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Structural Dynamics
Week 7: Module 04

Approximate Methods For Finding Natural Frequency

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Welcome to structural dynamics class. In this class we will study approximate methods for finding first natural frequency of multi-degree of freedom structure. So till now we have studied multi-degree of freedom structure how to find out eigenvalues and eigenvectors. So that is natural frequencies and mode shapes of the building we have found out.

Now usually first mode carries or first natural frequency carries lot of information like maximum mass participation will be usual in the first mode. So if we know the first mode without going to the TDS procedure of calculating matrices so we can find out the maximum forces and maximum displacements which are occurring on the building or due to the action of external forces.

So we will discuss two techniques in this class, two techniques of finding approximate natural frequency of the multi-degree of freedom system.

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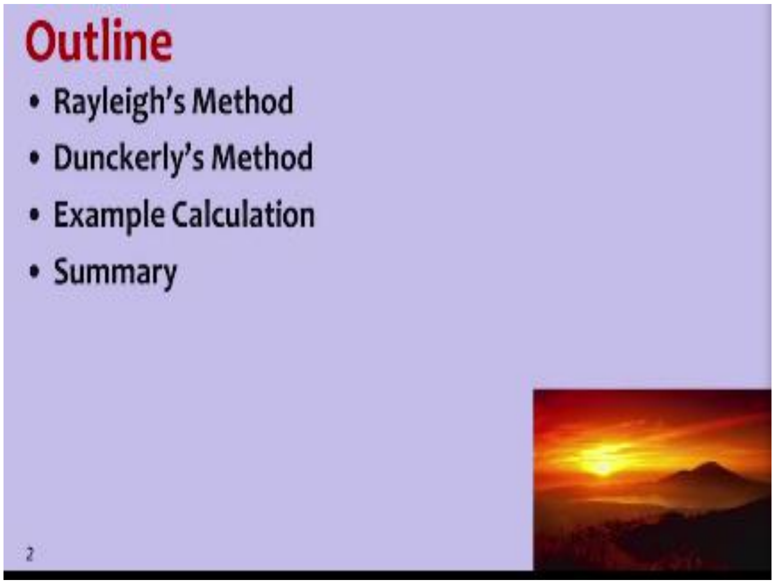

So we will discuss Rayleigh's method and we will discuss Dunckerly's method and we will solve one example problem in each method and find out how approximate these methods are compared to the natural frequency of the structure. First let us discuss about Rayleigh's method.

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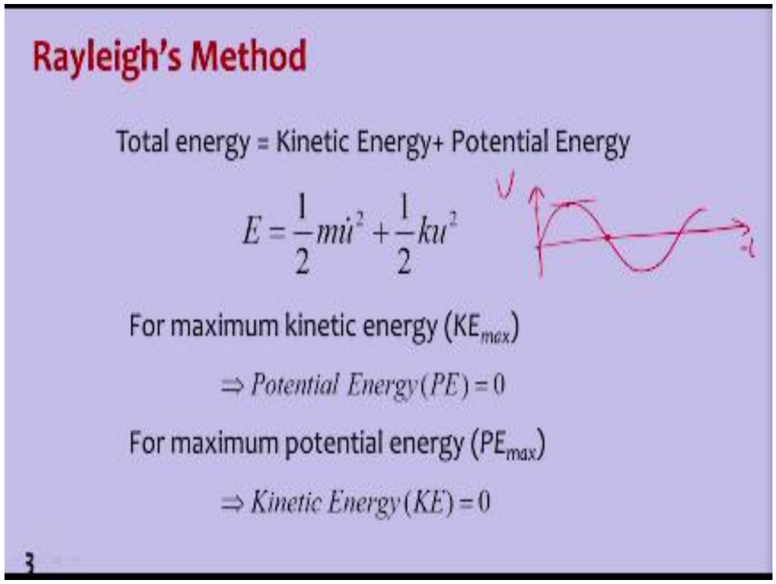

So in Rayleigh's method, so this is dependent on the energy. So if we look at the total energy we already know that total energy is the combination of kinetic energy of the system of vibrating system plus potential energy of the system. So potential energy and kinetic energy together is total energy. So if we write the equation for that it will look like this, so as you can see in this equation $E=1/2\ mu.^2$ and ½ ku^2, where m is the mass of the structure or mass of the system and u. is velocity and u is the displacement.

