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Structural Dynamics
Week 10: Module 02

Example Problem on Continuous system

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Welcome to structural dynamics class. So in this class we will study how to apply the continuous systems principles and solve a real world problems. So this problems could be in many ways say one is like a tower is vibrating, a tower is vibrating because of wind or a cantilever beam is vibrating because of say some force.

So you can assume that as diving board of a swimming pool and a person is jumping on that okay or maybe a sitting arrangement, a platform sitting arrangement or the slab in a stadium where people are sitting on the slab and then in excitement of seeing match, so then we do jumping on this one.

So when they sit it is a static load, but when they jump on it, then it is a dynamic load. So how this static deflection and dynamic deflection are related. So these are all real world problems, so how do we solve this real world problem using a principles of continuous systems. So this, suppose let us take an example of the slab itself a sitting arrangement in the stadium where mass is continuously distributed people load is also continuously distributed. Then how do we solve this problem.

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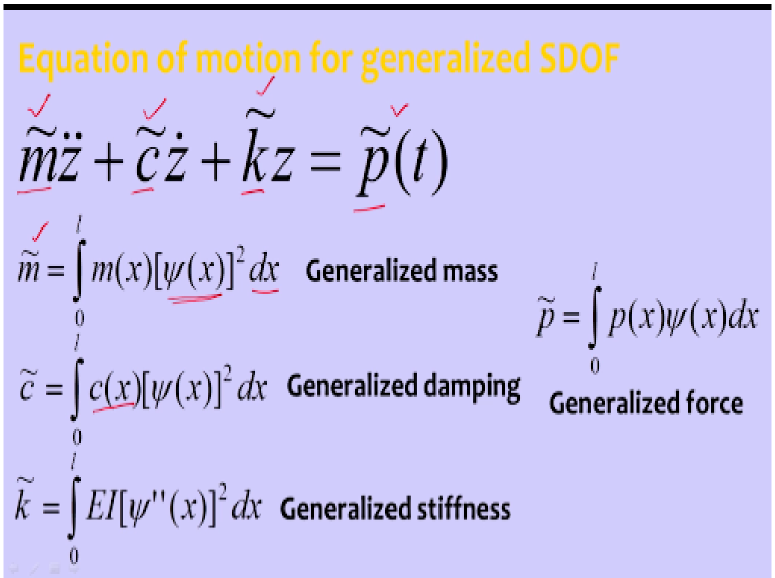

I'm formulating here a problem okay, so before that let me explain you what is a equation of motion for continuous systems. So equation of motion for continuous system is same as mu..cu.+ku=p so in this case m is a generalized mass, because it is not lumped at one location so we need to take into consideration the safe function which is a deflection profile of the member or the structure all that need to be considered.

So that is why we call it as generalized mass z.. is acceleration, c~ is generalized damping z. is velocity, k~ is generalized stiffness and z is the displacement function p~ is also generalized force. Now how do we get this generalized properties from the given structural properties say given structural parameters.

So what are the parameters of the structure geometry property, material property and end conditions three things are there. So if you look at this one m~ we can get integral of 0 to l that length m(x) so if mass is varying according to length. So for example, in this case so density that mass is higher at this location and slowly it is cross section is getting reduced.

So that means what mass is reducing here mass so that is why mass also can be a function of X. So mx $[\psi(x)]^2$ what is ψx, ψx is a shape function or we can also called it as a displacement or deflection profile and then dx so integral over the length will get the generalized mass, generalized mass means if this continuous system has to be made equivalent to a single degree of freedoms system what will be its mass, so that is m ~, then similar to that we have generalizing a damping value.

So damping value present at every location and then ψ x^2 dx this is generalizing damping, similarly we get generalize stiffness so stiffness is again 0 to l Eiψ `` x^2 dx is a generalized stiffness and also generalized force that is px ψ x dx integrated over 0 to l so we have generalized properties so generalized force generalized stiffness generalized damping, generalized mass all these we have. Now let us go into the applications so how do we make use of this and the solve the problem.

