NPTEL

NPTEL ONLINE COURSE

Structural Dynamics
Week 6: Module 03

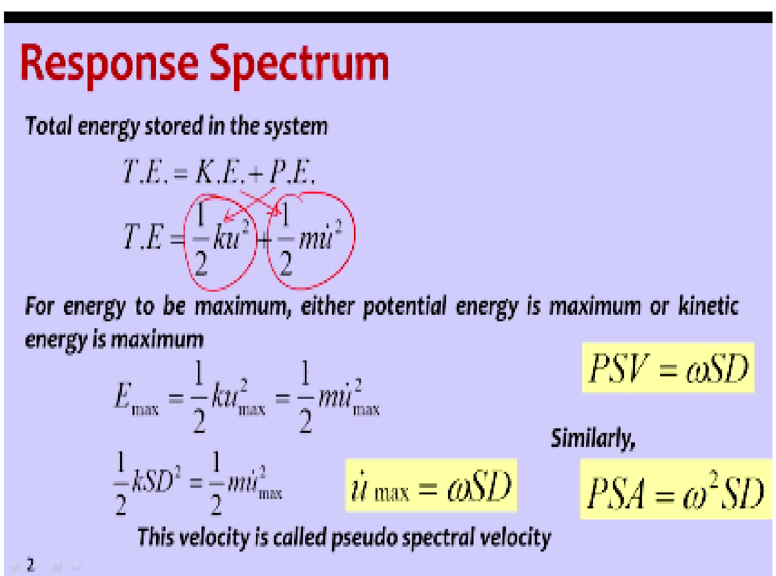
Development of Tripartite Plot

Ramancharla Pradeep Kumar
Earthquake Engineering Research Centre
IIT Hyderabad

Welcome to structural dynamics class. So in this class we will study how to construct tripartite graph. So this tripartite graph is a graph which shows displacement, acceleration and velocity all on the same plot. So it is a spectrum which all three values that is displacement, velocity and acceleration are shown in the same plot, so that is called tripartite plot.

So in the previous class we have discussed about how to construct response spectrum and its applications, its uses, and then what are the special cases of response spectrum we have studied. Now we will discuss about the construction of tripartite plot.

(Refer Slide Time: 00:55)



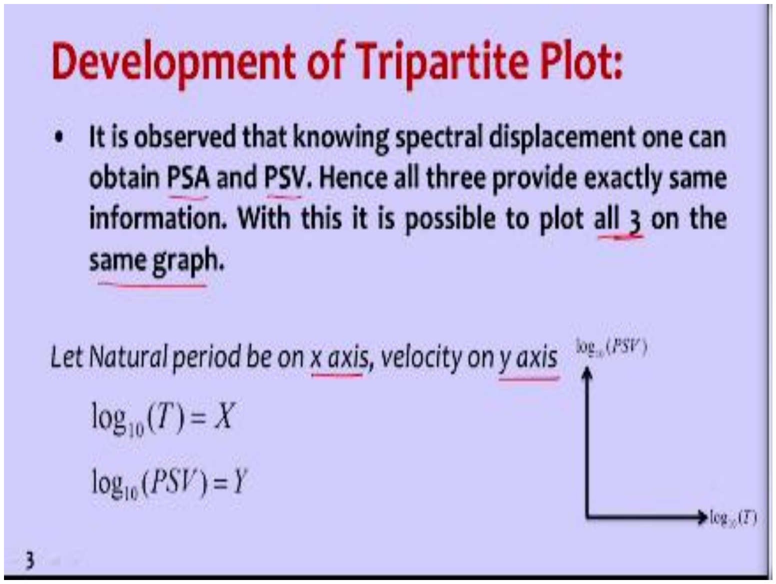
Now if we look at the total stored energy in the system that is equal to kinetic energy plus potential energy. Now kinetic energy is measured in terms of ½ mv^2 that is $U.^2$ and potential energy is ½ ku^2 ½ $mu.^2$ and ½ ku^2 so this is potential energy this is kinetic energy here it is something like this.

And for energy to be maximum either potential energy is maximum or kinetic energy is maximum. So E_{max} that is maximum energy is equal to ½ $kumax^2$ or ½ $mu._{max}{}^2$. So that means what if kinetic energy is maximum potential energy is 0, and if potential energy is maximum kinetic energy is minimum so like that. So that means what maximum kinetic energy is equal to maximum potential energy.

