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Structural Dynamics
Week 7: Tutorial 02

Eigen vector and Modal Orthogonality

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Namaste in today's tutorial we will discuss how to calculate the eigenvectors and how to check the model Orthogonality**,** so let us go directly into the problem.

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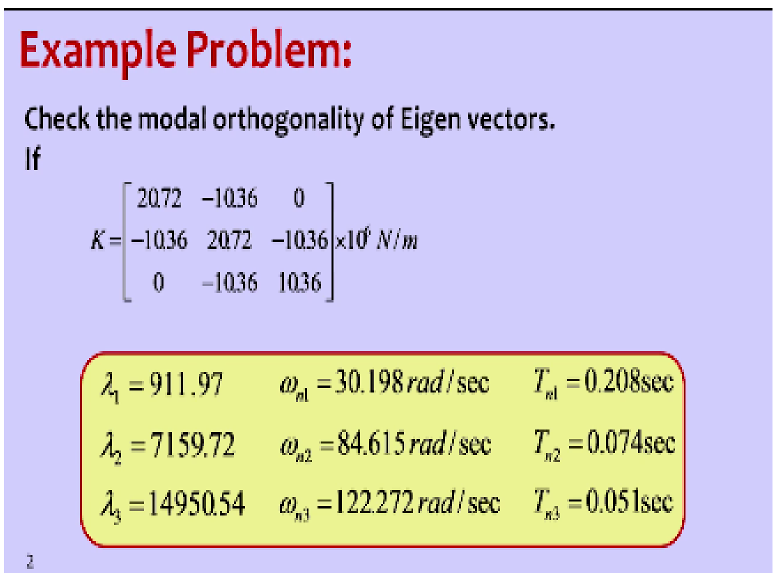

So here we have to check the model orthogonality of eigenvector if so here k is given to us the K matrix of three by three in nature is given to us and the λ1 λ2 and λ3 value and corresponding ω 1 and time period values are given to us, so why making use of this we will see how to calculate the Eigen vector and then later in second half we will check how to check the model orthogonality of a Eigen vector.

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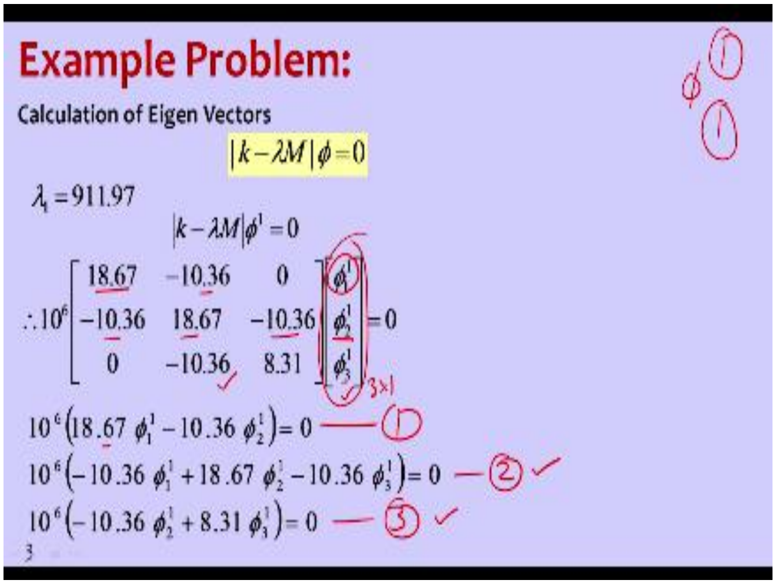

So first of all calculation of Eigen vector since we have to calculate the Eigen vector it is nothing but the K -λ M | K - λ| M into Ø = 0 will give us the Eigen vector, so if I put λ 1 as 911 .97 in this equation I will get this 5 matrix now here Ø 1, Ø11 so 1 indicates the mode number and this suffix indicate the floor level so Ø1, Ø 12 and Ø 13 so when I multiply this matrix with this one which is nothing but the k - λ M into Ø 11 vector or the 3 by 1 matrix.

