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Structural Dynamics
Week 10: Module 01

Vibration of Continuous Systems

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Welcome to structural dynamics class. In this class we are going to study vibrations of continuous systems. So continuous systems means where the mass is distributed continuously. So earlier we had studied single degree of freedom systems and then multi-degree of freedom systems. So multi-degree of freedom systems are something like where we can do the lumping of masses at different stages in the building.

So if you take a building like this, so you can assume that so this is floor 1, floor 2, floor 3, floor 4 and floor 5. So slab is mostly concentrated floor levels, mass is concentrated at floor levels or weight is concentrated at floor levels. So we can lump the mass here, lump the mass at this level, lump it here, here and here. So that is how we study a multi-degree of freedom systems.

But in continuous systems what happens is mass is distributed continuously throughout the structure, throughout the height of the structure. So for example, say a tower or a chimney okay, so communication tower or chimney so for that studying vibrations are understanding the solution or the response to the given ground motion or any wind vibration.

So we need to employ different techniques not the same technique as we have used in multi-degree of freedom systems or single degree of freedom systems. So let us understand how we find the solution for the equation of motion for continuous systems.

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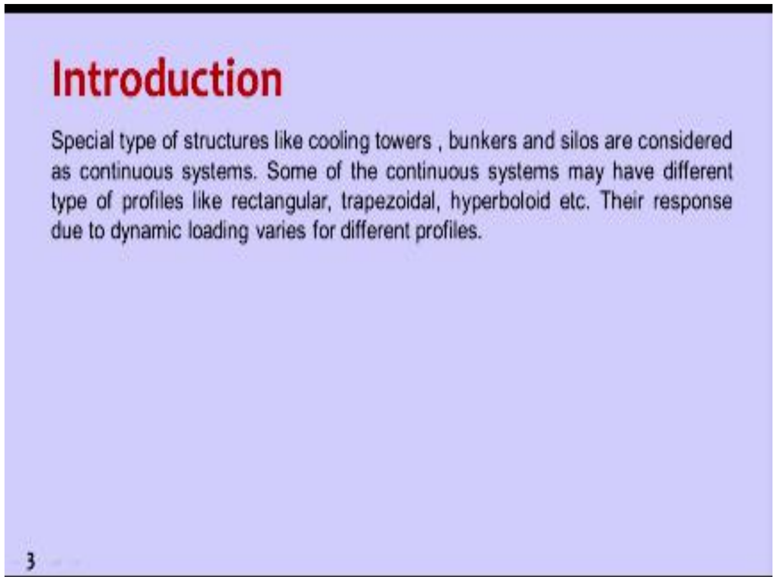

So a special type of structures like say cooling towers, bunkers, silos are considered as continuous systems where mass is distributed continuously. Some of the continuous systems may have different types of profiles like rectangular, trapezoidal, hyperboloid etc. So their response due to dynamic loading varies for different profiles. So what profiling or deflection profile so we need to understand that also first.

So as I discussed in the earlier classes in multi-degree of freedom systems so when a tower or a structure like this one tall building like this one is vibrating when it is vibrating when we want to say that displacement or deformation we need to take two variable quantities, one is at what place we are talking about, we are talking and what time we are talking.

So for that we need to tell in multi-degree of freedom systems space can be located as a first floor, second floor, third floor and fourth floor like that. And time is continuous, but in continuous systems this location also is continuous, time also is continuous so these things we have to discuss in this one.

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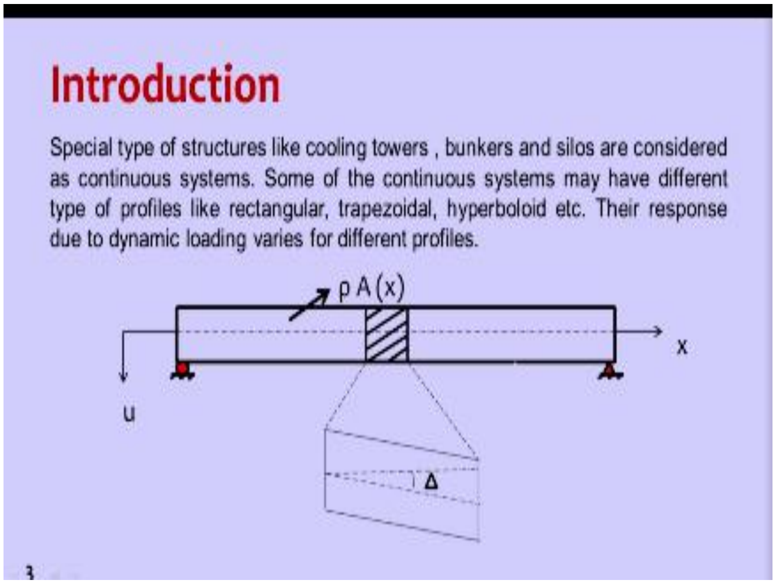

