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Structural Dynamics
Week 7: Module 01

Multi-Degree of Freedom System

Ramancharla Pradeep Kumar
Earthquake Engineering Research Centre
IIT Hyderabad

Welcome to structural dynamics class. So in this class we study about multi-degree of freedom systems. So multi-degree of freedom system is a close resemblance of the real systems. So if it takes a n story building we can convert that into any degree of freedom system. The three story building can be converted into three degree of freedom system if the vibration is along one direction.

So today we will study about multi-degree of freedom systems and how to formulate the equation of motion for multi-degree of freedom system and how to solve that equation, we will discuss those things today.

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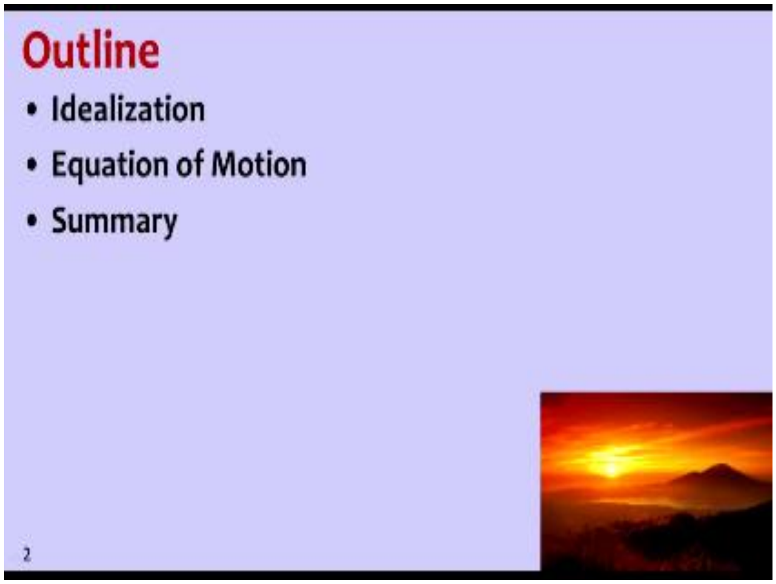

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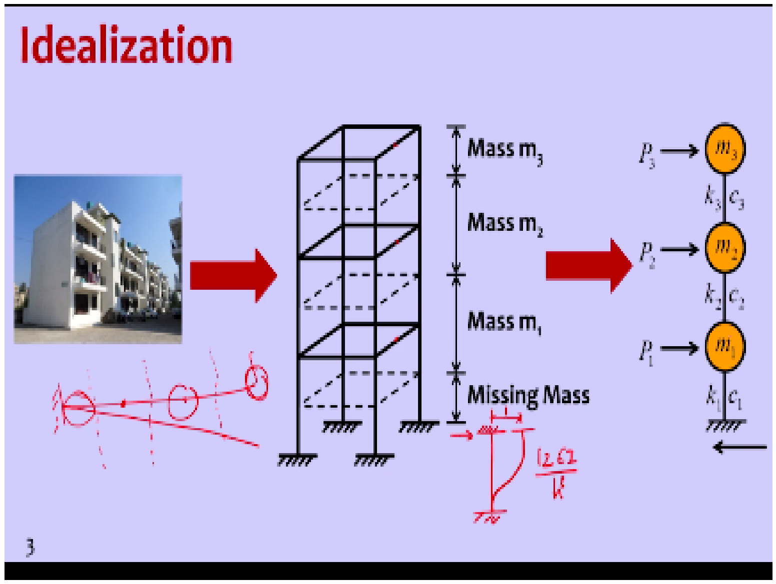

So if we look at this structure or a building you can see this one, this is a three story typical three story building. Now how do we convert this three story building into idealization system. So if we take this three story system so that can be converted into say framing system or supporting system and slabs. So here you can see the dotted line, so dotted line between two floors signifies that the total load or mass in between this can be concentrated at this level proof level at this floor level, this floor level, this floor level.

So mass 1 can be calculated by the summation say mass of this slab. So let us first calculate weight, weight of this slab beams at first floor slab level half the column weight of the first floor and half the column weight of second floor, and plus half the weight of wall load of the weight of wall load all put together will and the total weight we take that is W1 and if you divide by gravity then we get mass 1.

So like that we calculate mass 2, so mass 2 will come from weight of slab 2 plus weight of beams at this floor level, second floor level and then columns weight of columns at third floor half of the weight of columns at third floor, again half of the weight of columns at second floor, half wall load from floor 3 and half wall load from floor 2. So all put together we will get weight at this level, if we divide this by gravity we get mass at that level.

Similarly at a floor 3, so in floor 3 we do not have upper floor so that is why only half of the column of floor 3, half of the wall weight of that floor 3 and slab weight along with beam weight will come, so that divided by gravity we will get mass 3. So m1, m2, m3 are masses at individual floor level, so this is we are discussing about the lateral load not gravity load, lateral load.

