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Structural Dynamics
Week 9 Tutorial 1

Centre of Mass & Centre of Stiffness

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Welcome to the week 9 tutorial we will be discussing center of mass and center of stiffness how to calculate center of mass and center of stiffness in this tutorial. Basically we use the center of mass and center of stiffness calculations when we are telling the 3D analysis of structures or buildings so when we are lumping a mass as single degree of freedom system there is no change in center of mass and centre of stiffness of the structure.

But whereas when we deal with 3D analysis of structures then the torsional effect will be coming into picture. So in order to verify whether torsion is present in the structure or the building or not we need to calculate what is the center of mass I'm going to say the location of center of mass and the location of center of stiffness have to be known.

So the difference between these two will give you the eccentricity that is present in the building and then from that we get to know what are the forces that are acting on the given building to calculate the torsional effect. So with this introduction will go into calculating a simple problem where we get to know how to calculate center of mass and the center of stiffness.

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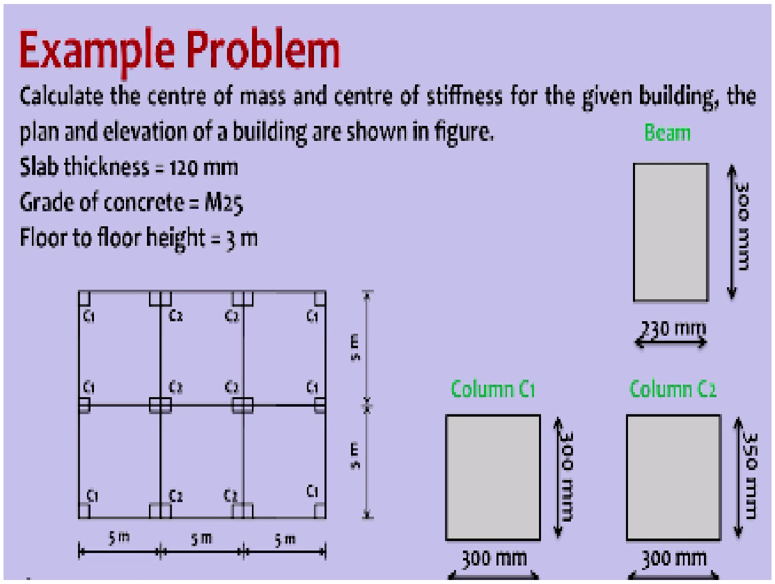

So the problem says to calculate the center of mass and center of stiffness for the given building the plan and elevation of the building are shown in the figure. So in the plan we have total 4x3 12 columns and of two types C1 which is of size 300/300 and C2 which is of size 300/350mm and the beams are of size 230/300mm and slab thickness is given as 120 mm and grade of concrete is assumed to be M25 and the floor to floor height is given as 3 meters.

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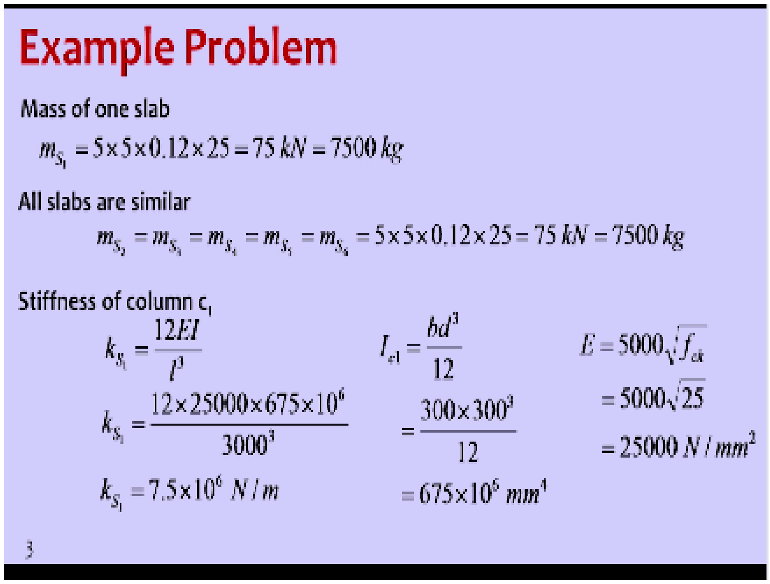

So with this introduction of the problem we go into calculating mass of this one slab so in this calculation particularly this problem we will be dealing only the slabs for calculating the center of mass. Basically in our building we have slabs and beams put together constitute more of the mass of the structure whereas columns are usually giving more importance to stiffness, stiffness of the building.

