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Structural Dynamics
Week 1: Module 06

Solution of Undamped Free vibration

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Now in this module we will discuss solution of undamped free vibration so this undamped free vibration means.

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Where force is 0 so there is no externally applied force and then when it will call as undamped so that means damping is 0 so undamped free vibrations. So the outline of this is discussion out the module is undamped free vibration and then how to find the solution of the undamped free vibration and then one example problem will solved in this module.

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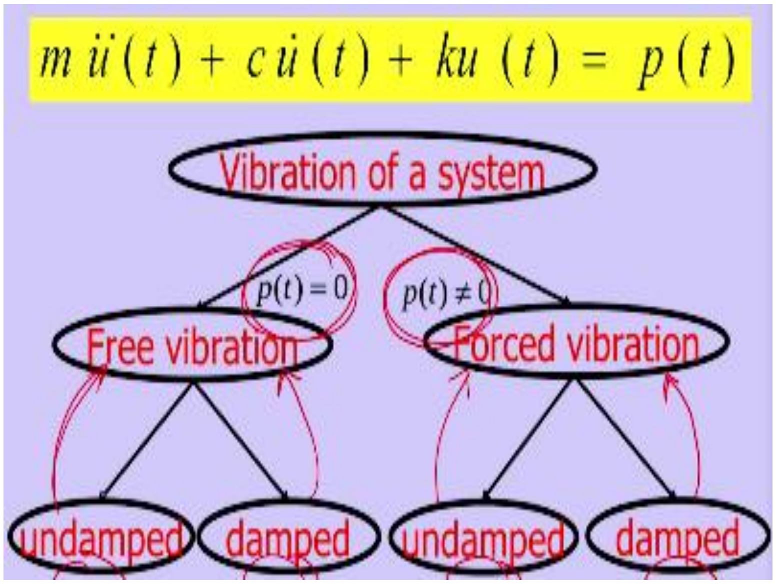

So this is a general dynamic equilibrium equation or equation of motion we can call it as, so the equation of motion of a vibrating system so the free vibration and forced vibration free vibration means where force is equal to 0 and forced vibration means force is present it is not 0 and in that 4 cases can be formed one is undamped free vibration so undamped free vibration damped free vibration undamped forced vibration and damped forced vibration.

So undamped free vibration means damping is 0 force is 0 damped free vibration means damping is present but force is 0 undamped forced vibration means damping is 0 but force is present damped forced vibration means damping is present and also force is present so this four cases will discuss out of it we are discussing this first case that is undamped free vibration. So how does equation of motion look like?

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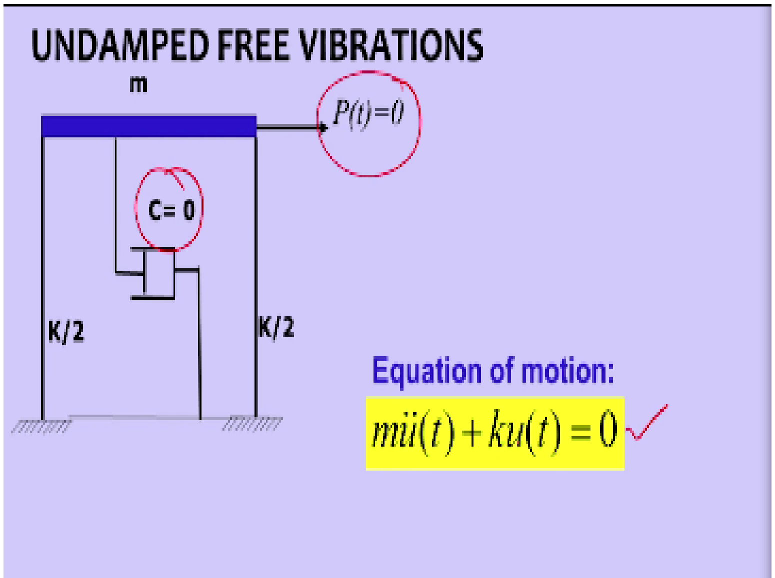

So it looks like this? mu double dot + ku is = 0 where damping term is removed from this one and force term is remove from this one as you can see from this picture so force is not there and damping is not there so when force is not there how the system is oscillating, so system will oscillate with initial disturbance so that initial disturbance can be in the form of initial displacement like this initial displacement is oscillates or initial velocity which is applied in the form of impulse or velocity.

So this is initial velocity or both initial displacements along with initial velocity like that so if we supply initial disturbances or the initial displacement or the initial velocity of both so systems will set to oscillate. Then how do we define the solution of this equation of equilibrium that is mu double dot + ku = 0 so for that we will discuss here.

