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Structural Dynamics
Week 9: Module 01

Three Dimensional Dynamic Analysis

Ramancharla Pradeep Kumar
Earthquake Engineering Research Centre
IIT Hyderabad

So welcome to structural dynamics class. So in this class we will discuss about three dimensional structural analysis. So this three dimensional structural analysis will enable us to analyze the structure in real, real conditions. So when we have a structure which is three degree of freedom on each floor. So if you take a real structure what we have is X direction translation, Y direction translation and rotation.

So these are three degrees of freedom which we have in three dimensional structure. So how to analyze that, how to formulate equation of motion for that and how to solve okay. So those things we will discuss in this class.

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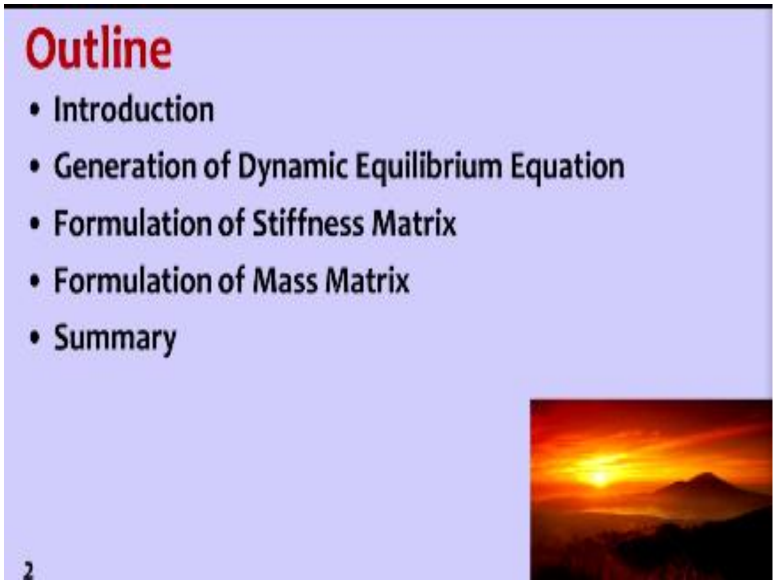

The first we will discuss about generation of dynamic equilibrium equation, and then how to formulate stiffness matrix, how to formulate mass matrix, and then we will discuss about the extensities what are present in between centre of mass and centre of stiffness.

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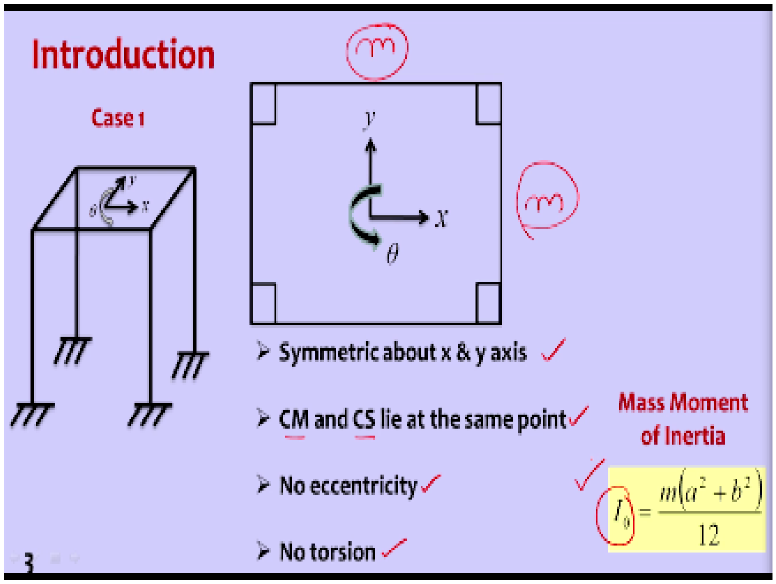

So if we look at this picture, this is a typical single degree of freedom system actually a single story structure. So a slab which is supported on four columns okay, so this slab can translate in X direction, it can translate in Y direction, it can translate or it can twist so that means it can rotate also. So if say it is symmetrical, so all the stiffnesses are same in X direction and also in Y direction.