So mass of slabs and beans put together make the mass of the structure. So assuming that we will be calculating mass of one slab which is calculated in this manner MS1, MS1 means I want to denote it as S1 the slab one which is of five meters by 5 meters which is given in this plan 5 into 5 the area and point 12 is the thickness of the slab and 25 is the unit weight of concrete.

So we get a 75kN which can be converted to7,500 kg. Similarly for all the slabs almost every slab in our plan is of same size. So S2 mass of S2, mass of S3, mass of S4, mass of S5 and mass of S6 are equally calculated in this manner in 25 is the area of the slab and 0.12 is the depth of the slab and 25 is the unit weight of concrete. So for each and every slab we got a mass of 7,500 kg.

So similarly we calculate stiffness as we have already discussed the stiffness of the building will be usually constituted by the column alone. So basically we are concerned about the lateral stiffness so we go over the lateral stiffness which is resisted by the column. So ks1 slab for the first column 12 EL/L^3 where E is calculated in this manner 5000 root FCK where FCK is the grade of concrete 5000 root 25 this is around 25,000N/mm^2 where 25 is the N/mm^2.

So E is calculated which is nothing but modulus of velocity and then I, I is moment of inertia for because we have two kinds of column C1 and C2 for C1 column BDQ/12 C1 is of size 300 by 30 0mm so 300 x 300^3 /12 which is nothing but 300^4/12 which is coming around 675 x 10^6 mm^4 so now substituting these two values into the stiffness calculation 12 x 25,000 x 675 x 10^6/ 3000, 3 is the floor to floor height.

So 3000 is an mm 3000^3 so which is coming around 7.5 x 10^6 Newton per meter so this is directly calculated the result which we got from 12 into the calculation which is shown here will be resulting in Newton per mm where you need to convert it to Newton per meter and which is equal to 7.5 x $10^{6.}$

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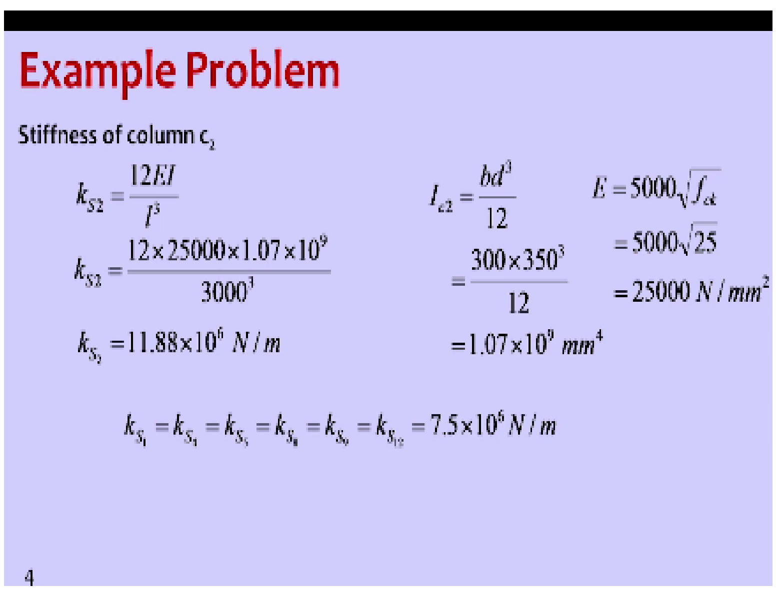

So we have another column for which stiffness is calculated in the similar manner E is again $5000\sqrt{fck}$ which is around 25,000 Newton per mm square and I is $bd^3/12$ for which the type of column 300 x 350 mm size so $300 \times 350^3/12$ which is coming around 1.07×10^9 mm^4 so now substituting E and I values into k, $12 \times 25{,}000 \times 1.07 \times 10^9/3000^3$ will give you around 11.88×10^6 Newton per meter. So these are the columns which we have.