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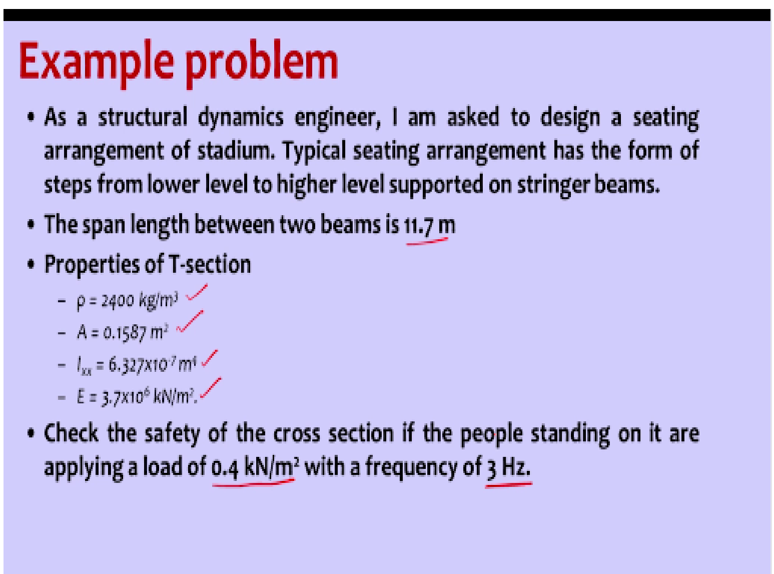

Now the problem is as a structural dynamics engineer I am asked to design a seating arrangement of a stadium, typical sitting arrangement has a is the form of steps from lower level to higher level supported on the stringer beam the span length between two beams is two support sis 11.7 meters and then properties of the t section so that means the cross section of the beam is T.

So that means density is given as so 2400 kg/m^3 area of cross section 0.1587 m^2 moment of inertia along its x – axis is 6.327×10^7 m^4 and then modulus of elasticity 3.7×10^6 kilo Newton per meter square, so using this properties this we need to find the deflection that is maximum deflection so check the same safety of cross section if the people standing on it are applying load of 0.4 kilo Newton per meter square. With the frequency of the hertz so now how these three hertz is coming let me explain you that.

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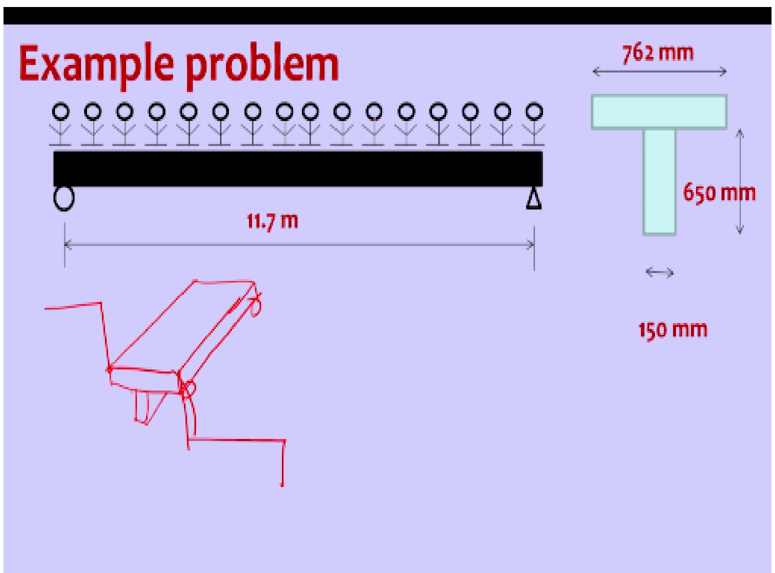

So this is the property so people are sitting on that something lie seating arrangement is something like this so we are talking about this one, so this is along this direction along this direction so this is what we are talking, okay so this beam is supported here supported here so that is by beam so now if I take the cross section of that is a T yeah it is something like this, okay T section.

Now people are standing here as I told you in excitement they are jumping stumping clapping all that so that is generating a harmonic motion of 3 Hz.