So what will happen is it will translate, so if I give motion in one direction it will oscillate in that direction only. So if you look at see this as a slab and then support it by four columns. So if I pull in one direction and leave it, it will oscillate. And if I pull in another direction and it will oscillate. And then if I twist it, it will twist, if I give rotation it will twist. So if I pull and it is oscillating in one direction so that means what it is symmetrical along X-axis.

If I pull in Y direction and it is oscillating only in Y direction it is symmetrical along Y-axis. So how it becomes symmetrical along X-axis of Y-axis, so if the stiffness of columns all supporting columns is same on this side and on this side and centre of centre is exactly in the centre. So the diagonal, intersection of diagonal is centre of mass and then centre of stiffness if both lie on the same point then if I pull it will translate in one direction only no rotation takes place.

And similarly in the Y direction also no rotation takes place. But if the stiffness of any column is different from the other columns, then even if I pull in one direction it starts twisting. So will illustrate more as I will show you modal and illustrate this in little more detail in the next class. Now let us look at this slab, if I give unit displacement or displacement of X direction it will translate in X direction.

So symmetric about X-axis and also about Y-axis, centre of mass and centre of stiffness lie at the same point. So we will discuss about how to calculate this centre of mass and centre of stiffness. And then if they lie at the same point that means there is no extensity, when there is no extensity there is no torsion in the building. So we can develop equation of motion for this one in a very manner.

Next so this is mass moment of inertia, so if the slab is moving in one direction then how much mass is contributing to this motion to entire mass vibrating in one direction. So that means what inertia force we will consider the total mass, if it is translating in Y direction entire mass is contributing, but it is rotating. So that means what all the points are not at the same distance, so here when we, when I'm translating an X direction all the points are moving with the same amount of displacement.

But when I translate all the points are not moving with the same amount of displacement, but they are moving with the same amount of angular rotation. So that is why we consider mass moment of inertia for that. So mass moment of inertia can be calculated as $m(a^2+b^2)$ so a and b are the sides of the slab, $a^2+b^2/12$, so this will come from the first basic principles of mechanics.

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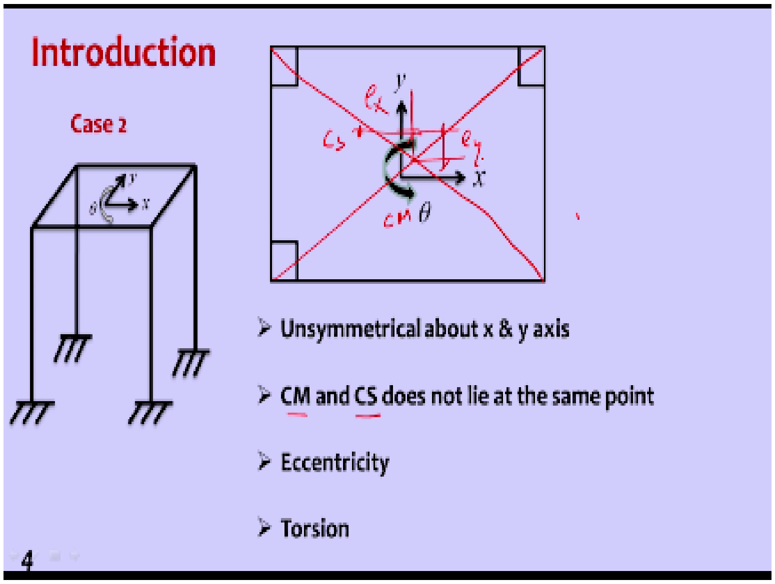

Then next one is case 2, where there is no symmetry, so that means what so purposely we have removed one column, if you can notice here one column is removed. So in this case even if I translate in one direction it will rotate and as well as translate in another direction also. So x-direction motion will translate in x-direction as well as y direction and rotation, so that what happens, so un-symmetrical about x-axis as well as y axis center of mass and center of stiffness does not lie at the same point.

And eccentricity is present in the structure and then torsion will take place so now what is centre of mass and centre of stiffness, center of mass is usually in uniformly distributed road so this is intersection of diagonals will give us centre of mass and now because only three columns are present so what will happen is centre of stiffness will be somewhere here this is centre of stiffness there is a centre of mass.

