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Structural Dynamics
Week 6: Module 02

Special Cases of Response Spectrum

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Welcome to structural dynamics class. In this class we will discuss some special cases of a response spectrum. So in last class we have discussed a concept of response spectrum, how response spectrum is developed and how we use the response spectrum in finding out the maximum displacement, velocity and acceleration values for given structure or before designing the structure.

And we also studied about how to find base shape and also bas movement. Now in this one we will discuss some special cases. So what happens if damping is 0 or damping is extreme value or what happens if natural period is 0 or natural period is extreme values.

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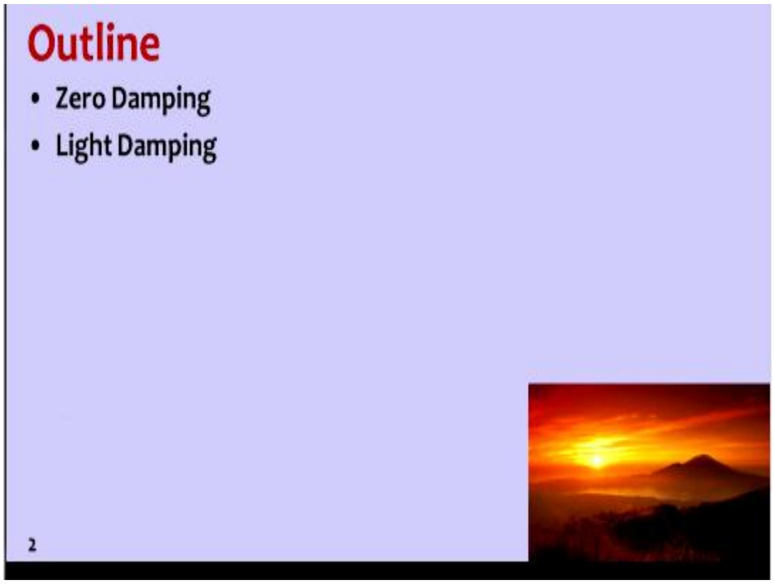

Okay 0 or infinity.

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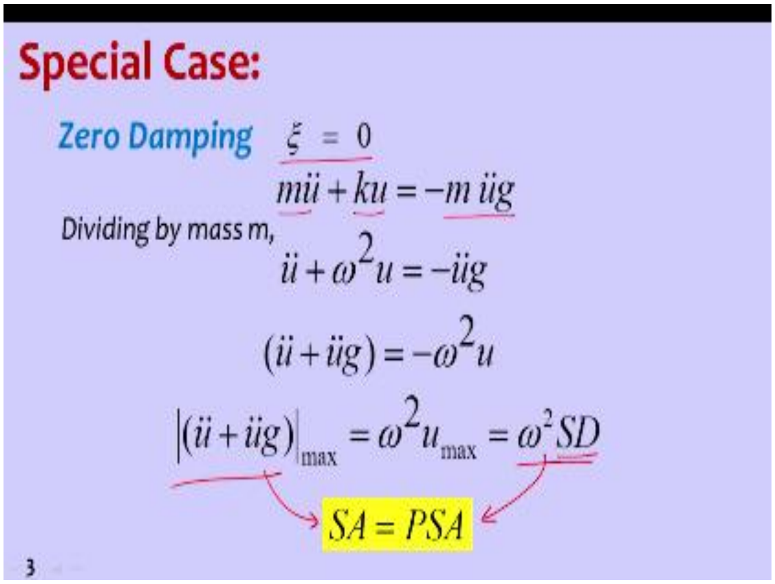

So special case one, so this case is about zero damping. So if we look at equation of motion mu..+cu.+tu=-mug.. so this ug.. is earthquake force u..g is the acceleration earthquake acceleration. So now mu..+cu is not there, because damping is zero so the second term is not there. So this inertia force plus stiffness force is equal to applied force, then if we divide this by mass what we get is acceleration plus this k/m is nothing but ω^2 u=u..g.