So in lateral load if we convert this into say a cantilever kind of thing so this is first floor this is second floor this is third floor point so the mass which is lumped here can come from half of this one the mass which is lumped at this point will be in this zone the mass which is lumped at this floor level will be from this zone.

So this much mass will not be considered so that is called missed mass so because the acceleration levels at that location will be less even when you look at the vibration of that so it will be something like this so in this acceleration levels will be less so that is why inertia force contribution in that zone will be less so now we are with this ideal I say 3 degree of freedom system.

So mass one at floor one mass 2 at floor 2 mass 3 at floor 3, now we need to calculate the stiffness one at this floor level so we already know that in the building with the share behavior so each Column will give a stiffness equivalent to $2Lei/L^3$ something like this okay, so by moving it with one unit so the forced required to move this by 1 unit is 12 Ei/l^3 or h^3 so in this direction we have four columns.

So 4 columns will offer stiffness that will become 4 times Ei/h^3 so this E and II should be for the individual column in that direction, so similarly we get k2 stiffness 2 to stiffness 3 and then C1 is damping at floor level 1 C2 is damping at floor level 2 C3 is damping at floor level 3, so now with this I realized 3 degrees of freedom system how do we convert this into equation of a motion.

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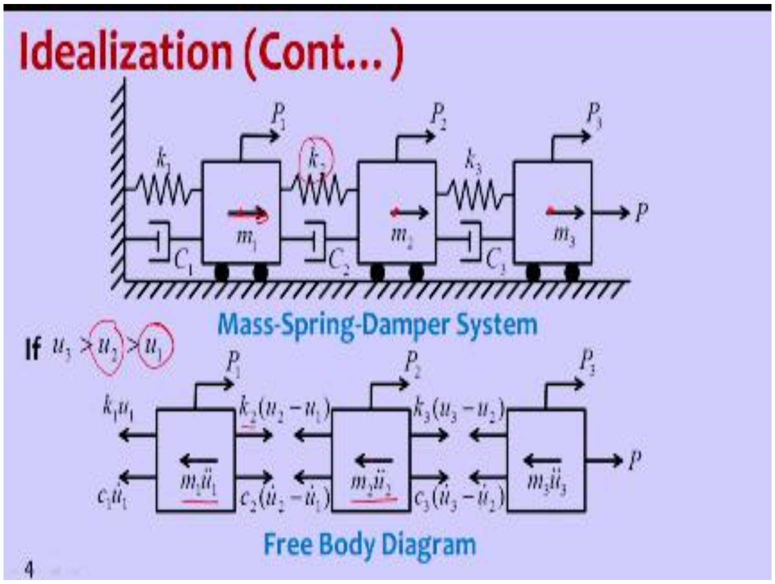

So this is mass spring damper system so the same thing we are representing in mass spring damper system, so mass 1 is represented here damping 1 and stiffness 1, mass 2 is represented here damping 2 and stiffness 2 mass 3 damping 3 stiffness 3 so these are relative properties between 2 floors.

So if the displacement at floor 3 and displacement at floor 2 will is same then there will be no floors in-between so displacement at floor 2 and displacement at floor 1 is same then there will be no force in-between so like that we have represented in this one springs so there is relative displacement between P2 mass 2 and mass 3 then only there will be force in this thing if both the motions are same then there cannot be any force or resistance from the offered from the spring or say the columns from there.

So in order to set this in motion we take this assumption first that is displacement rate point mass 3 displacement at point center of mass at point mass 2 and mass 1. So these are represented by displacement at mass 1, displacement in mass 2 displacement of center of mass 3.

So this u_3 is greater than u_2, u_2 is greater than u_1 so that means this displacement is higher than this so that there is elongation in this spring, and u_2 is greater than u1 so that there is elongation in this second spring so that means the con-floor columns and if u_1 is greater than 0 so that means there is elongation. Now if we take this into consideration and then draw free body diagram for each mass so let us see this mass 1.

So mass 1 is moving forward so with the displacement value u_1, so if this is moving with displacement value u_1 we already know a Duhamel's principle moving mass generates inertia force opposing the motion, so motion is in forward direction inertia force will be in the backward direction. So what is the value inertia force, it is 1 $\ddot{u}_1$, and then because this mass is moved in the forward direction of this displacement then there will be elongation in this spring.

So the elongation in this spring will force or will try to pull the mass back, so this is the pulling force that is k_1u_1 so the pulling force that is the resistance force will be equivalent to stiffness times displacements and correspondingly damping force that is proportional to the velocity so u_1 is displacement $\dot{u}_1$ is velocity and $\ddot{u}_1$ is acceleration, so now velocity is $c_1\dot{u}_1$ so opposing. Now this and then force p1 is acting yet.

