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Structural Dynamics
Week 8: Module 02

Response Spectrum Analysis

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Welcome to structural dynamics class. So in this class we will discuss about response spectrum analysis. So when do we want response spectrum analysis, so we do response spectrum analysis when we do not have time history available to us, and then for most of the buildings, so the response spectrum, design spectrum whatever is suggested in IS1893 is available.

So we can make use of that and find the maximum forces and maximum deformations which might occur in the structure using this response spectrum analysis. So response spectrum analysis uses almost same principles as time history analysis, but only instead of finding time history we will find the maximum values of the response.

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So let us go to the outline of the lecture, outline of the class that first we will understand the building details and how we do the idealization of the building into lumped mass systems, then we find out mass stiffness and damping matrices, then dynamic equilibrium equation, then how to decouple these equations, and modal response from the response spectrum, and then we find the total response at each floor, and then we will solve one example problem.

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So building details so it as an input so this is actual building.

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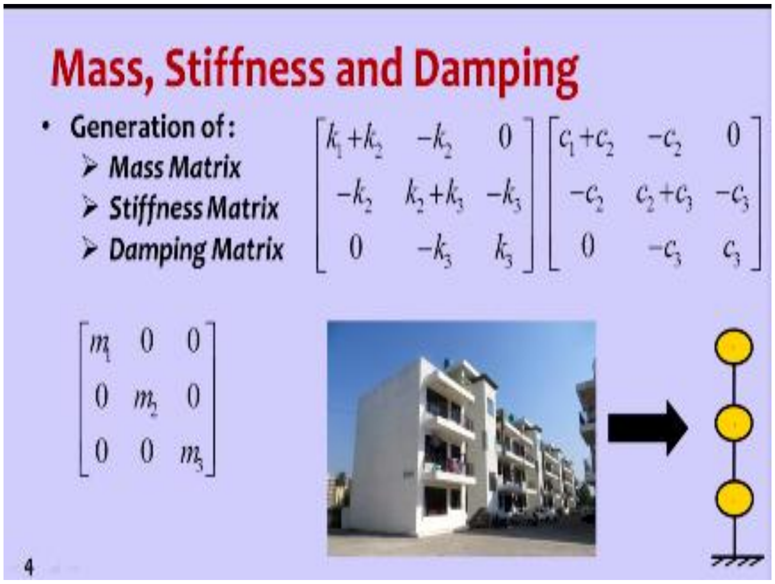

So we are converting this building into lump or mass system. So mass 1, mass 2, mass 3 at say floor height levels mass 1, mass 2, mass 3 and stiffness 1, stiffness 2, stiffness 3. So we generate mass matrix, stiffness matrix, and damping matrix. So we already know how to generate mass matrix that is M1, M2, M3 all terms will be in diagonal and off diagonal terms will be 0. And then stiffness matrix we know how to generate it and then damping matrix also we know how to generate it.

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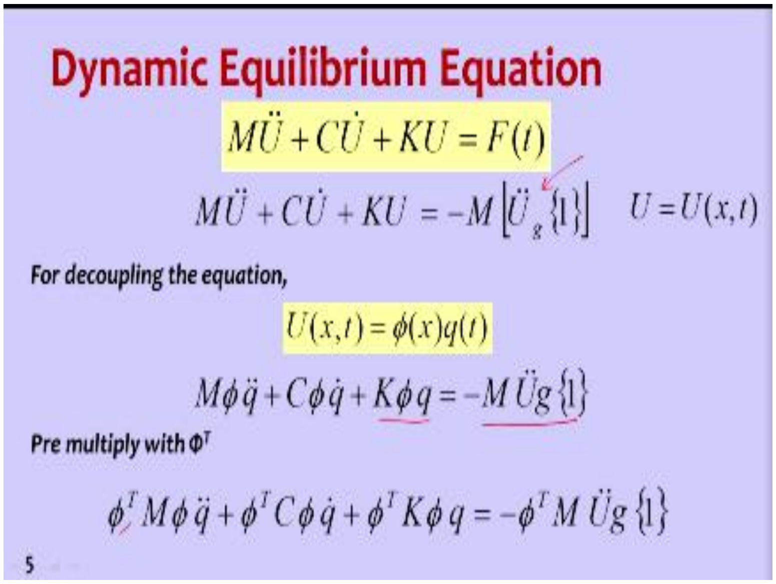

And this is general equation of motion so mu..+cu.+ku= applied force. So this applied force in terms of earthquake actions we use that as Mug.., so when Ug.. is available we do time history analysis okay. And if the spectrum of this Ug.. acceleration is available then we go for response spectrum analysis.