Then if we rearrange these terms acceleration and as ground acceleration together this is absolute acceleration is equal to – ω^2u. So the maximum value of this one should be equal to maximum value of this one. So that means what maximum acceleration is equal to ω^2 times u max, so what is u max, u max is a spectral displacement so we can say that spectral acceleration is equal to pseudo spectral acceleration in case of 0 damping, so what is pseudo spectral acceleration.

Pseudo spectral acceleration is, displacement spectral displacement multiply by frequency square that is pseudo spectral acceleration, it is not go for all the cases but in the special case is it same that is when damping is equal to 0 spectral acceleration is equal to pseudo spectral acceleration actually this is spectral acceleration, so the left hand side so that is equal to pseudo spectral acceleration. This is special case one and damping is equal to 0.

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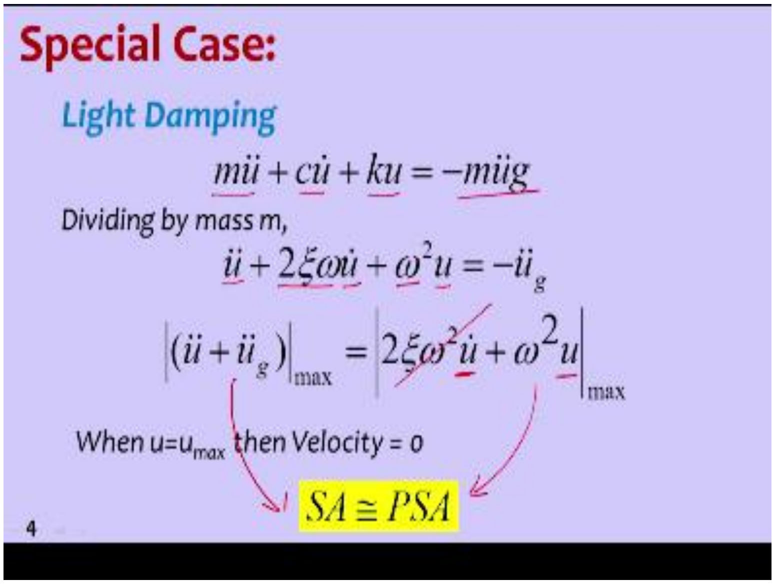

Now under light damping case that way damping is lesser value so there we cannot ignore this damping term so Mu.. + u.+ku = -Mug .. so if we divide this by mass then what we get is acceleration plus 2 ζω term into velocity into natural frequency square ω^2 into u = u..g now again if you rearrange this one the terms will be u.. + u..g maximum value should be equal to 2ζ ω^2 u.+ ω^2 u.

Now you can observe in this one, one important thing is this is velocity this is displacement and we are trying for maximum value, now one displacement is maximum velocity will be automatically 0 so that way what this term disappears so that means what, even in likely damping case a likely damped case spectral acceleration is equal to pseudo spectral acceleration, so that means what for 0 damping case and for likely damping case we can consider this one, acceleration is equal to pseudo spectral acceleration this is response acceleration I am talking about.

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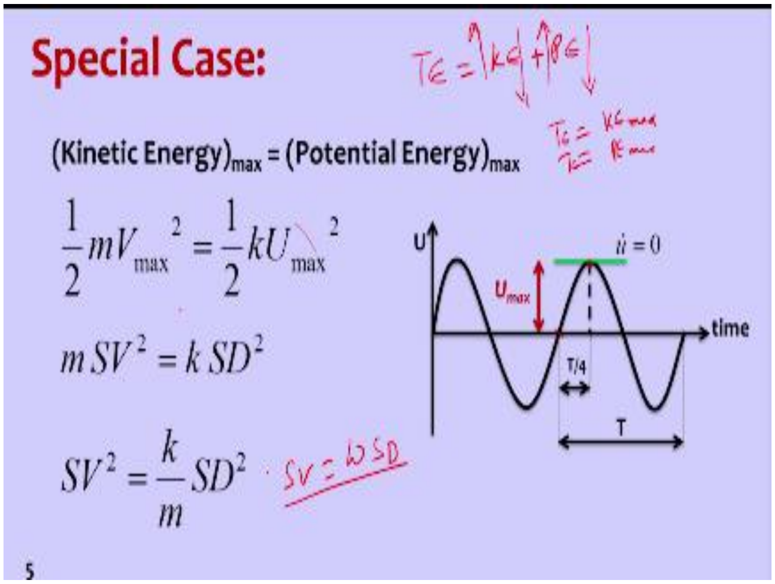