So ½ $mu.^2$ ½ ku^2 this is the total energy of the system. Now if say kinetic energy is maximum, so that is first component KE is maximum so potential energy will be 0, why, because when displacement is maximum, velocity is 0, and when velocity is 0, displacement is maximum it is something like this. So here the displacement response U versus time, so at this location displacement is maximum value and velocity here is 0.

So tangent of this curve is velocity, so velocity is horizontal here so that is 0, 0 slope here velocity is high, very high, high value where displacement is 0. So that means what for maximum kinetic energy that is KE_{max}, potential energy will be 0. And similarly, for maximum potential energy kinetic energy will be 0.

So at this location potential energy will be 0, this location is of maximum displacement, so kinetic energy will be 0 and this location is of 0 displacement, potential energy will be 0. So kinetic energy will be 0, potential energy will be 0. So kinetic energy is maximum, potential energy is maximum at these locations.

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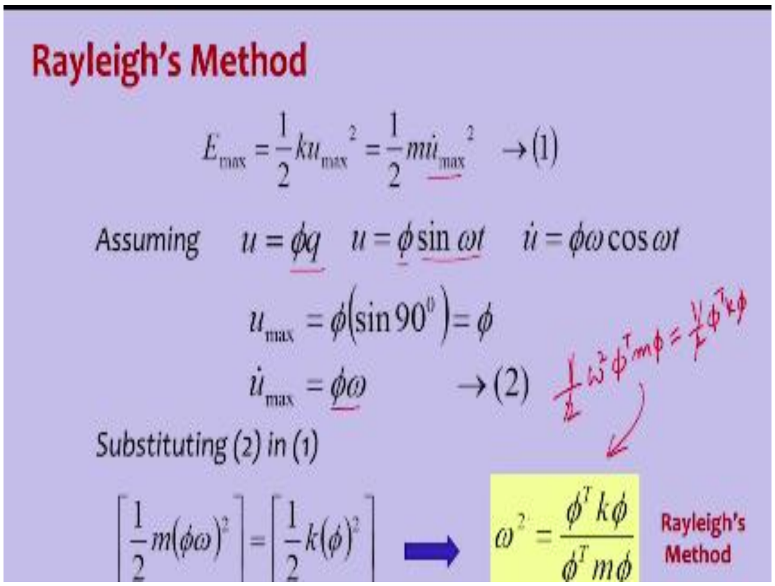

Now from this one using this principle we can write that, so ½ ku_{max}^2 that is ½ k displacement maximum square that is maximum potential energy that will be equal to maximum kinetic energy. So that means what total energy is equal to kinetic plus potential, but maximum kinetic energy is equal to total energy, maximum potential energy is equal to total energy that means two maximum values must be same.

Then here we are making an assumption of displacement functions. So since we are discussing about multi degree of freedom system so displacement is not of a point mass but a series of point masses like that, so we define u = Ø and Q so Ø is a spatial quantity which is described at a discrete locations of where center of mass is there.

And then Q is a time variation, so Q(t) and Ø(x) so that is u so displacement u so we can write u as Ø Sin ω t as a displacement function so u. that is velocity response is Ø ω Cos ω t so if we substitute these things and look at the maximum values so displacement maximum value will be Ø x sin ω t maximum value is maximum minimum value will be in the range of -1 to +1.

So if you take -1 or +1 the maximum value will be u max will be equal to Ø itself and then if it takes the u, max again this Cos value maximum value is again 1 so u. max = Ø ω so we have maximum displacement value and maximum velocity value so if we take this two things and substitute it back in the equation 1 so what we get is 1/2m u max so that is Ø ω^2 that is equal to 1/2k $Ø^2$.