So for decoupling this equation we use U=ϕ and Q, so ϕ is a base quantity which we call it as mode shape and Q is a time quantity. So if we substitute this in the above equation what we get is MϕQ.. CϕQ. KϕQ=MUG.., so if we pre multiply this with ϕ^T so this ϕ^T pre multiplication we are doing for decoupling these equations.

So damping matrix is coupled, stiffness matrix is coupled, so that is the reason why our equations three equations are coupled equations. So we need to decouple this decoupling is done by using the orthogonality property of the mode shape.

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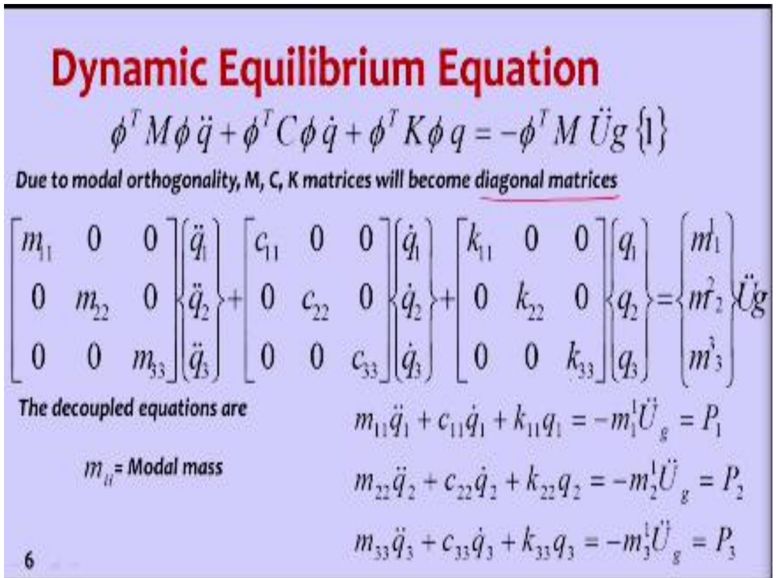

Now ϕ transpose MϕQ..+ϕ^T CϕQ.+ϕ^T KϕQ=-ϕ^T MU..G(1) is a vector. So due to modal orthogonality of M, C and Q will become diagonal matrices. So now M_{11}, M_{22}, M_{33} they are diagonal terms C_{11}, C_{22}, C_{33} diagonal terms K_{11}, K_{22}, K_{33} diagonal terms of stiffness matrix.

Now this is a decoupled equation so M_{11} x Q_1.. C_{11} x Q_1. K_{11} x Q_1 = M_1^1 x U..G so this decouple equations will look like this. Whereas this is corresponding to okay, so force1, force2, and force3 those in that degree of freedom sorry in that mode shape. So modal analysis is converting n degree of freedom system into n single degree of freedom systems.

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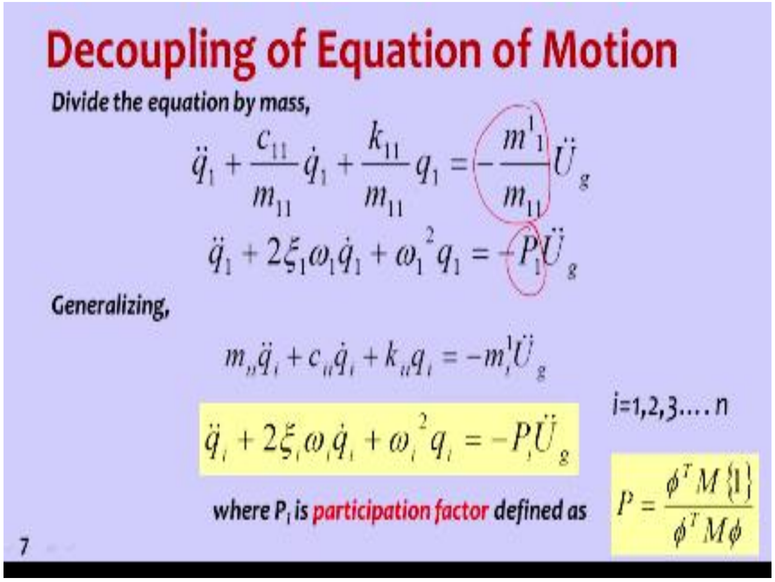