I will get this three equation first second and third so how we get let us calculate one equation so 18.6 7 x Ø 11 will give me first 1, -10 . 36 x Ø 1 2 + 0 x Ø 13 so since it is zero there is no need to calculate so this will give me the first equation, similarly second equation will have -10.36 x Ø11 + 18 .67 x Ø 12 and last term here we have a positive value or some value other than zero that is -10.36 x Ø 13.

We will get the second equation similarly third equation now here we have three equations and three and one we can easily find out.

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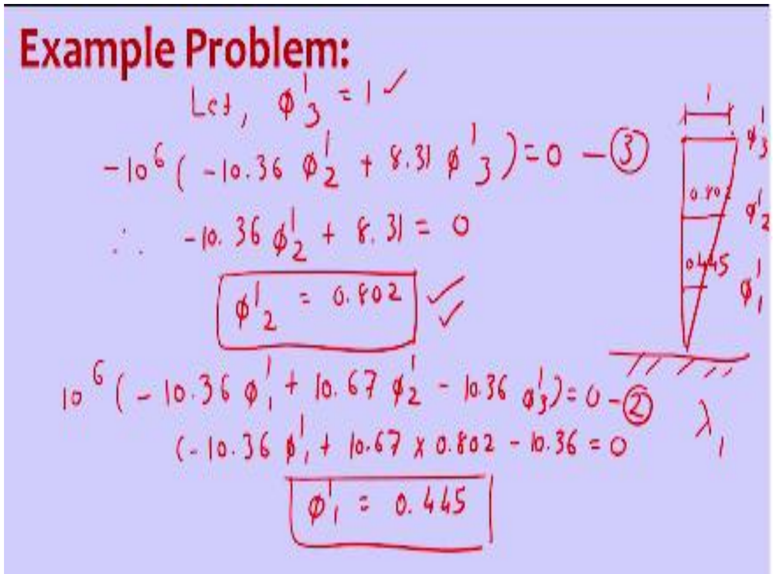

Now since I want the vector in some normalized form what I will do is I will assume the top value which is nothing but the Ø 13 is a one, now first of all I will substitute this one to get one more unknown value from equation 3 so our equation 3 is nothing but the -10^6 x -10.36 α 12 which is at the second floor plus 8.31 Ø 13 = 0 so this is third equation, now if I substitute this Ø 1 3 as a 1 I will get 10.36 Ø 12 + 8.31 = 0.

If I compute this one I will get the Ø 12 s 0.802, now let us see that what exactly physical meaning of α12 now if I plot this three-floor m1 m2 m3 so first of all for first more this is the third position or the top floor position Ø 13 assuming it as a 1 so this I will get the first point now Ø 12 will be over here as a pointed 0.802, now let us see how to calculate or how to get the Ø11 now in order to calculate Ø 11 we already have value of Ø 1 3 as a 1, Ø 12 as appointed 02.

Now we will substitute this in equation number 2, now what is equation number 2equation number 2 is nothing but the 10^6 - 10.36 Ø11 + 10.6 7 Ø 12 - 10.36 Ø 13 = 0, so this is our equation number 2 now substituting values 10.36 Ø11 + 10.67 x 0.802 which is value of Ø 1 2 – 10.36 = 0, so if I solve this one I will get Ø11 as a 0.445 so once I plot this 0.445 which is somewhere over here.

And I will connect all the dots so here I will get the values this will give me the mode shape, so here I we got the mode shape of a mode shape of a building so this is for Δ 1mode shape of a Δ1.