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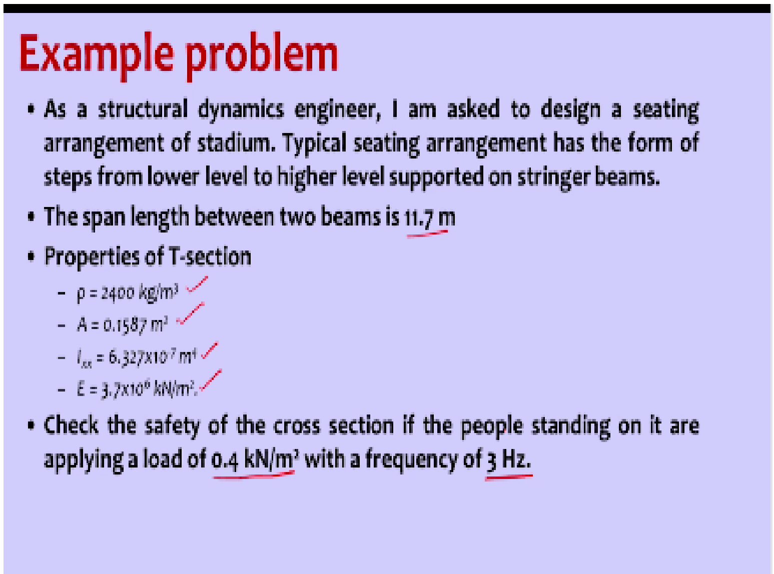

So now we need to put this into say mathematical form.

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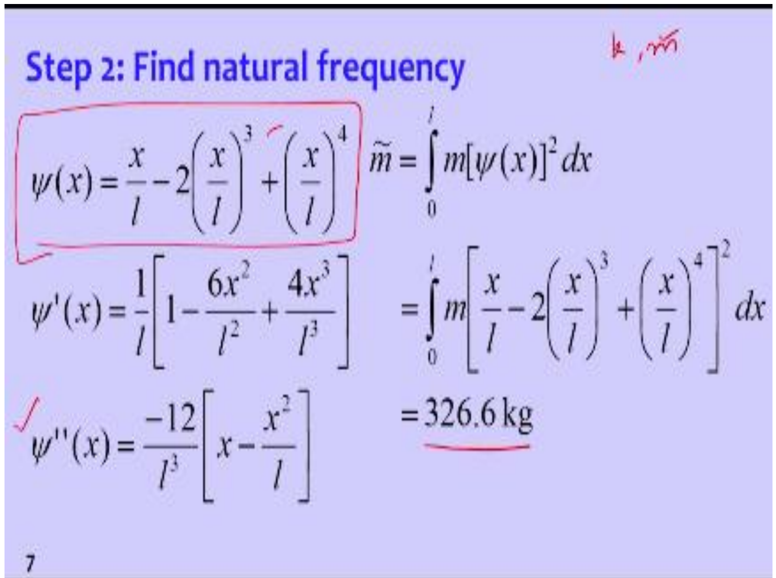

So now step 1, let us first find out the applied force, so we already know that applied force that is generalized applied force is integral 0 to l p(x)dx this is analyst applied force. So that value is say 0.4 kN/m^3 is given I am using that $0.4\text{x}10^3\text{x}0.762\sin6\pi t\text{x}\psi(x)$ okay, so in this one there is $\psi(x)$ term also, $\psi(x)$ okay, so this is a force which is applied sinusoidal, okay then $\psi(x)dx$ all put together we are getting this value 713.23 $\sin6\pi t$ so 6π is the forcing frequency.

Then step 2 let us find out natural frequency of the system, so for finding natural frequency of the system we need k, we need m so $\sqrt{k/m}$ is a natural frequency. So let us first find out what is say m, mass of the system so mass of the system is $m[\psi(x)^2]$ so ψ is given as ψ is a deflection profile is given by this function, so $\psi(x)=x/l\ 2(x/l)^3+(x/l)^4$ so this need to satisfy the boundary condition.

So at x=0 ψ=0 at x=l ψ=0 so then what we get is you can see this one, x is equal to this becomes 1 so this becomes 1, this becomes 1 so 1-2+1 is 0 so at x=l this is satisfying and at x=0 also it satisfying. So now what we get is mass so if you substitute this one in this one so 326.6kg is the mass and then we need to use X″ in calculation of stiffness.

