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Structural Dynamics
Week 8: Module 01

Time History Analysis

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Welcome to structural dynamics class. So in this class we will study about time history analysis which is application to multi-degree of freedom systems. So when a dynamic force is acting on the structure or in the form of say, earthquake or wind how do we analyze the structure and how do we find the response of the structure.

So the outline of this class is we will discuss about how buildings are converted into a lumped mass system which is idealized it is 2 degree, 3 degree, 4 degree freedom systems.

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And then how do we develop mass matrix, stiffness matrix and damping matrix for that. And then we formulate dynamic equilibrium equation and then these dynamic equilibrium equation is usually coupled equation. So now how do we decouple this equations, equation matrix and then so what is the concept of participation factor and then finally how do we find displacement when the input ground motion or input force is given to us.

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So first, so this is a building, so how we are converting this building into a lumped mass system.

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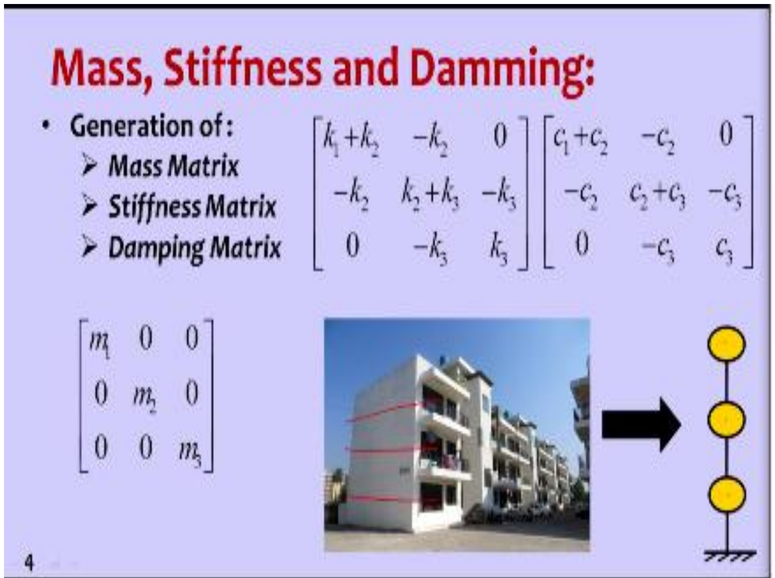

So as you can see this we need to say land dimensions okay, elevation dimensions so for fixing the mass of the structure at different, different floors and then stiffness of the floors. So mass of each floor okay, usually is lumped at slab levels or say if slab 1, slab 2 and slab 3. So this slab level something like mass from this to this location will be lumped at slab 1 and mass from this level to this level is lumped at second level and then remaining mass is lumped at third level.

And then this one will be the missing mass, so this will not be considered in this lumped mass analysis. So usually the acceleration of the inertia force generated at this place is lesser and then these forces we consider okay and m1, m2 and m3. So like that we have already studied how to develop stiffness matrix, mass matrix and damping matrix. So if there is any doubt please refer to previous lectures.

So this mass matrix m1, m2, m3 and the stiffness matrix k1+k2, -k2, 0 –k2, k2+k3, -k3 0, -k3, and k3 and this is damping matrix similar to stiffness matrix c1+c2, -c2, 0 -c2, c2+c3, -c3 and then 0, -c3, and c3. So this is mass matrix, stiffness matrix and damping matrix. So I already discussed in the previous class that all these matrices are positive definite. And because of which what roots we get are real roots.

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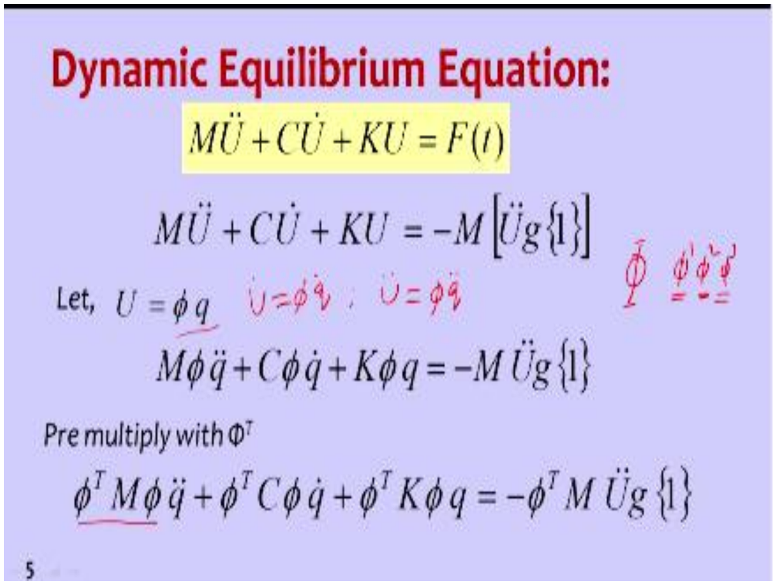