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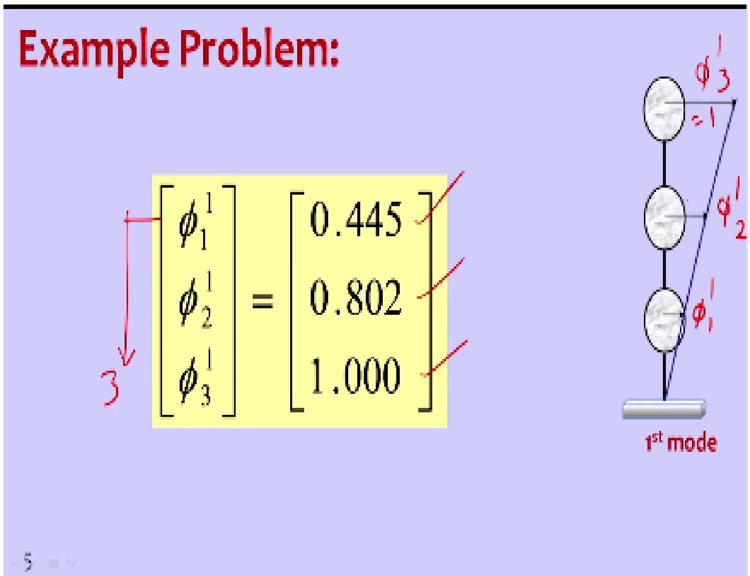

Now similarly so this is the our final answer 0.445, 0.802 one we are writing from first to third floor or the top floor top most floor if I draw the diagram Ø 13 Ø 12 and Ø 11 here we have Ø 13 as a 1 and 0.802 and 0.445.

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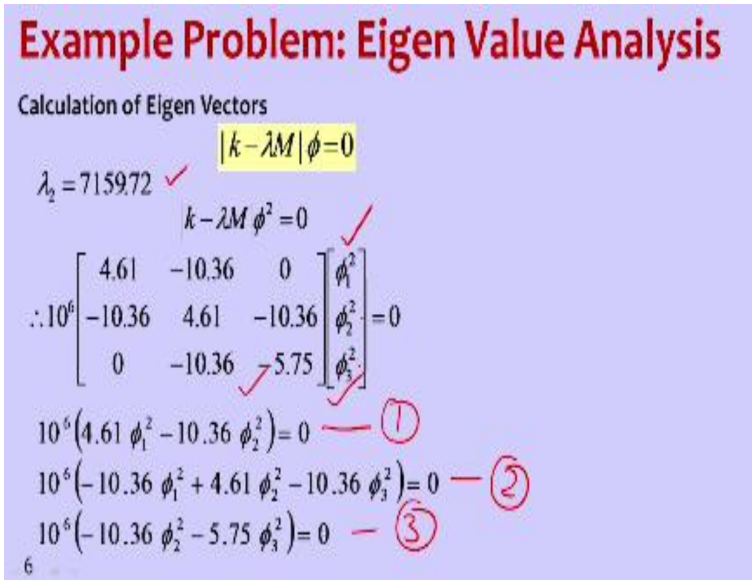

Now similarly we can calculate the board shape that is a Ø vector for second mode by substituting λ 2 as actual value of 7150 9.72 so by substituting λ 2 I will get this to matrix and again multiplying this 2 matrix with each other I will get equation number one equation number 2 equation number three again with the three unknown of Ø 21 Ø 22 and Ø 23.

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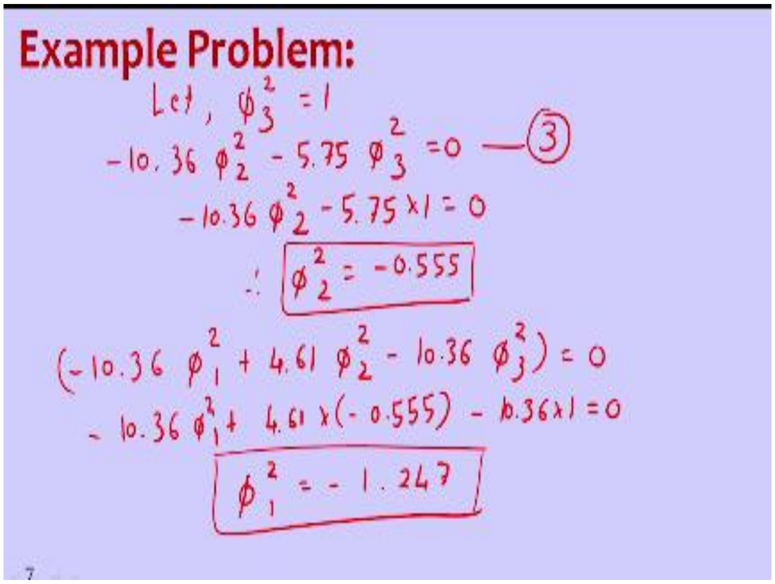