So if we rewrite this above equation ½ ku_{max} is nothing but SD spectral displacement so square of spectral displacement that is equal to ½ $mu._{max}{}^2$. So this ½ $mu._{max}{}^2$ if i look at it SD is here, so if i bring this m term below and strike off this two ½ and ½ so we will get K/M x SD^2. So if you take root under root K/M is ω and SD^2 root is SD. So u.max that is velocity maximum is equal to ω times the spectral displacement.

So this velocity is called pseudo spectral velocity it is like velocity but equal to pseudo spectral velocity. So similarly we can get pseudo spectral acceleration as ω square SD.

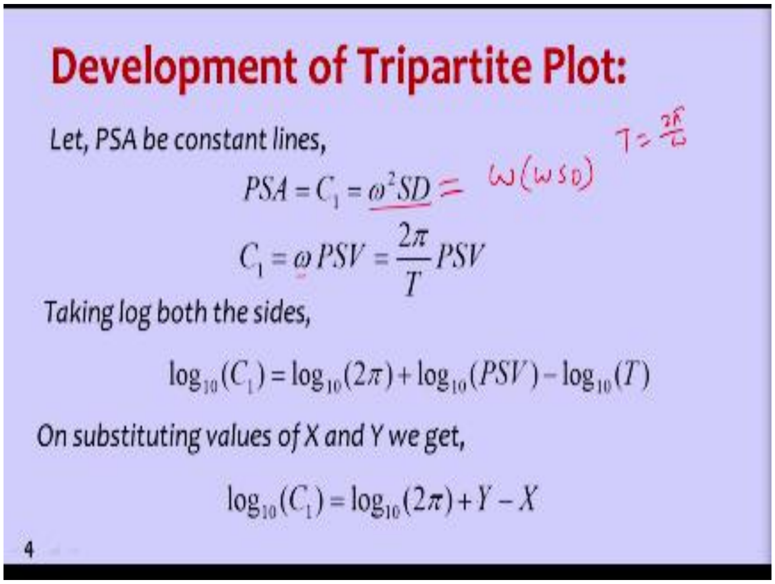
(Refer Slide Time: 02:46)



Now how do we develop tripartite plot, so it is observed that knowing spectral displacement one can obtain spectral acceleration and spectral velocity that is pseudo spectral acceleration and pseudo spectral velocity. Hence all three provide exactly the same information. So with this it is possible to plot all three on the same graph and how do we plot all three on the same graph.

So let natural period be on the x-axis and velocity be on the y-axis. So we plot on the log-log scale so log of natural period that is log(t) logarithm to the base 10. So log(t) is x-axis and log to the base 10 psv on y-axis. So velocity on y-axis and natural period on x-axis so we plot that.

(Refer Slide Time: 03:34)

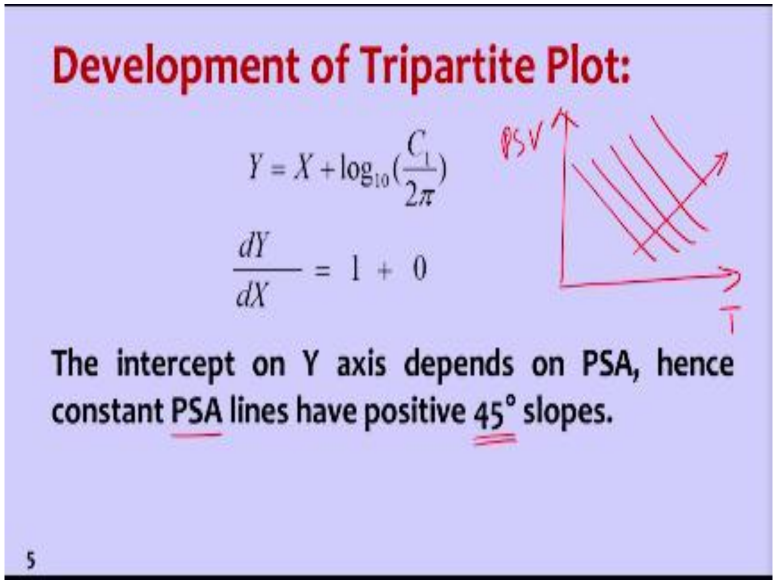


Then let us look at PSA be constant line, so the pseudo spectral acceleration let it be constant 1 C1 is equal to PSA= ω square SD so we have already seen that ω square SD. So this ω square SD so we write in a slightly different form ω square SD can be written as ω(ω SD). So we already know that ω SD is PSV, so C1 is equal to ω into PSV.