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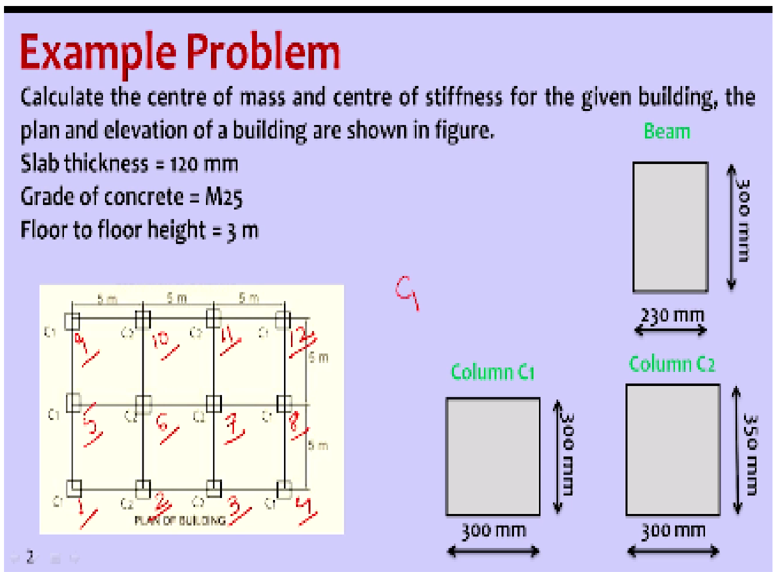

So if we go to the plan we have six columns of type c 1 and 6 columns of type c 2 so if I start numbering from here this is column 1 and this is column 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 so the first column where we have C1 type of column and second column we have C2 type of column third one is a C2 type of column and fourth one C1type of column so similarly for all other frames also we have Ø1 as C1 and Ø 8 as seven, so we have K1, K4 K5, K8, K9, K12 as c1 columns and k 2 k3 k6 k7 k10 k11 as c 2 columns.

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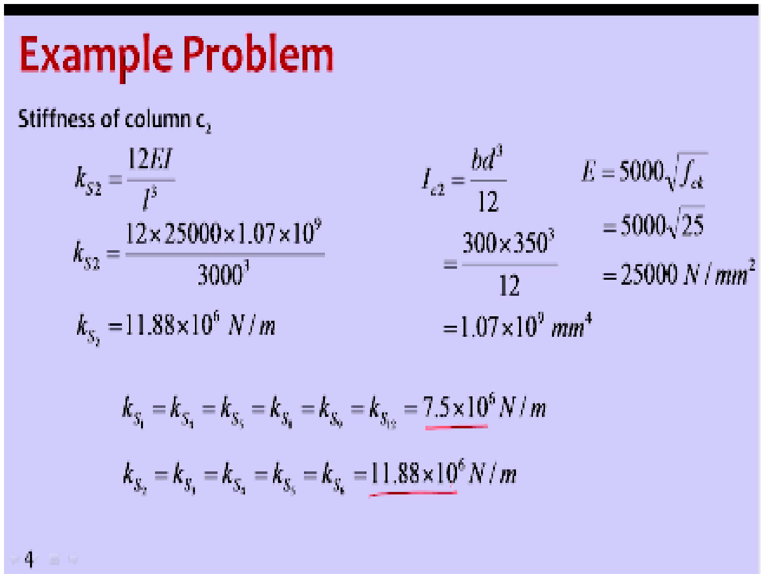

So here k as 1, k as 4, 5, 8, 9 and 12 as 7.5 x 10^6 Newton per meter and Ks2, s3, s4, s5, s6 as 11.88 x 10^6 Newton per meter.