Then let us look at the energy, so at any point of time total energy is constant so total energy is equal to kinetic energy plus potential energy but in the oscillating system in dynamics usually what happens is when kinetic energy is very high potential energy is very low, okay. When potential energy is very low kinetic energy is very high, so it is like that. So that means what, T_E total energy is equal to K_{Emax} like that total energy is also equal to potential energy max so that is the relationship here.

So kinetic energy maximum is equal to potential energy maximum so both maximums are equal, so if you look at this graph this is 0 displacement time but high velocity time, this is maximum displacement time but 0 velocity time so these are reverse of each other, so when displacement is maximum velocity is 0, when is velocity is maximum displacement is 0. Now if we apply that here $1/2mV^2$ is the maximum kinetic energy that is $V_{max.}$

If we take potential energy is $1/2kU^2$ so U_{max} okay, the displacement, then if we substitute it the spectral values mass into s spectral velocity square is equal to stiffness multiply by spectral displacement square, so from this what we can conclude is spectral velocity square is equal to k/m into spectral displacement square. If we take out the spectral velocity only then what we get is spectral velocity is equal to √k/m so that is ω spectral displacement.

So we can see this one so spectral velocity is equal to ω time spectral displacement so now if you look at.

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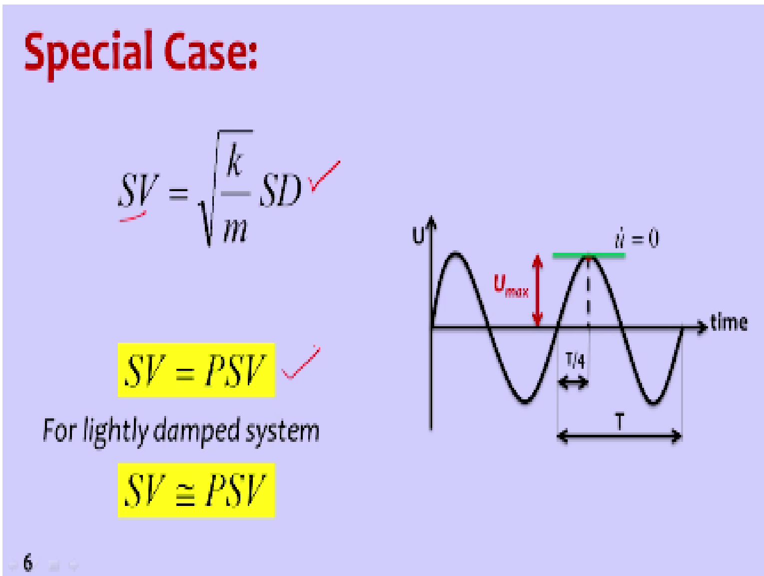

All the cases yeah this case special case we can see this one spectral velocity is equal to spectral displacement multiplied by ω so that is spectral velocity is equal to pseudo spectral velocity and this situation and for similarly for lightly damped case also same thing.

