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Structural Dynamics
Week 1: Module 05

Methods Solution of Equilibrium Equation

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In this module we will study methods of solution of equilibrium equation so the outline of this module is.

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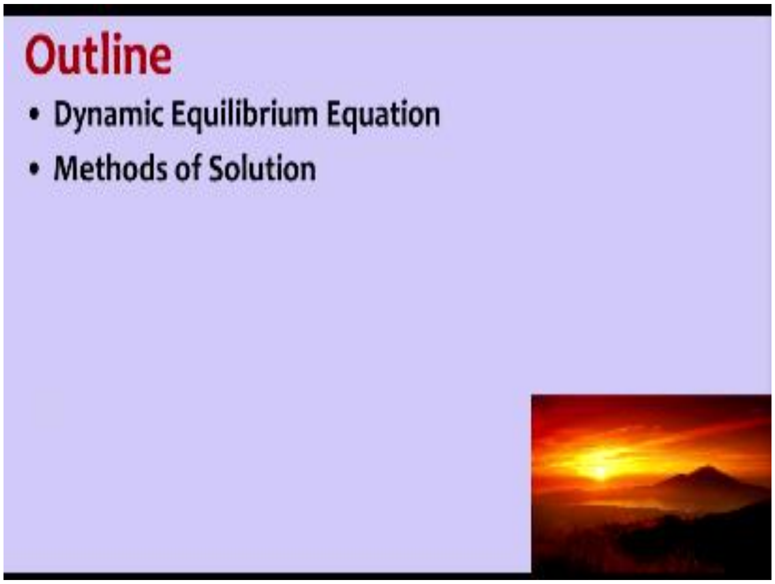

Dynamic equilibrium equation well discusses and then what methods are available for solving this dynamic equilibrium equation.

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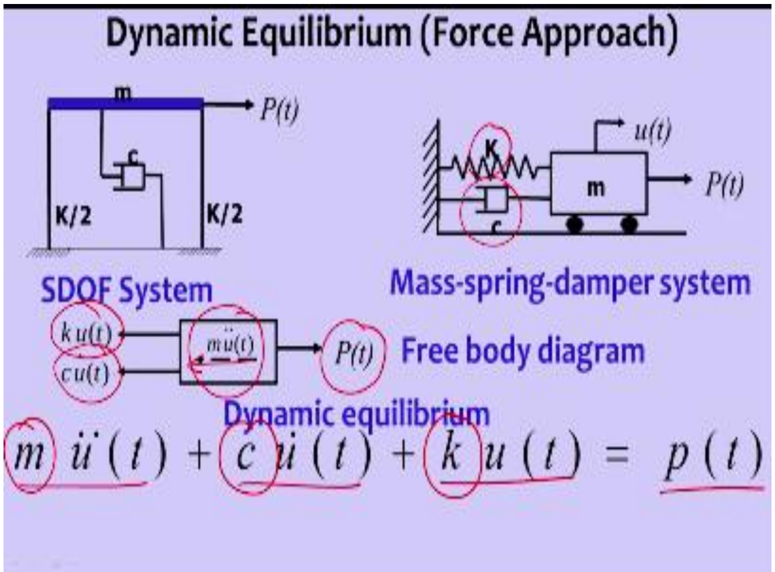

So dynamic equilibrium equation we discussed earlier in the earlier module that is by force approach so this is a single degree of freedom system and then that can be converted into mass spring damper system so where force displacement stiffness and damping all are represented, so stiffness is represented as a spring and which is resisting the motion and damping is represented as a dash pot which is removing the energy in vibration.

So this is stiffness and if you draw the free body diagram of the system of vibrator body then external force speed acting in the right direction and then as we discussed earlier according to develop Alembert principle moving mass generates inertia force opposing motion so that is why mass is moving in the opposite direction but inertia force is generated in the reverse direction so forward direction and inertia force is generated in the reverse direction.

So that how it is represented in the backside when mass is moving in the forward direction the resisting elements of which are columns or vertical resisting elements of the building are trying to pull it back, so that is represented by elastic force so that is equal to stiffness multiplied by displacement and damping force which is removing the energy from the vibrating body that is a damping C multiplied by velocity so if we write the dynamic equilibrium equation these three terms inertia term damping term and stiffness term which is all put together equal to externally applied force so in the structural dynamics and understanding the parameters mass stiffness and mass damping and stiffness is very crucial.