So the distance between this okay this is the eccentricity x and this distance is eccentricity y so because of this torsion take place in the structure.

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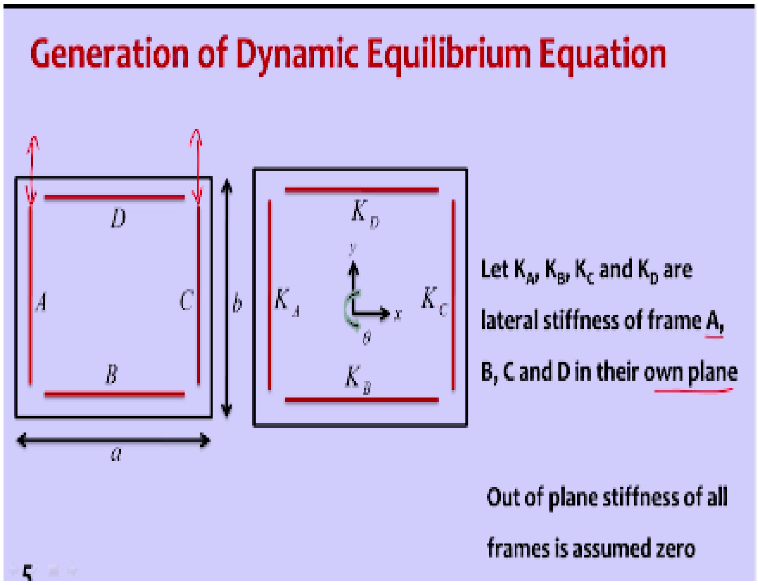

Now let us develop equation of motion first we need to find out how to develop stiffness matrix, okay. So this ABCD let us assume these are at frames okay so and each frame will act along its own plane but in the frame perpendicular to it its stiffness is treated as 0, so let Ka, Kb, Kc and Kd are lateral stiffness of frame ABC and D respectively in their own plane and then out of lane stiffness of all the frames assume at 0.

So that means what this frame will offer resistance only along this direction, this frame will offer only along this direction so that means what if my displacement is along y direction then k and Kc will give resistance, Kb and Kd resistance is 0 and if my displacement is in x direction then Kb and Kd will offer resistance, Ka and Kc are 0 and if I rotate all four frames offer stiffness for resistance.

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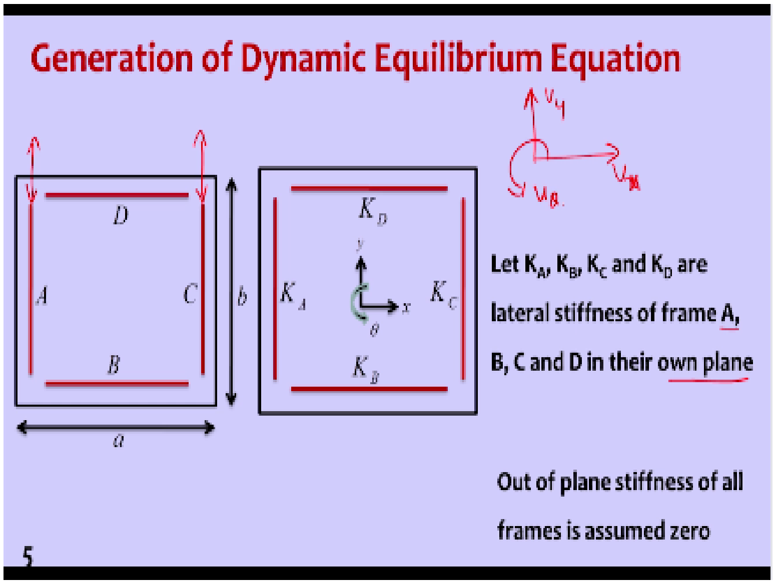

Now let us formulate stiffness matrix now how do we form stiffness matrix, we have say 3 degrees of freedom here so to the centre of mass we are writing 3 degrees of freedom so first degree of freedom T is ux displacement along that and second degree of freedom is uy and third degree of freedom is say uθ.

