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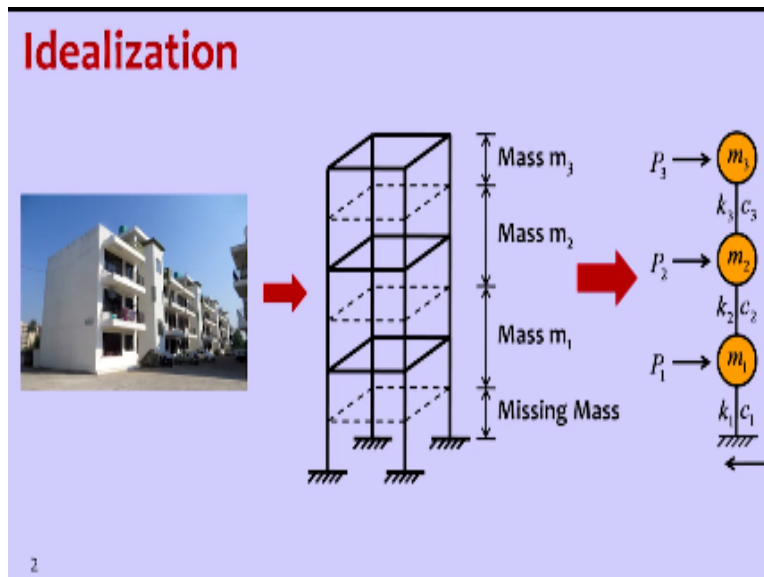
Structural Dynamics
Week 7: Tutorial 01

Generation of Mass Matrix

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For today's tutorial we will discuss on how to calculate the mass matrix for multi-degree of freedom system. So before we go to actual presentation or the today's problem we will just go through the, we will revise some basics of multi-degree of freedom system.

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Now if we have any building of G+2 we want to get the 3 degrees of freedom. So we want to calculate the mass at first floor, second floor, and third floor. So our objective is to get the mass at the particular floor level, so entire mass of a building we will concentrate on the three point or number of point. So here since you have G+2 building we will distribute our mass or calculate entire mass divided it into three parts that is first floor, second floor, and third floor.

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Equation of Motion

$$M\ddot{U} + C\dot{U} + KU = P_0$$

- Generation of :
 - **Mass Matrix**
 - **Stiffness Matrix**
 - **Damping Matrix**


$$\begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix} \quad \begin{bmatrix} k_1+k_2 & -k_2 & 0 \\ -k_2 & k_2+k_3 & -k_3 \\ 0 & -k_3 & k_3 \end{bmatrix}$$

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Now the basic, this is a basic equation of a motion $MU..+CU+KU=$ force so for this basic equation you, the displacement factor is an unknown that is U is unknown, and in order to calculate that displacement we want mass, stiffness and the damping. So all this three term in terms of a matrix. So apart from this three term today we will only discuss focus on the mass matrix.

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Example Problem:

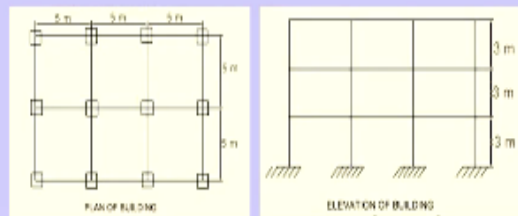
Generate the mass matrix for the given building, the plan and elevation of a building are shown in figure.