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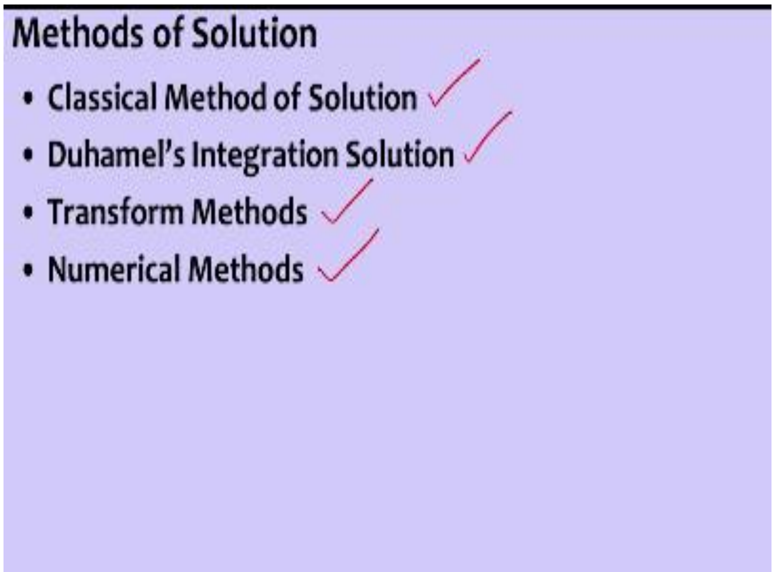

So now how do we solve this dynamic equilibrium equation which we derived just now so there are four methods of solving this dynamic equilibrium equation so the first one is classic method of solving dynamic equilibrium equation the second one is Duhamel's Integration Solution and third one is transform method and fourth one is numerical method so we will discuss in brief each method.

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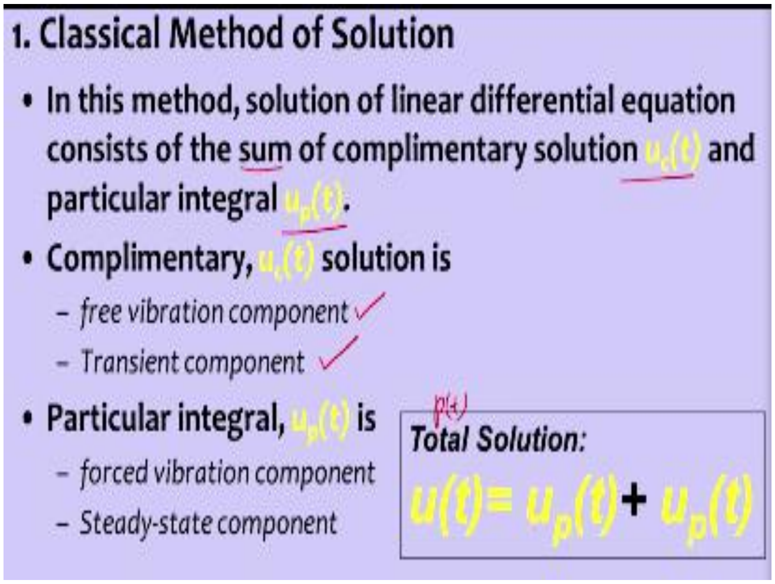

So let us discuss about the classical method of solution so in this classical method of solution so what we are going to do is a differential equation the solution of differential equation will have two components so one is called complementary function that is U complementary which is a function of time and then the second one is U particular which is a also a function of time so the sum of these two that is complementary solution and particular integral will give us the total solution.

So complementary function we can also call it as a free vibration component so this depends only on the structures natural parameters or natural properties of the structure decide this free vibration component this can also be called as transient motion or transient component of vibration and then particularly integral so particularly integral u P which is dependent on applied force so whatever force we are applying so that force is taken into consideration while solving this component.

This is also called steady state component so u complementary u particular u complimentary transient u particular is a steady state component so the total solution looks like this one so complementary solution and particular integral.