So for that let us take say a structure or a member something like a shear beam. So here we can see that, so this is positive u axis, u is the displacement. And then if we take the infinite decimal cross section or infinite decimal length which is of dx and ρ is the mass density AX is the area of cross section this is X-axis.

So throughout the length of the member or the height of the structure. So now let us write the equation of motion for this one.

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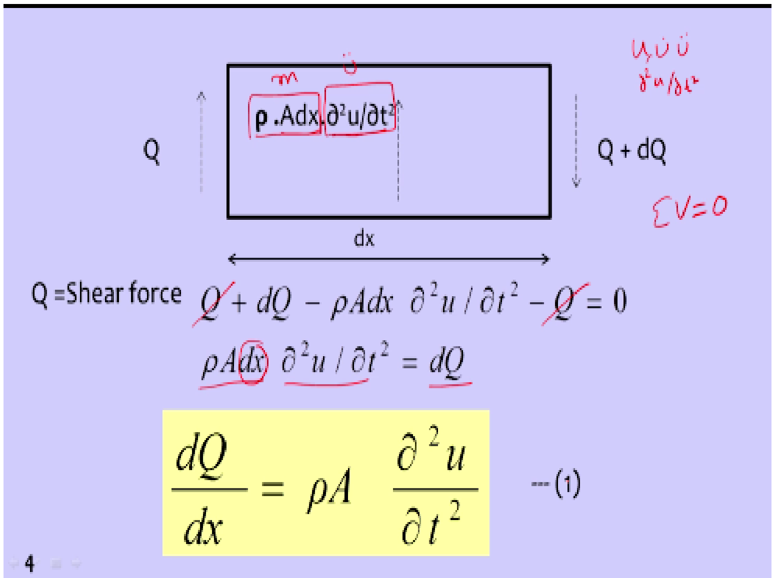

So how do we get so when the structure is vibrating so when it is moving down then we know length is dx the infinite decimal length. So we know that moving mass generates inertia of force oppose in the motion. So now what is inertia force, inertia force is mass times acceleration. Since I told you U is the displacement so U. will be velocity or u.. is a acceleration so u.. is $\partial^2 u/\partial t^2$ so this term is acceleration term and Adx, A is the cross section area and dx is a length so Adx is volume, volume multiply by mass density so we get mass.

So this term is representing mass and this term is representing acceleration so mass times acceleration n times u.. so this is inertia force which is opposing the motion, now if we write the shear force which is acting upwards and on the other side if we write so the sheer force will have some like additional value so q here q+ dq if we write equilibrium equation of this one so q+ dq downward minus ∂a dx $\partial^2 u/\partial t^2$ phase upwards.

Minus q upwards minus is equal to 0, so σv some of all forces in vertical direction should be equal to 0, so then see this q and q will get cancelled so we are left with so if we if I take tis terms in on to the right hand side so if I rearrange the terms ∂a dx is mass times acceleration is equal to dq, so change of shear so dq should be equal to that value, now if we write from this one so what we can do is rewrite this. Dx can go down dq/dx = ∂a $\partial^2 u/\partial t^2$ so this is equation number 1.

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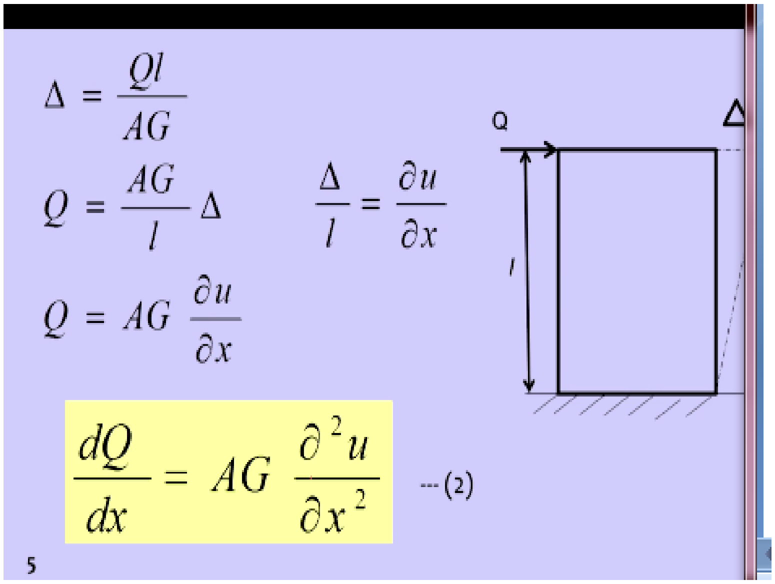