Now let us come to relative displacement between mass 2 and mass 1, now here if mass 2 is also moving by the same value as u1 then there would not be any force in this spring. So what we assumed here is u_2 is greater than u_1, so if u_2 is greater than u1 so that means there is elongation in spring 2. So because there is elongation in spring 2 it will try to pull these two masses together again back, so that is how this force is represented k_2xu_2-u_1. So it is pulling these two masses towards each other.

Similarly damping force that is $c_2.\dot{u}_2$-$\dot{u}_1$ so these forces are represented okay, and again here moving mass generates inertia force opposing motion so this is inertia force at second mass level, and then p_2 is external force applied at mass 2 level, and then when it comes to mass 3 again the same thing mass 3 and mass 2.

So k_3 x u_3 – u_2 c_3 x u_3.-u_2. So again this is inertia force so now we want to write this all this forces at one place so this first degree of freedom second degree of freedom third degree of freedom so let us write equation of equilibrium for each degree of freedom separately.

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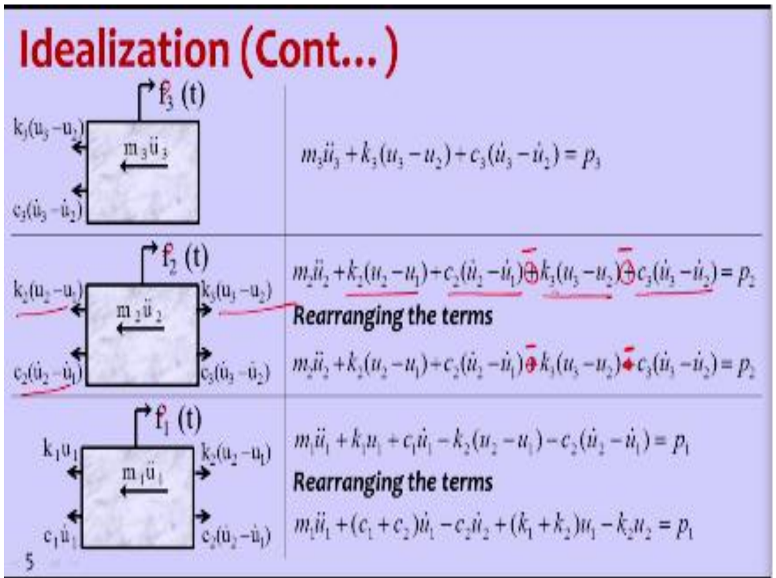

Now mass three so force t3 is acting there so pt let us represent this by p so p_3 t okay m_3u_3.. k_3 u_3- u_2 and c_3u_3.-u_2. So m if I am write equilibrium here m3 u_3 .. + k_3u_3 – u_2 c_3 u_3. –u_2. = force 3 acting at that level, now let us write the same thing for mass two the center of mass two so this is t_2 so m_2 u_2 .. k_2 u_2 – u_1 c_2 u_2. – u_1. So on the other side we have k_3 u_3 – u_2 k_3 u_3 . –u_2 . so if you write all in the form of algebraic sum equivalent to p_2 so that is inertia force so k_2 x this value where putting here and then this value we are putting here this value we are putting here and this value we are putting here.

So here u_3 – u_2 this should be negative sin, and this should be negative sin so it is not positive sin this is negative sin. So by rearranging these terms what we get is m_2u_2 .. k_2 x u_2 – u_1 + u_2 x u_2. – u_1 . And this is negative sin okay similarly we get p1 okay so m1u1.. = k_1 u_1 + c_1u_1. K_2u_{21}- 2 u_1 c_2 x u_2 – u_1. = p_1 so if you rearrange this one we get m1 u_1.. + c_1 + c2 x u_1.- E_2 U_2 . + K_1+K_2 X U_1 – K_2 x U_2.

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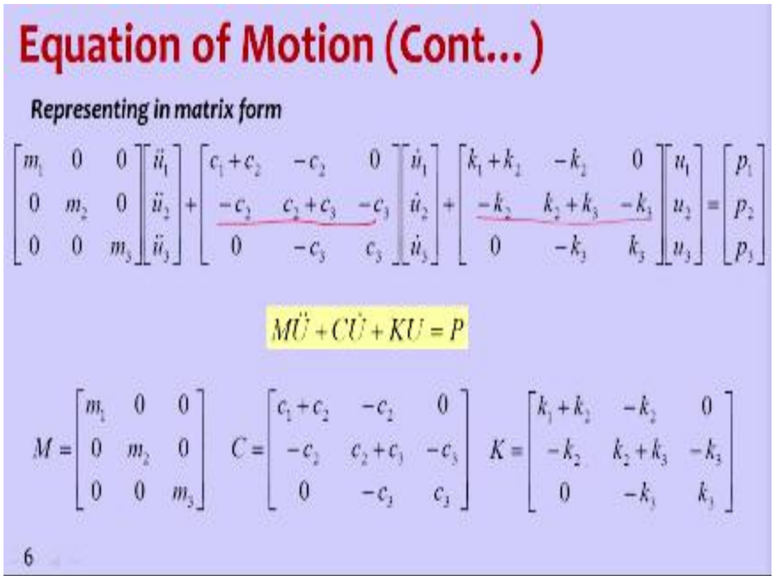