The way we have consider assume the value of Ø 13 here also we will assume value of Ø 2 3 as a one now rewriting the equation number three 10.36, Ø 22, -5. 75, Ø23 = 0 so this is my equation number three now10.36 Ø 22 which is unknown 5,771 x 1 = 0, so Ø 22 is equal to -0.555 so here you can observe that the value is less than one but it is having the negative sign similarly let us write the equation number 2 which is nothing but the 10.36 Ø 21 + 4.6 1Ø 22-10.36 , Ø 23 this is a entire bracket is equal to zero.

So now I will substitute each and every value one by one 4.61 x -0.5 Ø - 10.36 X 1 = 0 So I get Ø21s 1.247 so again negative value but greater than one.

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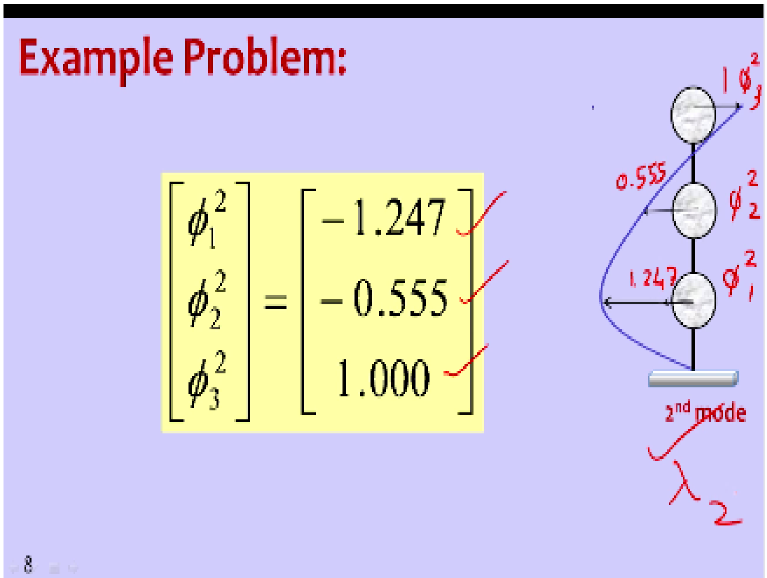

So when I plot this one you can see that I will get one on the roof top as a Ø 23 and then second one at the second floor which is Ø 22 because of negative sign it is on the left hand side 0.55 and then third value at the first floor which is highest if we just look at the absolute magnitude as a 1.2 47 but since it is negative we are drawing it on the left hand side as Ø21 this was the second mode.

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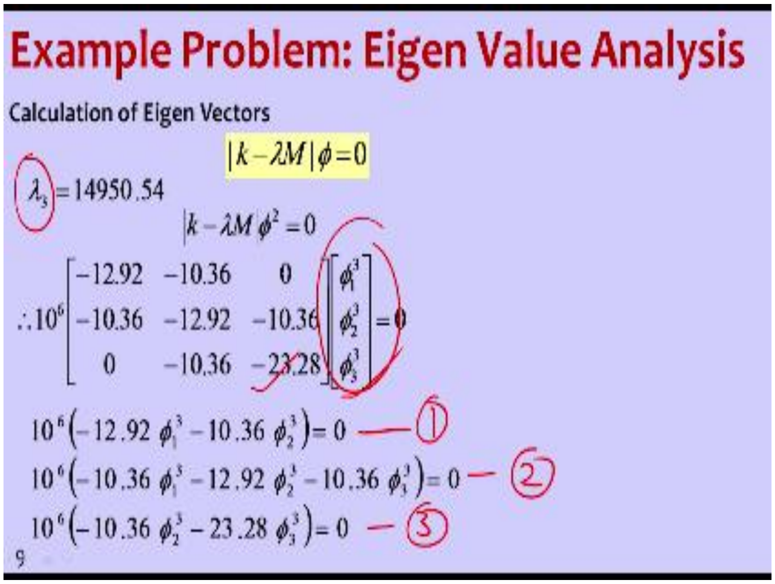