Slab thickness = 120 mm

Grade of concrete = M25

Brick Wall thickness (Interior, Exterior and parapet) = 230 mm

Height of parapet wall = 1 m

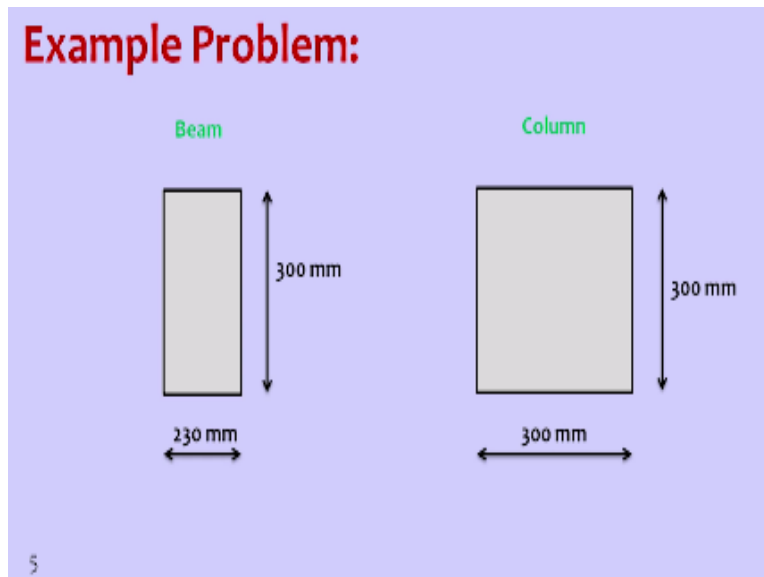


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Now today's problem is to generate the mass matrix for the given building the plan and elevation of building are shown in figure. So here we have building plan which is rectangle in nature having three base in X direction and two base in Y direction, and all the base are of 5m in length uniform in both the direction.

As well as we have 12 number of column which are square in nature and the floor to floor height of a building is 3m. Now apart from this detail we have grade of concrete as M25 brick wall thickness as a 230mm so for any building we may have different types of a wall. So here we have exterior interior and parapet wall, but there is no change all three type of wall have the uniform thickness of 230mm, and the height of parapet wall is 1m.

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So the beam cross section given for this problem is 230mm as a width, 300mm as a depth and the column size is square which is 300mm/300mm.

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Example Problem:

$$M = \begin{bmatrix} m_1 & 0 & 0 \\ 0 & m_2 & 0 \\ 0 & 0 & m_3 \end{bmatrix}$$
$$M = \begin{bmatrix} m_1 & 0 & 0 & \dots & 0 \\ 0 & m_2 & 0 & \dots & 0 \\ 0 & 0 & m_3 & \dots & 0 \\ \dots & \dots & \dots & \ddots & \dots \\ 0 & 0 & 0 & 0 & m_n \end{bmatrix} \quad n \times n$$

So our today's objective is to create, to generate the mass matrix where we will calculate this m_1 , m_2 and m_3 . And suppose if we have more number of degrees of freedom this is a universal matrix for n degrees of freedom where we have M_{ij} so m_{i+1} , m_{i+2} and so on all the diagonal element up to M_{nn} in the size of element or the matrix will be $N \times N$.

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Example Problem:

IS 1893 (Part 1) : 2002

7.3 Design Imposed Loads for Earthquakes Force Calculation

7.3.1 For various loading classes as specified in IS 875(Part 2), the earthquake force shall be calculated for the full dead load plus the percentage of imposed load as given in Table 8.

Imposed Uniformity Distributed Floor Loads (kN/ m ²)	Percentage of Imposed Load
Upto and including 3.0	25
Above 3.0	50

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Now in order to consider the live load or an imposed load for calculation of a suspect mass IS 1893 2002 have some guideline. So if i look at to the clause number 7.3.1 for various loading cases as specified in IS 875 part 2. The earthquake force shall be calculated for the full dead load plus the percentage of imposed load as given in table 8.

Now table 8 what it summarizes if imposed load is uniform in nature and the intensity is 3kN/m^2 less than or equal to 3kN/m^2 we have to consider that 25% of imposed load. Now the same impulse not having intensity more than 3kN/m^2 we have to consider the 50% of impose load for seismic mass calculation.

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Example Problem:

IS 1893 (Part 1) : 2002

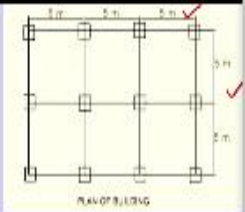
7.3 Design Imposed Loads for Earthquakes Force Calculation

7.3.2 For calculating the design seismic forces of the structure, the imposed load on roof need not be considered.

The second Clause on the same imposed load suggest us to avoid the imposed load on the roof top for the seismic mass calculation.

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Solution: Slab

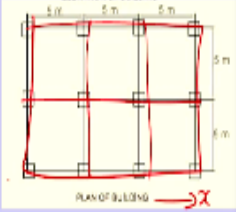
$$\begin{aligned} \text{Slab} &= V \times \rho \\ &= (15 \times 10 \times 0.12) \times \frac{25 \times 10^3}{9.81} \\ &= \underline{45,871.56 \text{ kg}} \end{aligned}$$


Now let us discuss how to calculate the mass of a building. Now in order to calculate the mass of an entire building we will divide it into small, small component. Now if we divide the building in different, different component we will get it as a four to five main element. Let us recall the in list all the element first one is a slab, second can be beam, third can be column, and fourth can be in film wall, though we have fifth one also as a footing but since putting will be under the ground and it will not shake, so it will not comes under the seismic mass.