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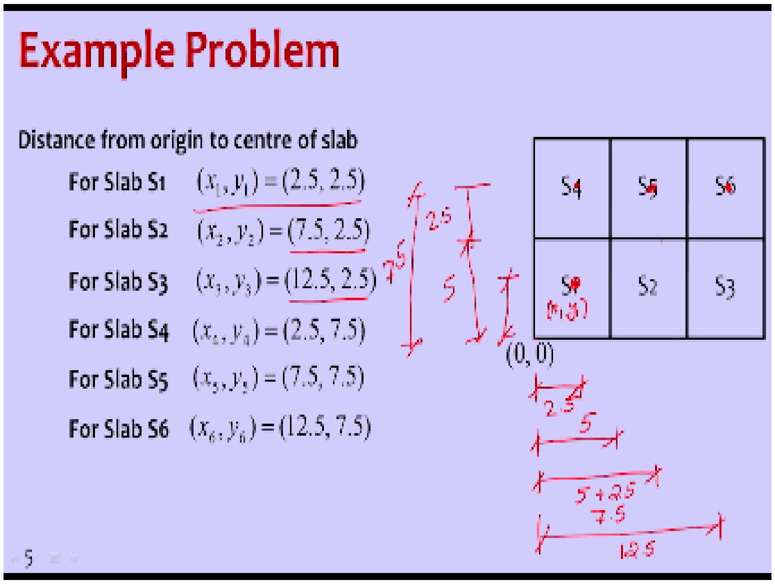

So keeping this in mind we need to calculate the distance from this origin assuming this as the origin we have assuming this as the origin we have slab 1, 2, 3, 4 and 5 so we need to calculate the centre of this slab such that the distance between the origin and the center of the slab in both x and y direction can be calculated for calculating the Xc value that means the center of mass value, so I here denote the x1y1 and similarly for other slabs.

So this x1y1 will give me the center of this slab 1 S1 so from origin I have 2.5 meters because my span is 5m, so 5m is the span center because it is a square column both size is 5m the center will be at 2.5 so x_1 is 2.5 and x_1 and $y_1$is also 2.5. Similarly starting from here to this point so I have this length as 5+2.5 which will be around 7.5 so this is 7.5 and 2.5 in y direction.

Similarly for x_3 and y_3 this is the distance so this will be 5+5 is 10, 10+2.5 which is coming around 12.5, so this is the slab 3 and similarly we calculate for slab 4 where we have different y value which is increasing from 5 to 2 7.5 this is again 2.5 so all put together will be giving around 7.5. So 2.5 and 7.5 and this location will be 7.5 and 7.5 this one will be a 12.5 and 7.5.

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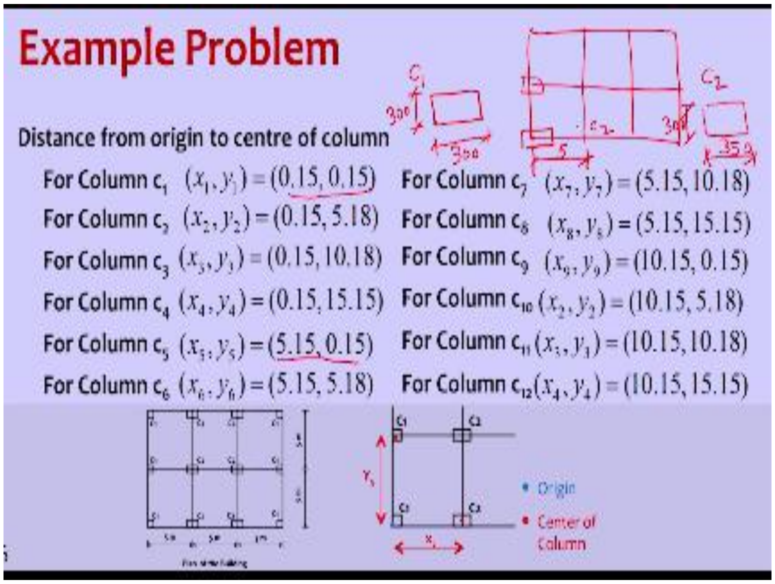