So since this Ø is a vector what we get is so this k x $Ø^2$ will become $Ø^{-1}$ k Ø and here ω comes out $Ø^{-1}$ m Ø it will become so with this we can write ω^2 this is ω^2 is coming from say ½, ½ gets cancelled m/k we can out here so this I'm rewriting this equation here ½ ω^2 $Ø^{-1}$ m Ø left hand side, right hand side is equal to ½ $Ø^{-1}$ k Ø so ½, ½ gets cancelled now from this one we can get so $\omega^2 = Ø^{-1}$ k Ø/ $Ø^{-1}$ m Ø.

So this is called Rayleigh quotient or natural frequency first fundamental natural frequency using Rayleigh’s method, now why are calling first fundamental natural frequency because we use here mode shape 1 first mode shape, okay. So then we can call it as first fundamental natural frequency instead of Ø we use a second mode shape so then we get second fundamental natural frequency.

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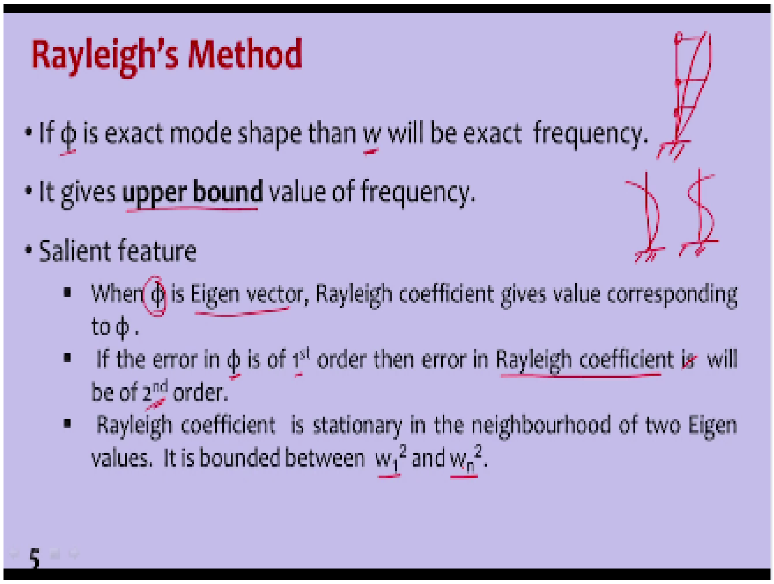

So let us discuss what are the properties involved in that, so if Ø is the exact mode shape then ω will be exact frequency if Ø is a exact mode shape then ω will be exact frequency, so here in this case Ø so let us take 3 degrees of freedom system so 1, 2, 3 we can assume any function so we can assume state line function, we can assume parabolic function we can assume say sin function.

Any function we can assume to get the first fundamental natural frequency this should be approximately near to the first mode, so if we approximately if we make it approximately neither the first mode then we get first natural frequency so if Ø is the exact mode shape then ω is exact frequency, so in this we are not evaluating mode shape we are assuming some mode shape, and then with that we are getting like approximate a fundamental natural frequency.

So any mode shape whatever we assume that cannot give the exact so the lower value of energy, so that means what any motion if we are assuming that will be upper bound value of frequency so that means what whatever is the original frequency we will get slightly higher frequency in the Rayleigh's method. So the salient features of this are when ϕ is Eigen vector Rayleigh coefficient gives value corresponding to that ϕ, so we can assume this as a first mode or we can assume that as second mode something like this or we can assume that as a third mode.

So first mode, second mode, third mode any motion if we are assuming as ϕ we will get corresponding natural frequency, and one important is if the error in estimating ϕ^r error in like assuming ϕ is a first order then the error in Rayleigh's coefficient will be of the second order, so that means what so there is an advantage even though if error is slightly higher but actual error in estimating the Eigen that ω that is frequency is of the lower order.

So Rayleigh's coefficient is stationary in the neighborhood of two Eigen values, so it is bounded between ω_1^2 and ω_n^2 so whatever motion if we are assuming so we will get in between that.