So we will focus on four element now let us calculate the mass of a slab. Now here we can see that mass of slab is nothing but for calculating any mass we need to multiply the volume into density. Here we have 15m in x-direction, 10 meter in y-direction so we will write it down $15 \times 10 \times$ thickness of slab so this is the volume, density is nothing but it 25kN/m^3 this is a density of a concrete $\times 10^3/9.81$. This will give us the mass of a slab as 45,000 870 1.56 kg.

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Solution: Beam

$$\begin{aligned} \text{Beam} &= 0.23 \times 0.3 \times (15 \times 3 + 10 \times 4) \\ &\quad \times 25 \times 10^3 \\ &\quad \underline{\quad 9.81} \\ &= \underline{14,946.48 \text{ kg}} \end{aligned}$$


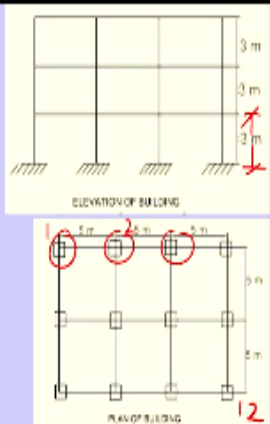
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Now let us move to the beam now beam is the element which supports the slab and generally we provide beam just below the walls. Now since we provide beam just below the walls it will be run throughout the floor. So here if we look at the diagram we have beam at the periphery as well as inside, inside the building. The way we have calculated the mass of a slab similarly mass of beam will be nothing but the, first of all we have to calculate the volume.

So for one beam it is 0.23 is a cross-section 0.3 is a depth multiplied by length. Now here length is let us consider entire length, so first of all we will, this is the 15 meter is a length in X direction, so $15 \times 3 + 10 \times 4$ is a length. So this will give us this product, will give us the volume multiplied by $25 \times 10^3 / 9.81$. So this will give us the 14,946.48kg as a mass of a beam.

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Solution: Column

$$\text{Column} = 0.3 \times 0.3 \times 3 \times \frac{25 \times 10^3}{9.81}$$
$$= 8,256.88 \text{ kg}$$


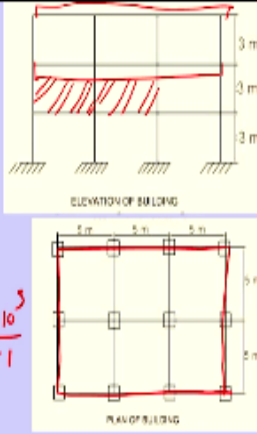
The diagrams show a three-story building. The 'ELEVATION OF BUILDING' shows three floors, each 3m high, with columns at the corners. The 'PLAN OF BUILDING' shows a 3x3 grid of columns with 3m spacing between them. The columns are numbered 1 to 12. A red square indicates a 0.3m x 0.3m cross-section for one column.

Now let us move to the column, now column is the element structural element which support the beam. So it will generally at the end of a or at the intersection of a various beam. So here if you look at the diagram we have square column at all the junction so if we count it 12 up to the 12 and the cross-section of column is 0.3/0.3 also the height of floor to floor height of a column is 3m.

So let us calculate the mass of a column which is nothing but the 0.3/0.3 is a cross section x height x density/9.81 will give me the total mass as 8,256.88 kg. So this is a column.

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Solution: Wall

$$W_{\text{all}} = 0.23 \times (3 - 0.3) \times (15 \times 3 + 10 \times 4) \times \frac{20 \times 10^3}{9.81}$$
$$= 1,07,614.68 \text{ kg}$$
$$P. W_{\text{all}} = 0.23 \times 1 \times (2(15 + 10)) \times \frac{20 \times 10^3}{9.81}$$
$$= 23,445.46 \text{ kg}$$


The diagrams show the building's elevation and plan. The elevation view shows a three-story building with a total height of 9m (3m per floor). A parapet wall is shown on top of the third floor, extending 0.3m above the floor level. The plan view shows a rectangular building with a total width of 15m (5m + 5m + 5m) and a total depth of 10m (5m + 5m). The parapet wall is highlighted in red in both views.

Now similarly for wall, so wall here we have two type of wall interior, exterior as a parapet. Now interior, exterior can be clubbed as a one because they have same thickness, but height of exterior, interior wall and the height of parapet wall are is different, so we will consider this wall, we will calculate mass of this two wall separately.