So if we divide by mass we get Q1..+C_{11}/M_{11} Q..+K_{11}/M_{11} x Q_1 = M_1 with this modal mass divided by M_{11} x U.. So this one is modal participation factor generalizing that what we get is qi..+ 2 ζi ω I qi. + ωi^2 qi = pi ui .. g, so this PI is participation factor how much mass is participating in that more that much force will be applied in that mode. so it is defined as Ø transpose M 1 by Ø transpose M Ø, so we discussed in detail about what is participation factor in the previous lecture. So that is time history analysis you can refer to that for getting the clarity on participation factor.

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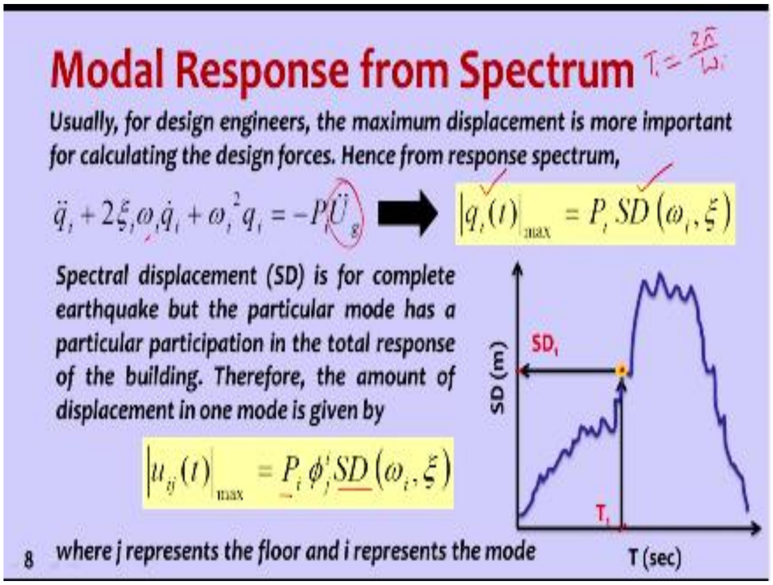

So usually for designers design engineers the maximum displacement is of more important for calculating the design forces so hence from response spectrum we get these values instead of time history so we get say Q I.. + 2 ζ I ω I q i . + ω I^2 qi = pi u..g, so u..g is a ground motion P is the mass participation in that mode shape, so now what we get is maximum value of Q what we want is so whatever is the natural period or natural frequency of that mode from that we study response spectrum.

And get the displacement value maximum displacement value if we multiply that maximum displacement value with participation factor we get maximum displacement in that time history response, so let us assume that this is a spectral displacement curve of this acceleration time history so we have acceleration time history so from that we are converting that into R we are calculating a response spectrum for that.

So how to convert out of draw response vector they have discussed in response spectrum lecture you can refer to that so this is a spectral displacement curve so natural Ω is available with us so T = 2Π/ ω, so Ti means T 2Π/ ω so if we put first mode ω we get first more natural period so we take that first more natural period and then hit the curve at the location and then read a spectral displacement value.

So that spectral displacement value we use here so that is the maximum time history response in that mode shape and there is a mass participation in that mode shape which is given by P 1 so spectral displacement we multiply with participation factor to get the maximum displacement which it is contributing to the total response, so spectral displacement is for complete earthquake this one is for complete earthquake.

But the particular mode shape mode has particular participation in the total response of the building this we discussed in detail in the previous lecture of time history analysis so each mode shape is contributing some extent to the total response so that depends on participation factor so particulars particular participation in the total response of the building therefore the amount of displacement in one mode Is given by.

So this equation SD x participation factor this is a maximum response of that mode shape or in that mode x mode shape values at different degrees of freedom so then we will get u_{IJ} (T) maximum value so it is something like that, so J is represent the floor and I represents the mode.