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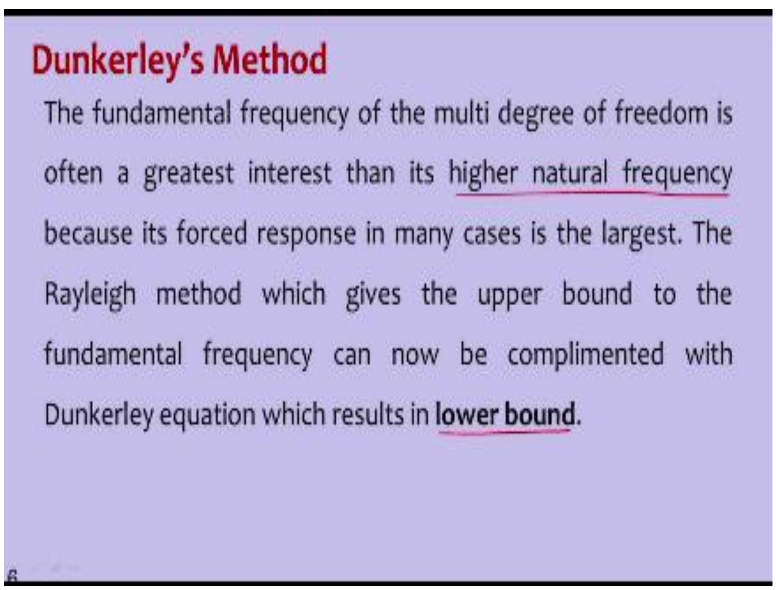

So the next method which is Dunkerley's method so this Dunkerley's method is a lower bound method so Rayleigh's method is the upper bound method. So let us discuss about second method that is Dunkerley's method for finding approximate natural frequency, so in this the fundamental frequency of multi degree of freedom system is often of greater interest than its higher natural frequency, because it is forced response in many cases is the largest.

So the Rayleigh's method which gives upper bound to fundamental frequency it can now be complimented with Dunkerley's equation which is lower bound method. So what is the concept of this Dunkerley's method.

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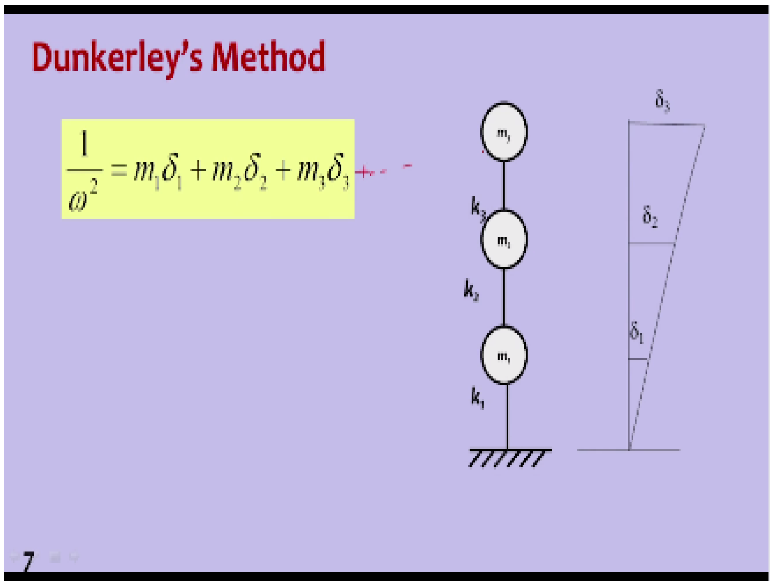

So it works with the flexibility coefficients so one simple equation is there in Dunkerley's method that is $1/\omega^2=m_1\delta_1+m_2\delta_2+m_3\delta_3+$ so on like that it depends on number of masses. So if you take three degree of freedom system or three storey building and we lump the masses at this center of mass locations or its lab locations and take stiffness 1, stiffness 2, stiffness 3 has column stiffness and then estimate this δ_1, δ_2, δ_3 so this δ_1, δ_2, δ_3 are flexibility coefficients.