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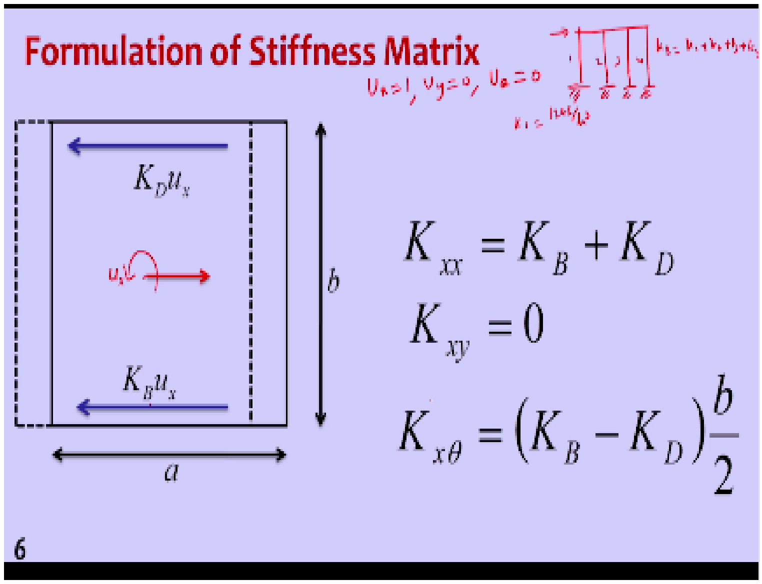

So if we give displacement that is ux as 1, uy as 0 and uθ as 0, so if we give 1 unit displacement along x direction then what happens is, the resistance comes from frame B and frame D, so stiffness of frame B is Kb so stiffness of frame D is Kd, how do we get this frame stiffness, so depending on say number of columns in that so a four columns are present so something like this your offer so 1, 2, 3, 4.

So Kb = K1+ k2+ k3 + k4 now how do we get k1, k1 is nothing but $12Ei/h^3$ of this column next week this column next week that is how we get total stiffness of Kb similarly Kd so we are giving displacement of Ux in this direction right side direction then the resistance force will be Kb and Kd together, so if this is unit displacement then Kb + Kd is a force so that K xx force is nothing but Kb and Kd.

Then what is force along say y direction because there is no force available in that direction, no resistance in that direction 0, then θ so θ direction rotation, rotation is in this so Kx θ, so Kx X direction, Kx y direction, Kx θ direction so that is you can see this one clockwise so if you take moments of the forces at this location Kb x B/2 so this is B/2 lever arm, Kb x B/2 minus anti clockwise x B/2. This is Kx θ so this is one column of stiffness matrix.

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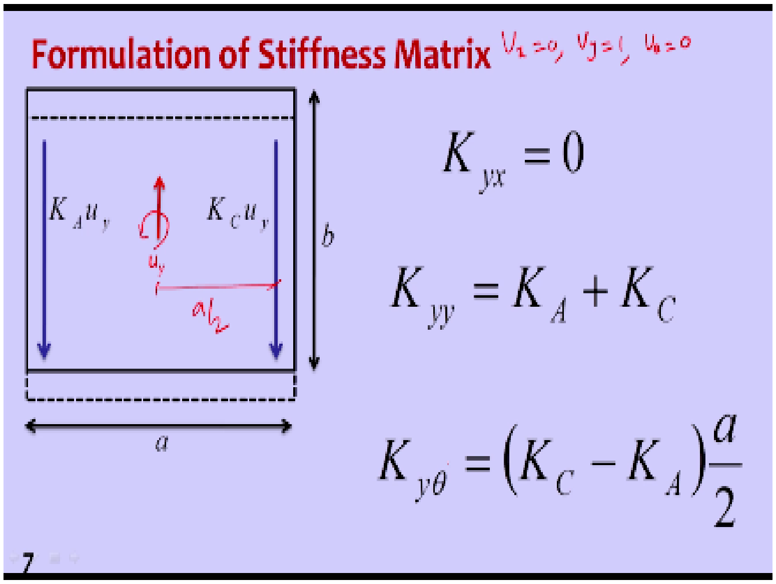