Now this is a dynamic equilibrium equation for the multi-degree of freedom system. So whereas where these three terms are inertia force, damping force and stiffness force this externally applied force F. Now if externally applied force is coming from earthquake so we already derived how earthquake force or equation of equilibrium for earthquake actions. So MU..+CU.+Ku=-U..G(1) because it is a matrix and force is a vector.

So this vector 1 is 111, so it depends on number of degrees of freedom. So if n degree of freedom system is there, so this is n rows and one column vector. So we assume solution to be U=ϕ and q. So this ϕq I will explain you once again. So if I take say this as a structure it is a 3 degree of freedom structure say mass is lumped at this location, mass is lumped at second location, mass is lumped at third location.

So U is the displacement of entire system, but where we are defining displacements at mass one point, mass second point and mass third point. So where system is vibrating because of the action of external forces, so we have to tell which location and at what time. So which location at what time, so in this equation U=ϕq, so ϕ is dependent on X that is location and q is dependent on time.

So U=ϕq, so since it is discrete system x takes only three values like is four value, 2nd floor value, 3rd floor value that means it is 3 meters or say 6 meters or 9 meters assuming that each floor to floor height is 3 meter so 3 and 3 so this is 3 meters this is 6 meters and this is 9 meters, so only three values it takes discrete values, then when we substitute this u = Ø Q in the above equation what we get is.

M Ø Q.. so why because Q is a time variable so what we get from this equation is let me so u. = Ø times Q. similarly u.. = Ø times Q.. So like that and Ø Q.. + CØ Q. + K Ø Q so we are substituting u here u. here and u.. in this equation and we are getting so this total equation is equal to –mu..g then if we pre-multiply this with $Ø^{-1}$ what will happen is $Ø^{-1}$ m Ø x u..+$Ø^{-1}$ C Ø . Q. + $Ø^{-1}$ k Ø .Q = $Ø^{-1}$ m u.. 1.

So why we are multiplying this with $Ø^{-1}$ so Ø is a total Ø is a model matrix and each Ø so Ø 1, Ø 2 and Ø 3 this is more shape vector so mode shape has a property to take up the coupled equations because of orthogonality, so in the previous lecture in the previous classes we have discussed these mode shapes are orthogonal to each other so each mode shape is perpendicular to other mode shape.

If you look at graphically you can easily what assume or visualize that each mod shape is perpendicular to the other mode shapes so using that principles orthogonality of the mode shapes principle if we pre-multiply and post multiply mode shape with matrix then what we get is a diagonal matrix of m diagonal matrix of c diagonal matrix of k.

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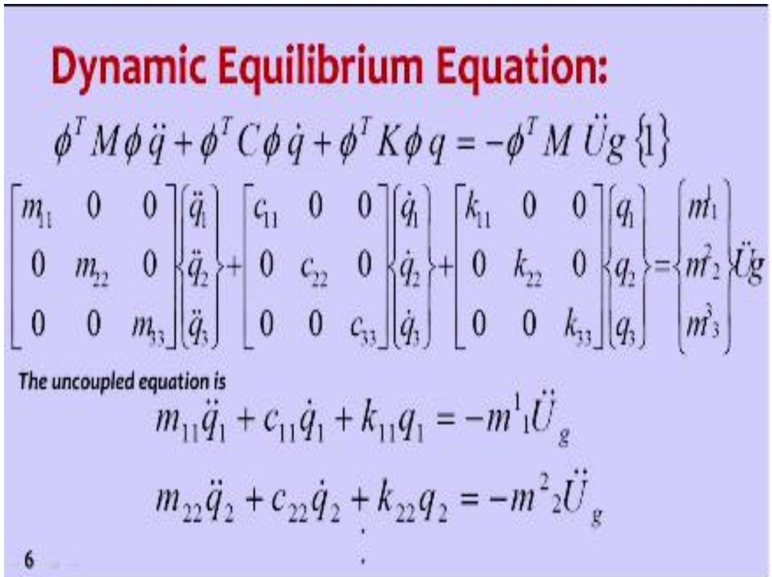

So that will decouple the coupled equations so if you look at this one mass matrix, damping matrix and stiffness matrix so this is a general equation of motion for multi degree of freedom system, so we are pre-multiplying and post multiplying with the mode shape or model matrix so if we do that this uncoupled equations so like this m11q1.. + c11q1. + k11q1 so this dot decoupled so uncoupled equations -m11 u..g.