So if we write this things in the matrix form what we get is the matrix m_1 x u_1.. c_1 + c_2 x u_1 . + k_1 + k_2 x u_1 = first equation p_1, m_2 x u_2.. + if we take this one – c_2 x u_1. + c_2 + c_3 x u. so something like this multiply by this vector this row multiplied by this vector that will be we will get the P2 equation. And this P3 equation so this the same thing we are writing in the matrix form so if we rewrite that MÜ+ CU.+KU=P. so this is similar to equation of motion to single degree of freedom system were as MC and K so M was representing a point mass there here it is the matrix.

And C was representing damping here it is representing damping matrix and K of one floor here it is representing stiffness of the structure which is in the form of this matrix and P was a cores acting at the center of the mass here it is acting a vector which is acting at 3centers of mass. Now mass matrix M1 M2 M3 is a diagonal so this clearly shows that the diagonal terms are 0 so it is uncoupled from the point of your mass and coupled from the point of the damping and coupled from the point of stiffness so stiffness matrix sorry damping matrix and stiffness matrix.

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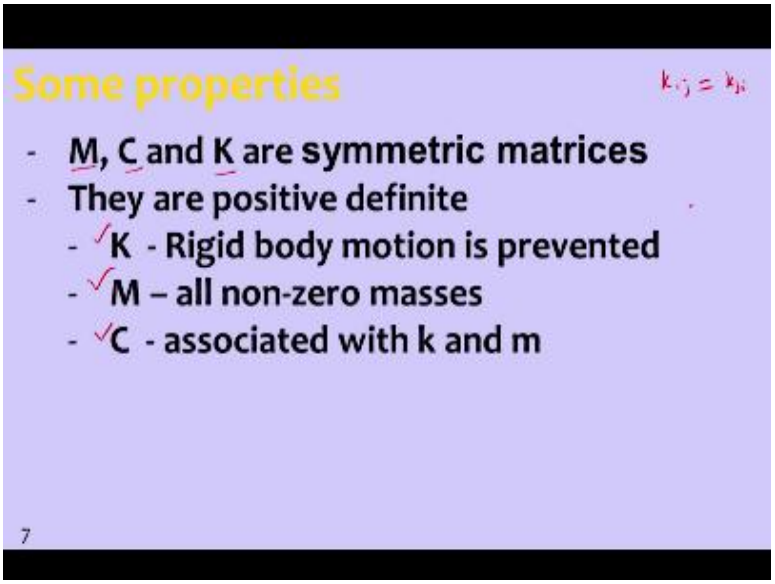

So there some important properties of this matrices so mass matrix M , C matrix and then K matrix are symmetric matrices. So symmetric matrices means what it's something like KIJ=KJi it means I through J column term =J through I column this is a symmetric matrix they are positive definite all these matrices are positive definite so that means the deterge ant is not 0 it is positive value so K is representing a rigid body motion so rigid body motion is prevented that’s why it is positive definite K.

And M is becoming positive definite because there are all 0 terms are there though M1 M2 M3 are some real masses they are not fictions masses there they are real mass and then C as the associated with K and M this is becoming positive definite so these three mass matrix stiffness matrix and damping matrices all three matrices are symmetric and positive definite because we are dealing within the real systems so in summary what we have discussed.

In this class so we have discussed how a complex system can be idealized as multi degree of freedom system and then from multi degree of freedom system how to generate equation of motion by giving the relative displacement. Displacement of say upper floor is slightly higher than the lower floor so that’s how we have drawn on the free body diagram and then took all the associated forces and wrote the equilibrium of the each point mass and t5hen finally convert that into a matrix. So this matrix MÜ+CU.+KU=P is exactly similar to single degree of freedom system.

So single degree of freedom system properties are mass stiffness and damping properties of one floor is here multi degree of freedom system it is a matrix which is represented by representing all the cores then displacement velocity and acceleration are related to one point mass in single degree of freedom system were as in multi degree freedom system it is a vector and force in single degree of freedom system is acting at the center of mass of the point mass and force vector here is or acting at all centers of masses and we will study the solution so this one how to solve a numerically multi degree of freedom system in the next class.

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