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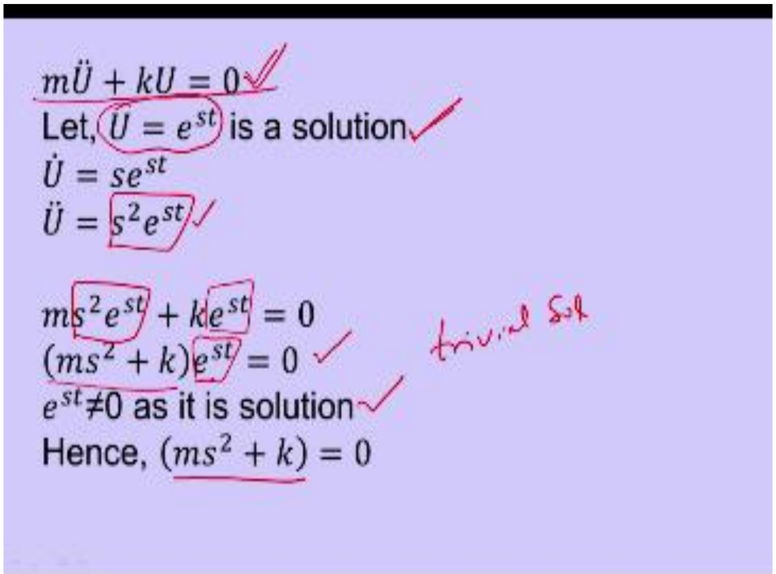

So how to form a solution so this is mu double dot + ku = 0 so this is the basic equation of motion so we want solution we are interested in the solution which is u so u we assuming it has u = e^{st} is a possible solution, so this we are assuming that e^{st} is possible solution, so this is the usual solution taken for simple harmonic motion problems. So if u = e^{st} is the solution then it is derivative that is u dot that is velocity is so if you differentiate this one we get s x e^{st} and then if you double differentiate the same then we get s^2 e^{st}.

So we substitute u and u double dot in the original equation so if we substitute that as you can see here so u double dot as s^2 e^{st} we are substituted and then u as e^{st} we are substituted so that is m u double dot + ku = 0 so in this we can take e^{st} is common and m s^2 + k x e^{st} = 0, so from this equation we can clearly see that for this equation to become 0 either fasted term should be 0 or second term should be 0 mathematically speaking.

So if second term is 0 so that means what second term is nothing but the assumed solution so we cannot make this second term 0 so if second term is 0 that will become a trivial solution, so for non trivial solution this e^{st} which is a assumed displacement response it cannot be 0 so that means what if e^{st} cannot be 0 so then that means bracket term should be equal to 0that is ms^2 + k= 0 so this bracket term should be equal to 0. So if this bracket term is 0 then what is going to happen let us see this next slide yeah.

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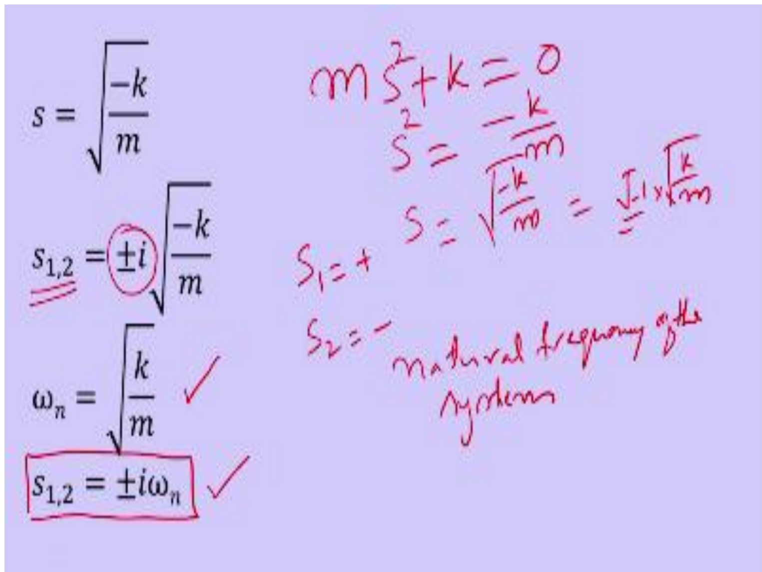

So if the bracket term is 0 so which is m $s^2$3 = k= 0 so it means s^2 = - k/m then s will become √=k/m so this √-k/m if we take out this -√I √-1 x √ k/m to separately with the symbol plus or minus because it is in the inside the √ it is coming out so then we can have 2 answers top this one s1 and s2. So that s1 is with = symbol and s2 is with – symbol and then this √k/m can be represented by as ω n which is call natural frequency of the system.