Now let us take like from the shear relationship so if we apply shear force q and then deflection is Δ so what we can write is Δ deflection = ql/AG so q is a shear force l is a height or length of that member so ql/AA is a cross section area G is a shear modulus, so from this we can write q = AG/l x Δ so we already know that Δ/l so if we take from this one Δ /l so Δ is say deflection l is the length.

So this we are writing so infinite decimal defection by infinite decimal length so ∂x is infinite decimal length of the member and ∂u is a infinite decimal deflection of the this Δ x length, so if we substitute Δ/l as ∂u. ∂x so we get q = AG x ∂u/∂x so from the above equation one if you look at this equation we need dq/dx, so that means what if we differentiate this once what we get is dq/dx = AG x $\partial^2 u/\partial x^2$ then substitute in the earlier equation, equation number 1.

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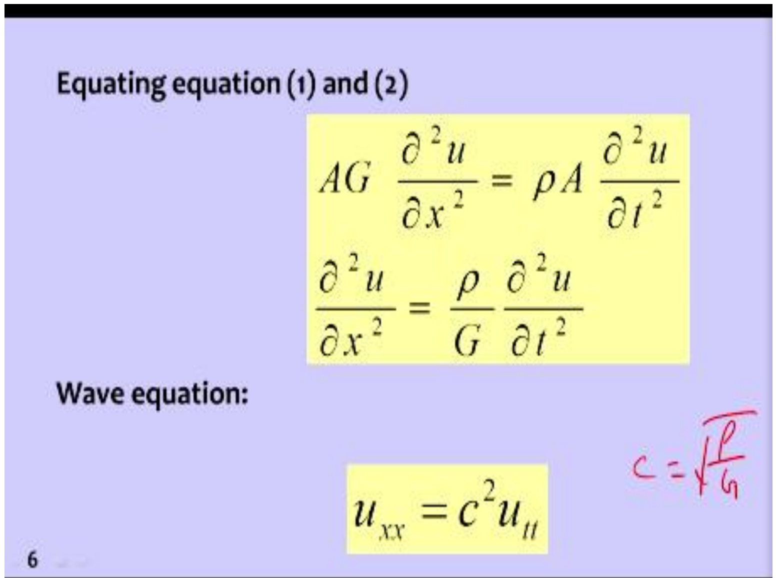

Then we get AG Δ^2 u/ Δ x^2 = ρA Δ^2u/Δ t^2 so that means what we get is if we rearrange these terms Δ^2 u/Δ x^2 = ρ/G Δ^2 u/Δ t^2 so this is wave equation, so uxx = C^2 u_{tt} actually c represents wave velocity which is equal to √ ρ/G now we need to solve this so this is a differential equation now governing differential equation of vibration of a shear beam okay shear beam which can be equated to a τ or any building okay high raise structures like that so now we need solution.

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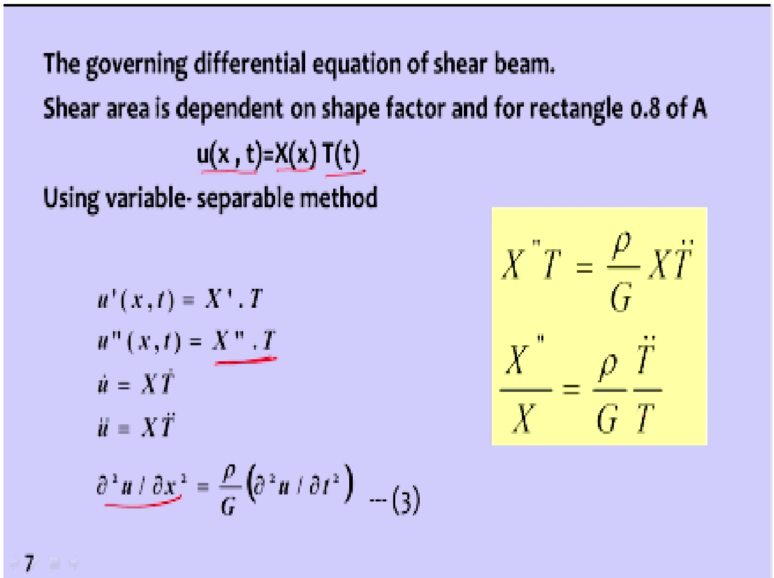