Similarly we write 2nd column in 2nd column writing 2nd column what we do is, Kx = 0, Ky = 1 and sorry ux = 0, uy = 1 and uθ = 0, so we are giving one unit displacement along y direction so K a x uy, Kc x uy so if u_y is 1 then what we get is so along x axis there is no force so that is why 0 along y axis two forces are there so K_A and K_C so we add that and then along θ axis again along this direction we need to so (KC) a/2 this is a so half of a is this is a/2, so K_C into a/2 minus this is anti-clock wise direction K_A into a/2 this is $K_{y\theta}$ so this second column of stiffness matrix.

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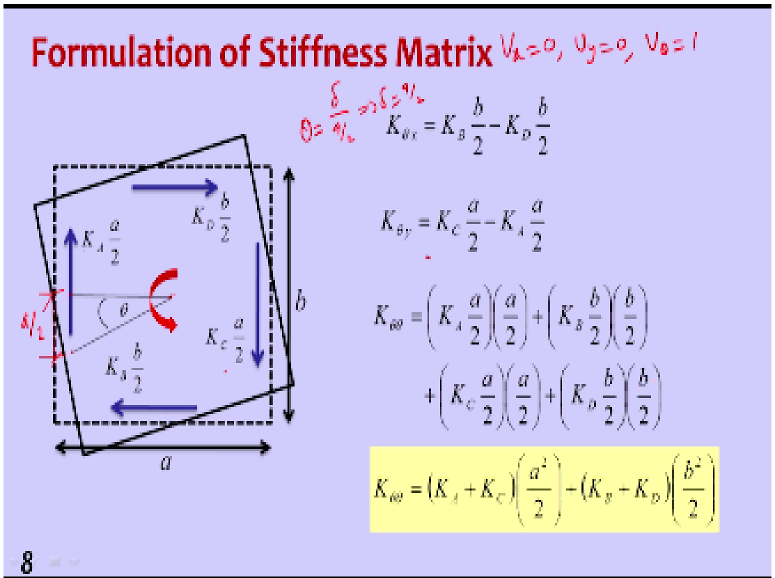

Now what it comes to third column of stiffness matrix so how do we generate that $u_x=0$, $u_y=0$ and $u_\theta=1$ one unit rotation we are giving. So if we give unit rotation what happens is so this point comes up to this so one unit rotation θ is one unit, so then how do we get this value. So angle is equal to arc/radius, so radius is say a/2 so θ=arc/radius so this is δ/a/2 so if θ is 1 implies δ=a/2 so this value is a/2 so that means what K_A into a/2 is a force in this direction.

Similarly, K_C into a/2 is force in this direction, here so since it is rotating an anti-clock wise direction all the forces will resistance will come in clock wise so that means upward right side, downward left side. So this K_B into b/2 so because lever arm is b/2 here so b/2 again K_C into a/2 K_B into b/2 so these are the forces in the direction corresponding directions. Now equilibrium along x,y and θ direction force equilibrium so we can get $K_{\theta x}$ direction x direction forces are K_B into b/2-K_D into b/2 then y direction Kθ in y directions K_C into a/2 K_A into a/2.

Then $K_{\theta\theta}$ θθ direction is what, so the force we need to generate moment at this points so that is because of the rotation. So K_A into a/2 into again a/2 from her to here, so like that K_B into b/2 again b/2 K_C into a/2 again a/2, K_D into b/2 into b/2, so we have $K_{\theta\theta}$ wrote here.

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Formulation of Stiffness Matrix

$$K=\begin{bmatrix} K_B+K_D & 0 & (K_B-K_D)\frac{b}{2} \\ 0 & K_A+K_C & (K_C-K_A)\frac{a}{2} \\ (K_B-K_D)\frac{b}{2} & (K_C-K_A)\frac{a}{2} & (K_A+K_C)\frac{a^2}{2}+(K_B+K_D)\frac{b^2}{2} \end{bmatrix}$$

So then final stiffness matrix is this, okay all terms placed here that is giving final stiffness matrix.