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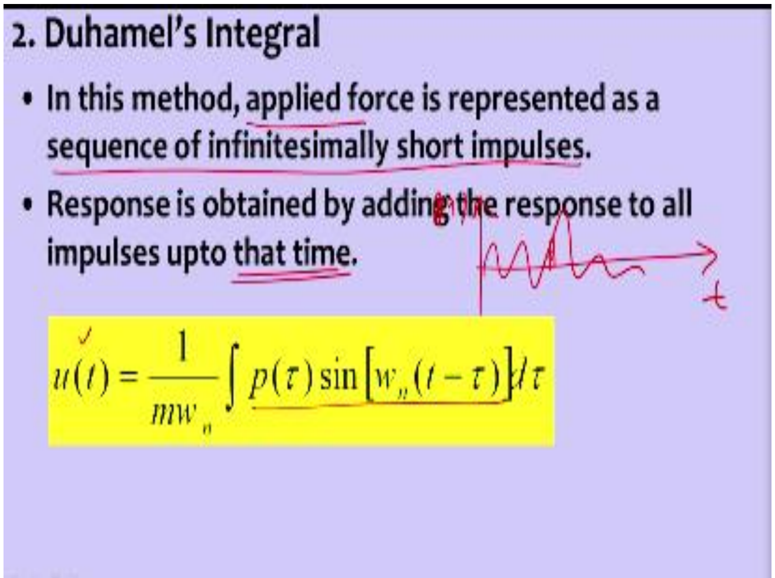

And the second method is called Duhamel integral so in this Duhamel's integral so what we do is whatever externally applied force is there so that is divided into several components so in this method applied force is represented as a sequence of infinite a similar short impulses so for example if applied force is there like this one okay as input motion then this can be divided into small components and then each component is taken separately and solved and later all those things can be added by using super position principle.

So the response is obtained by adding the response to all impulses up to that time so this is in the Duhamel's integral so a typical equation of do a Duhamel's integral is this shown here in this one so this is a solution of one impulse and then we are integrating over many, many impulses so this is a total response so this is a response due to one impulse and that is integrated over all the impulses up to that time so this is a second method of solving dynamic equilibrium equation.

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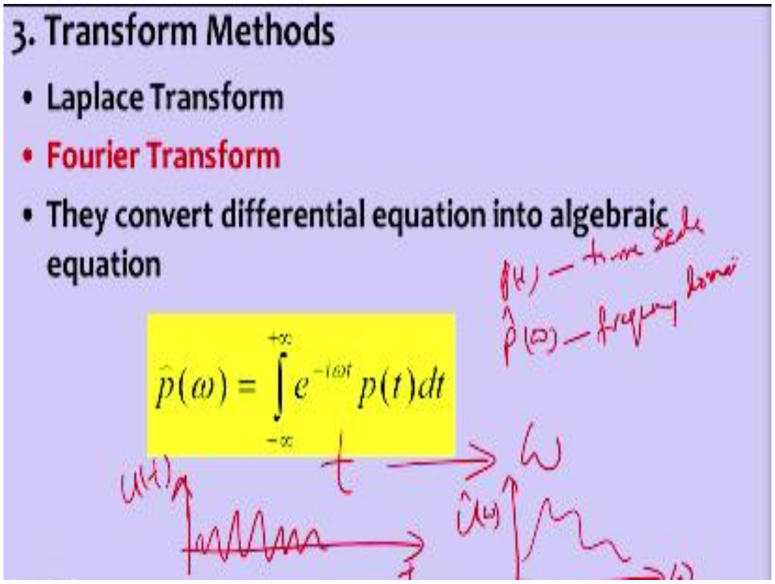

Now there is a third method so the third method is like transforms either Laplace transform or Fourier transform so usually Fourier transforms are used extensively in structural dynamics so what we do in this Fourier transform is convert the equation which is a differential in nature that is converted into algebraic equation so while converting this differential equation into algebraic equation so what happens is the time V equation which is present in the time domain gets converted into frequency domain.

So here like equation is represented in on the time scale amplitudes are represented on the timescale in this one amplitudes are represented on the frequency scale, so Û (ω) so in this one what we get is what is the energy or what is the amplitude of vibration at different frequency levels whereas in the time history analysis which is coming from the differential equation approach or the classical method we get the time history at each time and here we get amplitude of vibration at each frequency and yeah this is how we convert. So this P of T is a force which is the signal in time scale and peak F of Omega is a signal in frequency domain this is in frequency domain.