So then that gives as 2 √ to this equation that is s_1 = + iωn and s_2 - iωn so this 2 values are there s2 and s2, so now upon substitute in this s1 and s2 in the back in the equations.

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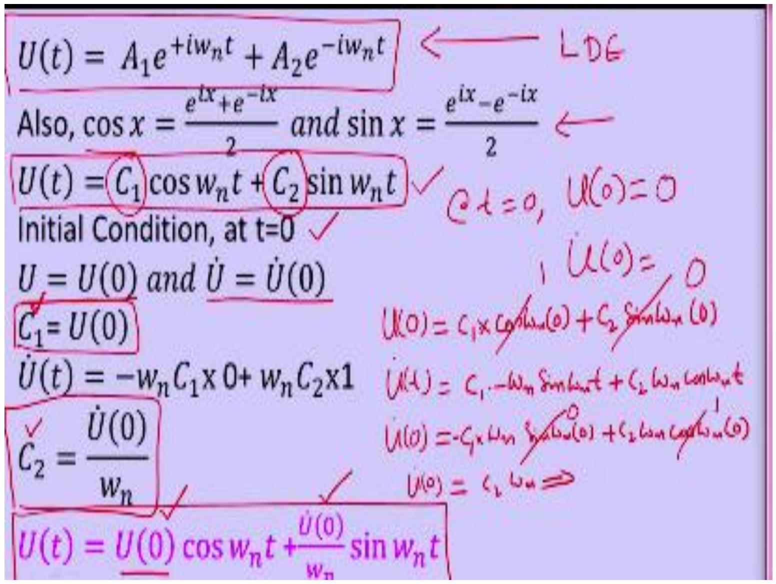

So we get this so u(t) A1 $e^{+i\omega n}$ and A2 $e^{-i\omega n}$, so this is the form of solution of linear differential equations with constant co efficiently linear differential equation with constant co efficient so this is the solution form of that, so this is complicated so we need simplified solution so there we use the trigonometric function which is cos x can be represented as in the form of exponential term that is $e^{ix} + e^{-ix}/2$ and sin x can be represented as $e^{ix} – e^{-ix}$ / 2so this cos x and sin x along with this exponential terms can be rearranged and this solution can be written in the form of t_1 cos $\omega_n t + c_2 \sin \omega_n t$.

So in this equation so we have 2 constant C1 and C_2 so we need to get read of this constants C_1 and C_2 for getting a complete solution so this C_2 and C_2can be calculated or can be found out by using initial conditions and we have two initial condition so that is act time t = 0 what is status of displacement a time t = 0 what is the status of velocity so a time t = 0 what is displacement and time t = 0 what is velocity so this is something like this so time u at u =-0 is first initial condition and u dot at u dot at time t =0 is second initial condition.

So if you substitute these initial conditions in the equation we will get C_1 and C_2 it something like if you substitutes a u0 so let initial condition that u displacement a time t = 0 is 0, so if you substitute this so what is going to happen is time t = 0 cos 0 and sin 0 so sin0 cos 0 is 1. So if sin 0 let me write here u (0) = C_1 x cos ωn x 0 + C_2 sin ωn x 0, so this gives as sin 0 is 0 so this entire term goes to 0 and cos 0 is 1 so that gives as C_1 = u0 so this c1 as constant is evaluated.

Now we need c2 so C_2 is having a term as we like in second initial condition is having u dot but we have solution u so we need to do the differentiation of this solution so u. of t = single term will remain like that then differentiation of cos will be – ω n sine ω n t + differentiation of the second term that is c2 will remain like that ω n cos ω n t now we need to apply the second initial condition in this one.

So the second initial condition is u.0 so u.0 = 0 if here using then c like ω or C_1 x ω and with – symbol and then sin ω n 0+ C_2 ω n cos ω n x 0 so again here you can see sin 0 is 0 it is entire term goes to 0 cos 0 is 1 so that means what u.0 = C_2 ω n this implies C_2 = u./ ω n so we got c1 constant and we got C_2 constant and C_1 constant so if you substitute this c1 and C_2 constants in the original equation that is a solution.