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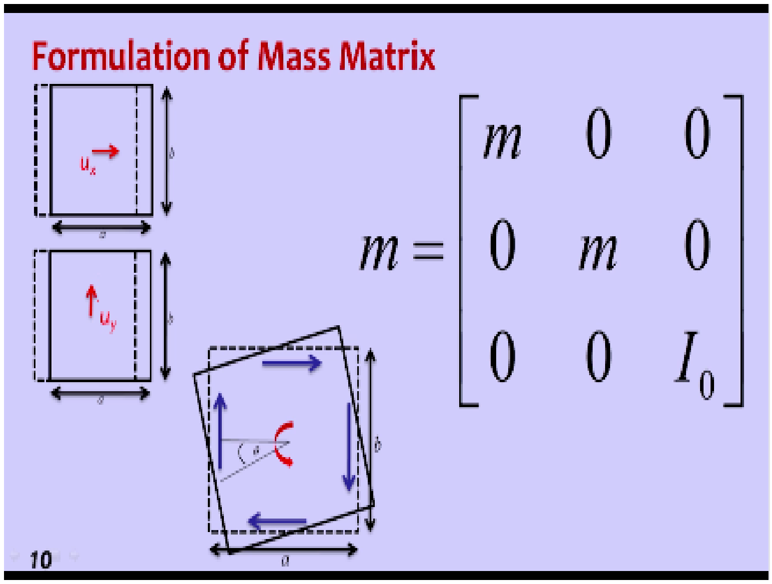

So now in this one what is happening is so mass matrix when mass is moving in say x direction entire mass is having the same acceleration value. So that is why whole mass is participating, in y direction.

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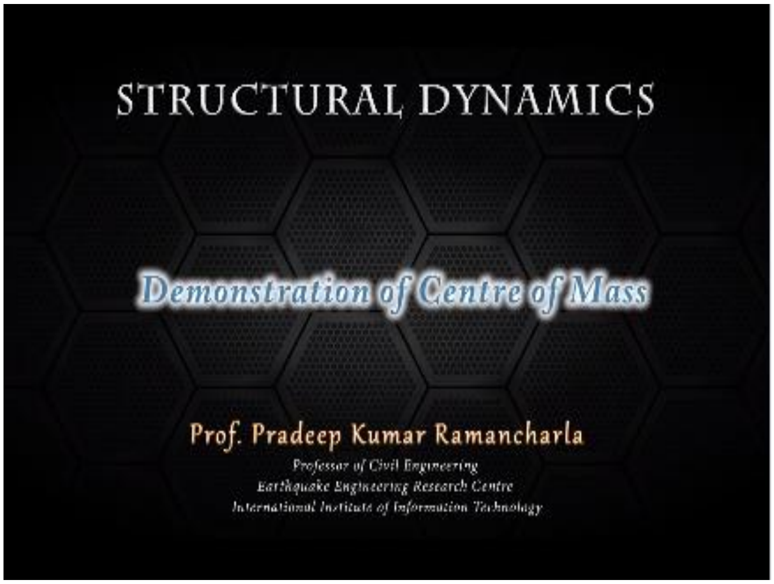

So in this demonstration we will discuss about center of mass, so what is center of mass. So in a building when mass is distributed uniformly center of mass will lie at the intersection of diagonals, so if I move this building it will move straight because I am applying force at the center of mass, but if I am adding some mass additional mass at some location here okay, then again I start moving this one this building is twisting.

So why this building is twisting, why not the earlier case. In earlier case mass is distributed uniformly in this case additional mass is present here that is why building is twisting, so in 3D analysis of buildings we need to find out first center of mass so when center of mass is located at the center and center of stiffness is also located at the center then we will not have of diagonal terms in stiffness matrix but if centre of mass is slightly shifted.

And then center of stiffness is at some place then even if we start translating or motion in translation direction then building will twist so we will have of diagonal terms in the stiffness matrix so, so we need to as far as possible we need to maintain center of mass and centre of stiffness in a close proximity.