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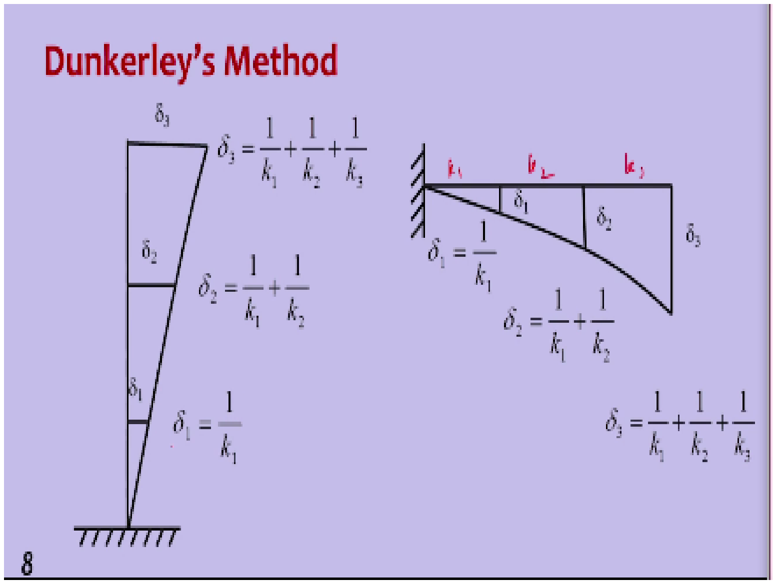

So now how do we estimate this δ_1, δ_2, δ_3 flexibility coefficients as you can see in this one so this is a structure we need to find out δ_1, δ_2, δ_3 so δ1 is like this there is k_1 here, k_2 here, k_3 here it is written here so δ_1 is $1/k_1$, δ_2 is $1/k_1+1/k_2$, δ_3 is $1/k_1+1/k_2+1/k_3$ it is something like this spring's are attached in series and where you are working so below that masses are 0 so in this getting λ3 flexibility co efficient so m2 and m3 are assigned 0 and then we have three springs attached ion series so what we get is the from this one this is the equivalent stiffness so 1/ k1 ++ 1/k2 + 1/k3 is a equivalent difference what we are getting.

And for λ2 m3 is 0 m1 is 0 so m2 is present so when m2 is present when m3 is 0 so anyway the upper will be eliminated so the equivalent stiffness of k1 and k2 will be 1/k1 + 1/ k2 so we get λ2, this is the flexibility coefficient, and for finding λ1 we have m20 and m30 and we have only m1 so since only m1 and k1 is there so we get 1/k1 as a flexibility coefficient.

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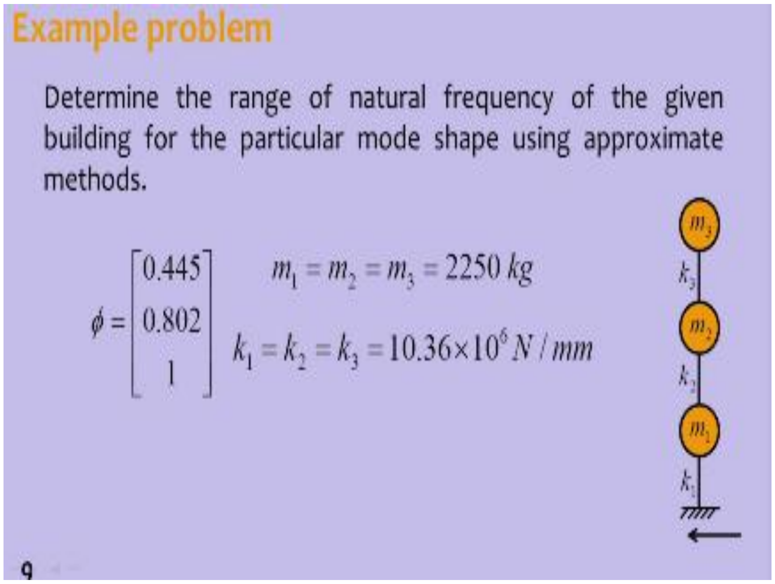

Now let us solve one problem and understand Rallies co efficient Rallies method and Dunkerley's method so in this one determine the range of natural frequency of the given building from the particular mode shape using approximate method, so building is given so masses are m1 m2 and m3 so all three masses are say 2250 kg which is the same example which we are carrying from the previous class so k1 k2 k3 are 10.36 x 10^6 n/mm.