So we get this so from this you can see this so the solution of undamped free vibration so in this u0 is the initial displacement and u.0 is a initial velocity so from this equation we can clearly see that if we supply initial displacement or initial velocity or both then only undamped free vibration then only there will be oscillation in the system. Otherwise if initial displacement is 0 or an initial velocity is 0 system cannot oscillate system will be in the static position only.

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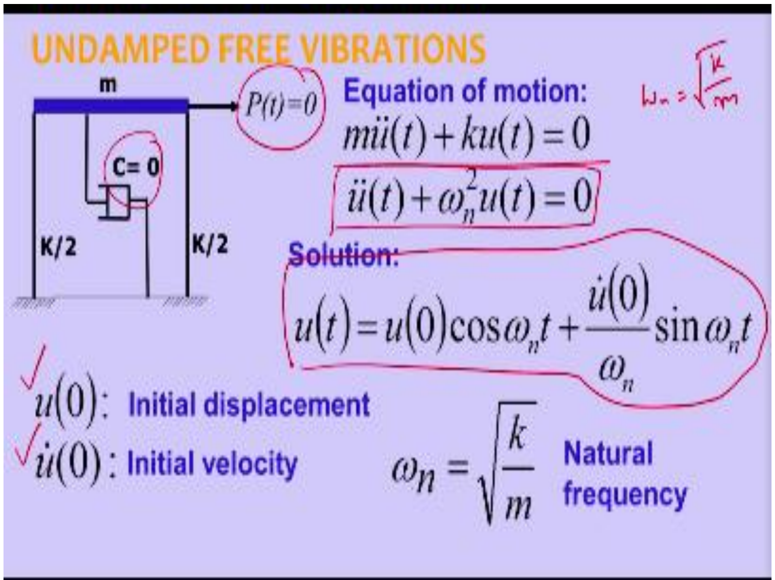

So now this slide shows the overall summary of undamped free vibration so this is system where damping is 0 and force is 0 then equation of equilibrium is mu double dot + k_u =0 if we divide this equation by m so that will give us u double dot + k/ m we have already discussed it that is equal to frequency ω n = √ k/m so if k/m so if k/m is nothing but ω n^2 so this is the equation of the motion the solution of equation of motion is this. Solution of the equation of the motion is u0 cos ω nt + u dot 0/ω n x sin ω nt so ion this u 0 is initial displacement u dot 0- is initial velocity and natural frequency which is ω n = √ k/m.

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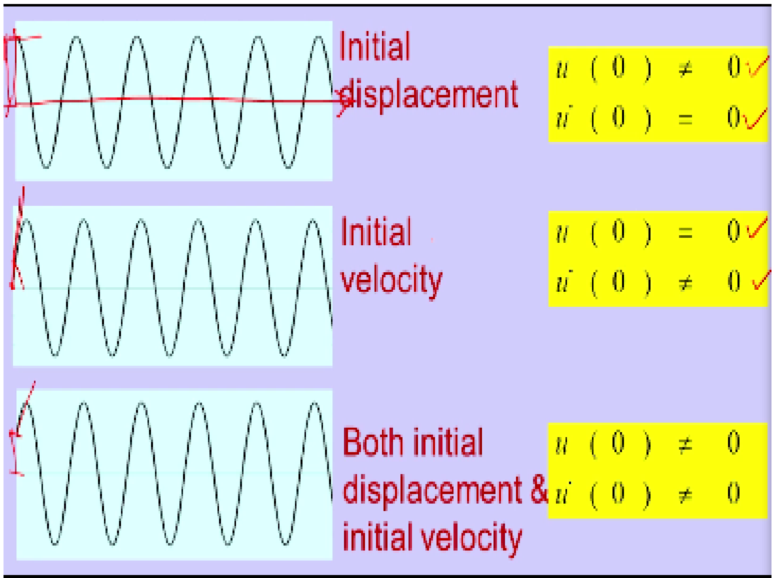

Yeah again initial displacement yeah you can see here a time t = 0 this is the time scale a time t = 0 initial displacement is having some value it is non 0 value, okay some value is there but initial velocity is that is a tangent is horizontal to x axis that means initial velocity 0 initial displacement is non 0 in the second case initial velocity is present initial displacement is 0 and initial velocity is non 0 and here in third case initial displacement is there also initial velocity is so this 3 cases are there.

Initial displacement initial velocity initial displacement along with initial velocity now with initial velocity.