So here m22 u..g so this is second equation so this is first equation second equation 3rd equation so in a way what we have done here is n degree of freedom systems we converted into n single degree of freedom systems if you look at each equation it is a single degree of freedom system with changed coordinates.

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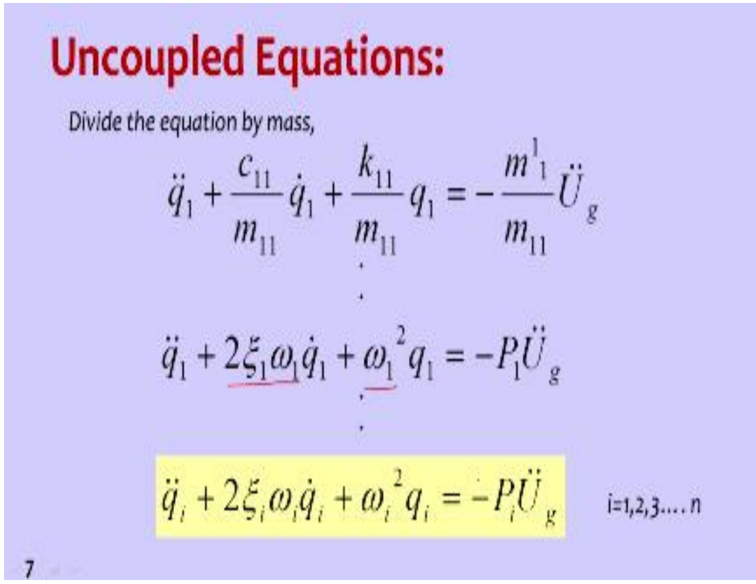

So displacement function is not now any more in terms of u now a displacement function has got converted into q when we take one equation and then divide that equation by m its own mass so Q1.. + C11/m11 q1. + k11/mm q1 = -m//m11 u.. so this is a ground acceleration this is a mass which is present in that first mode and this is a mass term in the mass matrix so with this what we can write is so C11/m11 is nothing but 2ζ 1 ω1, K11/m11 is ω 1^2.

And this m1/m11 is mass participation now the equation of motion for the uncoupled equations that is q1.. + 2ζ i ωi qi.. so qi. + ωi^2 qi = - Ø..g so this is the signal degree of freedom system we converted n degree off freedom systems into n single degree of freedom systems so out of is this I so we can write I = 1, 2, 3…n number so what is participation factor.

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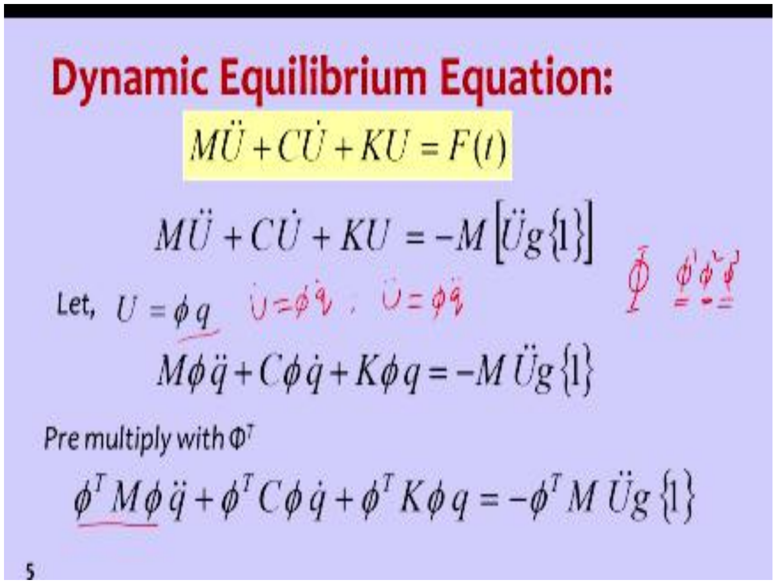

Participation factor is like Ø $^{-1}$ m1 / Ø $^{-1}$ m Ø so if we look at this equation here.