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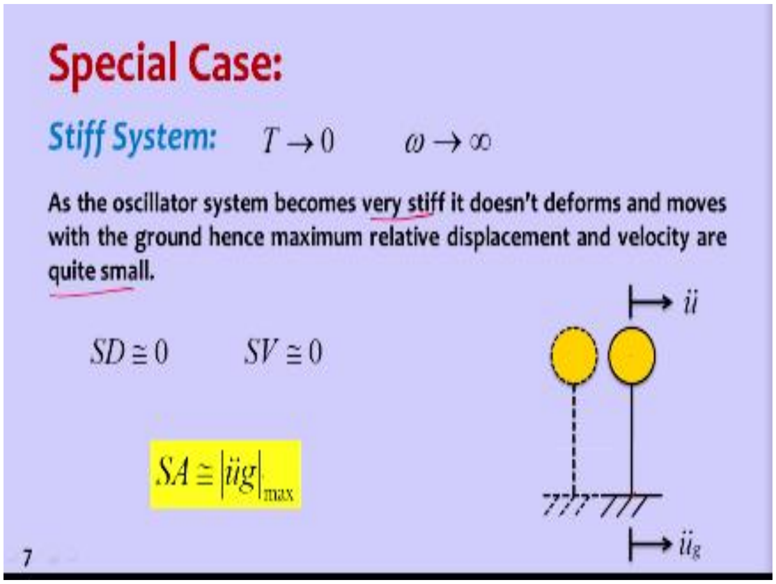

And for stiff systems now we discussed about damping now let us take the extreme values of period so for stiff systems that pays what so t= 0 so that means reject system so ω natural frequency is infinities so they are reverse of each other so as the oscillator system becomes very stiff it dopes deform and moves with the ground hence maximum relative displacement and velocity is quite small so it is something like this if you look at this one is very stiff system.

So whatever motion I give so base motion is equal to the motion of the mass at the top same motion now change in the motion so you can see this there is no relative displacement between base and the top, so that can be illustrate from the by seeing this figure so this one it is very stiff system if I move this okay by some acceleration same is the movement so there is no relative moment between the base and the mass.

So that means what spectral displacement is approximately 0 spectral displacement is what deformation ion the system is spectral displacement that is approximately 0 so that is no and spectral velocity is also there is no relativeness velocity also comes from the relativeness of base motion and the motion of the centre of mass so that means what spectral acceleration is exactly equal to ground acceleration. Whatever ground accelerations given that is equal to spectral acceleration.

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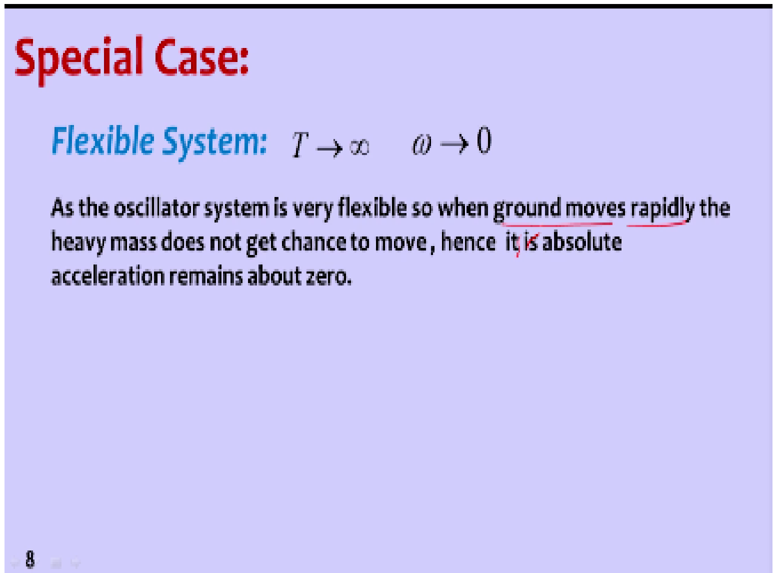

And another special case is even t is infinity very, very flexible structure okay that means ω is 0 so as an oscillator system, system is very flexible so when ground moves rapidly the heavy mass does not get chance to move it is absolute acceleration remains 0 it is something like this. So when I'm moving the base here mass is very heavy it is staying here only base is moving but the mass is staying here so why because it is heavy mass it is not getting chance to move because the rapid movement of the base.