Let us consider calculate the mass of all exterior and interior wall. So the total mass of wall is nothing but the 0.23 is a thickness, now 3m is a height but apart from 3m height suppose we have this is a beam of 0.3m so wall will be only up to 2.7m height for all infill panel. So hence we will deduct the beam depth from the total height, and then the way we have calculated at the length of a beam same will be the length of a wall.

So $15 \times 3 + 10 \times 4$ so one is for X-direction another one is for Y-direction multiplied by, now here the density of a wall will be 20 kN/m^3 so $10^3/9.81$ will give us the total mass is a 1,07,614.68kg. So this is a mass of a wall this walls are interior and exterior. Now if we calculate the parapet wall mass it is nothing but the 0.3 thickness.

Now here on rooftop we do not have anything so it will be height of wall will be 1m into length, so as we know parapet wall is provided for protection which is, which will be at the periphery. So the length will be perimeter 15+10 this is the length and again density $20 \times 10^3 / 9.81 = 23,445.46$ kg. So this is the total weight of a parapet wall.

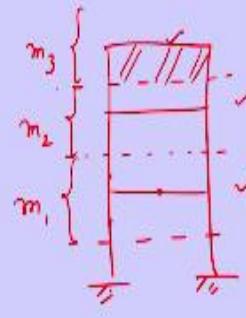
We want to get the final calculation of m_1 , m_2 and m_3 . Now if we think, if we assume it very carefully we can see that typical floor have the same mass and the top floor it will be having either more or less mass because if we consider half of the mass from top and from bottom if you will consider half mass, so for top floor we only have parapet wall up to 1m on the above floor whereas on the space between the third floor and second floor we have the half column.

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Solution: Total Mass

Typical floor
 $(m_1, m_2) = \text{Slab} \left(\frac{1}{2} l_b + \frac{1}{2} l_w \right) \times \text{Column} + \text{Wall} + \text{Beam}$
 $= 176.69 \times 10^3 \text{ kg}$

$m_3 = \text{Slab} + \frac{1}{2} \times \text{column} + \frac{1}{2} \times \text{Wall} + \text{beam} + \text{Parapet wall}$
 $= 136.215 \times 10^3 \text{ kg}$



So let us see how to calculate the mass of typical floor, so this is it for typical floor we have entire mass of slab, suppose this is our building under consideration we are talking about this one. Now we have to consider the only half so portion between this one will act as m_1 this will as m_2 , and above as a m_3 .

So here we are talking about m1 and m2 so total mass of a slab plus half of the column below plus half of the column above. So slab plus half for below floor, half floor above floor this is per column it will be plus, plus entire weight of a wall plus weight of a beam. So it will give us the mass as 176.69×10^3 kg. Now if we consider the calculate the mass of m3 it will have weight or the mass of slab plus half mass of column plus half mass of wall which is in this area plus mass of a beam and mass of parapet wall.

So sum of all this will give me the total mass on the rooftop which is nothing but the 136.215×10^3 kg. So this is how we have calculated the mass of first floor, second floor and the third floor.

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Solution: Final Matrix

$$M = \begin{bmatrix} 176.69 & 0 & 0 \\ 0 & 176.69 & 0 \\ 0 & 0 & 136.22 \end{bmatrix} \times 10^3 \text{ kg}$$

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And when we put this into matrix it will look like m1, m2 and m3. So we are just taking out 10^3 outside and all the unit of this one is in kg. First of all we have calculated the mass of a slab now this mass of a slab can be of dead load in nature or live load in nature. So for dead load the typical process is nothing but to calculate the volume of a slab, and then multiply by density, then for impose load IS1893 tell us that if the intensity of live load or the impose load is up to 3 kN/m^2 we have to consider the 25% of mass.

And in case of it is more than 3 we have to consider only 50% and we have to neglect the impose load on the rooftop now once the calculation of mass of a slab is finished we can go with the beam after that column, and then at the end for wall. Now wall can be of parapet interior, exterior if all of them have same thickness we can go calculate the mass in the single shot, but parapet we have to calculate it separately because it has a height of 1m.

And in order to calculate the mass of interior and exterior wall we have to divide them to from total height of a wall we have to divide the depth of a beam. So once we do that we will get the actual mass of a wall between the two floors and then for corresponding m_1 m_2 which is nothing but the typical flow we have to consider the half of the wall load and column load for above and below floor and on the roof top floor we have to consider half of the column and the wall load for below floor and for top floor we have to consider the parapet wall, thank you.

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