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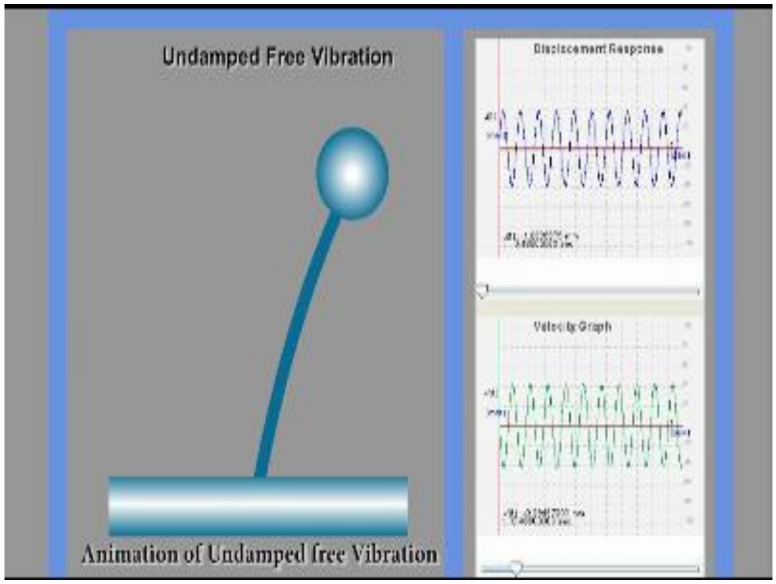

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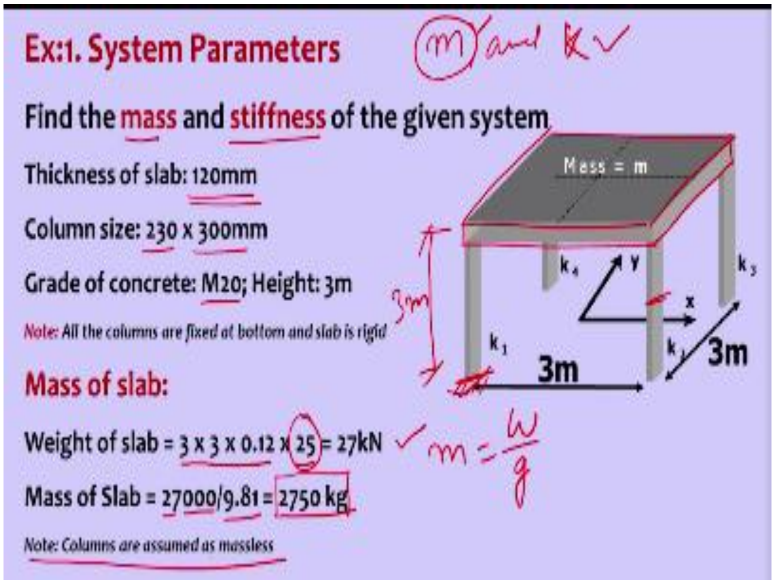

Now let us try to find out the system parameters so which is very much essential in linear single degree of freedom system for finding the solution so the 2 important system parameters in undamped free vibration or mass and stiffness that is m and k so mass and stiffness so if we look at this structure as slab supported on four columns so thickness of the slab is given is 120mm thick and column size is 230/ 300, so 230 on one side 300 on the other side 230 / 300 mm column size and the grade of concrete used as a m20 grade concrete material grade.

And height given as 3meters so a note is given all the columns are fixed at vast fixed as there here fixed it at vast slab is reject in its plane so there are more internal deformation in the slab internal deformation are not consider in the slab then how do we calculate the system parameters, so first of all we need to calculate mass of the slab for calculating mass of the slab we calculate the weight of the slab first so weight of the slab is we can get is through volume multiplied by weight density. So volume is 3m 3m / 3m / this 120mm thick.

So this is the slab volume, volume of the slab concrete multiply by weight density so since this is reinforce cement concrete slab so 25 km/m^3 is a weight density so that gives us 27kN as a weight of the slab. So mass of the slab is equal to weight/gravity so gravity value we are taking it as

9.81m/sec^2 and the 27kN that is multiplied by 1000 we get 27000N/9.81 so we get 2750kg is the mass of the system, so this is the first parameter.

Now we need to find out second parameter stiffness one note that columns are assumed as mass lasses, so usually columns contribute to the slab mass but in this case we are ignoring the mass of the columns, columns are contributing only for stiffness.