Now the same way when we go for the third calculation we get the two matrix this 2 matrix multiplication of this two matrix will lead us to a the equation where we have three unknowns this time path.

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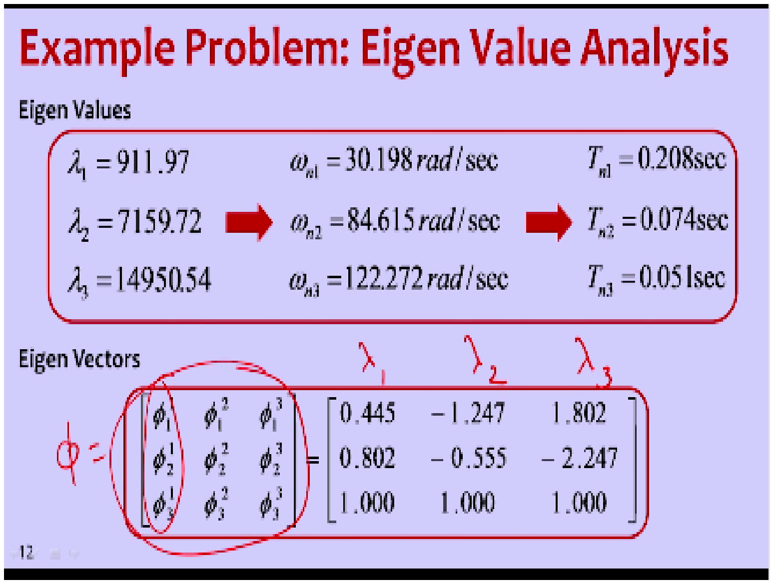

When we assume α 33 as a 1 and rewriting the third equation 10^6 – 10.36 532- 23.28 Ø33 = 0 this was my second equation sorry third equation when I substitute again 10.36 Ø 32 is equal to 3.28 x 1 so here I will get Ø32 s - 2.247 so this is a value at the second floor, now when I substitute for 1st floor in equation number 2, -10.36 Ø31-12. 92 Ø32 - 10.36 Ø33 = 0 this one is second one, - 10.36 Ø31 - 12.92 x Ø32value as - 2.2 47 – 10.36 = 0, Ø31 = 1. 802 so here we also got the values of Ø for third one in first and the second floor.

Now when I plot this one I will get on the this kind of vector in this kind of diagram, so this was theλ1 λ2 λ3 value corresponding ω values and the corresponding time period and this is the eigenvector which was for λ 1λ 2 and λ 3, so this is a Ø matrix so this can be called as a vector an entire matrix we can say as a Ø matrix.