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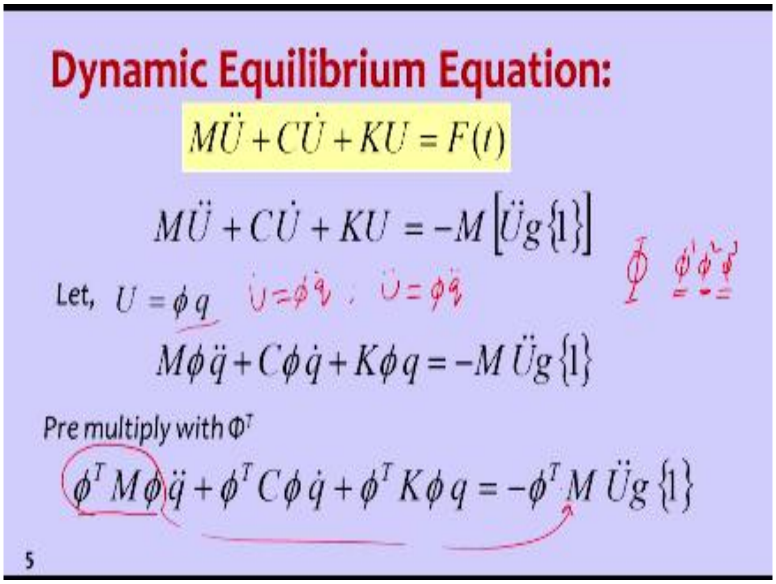

So $\phi^T M\{1\}$ by if you divide this entire thing here so the left from here if you divide this entire thing here it will become as $\phi^T M\{1\}/\phi^T M\phi$ so that is what it is.

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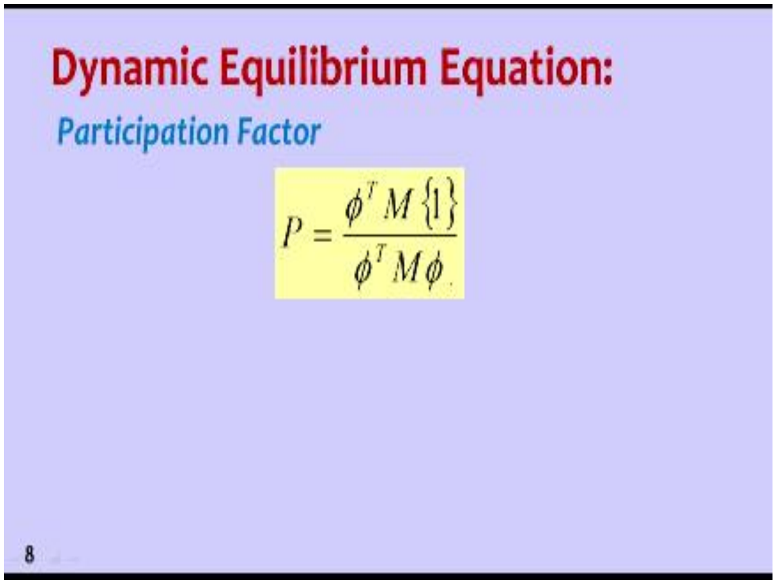

This is participation factor, now let me show you a small animation of participation factor.

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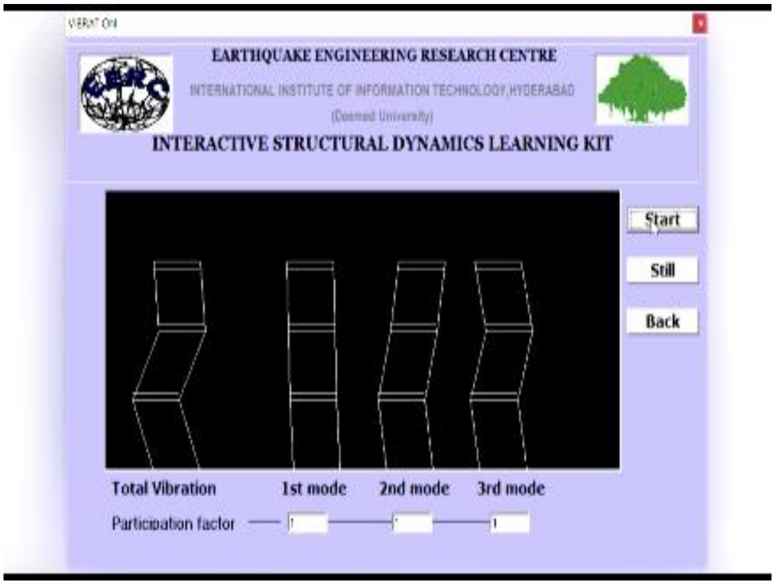