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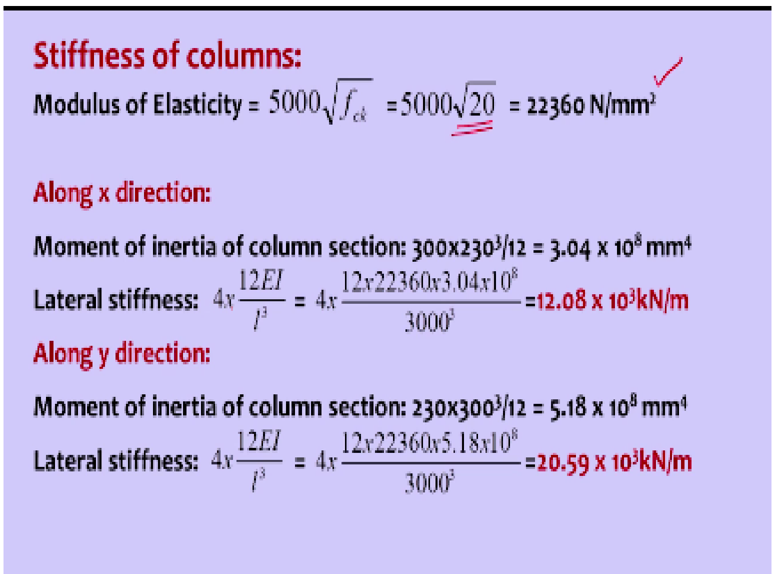

So how to calculate the stiffness of the column, so first of all we find out the modulus of elasticity of the material, column material that are concrete material since concrete material is given as m 20 grade concrete so according to IS456 2000 we calculate modulus of elasticity has 5000 under $\sqrt{f_{ck}}$ so which is 22360 N/mm^2 that is modulus of elasticity and then moment of inertia of column section i is 300x230^3/12 this is along one direction and lateral stiffness.

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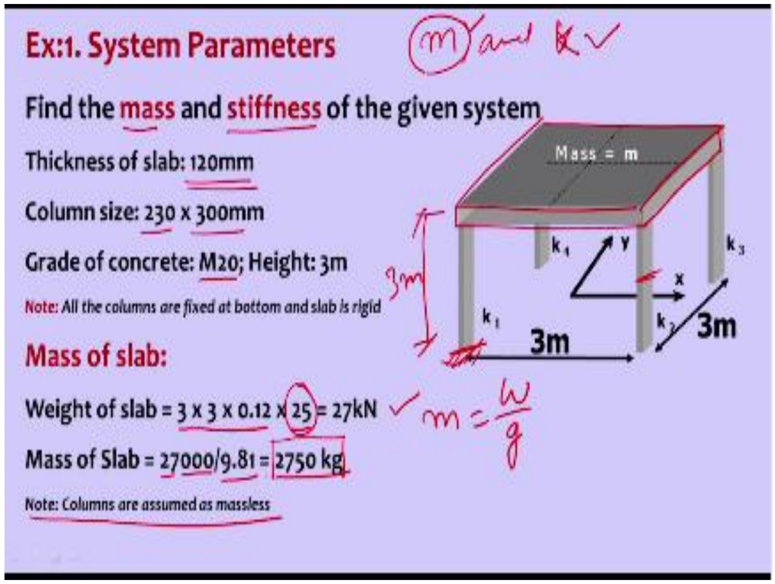

So one column, so if we take stiffness of this column along this direction if it is moving like this in this direction so one unit, so this we have already discussed in detail so lateral stiffness of this can be quantified like $K_1=12EI/h^3$ in this one E is a modulus of elasticity or the concrete material I is the moment of inertia along the direction of motion and h is the height of the column, h is the height of the column.

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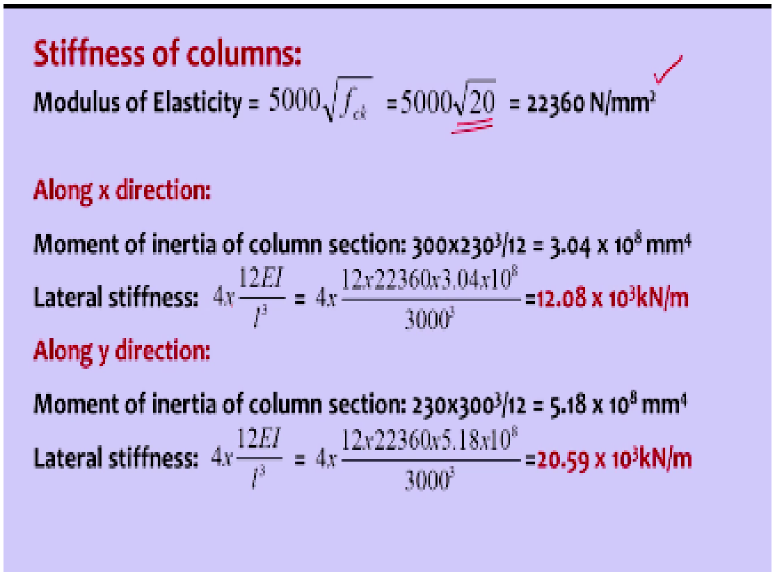

So like that we are evaluating because four columns are present in x direction four columns are resisting the opposing the motion along x direction again four columns are opposing the motion along y direction. So what we do is when we are calculating along x direction so we will take moment of inertia accordingly in that direction. So if you look at this one four times multiplied by $12EI/l^3$ l is the length of the column or height of the column so this is stiffness of one column multiplied by four columns.