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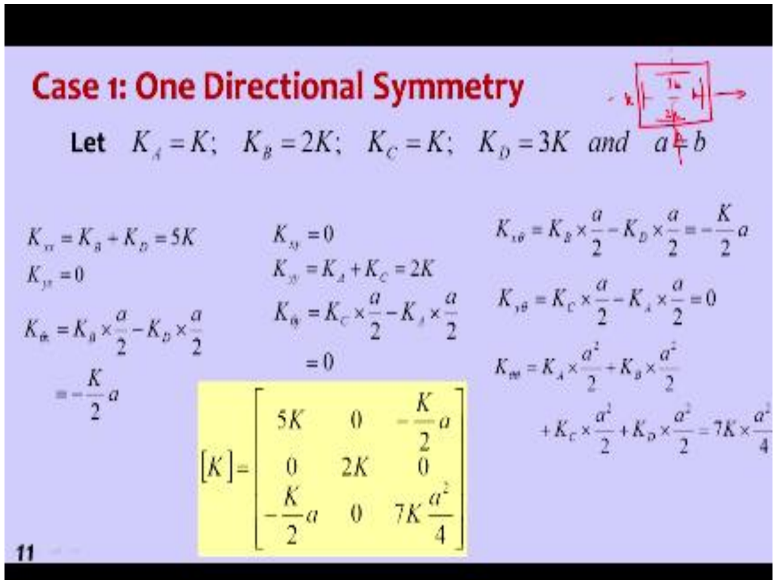

So let us solve one problem were one direction symmetry is there so Ka=K; Kb=2K; Kc=K; Kd=3K so in that so nits something like this problem is given in this form Ka Kb, Kc, Kd you can see along this direction wide direction there is symmetry along the next direction there vis no symmetry why there is no symmetry because here 2K and 3Kso no symmetry is there because center of stiffness will shift along this direction but when it comes to Y axis so K and K both are the symmetric about y axis un symmetric about x axis so if we formulate K axis K Yθ using in the previous formulation so we get this stiffness matrix.

So as you can see of diagonal terms are present so that means what when of diagonal terms are present if we move in one direction so of un symmetry it will rotate i8n the other direction also it will translate in the other direction also but if we give motion along wide direction it will not translate but if we give motion along X direction it will translate along with x and y but if we give more motion along the y direction it will translate only along y direction so that what is the directional symmetry.

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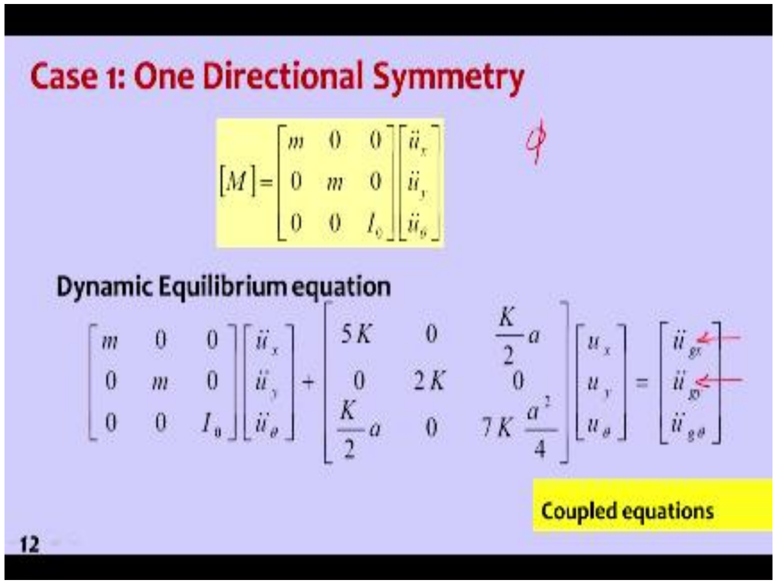

So now we have M mass matrix acceleration vector stiffness matrix displacement vector and earth quake ground motion so in this one so symmetry is if I apply if I give only the x direction acceleration it will displace the x direction as well as the y direction as well the t direction but if I give y direction only not x direction this is one this is zero it oscillates in y direction only that is what is given that is what we can understand from the dynamic equilibrium equation.