So like what is participation factor in multi degree of freedom system, so if you look at this animation here, this is a total vibration of building and this is the first mode, second mode and third mode so what modal analysis does is n degree of freedom system is getting converted into n single degree of freedom systems. So you can see this is the first mode this is second mode and this is third mode so if I stop it here so you can see this in time history analysis what we want is this is the total response, total response.

So this has two parts in it one is ϕ and second quantity is q so q is a response of that mode and ϕ is the response, okay at the location so two quantities are there. So now if we want response at this location you can look at the arrow, okay. We need to add this plus this plus this, so first mode response at this location, second mode response at this location top location, third mode response at top location so if we add these three things at time instant we will get total response at that time instant.

Similarly second floor if we take so this response plus this response plus this response if we add all three we will get this response at that time instant. Similarly, first floor level so it is a first mode first floor second mode first floor third mode first floor so we can add all these three tings to get floor response at that time instant. So in this one we need to understand one more important thing as you can see here so if I stop it, so here this is first mode this is second mode this is third mode, in this one I have given here equal weight ages so 1,1,1, weight ages I have given. But this 1,1,1, weight age usually will not be there in the real structures. In real structures first mode participation will be higher, second mode participation will be slight lower, third mode participation will be much lower so that depends on this.

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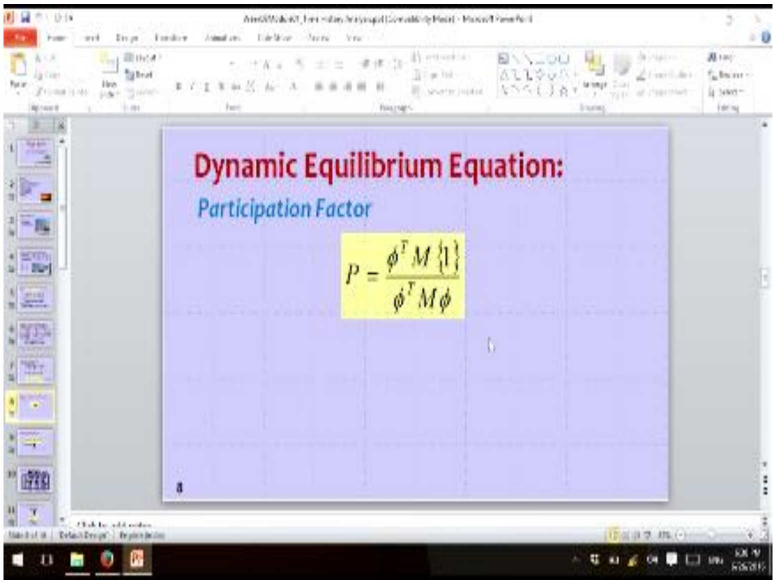

Participation factor, so this participation factor is telling us that how much mass is participating in each mode so that means what how much mass is participating like for developing inertia force in that mode, so that information is given by this.

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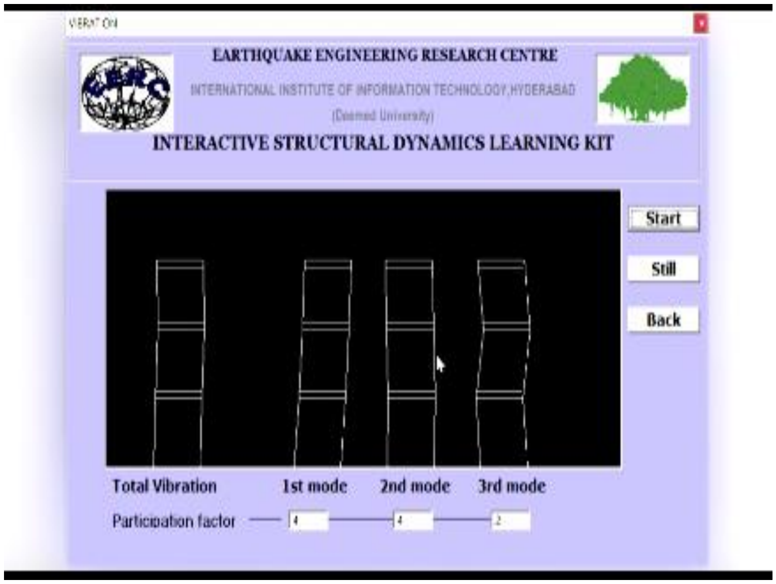

Now if we look at the diagram animation here, so now what I am doing you say, first mode participation is say 60% so first mode participation is a 60%, second mode participation is a 30% and third mode participation is a 10%, so if we look at this one then if I start the total response you can clearly see that the total vibration is more near to the first mode and then less likely that these two things this two things are also adding you can see this slide shade of second mode and third mode, okay.