So E is modulus of elasticity that is 22360xI along one direction so 3.04×10^8 divided by height is 3m so $3000mm^3$ so we get 12.08×10^3kN/m that is lateral stiffness along x direction and then moment of inertia of column section along y direction is obtained as $5.18 \times 10^8 mm^4$ and accordingly if we calculate stiffness so that is 4 times $12EI/l^3$ we get 20.59×10^3 so in this we can observe one thing is here.

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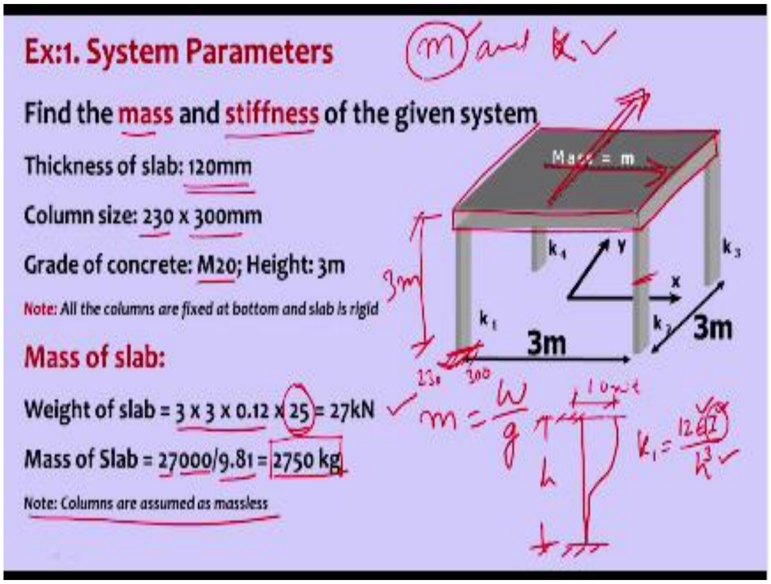

It looks like you can see column size is 230/300 so columns are 300mm depth along y direction and size of column along x direction is 230 so that means what along x direction structure id little flexible and along y direction structure is little stiff this is the relative stiffness so along x direction it is flexible relatively along y direction it is stiffer relatively, so same thing we can see in the stiffness also.

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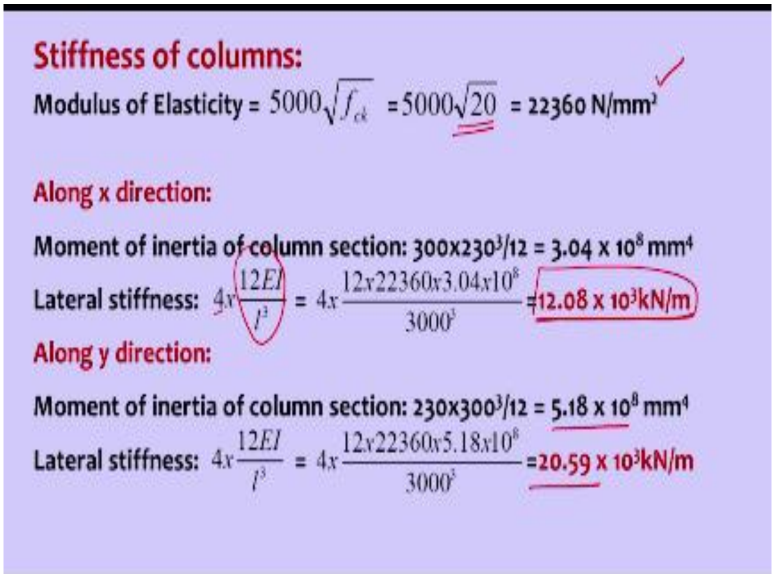

Sp along x direction we are getting stiffness that is residence to deformation as $12.08x10^3$ kN/m and along y direction $20.59x10^3$ kN/m along y direction so this is the stiffness calculation along x direction and y direction.