So now we are assuming this as a mode shape as this is a assumption so one pointed and 0.445 this is our assume mode shape.

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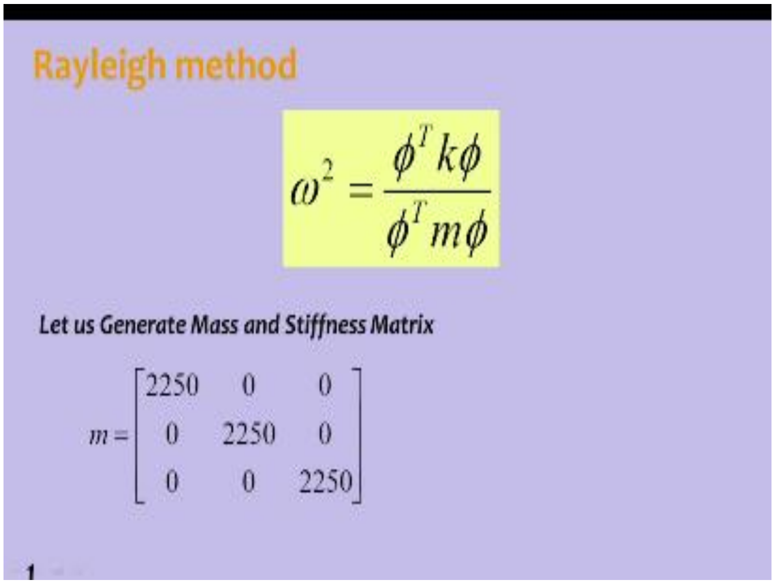

Now in Rallies method we know rallies coefficients that is ω 2 = φ^t k φ / φ^t m φ so upon substitute these values and mass matrix and stiffness matrix.

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$$\phi^T k\phi = \begin{bmatrix} 0.445 & 0.802 & 1 \end{bmatrix} \times \begin{bmatrix} 20.72 & -10.36 & 0 \\ -10.36 & 20.72 & -10.36 \\ 0 & -10.36 & 10.36 \end{bmatrix} \times 10^6 \times \begin{bmatrix} 0.445 \\ 0.802 \\ 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0.445 & 0.802 & 1 \end{bmatrix} \times \begin{bmatrix} 911680 \\ 1647240 \\ 2051280 \end{bmatrix}$$

$$\phi^T k\phi = 3778064.08$$

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If we substitute these values so what we get is so φ t m φ we need to work it out so φ^t is a vector so transpose of that and this is the column vector this is the ρ vector this is as a 3/3 matrix, so we got $\varphi^t k \varphi$ value as 3778064.08 that is the total value of that.

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$$\phi^T m\phi = \begin{bmatrix}0.445 & 0.802 & 1\end{bmatrix} \times \begin{bmatrix}2250 & 0 & 0 \\ 0 & 2250 & 0 \\ 0 & 0 & 2250\end{bmatrix} \times \begin{bmatrix}0.445 \\ 0.802 \\ 1\end{bmatrix}$$

$$= \begin{bmatrix}0.445 & 0.802 & 1\end{bmatrix} \times \begin{bmatrix}1001.25 \\ 1804.5 \\ 2250\end{bmatrix}$$

$$\phi^T m\phi = 4142.77$$

$$\omega^2 = \frac{\phi^T k\phi}{\phi^T m\phi} = \frac{3778064.08}{4142.77} = 911.97$$

$$\omega = 30.2\ rad/\sec$$

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And then similarly we evaluate φ t m φ value and then we get this 4142.77 so if we calculate ω^2 of φ^t kφ/φ^t mφ the value is 911.97 then if you take √ that is 30.2 rad/second so this is natural frequency of first fundamental natural frequency using rallies method you can observe that this value is almost near to the exact natural frequency from the previous class you can observe because we have assumed the same mode shape.

So in mode shape what is given first fundamental natural frequency we have used that because of that shape we are getting a like almost very near to the natural frequency.