So how do we get solution of this one, so the governing differential equation of shear beam is known, so shear area is dependent on the shape factor and for rectangle 0.8 of A is a shear factor. Now we are using say, u which is displacement function as a function of X(x) and T(t) okay, we need to use in this one variable separable method of solving differential equation for finding the solution of this governing differential equation.

So in this one if I differentiated once with respect to x so I get X' T, if I differentiated twice I get X'' T because T is not the function of space so that is why I get $X'T$ and $X''T$ here. So again similarly if I differentiated with respect to time $X\dot{T}$ $\ddot{u}=X\ddot{T}$ so I have so two quantities. Now if substitute in the governing differential equation that will $\delta^2u/\delta x^2=\rho/G(\delta^2u/\delta t^2)$ if I substitute this in the equation what I get is this term in place of this one. So I get $\delta X''T=\rho/GX\ddot{T}$ so I can rearrange this one $\ddot{X}/X=\rho/G\ \ddot{T}/T$.

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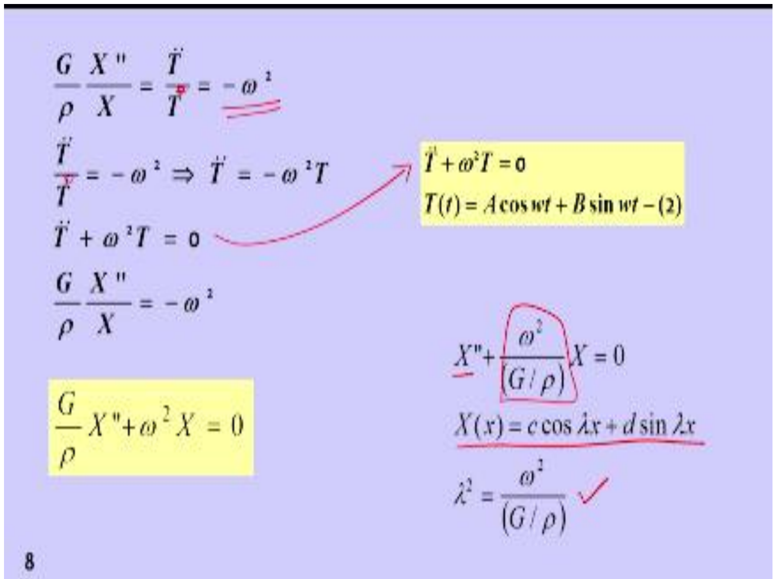

So what I do is this $\rho/G\ X''/X=\ddot{T}/\dot{T}$ $\ddot{T}/T$ not $\dot{T}$, so $\ddot{T}/T=-\omega^2$ so I am equating into some constant, so then if I do rearrange this one $\ddot{T}/T=-\omega^2$ so that means what $\ddot{T}/T=-\omega^2$ that means $\ddot{T}+\omega^2T=0$ this is one equation. $G/\rho\ X''/X=-\omega^2$ so this one is something like single degree of freedom system equation of motion and the solution for that is known to us so $T(t)=A\ \cos\omega t\ B\ \sin\omega t$ if so we already know this one.

Now let us solve this one second equation which is in space variable, so $G/\rho\ X''+\omega^2X=0$ now if we divide the second term by the first term and first time and again first term so what we get is $X''+\omega^2/(G/\rho)X=0$ so the possible solution of this could be $X(x)=C\ \cos\lambda X+ D\ \sin\lambda X$ so this λX will come from this function. So $\lambda^2=\omega^2/(G/\rho)$ so this, then.