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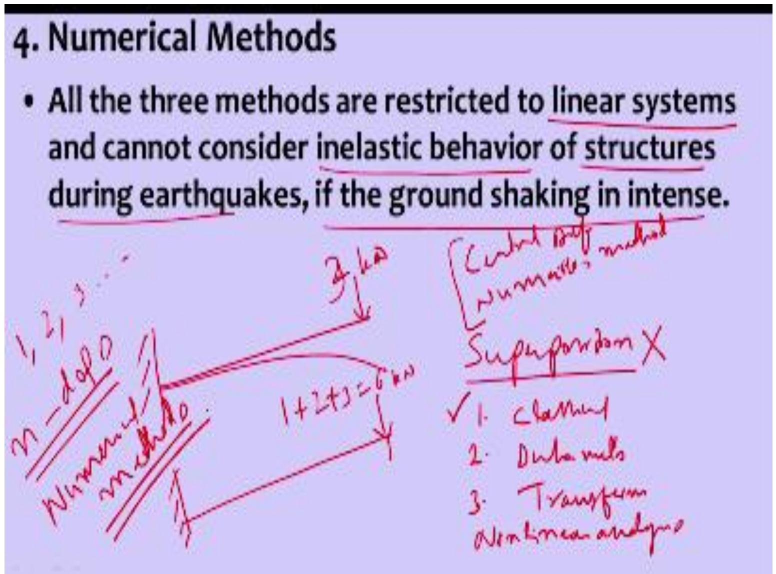

The fourth method is numerical method so this numerical method is mainly used for dealing with the nonlinear systems soother three methods are applicable only for linear systems where a principle of superposition is applicable superposition is applicable so principle of superposition what it says is say for example if I apply a load of 1 kilo Newton and I get some deflection and later I apply a load of 2 kilo Newton and I get some deflection and later I apply load of 3 kilo Newton and I get some deflection super position principle says that all these loads put together 1+2+3.

If I apply all at a time also I can get the same principle so that is a principle of superposition so where principle of superposition is applicable there we can use a classical method second Duhamel integral and third we can use transforms okay, when it comes to nonlinear analysis the superposition principle cannot be applied so we need a more powerful technique for solving dynamic equilibrium equations so that we then we rely on numerical methods.

So all three methods are restricted to linear systems and cannot consider inelastic behavior of the structures during earthquakes so if the ground shaking is intense so we use nonlinear methods so we have several nonlinear methods so for example say we have central difference method or

New mark's new marks method we discuss and the another advantage of this numerical method is all these other method especially classical method.

We can use this classical method when we are idealizing the structure into single degree of freedom system two degree of freedom system three degree of freedom system like that if number of degrees of freedoms are increasing enormously so then it is difficult to handle using methods which are described so then it we need to rely on numerical methods numerical methods so that is the advantage of these numerical methods.

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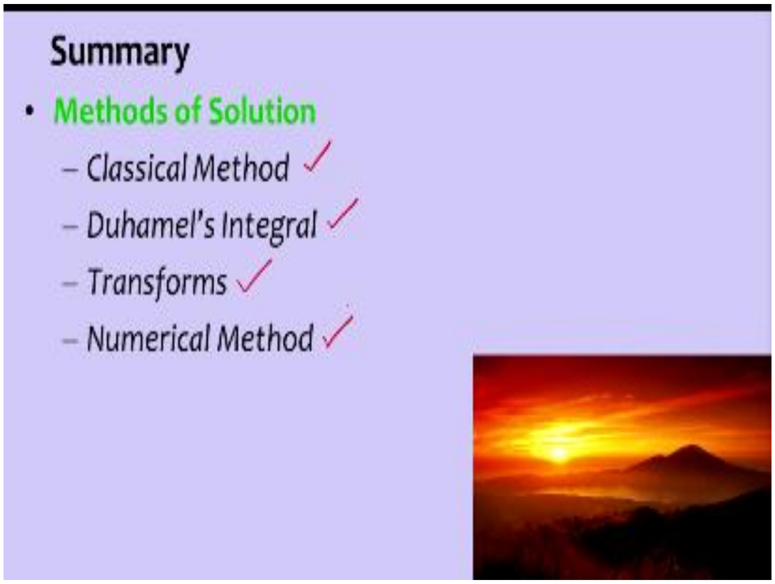

And then in summary what we have discussed in this module that is methods of solution of dynamic equilibrium equation so classical method do emerge integral method transforms and numerical method.

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