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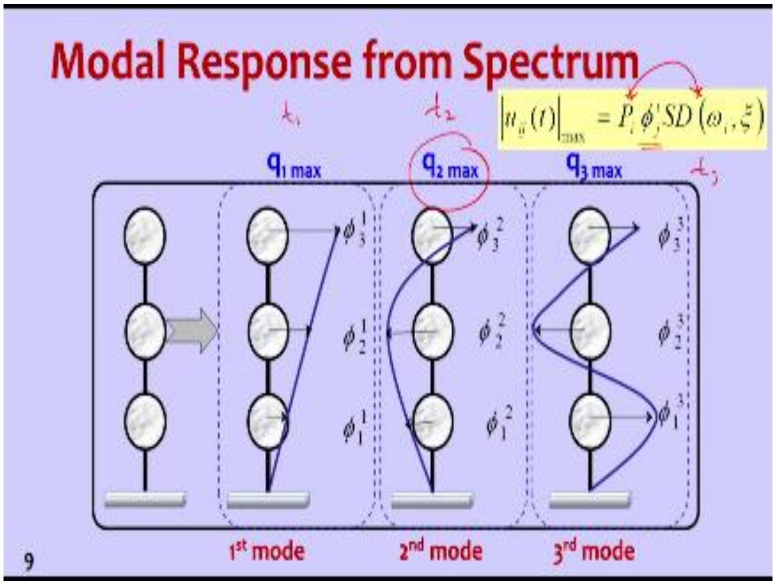

Now if you look at this one h mode shape so q1max q1 max is this multiplied by this so these two constitute that value q1 max participation factor in that mode shape multiplied by spectral displacement of that mode shape so this is maximum time history response in that mode and then if we multiply with same mode values so we get Ø 1 Ø 2 Ø 3 of first mode Ø1 Ø2 Ø3 of second mode Ø1 Ø 2 Ø 3 of third mode.

So if we multiply this q1 max with this one so we get this maximum values now how to add them. So we know that this maximum value is occurring at say some t1 time and this is occurring at t2 time this is occurring at t3 time so we cannot directly add in time is t analysis we were adding at every time instant all these values directly, but in response spectrum analysis we cannot add them directly. So we want but we want most conservative estimate so there are several techniques in adding this modal values.

So we have two techniques which we discuss that is SRSS methods that is called square root of sum of squares method and then second one is CQC method complete quadratic combination, so we discuss SRSS method here.

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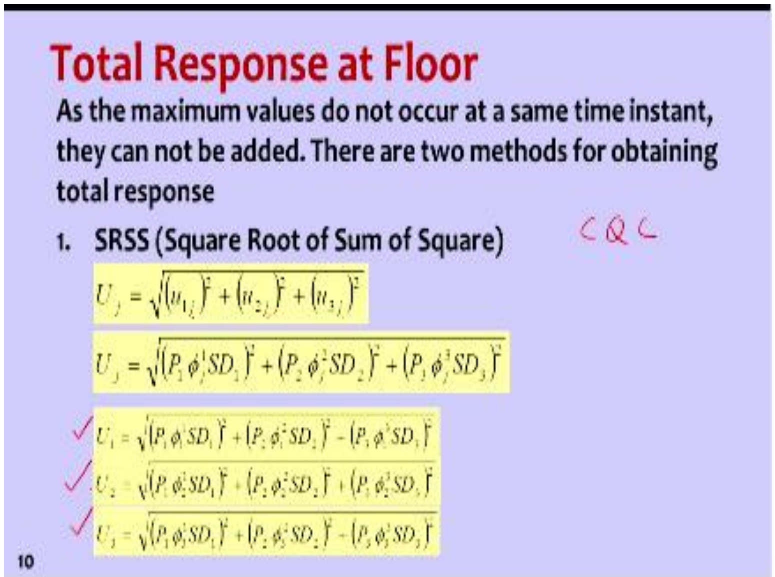

So SRSS method is used when there is a distance between the mode shapes or eigen values, so if eigen values are very well separated then we can go for SRSS method if eigen values are in the vicinity of 10% then we cannot go for SRSS method we have to go for CQC method. So in the vicinity of 10% means what say if I in first eigen value is say 30 and the second eigen value is say around say 33, so then we have to go for CQC method.