Sometimes it happens that first mode itself is a dominating one so maybe this is only 15% say 0.15 and 0.15 and the second mode is only .0, and third mode is only 0.5% that is 0.05. So now if you look at it so first mode and this are matching almost matching and the total response is very near to this one even though second mode and third mode are present, but they are less likely contribution or dominating the overall response.

So in some cases what happens is first mode may not contribute much or say is contribution is slightly lesser or equal so 40% is first mode and then say again 40% is the second mode and then 10% or 29% is a third mode. So in this one if we look at this now first mode and second mode both are dominating the total response, so when we look at the multi degree of freedom system say n degrees say some 10 storey building, 20 storey building it will have large number of mode shapes.

So how many mode shapes we need to consider in doing time history analysis of say 20 degree of freedom system say 20 storied building so a usual suggestion is like were mass participation is more than 95% that many modes you consider so mass participation more than 90% means what first mode is participating giving say 60%.

Second mode is giving say 20% third mode is giving say 8% forth mode is giving say 2% 3% say something like that so what we do is first mode second mode third mode forth mode we add so we add mass participation as like if it is crossing say 95% then we stop at that point we need not take all the mode shapes into consideration so if we take first few mode shapes were total mass participation is becoming more than 95% we can do the analysis.

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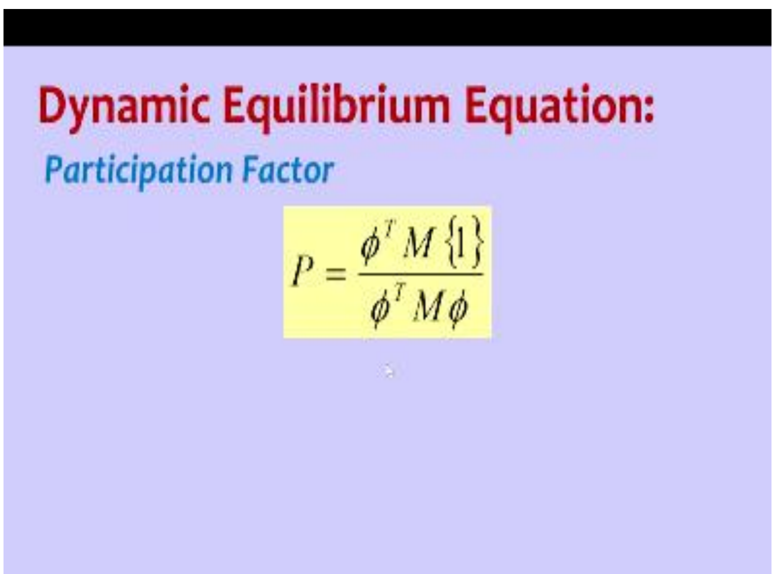

So that is what the participating factor.

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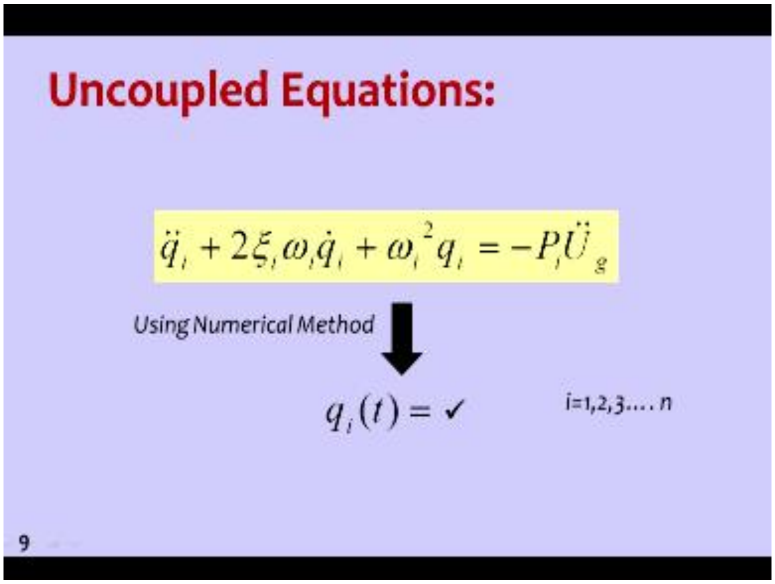