So now to come out of this one we need to do decoupling of this equations so how do we do decoupling so decoupling means again we find the mode shapes and pre multiply into post multiply then we solve the problem and then back calculate it.

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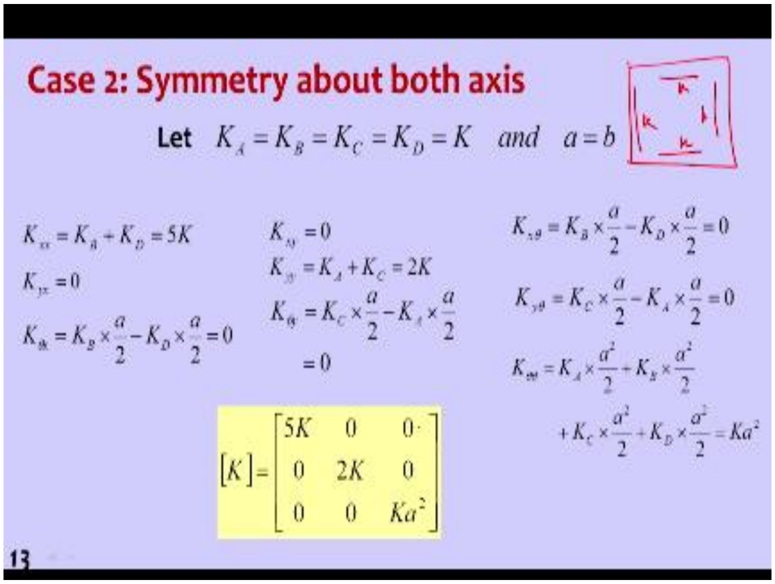

Now symmetry is there along both axis that means what all stiffness are the same so when all stiffness are same so it is something like this Ka Kb K c and Kd so all values are K there is symmetry in both the directions so what we get is so 5k 2k Ka 2 this is stiffness matrix.

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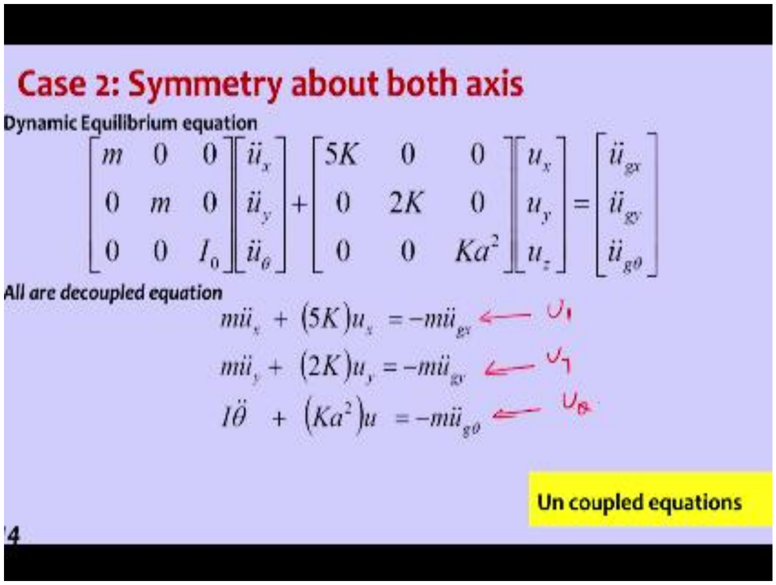

Mass matrix is there so as you can see the equation will be MÜ x 5K *x so each equation is independent and you can see this one so you can solve this equation to get Ux you can solve this equation to get Ui you can solve this equation to get Uθ this cannot happen in the earlier equation is where one dimensional symmetry was there.

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In this class what we have studied so we have studied how to formulate the stiffness matrix for 3d degree of freedom system so that is three dimensional buildings and then one directional un symmetry will affect the motion of the other direction so that we have studied and how to decouple this one decoupling can be done by using n again the same principles.

As the multi degree of freedom systems that is U+ Л*q as far as possible symmetry should be ensured in buildings for not to have torsion while earthquake ground motion is applies on to the buildings so we will solve one problem and understand this one and also we will solve one example problem of how to find out center of mass and center of stiffness.

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