So if first eigen value is 30 and second eigen value is say 40, 50 or 60 then we can use SRSS method that is called square root of sum of squares. So the total response at each floor that as the maximum values do not occur at the same time, at the same time instant they cannot be added, therefore two methods for obtaining total response so one is SRSS method, second one is CQC method which we are not discussing in this class.

So that is $u_j=u_{1j}^2+u_{2j}^2+u_{3j}^2$ so that means what so j is the floor level, so first mode first floor level maximum value first floor level second mode this is floor that is telling, so first floor level second mode maximum value, first floor level third mode maximum value so you add so that means what p_1 j_1 first mode as $d1^2$ so you add all this so u_1 u_2 u_3 we get, so this is the maximum displacement of first ground floor, maximum displacement of second floor, maximum displacement of roof in 3 degree of freedom system.

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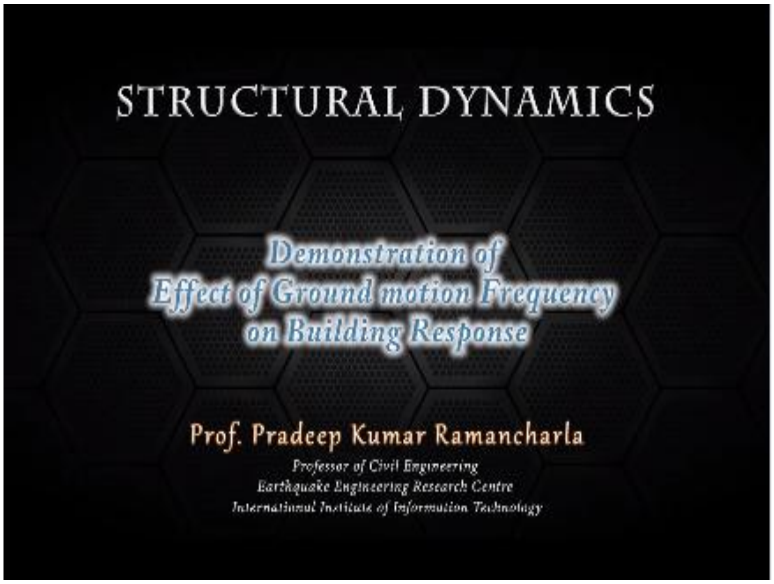

In this demonstration we will discuss about how frequency of vibration affect the behavior of buildings. So in this demonstration we have two types of buildings one is short building another one is long building, so long building will have more natural period and short building will have less natural period. So when I am moving the ground slowly so tall buildings get affected if the motion is something like jerk only short building get affected as you can see in this one so when the motion was just in the form of jerk this short building fell down and when the motion was very slow tall building fell down.

So this let us see the natural period of the buildings in another demonstration, this is simple pendulum experiment so here pendulums which are hung from the top and as you can see length of each pendulum is different. So if I give a displacement to this pendulum and raise it so it is oscillating so natural period of this pendulum is one two and fro motion. Now if I do give the same thing to another pendulum and release it as you can see this pendulum is oscillating in a faster manner compared to this pendulum.

So that means what time taken for longer pendulum to go one two and fro motion is large compared to shorter pendulum, so that means what short buildings will have less natural period and long buildings will have more natural period so if we come to this demonstration again when ground motion is having long periods tall buildings get affected and when ground motion is having short period something like this short buildings will get affected.

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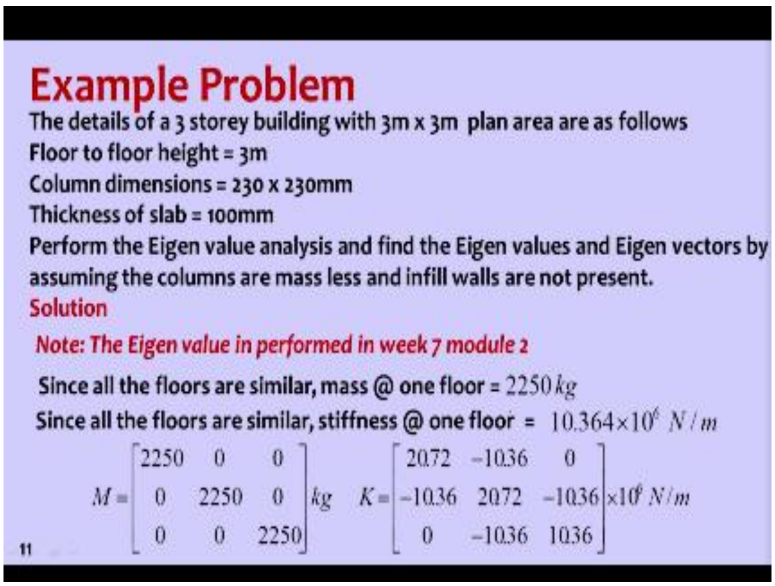

