism meter the concept what we have used is was un damped forced vibration of single degree of freedom systems.

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