So this Ω can be written as 2л/t so we already know that t = 2л/ω using this relation we write 2л/t PSV. So then taking log on both sides left hand side and right hand side log out to the base 10 C1 is equal to this loglog$_{10}$PSV-log$_t$ then on substituting the values of X&Y we get so what is X, X is logt and then why is logPSV. So if we substitute that value log$_{10}$ constant C1 is equal to log$_{10}$ 2л+y-x so this is Y and this is X.

(Refer Slide Time: 04:48)

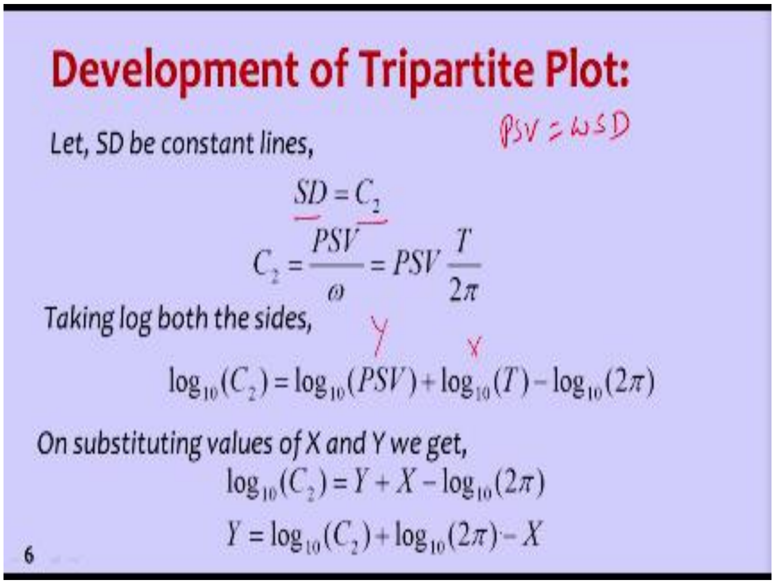
(Refer Slide Time: 05:52)



Now if we write this equation in the form of y=MX+C so what we get from the earlier equation is y=x and this values so C1/2л. So y=x+$\log_{10}$C1/2л so this is constant. So what will happen is if you differentiate this one dy.dx we get 1. So 1 means positive 45º slope so dy/dx slope is dy/dx 1 means positive 45º slope.

So the intercept on Y-axis depends on PSA, hence PSA lines have positive 45º slope. So in tripartite plot so if you take say this as T logt and this as PSV, so what we get is so these lines on this if we measure this one so they on these lines this positive 45º lines show. PSA acceleration values pseudo spectral acceleration values.

(Refer Slide Time: 05:52)

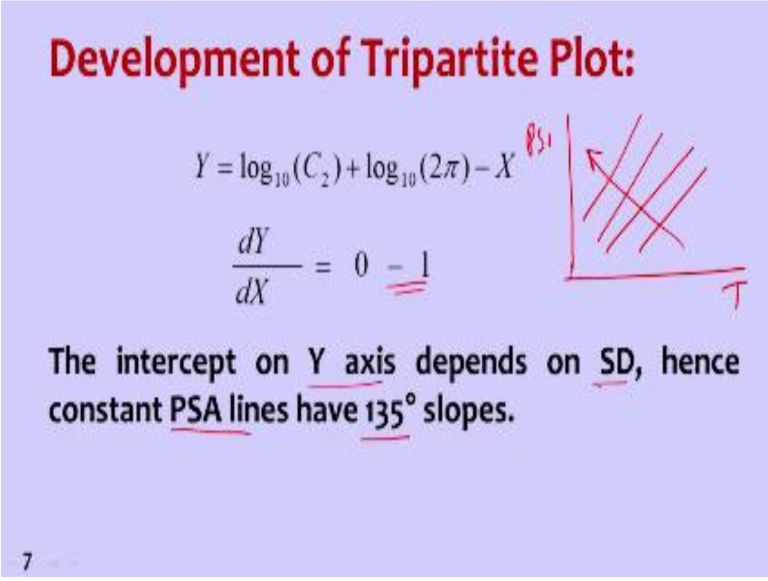


Similarly if we plot say displacement so SD to be constant lines so spectral displacement is equal to C2 constant 2. So now we derive spectral displacement that is C2=PSV/2 so we already know that SD PSV is equal to ωSD, so SD is PSV/ω. So PSV/ω we can write this Ω as t/2л so we write that so what we get is PSV(t/2л).

Again if you take log on both sides so $\log_{10}$C2=$\log_{10}$PSV+$\log_{10}$ of natural period t-$\log_{10}$2л. So again if we substitute x and y values in this one so what we get is x is this one and y is this one. So we get $\log_{10}$C2=y+x-$\log_{10}$2л, so again if we write in the form of y=mx+c, y = so X - X plus this entire thing is constant.