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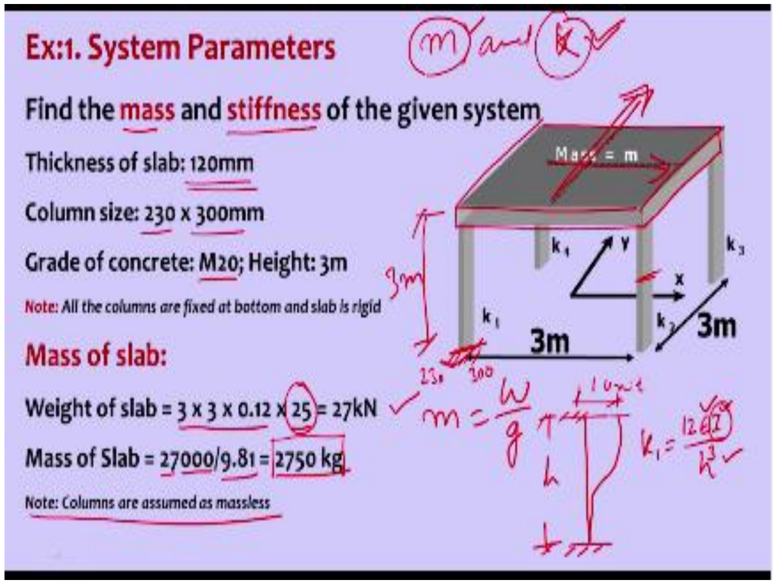

So that completes the calculation of stiffness, along x direction along y direction so if structure is vibrating along x direction we can use mass as well as stiffness along x direction. If structure is oscillating along y direction then we can use mass and stiffness along y direction.

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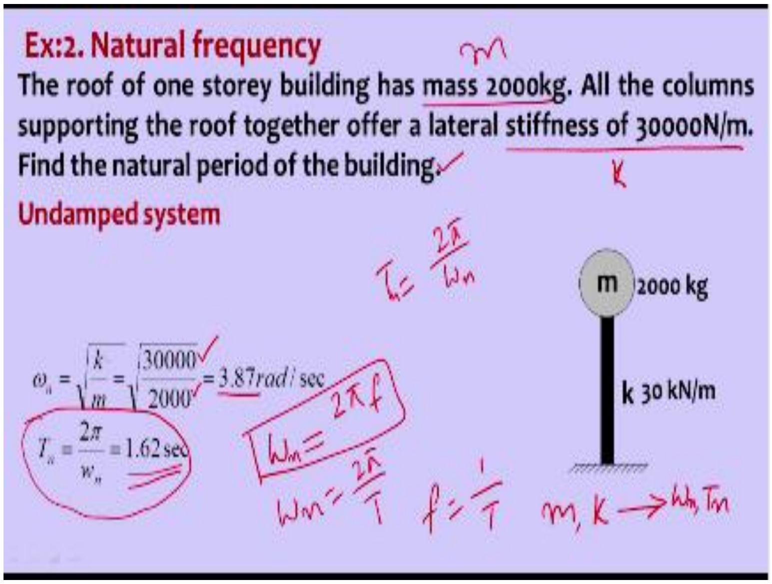

In this example, we will learn how to evaluate natural frequency of the system where parameters of the system are given to us like question reads like this the roof of one storey building has mass 2000kg, mass is one parameter which is given already 2000kg. All the columns supporting the roof together offer a lateral stiffness of 30000 N/m so it is 30 N/m or 30000 N/m this is second parameter, so mass is given to us and stiffness is given to us.

So we need to find out natural period of the building, so natural period of the building and natural frequency of the building are related in this form $\omega_n=2\pi f$ so this frequency is number of cycles per second, so in one second how many cycles are taking place that is represented by frequency f and natural period is time taken for 1,2 and 4 or one full cycle, so they are related $f=1/T$ or $T=1/f$ so in this relationship so $\omega_n=2\pi f$ we can also write this as $\omega_n=2\pi/T$ or $T=2\pi/\omega_n$ so T_n.

So we know how to calculate ω_n which is under $\sqrt{k/m}$ so k we know 30000 N/m and then m is 2000 kg so we get 3.87 radiance per second. So this is circular frequency so this 3.87 $2\pi/3.87$ we get 1.62 seconds, so natural period of the system is 1.62 seconds so that is what is asked to calculate. So system parameters are mass and stiffness are essential for finding natural parameters nor natural characteristics of the vibrating system that is natural frequency or natural period.

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