So once we get the local center of masses of each slab we will to know the stiffness for stiffness calculation also we need to know the center of stiffness of the particular slab or the column. So we have total of twelve columns, first column we have 0.15to 0.15 where we have the plan like this, so this is around 5 meters and this column is placed like this which is of size 1, c_1 type of column has a cross-section of 300/ 300which is nothing but 0.3/0.3 so this is a pipe c_1 and c_2 type of column has 350 mm as one of dimension and 300 mm s another dimension.

So c_2 so when we are calculating at this location this is origin and from here we have 0.15 that is 150 mm and then this is in X direction as well as in y direction, +5 mm will add up in calculating c_5 column this is c_5 so this is how we calculate center of I mean to say the center of each column this 0.15 and 0.15 where for c_2 column with this is c_2, c_2 column 0.15 and 5.18 is the in X direction, similarly for all other columns.

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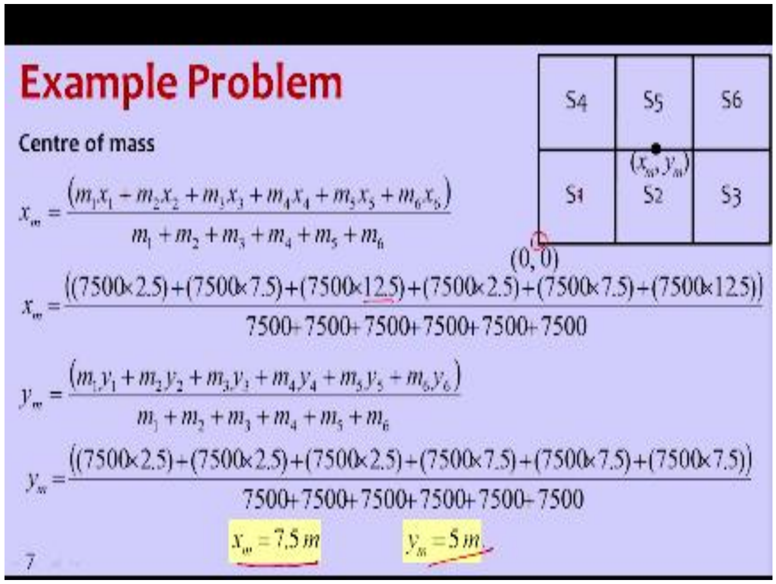

So in calculating center of mass we have calculated the distance from the origin to the center of slab so there we calculate 7500 as mass of one slab and x_1 is 2.5+m_2x_2 which is 7500 in 27.5+m_3 x_3 7500x12.5 these are the values which we have already calculated plus 7500x2.5+7500x27.5 +7500x12.5 this is for 6 slab and then adding up all the masses together 7500x6 because all this slabs mass is similar and when calculating y-coordinate we have the same formula $m_1y_1+m_2y_2+m_3y_3+m_4y_4+m_5y_5$ and m_6y_6 by total mass of the all slabs put together.

So y1 is for 2.5 slab one of 2.5 slab 2 for also 2.5 y coordinate and slab 32.5 whereas slab 4, 5 and 6 have7.5 which adds 5 and 2.5. So all put together we can calculate x cm of value7.5 meters from this origin and y cm as 5 meters from the same origin so our center of mass regard it as 7.5 meters and 5 meters similarly.