Now let us solve one example problem in this so the details of a three-story building with three meter by 3 meter plan area are, are as follows floor to floor height 3 meters column dimensions to 230 by 230 thickness of the slab 100 mm so perform Eigen value analysis and find out iron values and eigenvectors by assuming columns are mass less and in field walls are not present so let's first the existence all floors are similar mass is 2250 kg. So we are carrying the same example in the this also this case also stiffness matrix we know and then mass matrix also we know so Eigen value performed in like week 7 module 2 so we are using that.

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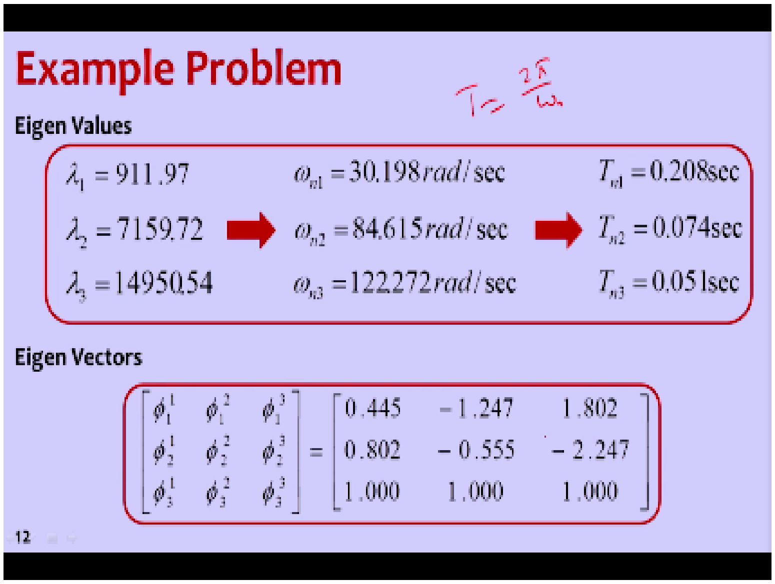

Now Eigen values we are carrying from that example so you can refer to the week 7 module 2 lecture so Eigen values 1 2 & 3are 911.97, 7159.72 so14950.52 – so these are Eigen values so if we take square root of that we get natural frequencies so30.198 radians per second first natural frequency 84.615 radians per second, second natural frequency 122.227 radians per second third natural frequency so corresponding natural periods also we can calculate by using 2Л by Ω T is equal to 2 Л by Ω n we can calculate that and then Eigen vectors are known values to us so we have already calculated this one in the previous class.

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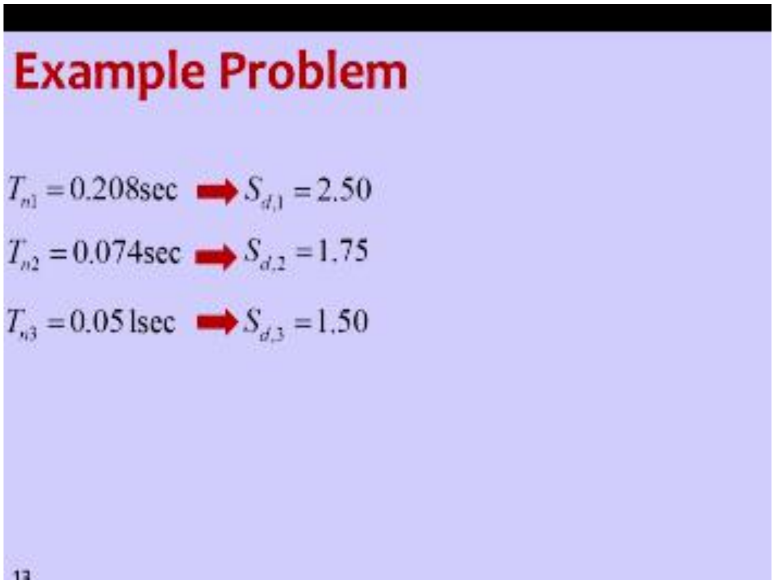

So we get natural period 0.2 seconds0.07 seconds and 0.051 seconds so three natural frequencies and corresponding SD values from the response spectrum chart is virtual displacement 1:2.5 units that is centimeters and 1.75 centimeters and SD three 1.5 centimeters.