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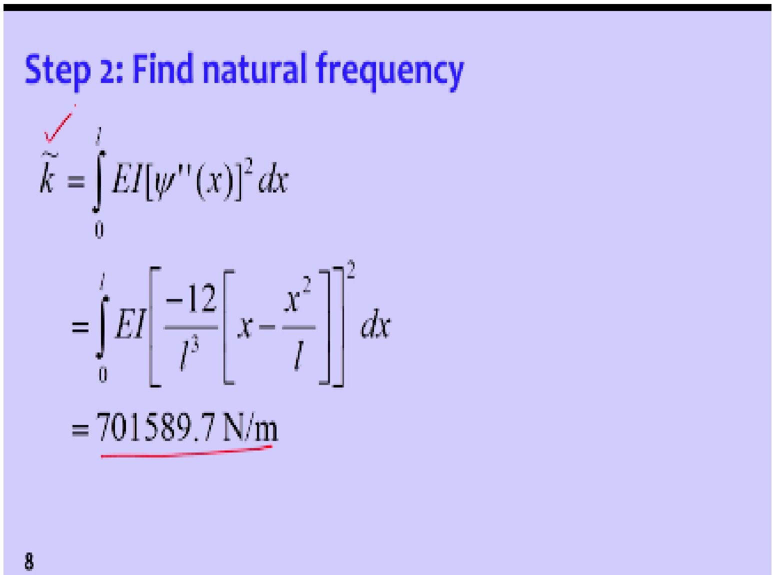

So now calculation of stiffness we get like this EI is known value ψ'' we derived it in the earlier slide so what we get is k=701589.7 N/m as a generalized stiffness. So generalized mass we calculated.

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Step 1: Applied force

$$\tilde{p} = \int_0^l p(x)\,\psi(x)\,dx$$

$$= \int_0^l 0.4\times 10^3 \times 0.762 \sin 6\pi t \times \psi(x)\,dx$$

$$= 713.23 \sin 6\pi t$$

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Generalized stiffness we calculated.

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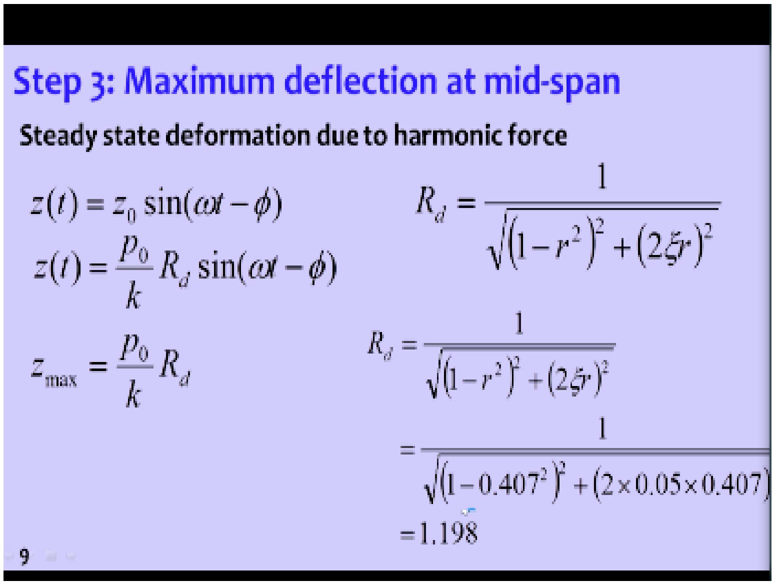

And then what we get is ω=√k/m we substitute value of stiffness and mass so generalized mass generalized stiffness we get 46.25 rad/s, then we get r that is frequency ratio ω/ω_n 6π by this one how we get 0.407 is frequency ratio that is r. Now the maximum deflection is at mid point so for that let us look at the study state deformation due to harmonic force so we have two displacement one is complementary and other is particular integral so we are more interested in particular integral cause this gives the maximum deflection so zt=z0 sin Ω at t-5 with face z 0 is the amplitude so p0 /k rd this we already know okay so you can refer to single degree of freedom system maximum dynamic amplification factor lecture.

So now z max is side part will disappear maximum value of this can be +1 so if we remove that we get z max= p0/k rd so rd is we know that 1-r square whole square +2 ξr whole square now if we substitute r value in this one we get 1.198 is rd.