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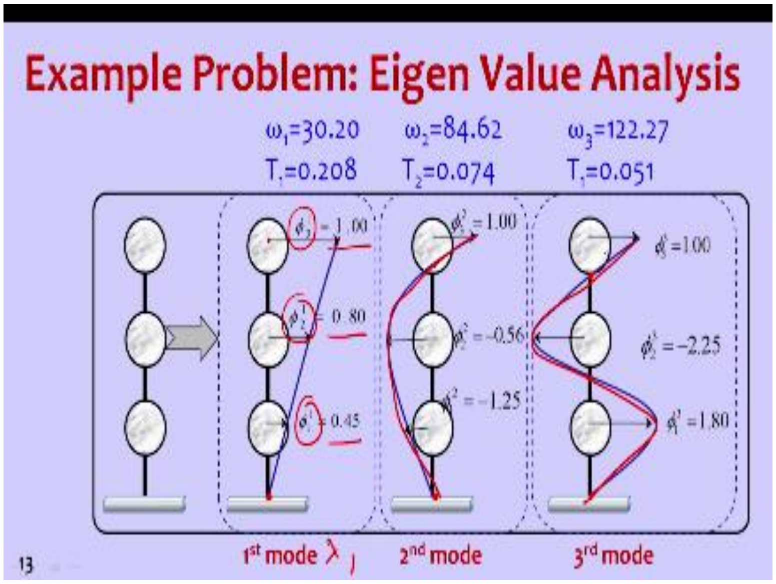

So this is a physical meaning of it so for λ1 Ø vector consists of Ø13, Ø12, Ø11 with corresponding values the magnitude will give us from center of mass of that floor how much far it is from center of center of mass of that floor and the positive and negative sign will give us about the in which direction it is and when we when we draw it we will get the mode shape so these are the mode shape for third one.

So here we can see that the for the first mode it will touch the vertical axis only at the 1 point for second one it is touching at the first point and at second point and for third mode we are touching vertical axis at the bottom at the middle one and the third one, so this can be a cross check for the calculation of Ø values now let us see how to check the orthogonality of a Eigen vector.

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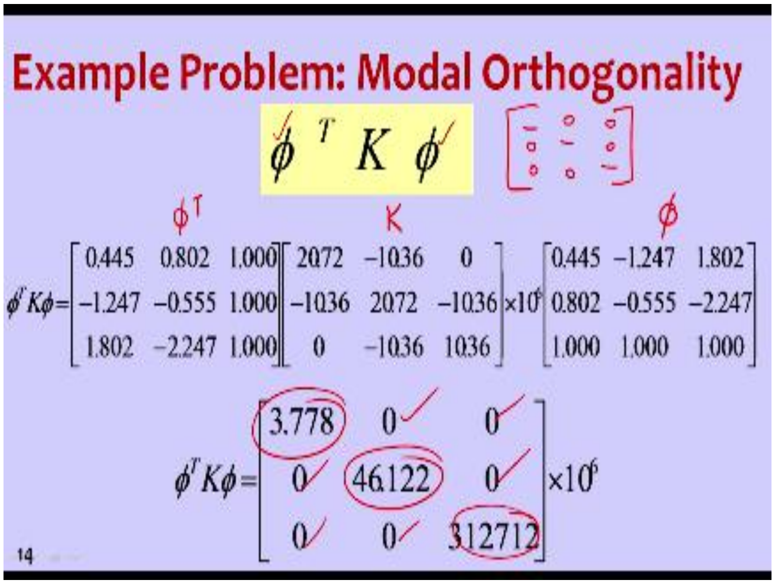

Now when we see when we have to define the model orthogonality of a matrix we have to prove that Ø$^{-1}$ K Ø should be diagonal matrix what is a diagonal matrix, diagonal matrix means we only have diagonal element as a sum value nonzero values rest all values are 0 so if we can prove this we can say that the Ø matrix is orthogonal.

So here now I am substituting the this is the Ø matrix which we have just now calculated and this one is a Ø transpose and this is a K matrix, so when I multiply this Ø $^{-1}$ K or and K into Ø ultimately I will get the answer as this matrix., so this has a diagonal element as a nonzero value and all non diagonal element as a zero by this we can prove that the modal orthogonality exist.

Till now we have checked the manual method to find out or to check the model orthogonality where we have computed the Ø $^{-1}$ K Ø, so product of Ø$^{-1}$ K Ø gave us the diagonal element or the diagonal matrix so that is a 1 part or the one way to prove that the model orthogonality now we will see how to check the or the prove model orthogonality for graphical method.