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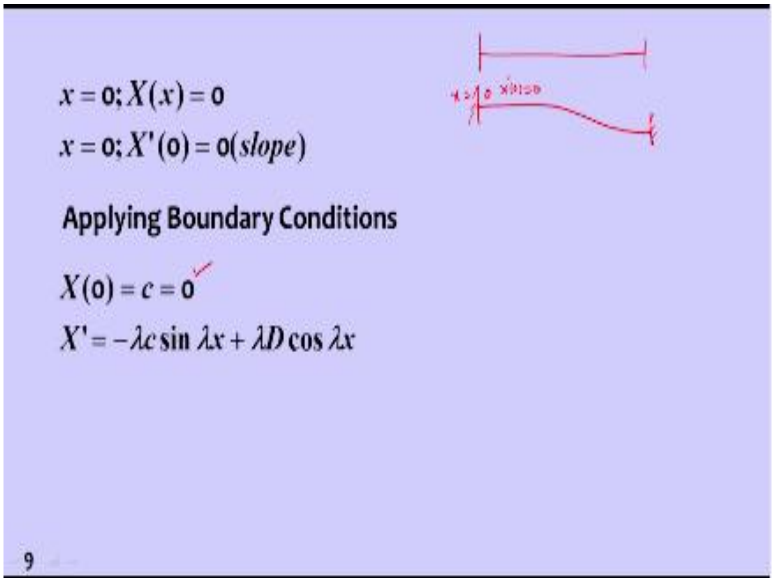

Let us find out the constants in this one C and D, now we know if at x=0 X(x) should be equal to 0 so that means what boundary condition will be applied there so at x=0 and X′ also should be equal to 0 because we are talking about the shear beam. So in shear beam what happens is deflection is 0 and slope is also 0 it is something like this beam is of this nature and what happens is you can see here at x=0 deflection is 0 and at x=l or x=0 you can get slope also has to be equal to 0 x′=0 x also should be equal to 0 at this point. So substituting the first boundary condition x(0)=0 so that means C constant is 0, okay in this one.

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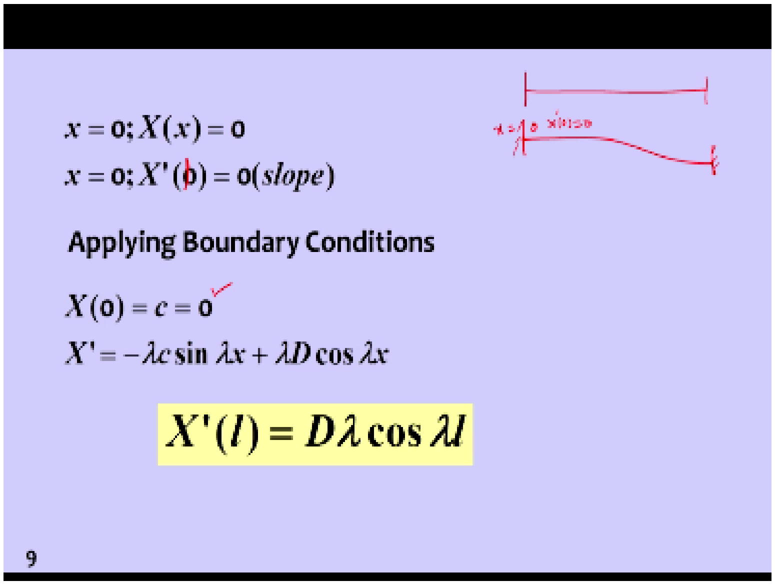

Okay C constant is 0 and then x prime x prime to differentiate it once so then what we get is –λc sinλx+λD cos λx. So since x is 0 we are left with only x prime = Dλcos λL so this Dλ cos Dλl we need to find out because this x prime L is 0 sorry the bounding foundation what we are using in this location is x prime L not 0.