Now so if this is a equation of motion for that single degree of for that mode shape for one mode shape so if we using numerical methods we can solve this one either you use central difference method or New mark's method so either you use central difference method or new mark's method and solve this so we can find out the solution easily for q1 q2 q3 so all these are responses time history responses for that mode shape so using the numerical technique we can find out qi so I can be 1 2 3 as many degrees of freedom are there that many time history responses we find out for a given component of earthquake ground motion.

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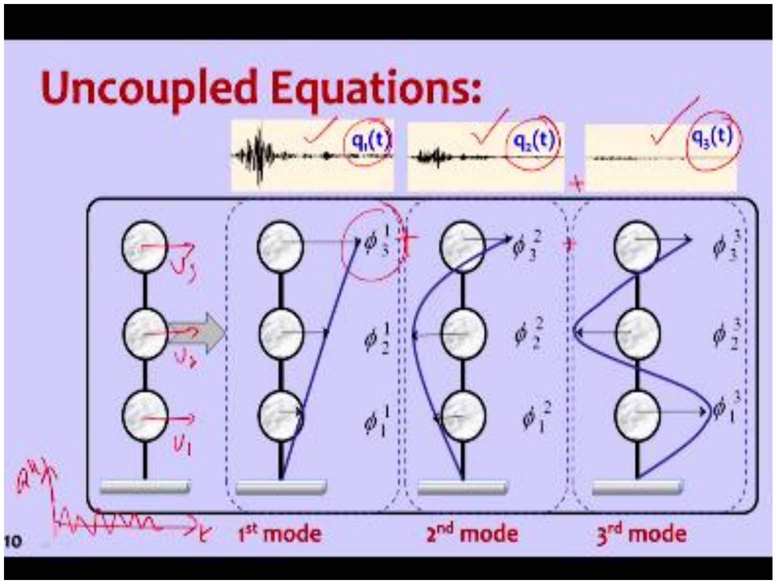

So as you can see this one so this a total response so the response here is so this is U1 U2 this is U3. So the 5 1 what we found out just now the q1 what we found out just now is this q so we are giving input ground motions some were here t and acceleration so with this one we decoupled equations in three mode shapes n degree of freedom system is converted into n single degree of freedom system with this mode shapes so mode shape 1 mode shape 2 mode shape 3 as you can see here.

So when we do that so we got time history response of first mode time history of second mode time history response of third mode. So in this participation factor is included in it so now we want to find out U1 U2 U3 so how do we get U1 U2 U3 so q1*Л3 of first mode + addition of q2 *Л3 of second mode + q3 *Л3 of third mode if we add all three we will get U3.

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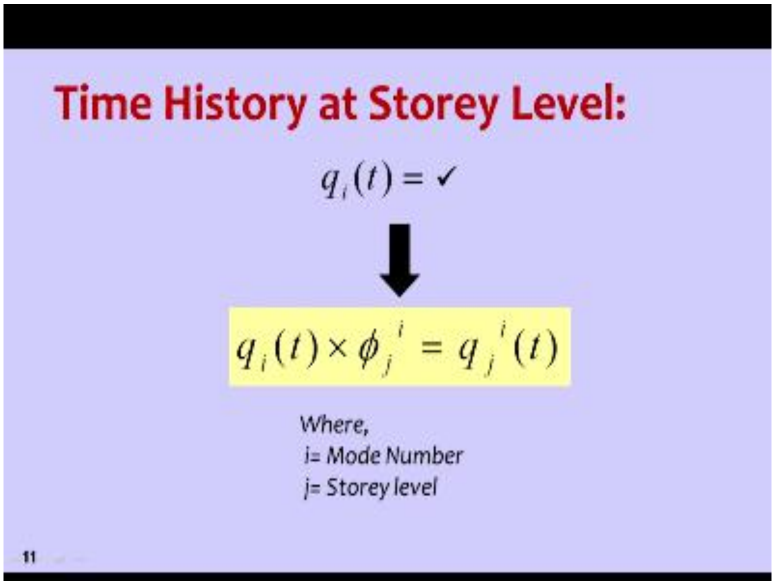

So that is a final solution so it is something like that okay so qi Лij.