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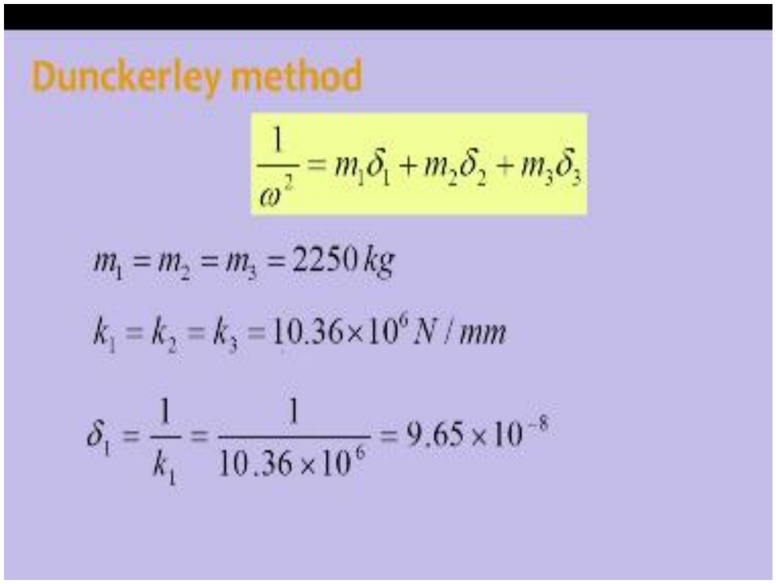

Dunckerley method so in dunckerley method the equation is $1/\Omega^2$ $=M1\delta1 + M2\delta2 + M3\delta3$. M1 M2 M3 are all are same 2250kg and k1=K2=k3=$10.36*10^6$ N/mm. so if we use that fallibility quotations what we get is 1/k1 value $9.68*10^{-8}$

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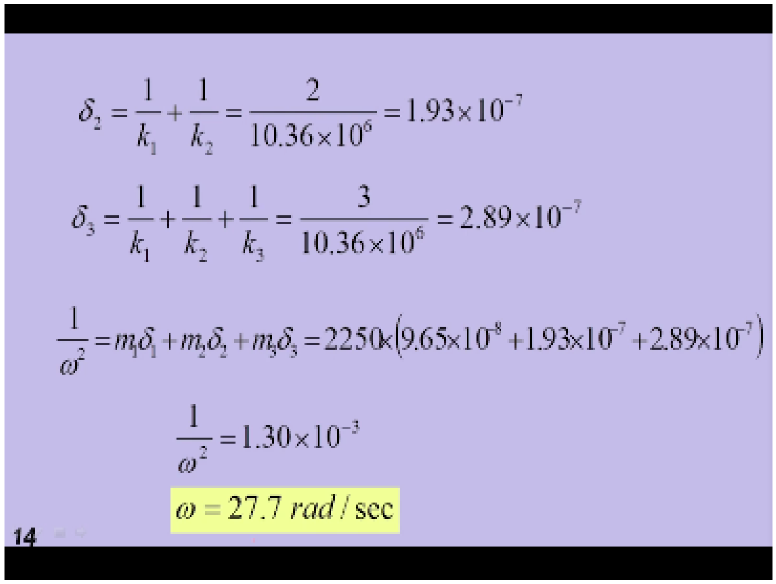

And δ2 value 1/k1+1/k2 value so then we get $1.98*10^{-7}$ and1/k1 δ3 s 1/k2+ 1/k3 so δ3s is $*2.89*10^{-7}$ so if we substitute all these values in the equation so we get 1/1Ω2 as $1.*10^{-3}$ and Ω as 32.7 radius per sec as you can see this is a lower bound value so this value is lesser than the exact natural frequency and rallies method natural frequency will be higher than the exact natural frequency so one can easily find out the exact natural frequency will be in between this rallies coefficient and lower bound method

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So in this class so what we have studied in the summaries that we have studied that how to find out the fundamental frequency which is quite often used for finding the maximum forces and maximum deformations which might occur on the structure might undergo so for that we need first fundamental frequency so we studied how to find out this fundamental natural frequency using two methods.

So one is rallies coefficient which uses the energy in the mode shapes and then Dunckerly's methods which uses the flexibility coefficients we have solved one example problem for understanding the same.

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