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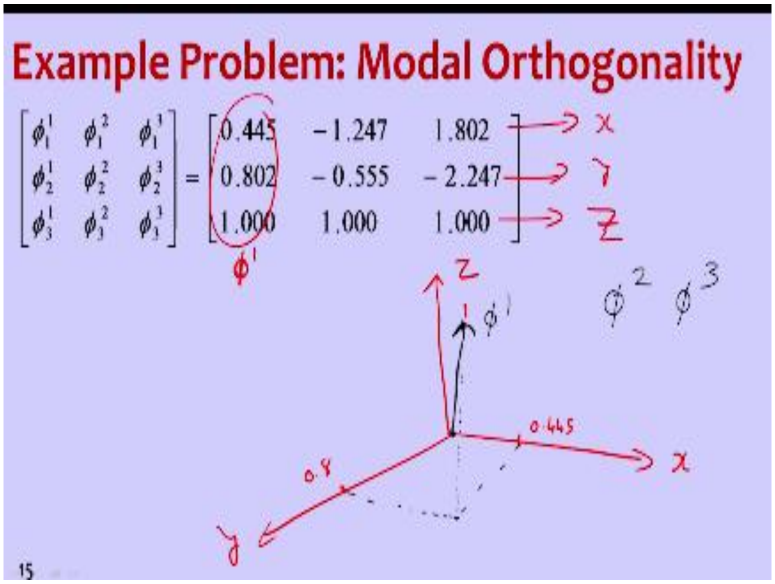

Now here we have Ø matrix which we have already calculated every one of you know now when I suppose these are the three axis this is X, this is y, this is z now you can consider this row as X this row as a y, this Row is a Z now let me plot a single point so each vector will give me one single point let us say α1 now I will connect I will first of all I will plot this α 11 in a graph or in a space and then I will connect it from the origin ,origin is nothing.

But the 000 now let us say x value for Ø 1 so Ø 1 in X Direction is around 0.45 say this is 0.445 then in y direction it is 0.8 which is somewhere around here 0.8 and in, in that direction it is one, I have to find out that in space so this was X Direction here it was y direction now if I go vertically Z direction up to 1 so I got this point as Ø 1.

Now if I connect it through origin till that point I will get this vector from origin it is going outward now when I compute the same thing same point for Ø 2 and Ø 3 let us look graphically how it will look.

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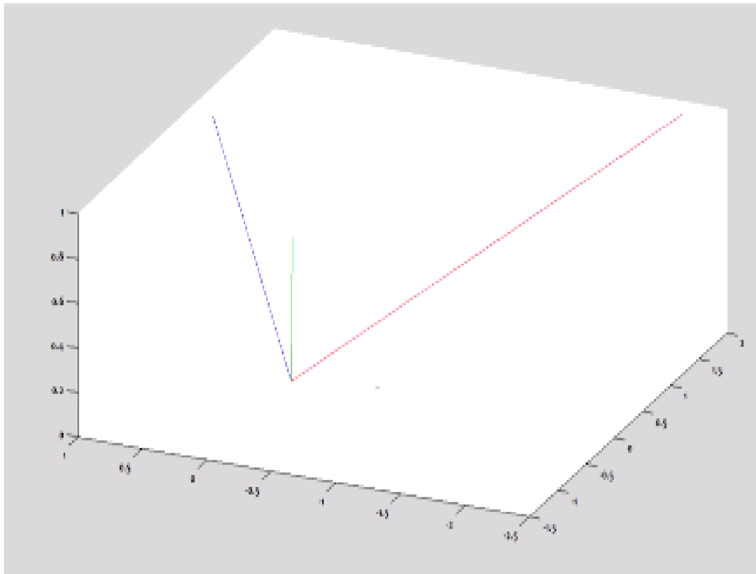

Now here you can see these are the three vector the way we have generated Ø 1 I am considered plotting Ø 2 and Ø 3 so when I plot this in any floating software available you can see that from origin how they are going and how they are orthogonal to each other they are perpendicular to each other and they will not cross each other in the space, so if we can prove this or if you can draw this is also one of the method to check the model orthogonality.

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