(Refer Slide Time: 06:58)

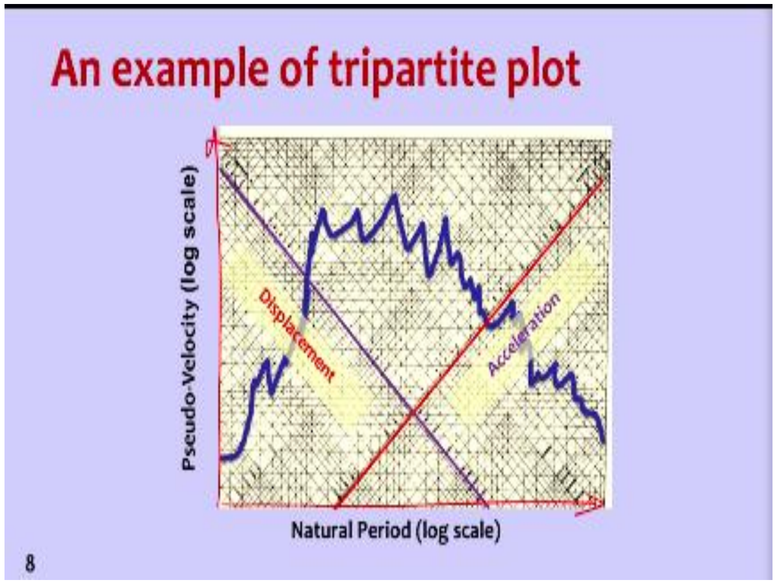
(Refer Slide Time: 07:32)



Now if you take the differentiation of this term dy/dx so we get - 1 so that means negative 45º slope. So the intercept on Y axis depends on SD, hence constant PSA lines will have 135º slope, so 135º positive slope or - 45º slope both are same. So that means what when we draw this one we have this measurement on this scale this is T this is PSA.

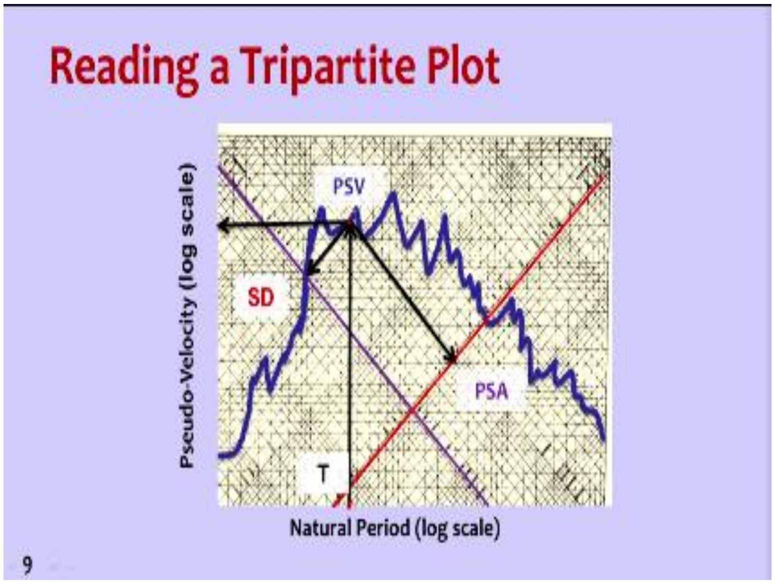
(Refer Slide Time: 07:32)

(Refer Slide Time: 07:52)



Now if we look at the plot and spectral displacement is spectral acceleration on the same plot we get plot something like this. So natural period is on the x-axis velocity is on y-axis and a - 45º line we get a displacement and + 45º line we get acceleration.

(Refer Slide Time: 07:52)



So how do we read this one, so first we find out the natural period of the building or structure and then go and straight away hit the curve. And then measure it horizontally on the y-axis that is PSV, so we get PSV value there and from the same point you drop onto the acceleration line constant acceleration line we get PSA value there and then on displacement line if we draw perpendicular there we get displacement line.

So like this we can measure from the same plot de spectral displacement, pseudo spectral velocity and pseudo spectral acceleration.

(Refer Slide Time: 08:30)

So in summary what we have understood in this class is how to construct a tripartite plot which gives all three measures that is spectral displacement, pseudo spectral velocity, and pseudo spectral acceleration on the same curve.

IIT Madras Production

Funded by

Department of Higher Education

Ministry of Human Resource Development

Government of India

www.nptel.ac.in