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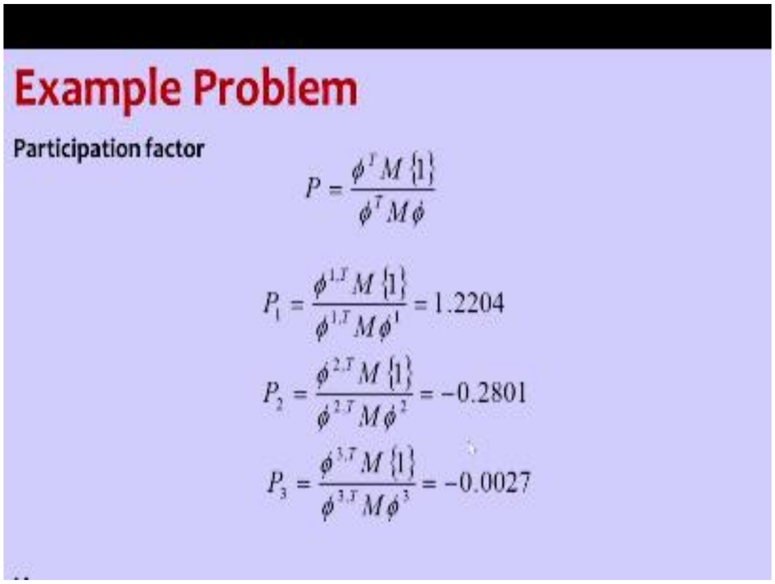

Then participation factor we can find out so a participation factor one from the given mass matrix and vectors so first second third Eigen values are modal vectors 1 .22 -28 and -.0027 seven so usually if you add mass participations of all three it should be equal to one so in this case it is equal to one.

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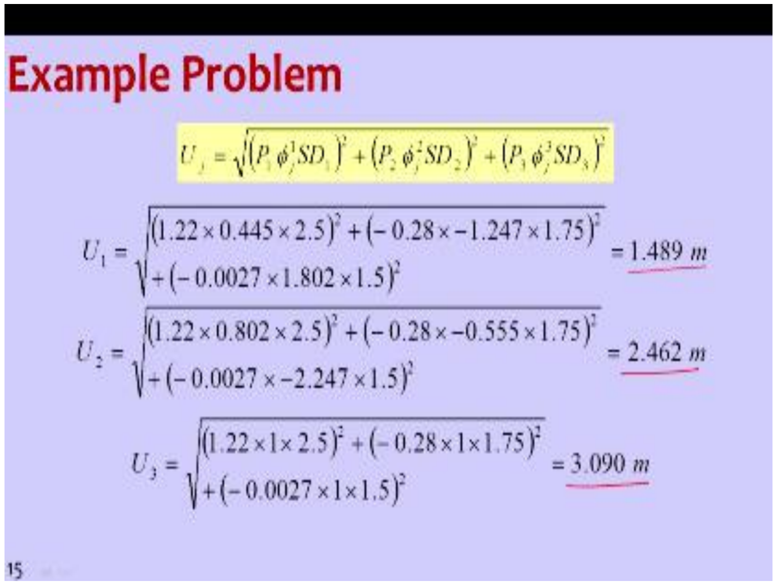

And then now we apply the equation of s RSS method so we substitute all the values we get so one point four eight nine meters displacement at the ground floor two point four six two meters at the second floor three point zero nine zero meters third floor so this is total displacement that is a maximum displacement in response spectrum method so we can compare this with a response maximum response of time history analysis which is usually lesser than response vector method because response vector method values are more conservative values.

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So in summary what we have discussed in this class is response spectrum method of analysis so usually Time mystery response is done for important and critical structures when time history is available if time history is not available and only response spectrum is available to us so that also can give us fairly good results so results in terms of maximum displacement and maximum forces what might occur on the structure.

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