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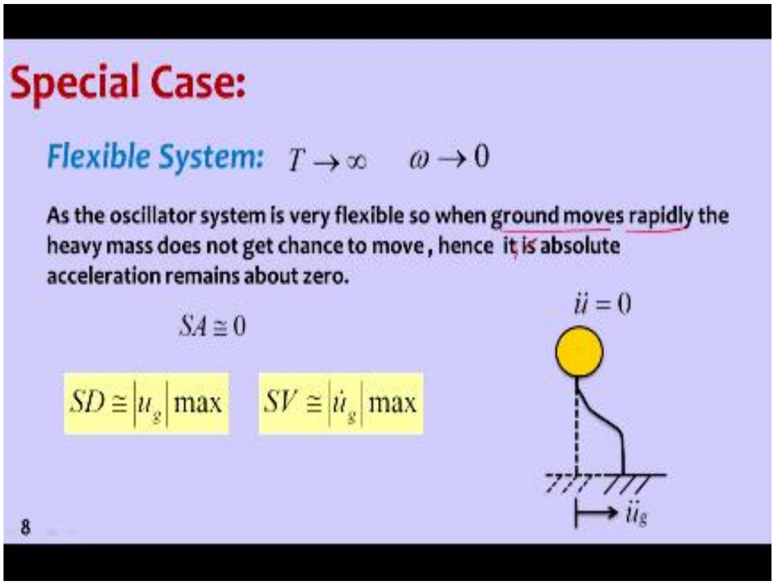

How does that look like looks like this one very heavy mass base is moving without the motion of the mass so that means what acceleration of the mass is zero were as spectral displacement is Ug max whatever is the ground displacement that is maximum value and then the spectral velocity is also the maximum value.

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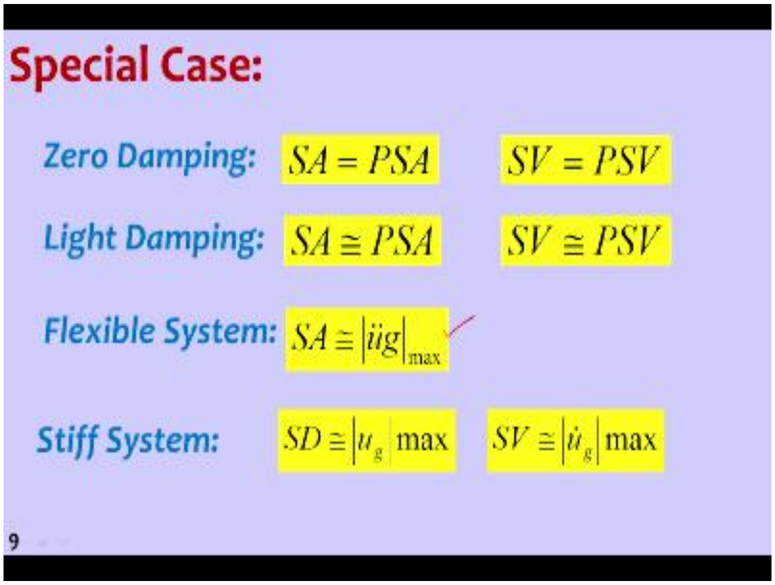

Ground velocity value so for zero damping case spectral acceleration = studio spectral acceleration spectral velocity is equal to studio spectral velocity for light damping case spectral acceleration is equal to studio spectral acceleration spectral velocity is approximately is equal to studio spectral velocity for flexible systems acceleration is equal to maximum acceleration ground acceleration for stiff systems.

Spectral displacement is equal to maximum ground displacement spectral velocity is equal to maximum ground velocity.

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So in this class we have studied about the special cases of response spectrum which can be made use in the application of the response spectrum to design of structures.

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