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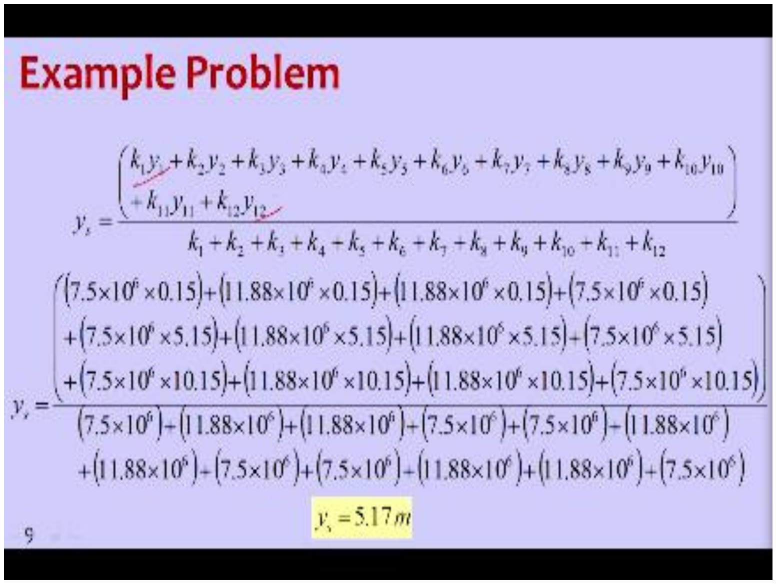

We can calculate center of stiffness in the similar fashion so now x 1 is the distance from origin to the center of the column so k1 x 1 is 4 bar first column similarly we have 12 columns so columns 12 we have so each column will get one component of k1 x 1 so for we got a stiffness of $7.5* 10^6$ in 2.15 + similarly for all other columns we get this equation and by the total stiffness of the structure or the total stiffness of the columns so $7.5 * 10^{10} * 6\ 10^6$ and $11.\ 8 * 10^{6}$ so we have 6 as even columns and 6 c two columns7.5* 6 we can also write like that and 11.8 * 6.

So we end up getting a value of 7.68 this is x coordinate similarly we can calculate for y coordinate also so x1 y1 is a component of first column and till a calculating 12 column k 2 y2and adding up all the stiffness's from one to twelve will be in the denominator so similarly we got y, y s as 5.17 meters so now.

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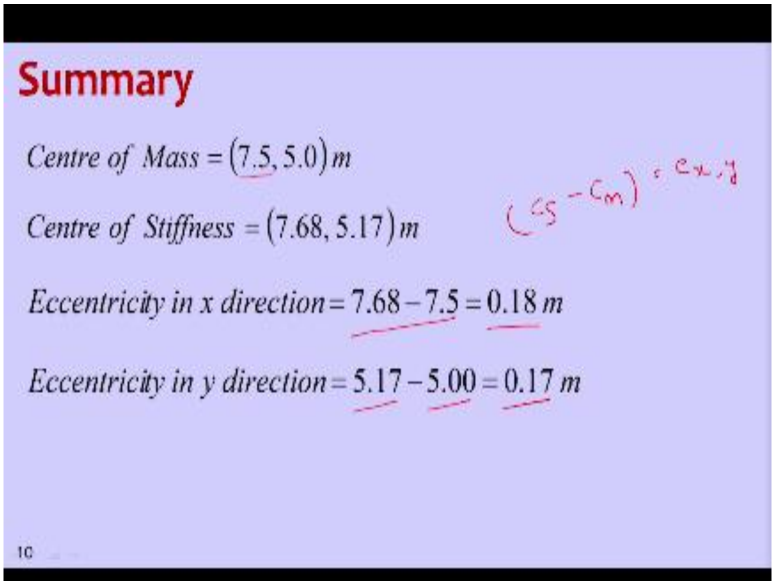

As a summary so this is the center of mass but we got 7.5 and 5.0 center of stiffness as 7.6 68 and 5.17 so in calculating a centricity to get a tensional moment so a centricity in x direction will give you like this7.68 - 7.5 that means center of stiff difference between center of stiffness and center of mass will give you the Centricity so if it is in x direction or y direction any of the directions we can find out so 7.68 and 7.5 you result up at 0.18 and 5.17and 5 you will get as a centricity in y direction as 0. 17.

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