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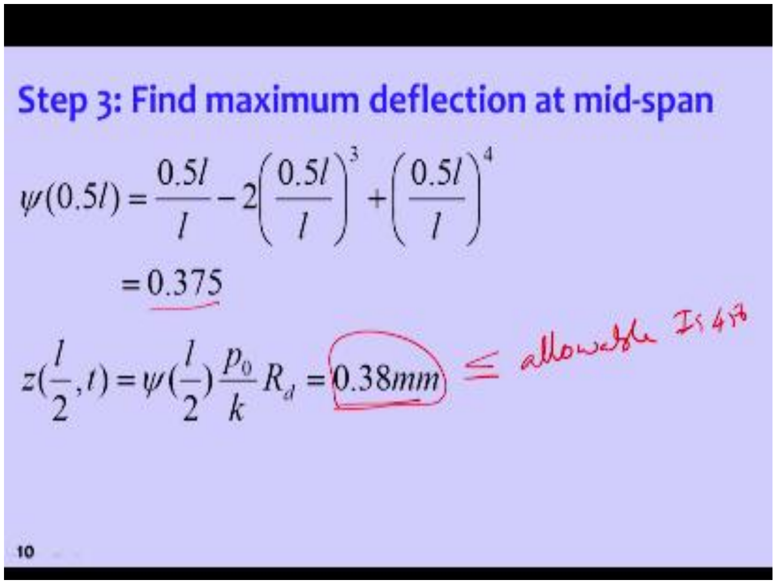

Then maximum deflection is at mid point so maximum deflection at mid point is so this is according to deflection profile we get 0.375 so in deflection profile equation you substitute L=X =L/2 so then we get 0.375 then maximum deflection will be will occur at say peak value of z and at L y 2 so that means what sin at L/2 p0/k rd so p0/k into rd if you substitute these values p0 /k rd value and Л=L/2 value so if we substitute finally what we get is 0.38mm as a deflection because of the loading applied by people in dynamic manner this is the maximum deflection.

So we need to check whether it is less than or equal to allowable, allowable deflection by using like first fundamentals of IS 456 limits okay by using limits of Is 456.

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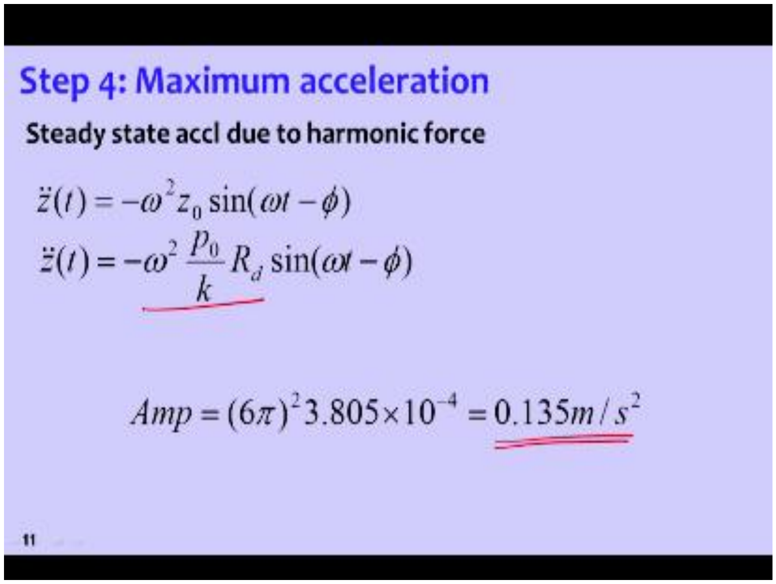

Okay now study state acceleration due to harmonic force so that is acceleration = -Ω square z0 sinΩ t –y this is only two time different of z we will get this value so if I substitute that value amplitude of vibration = amplitude of 6Л2 into p0 /k that value is rd into p0/k that is 0.3805into 10^{-4} so maximum acceleration is 0.135 meters per second square so found out acceleration we found out this displacement if we check if it is whether it is less than the specified limits or more than the specified limits we will get whether structure is safe or not.

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So in summary what we have discussed in this class is a application of continuous systems so in continuous systems we write generalized equation of motion and then we find out properties such as generalized mass generalized damping and generalized stiffness and as well as generalized force so again by using the fundamentals of the single degree of freedom system we can find the maximum deflection maximum deformation using the same principles by using generalized properties of the structure.

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