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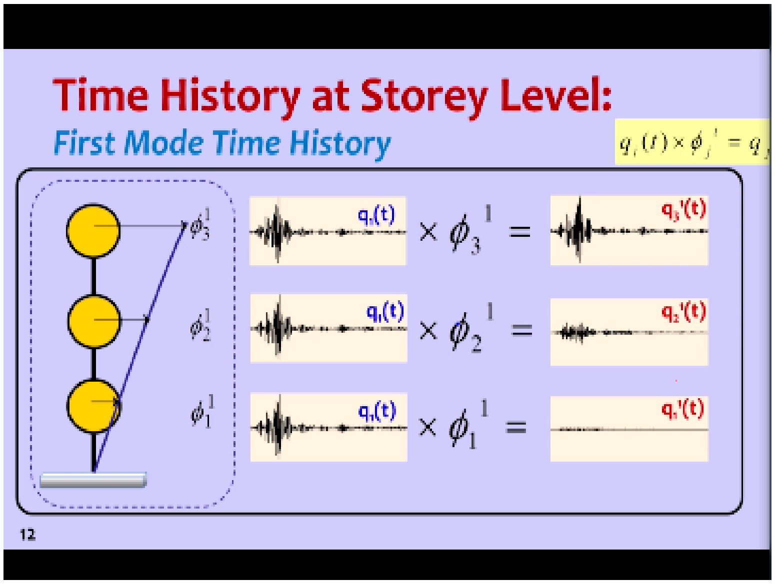

Now what we get is this so first mode you can see this one so q1, q1, q1 all so we get q1(3) q3(1) q2(1) q1(1) .

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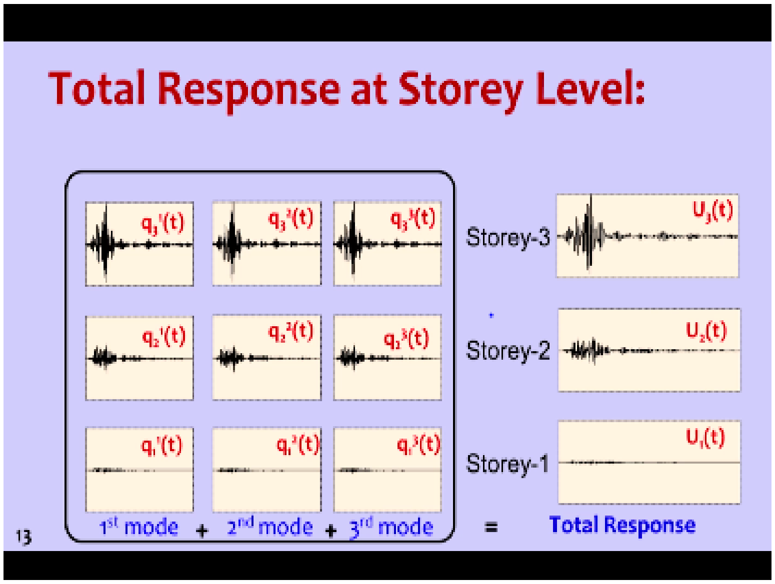

And then we get this so storey 1 U1 storey 2 U2 storey 3 U3. So q1 of first mode q1 of second mode q1 of third mode if we add all three we get U1 q2 of first mode q2 of second mode q2 of third mode we displacement of second storey q3 of first mode q3 of second mode q3 of third mode we get displacement third floor so first mode second mode third mode response addition of that is equal to total response.

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So in summary what we have studied in this class so in this class we gave studied how to decouple the coupled equations so what we have use is Л matrix foe Л vector which has an i8mportant proprietors and orthogonality so if we pre multiply and post multiply Л matrix with amass matrix or stiffness matrix or damping matrix it will decouple the equations so that means what it will remove the of diagonal terms so leaving us with the single equation so, so three degree freedom system converted into three degree of freedom system.

So what we call this as model analysis so model analysis means converting n degree of freedom system into n degree of freedom system so then we have after converting that we have studied how to find out the time history response if ground acceleration is given so and finally we have added this by using model participation factor so this model participating factor comes from Л^t By Лt m 5 so this model participation factor we will tell how much mass is participating in the first mode vibration how much mass 9is participating in second mode vibration. How much mass is participating in third mode vibration.

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