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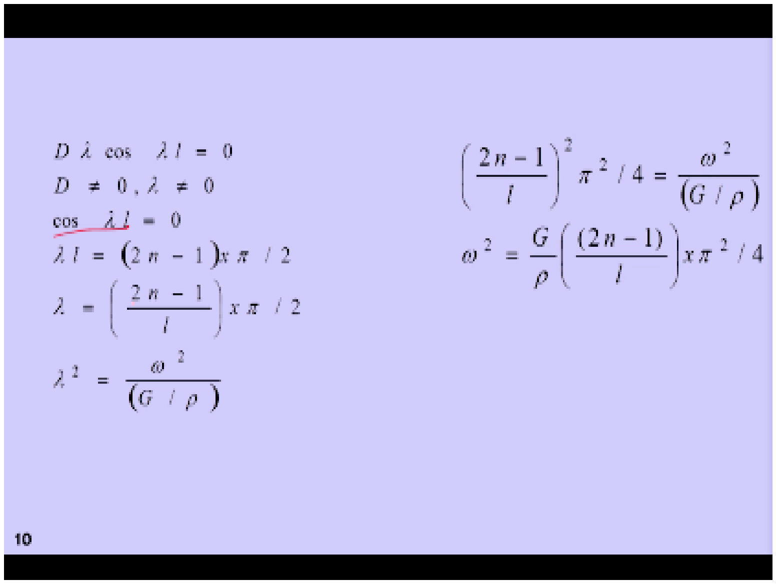

So $D\lambda\cos \lambda L = 0$ $D \neq 0$ for non solution λ also cannot be 0 so what should be zero is $\cos \lambda L$ should be =0 so that means what λL is (2n-1)into Л/2 so λ is 2n-1/L into Л/2. So λ^2 we already know Ω^2/ g by row so if we equate these two so what we get is Ω.

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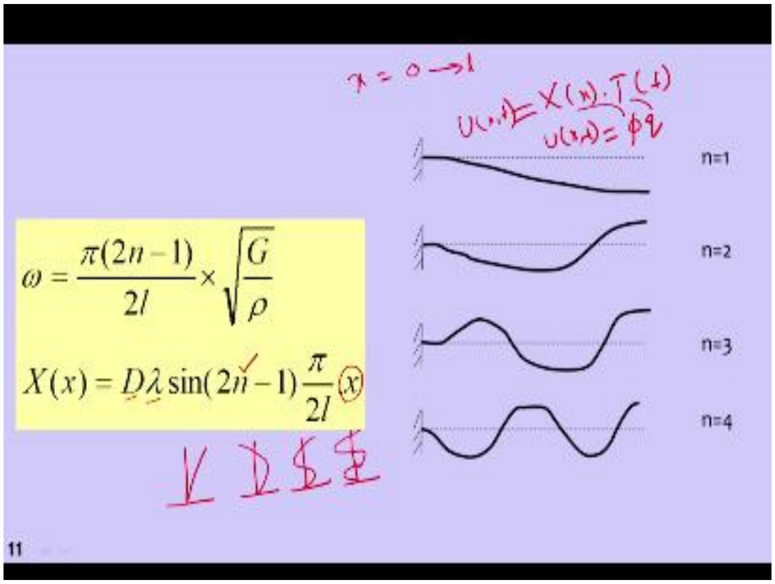

So the Ω value we get so then back substituting Ω Л/2 2n-1 into L by under root g by row so x of x finally the value is D multiplied by λ multiplied by sin 2n-1 into Л2L into x. so if we substitute x values at different, different points f 0 to l s y X = ranging from 0to l so if we substitute this one we can draw the mode shapes as you can see if N =1 then we get this so this is = to first mode shape if n =2 so this is second mode shape if n=3 this is third mode shape and n =4 this is forth mode shape so first mode shape second mode shape third mode shape and forth mode shape.

And this one is similar to our mode shapes in multi degree of freedom system so if you just rotate this one and look at it so this is first mode shape of the structure this is second mode shape of the structure this is the third mode shape of the structure and this is the forth mode shape the structure so like this so first second third fourth mode shape as we got in multi degree freedom system in continuous system also we can get as many number of mode shapes as possible we only need to substitute value of n and evaluate at any point .

So this mode shape will give you give us a different profile of the structure and then we need to solve the time function so that is say x of x we are getting and we need t of t so th9is is a overall displacement so what we have discussed in multi degree freedom system is x, t=5q so this t is representing q term and x is representing Л term so now how do we use this Л term that is the deflection profile and find the response of continuous system so that we will discuss in the next class.

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So what we discussed in this class is vibrations of continuous systems so vibrations of continuous systems cannot be solved by single degree freedom systems equation of motion or multi degree of freedom system equation of motion so what we need to do is we need to write an equation of motion to the continuous system which will be in a generalized coordinates so in that we have understood how to write the equilibrium equation for a share beam.

And then how to find a solution of that and then hoe to n evaluate that the modes of the share beam that's what we have understood in this class in next class we will try to find out how we will make use if this mode shape and write the equilibrium equations in the generalized coordinates.

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