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Structural Dynamics
Week 6: Module 01

Response Spectrum

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Welcome the structural dynamics class. In this class we will discuss about response spectrum. What is response spectrum, response spectrum is a plot of maximum response of linear single degree of freedom oscillator for a given component of earthquake ground motion. So on the x-axis we have natural of period and on y-axis we have response quantity. So this response quantity can be maximum displacement, maximum velocity or maximum acceleration.

So we can plot response spectrum for displacement, for velocity and for acceleration. Now what is this concept all about?

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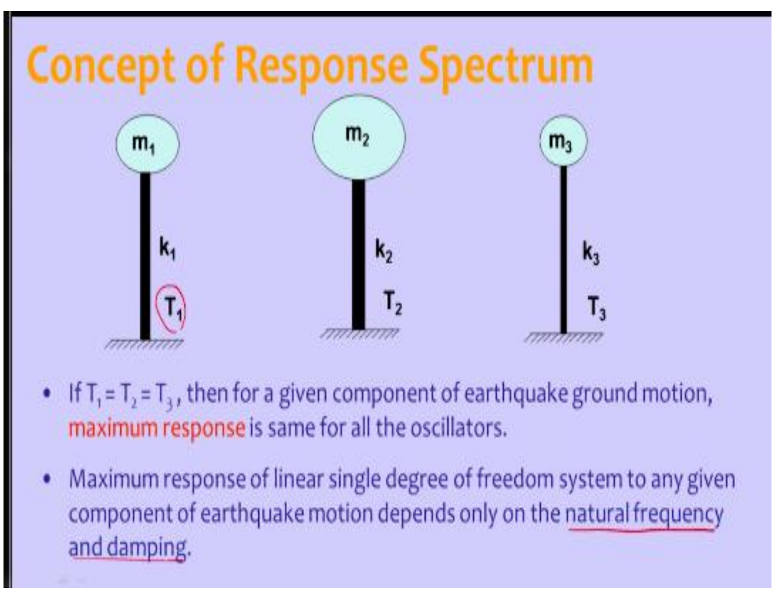

Let us look at the slide and understand. So here you can see three idealized structures. So first structure has mass 1 stiffness 1 and then this structure if we calculate natural period of this structure it will be T1. So T1= 2л root m1/k1 so that is T1, now second structure is again idealize structure mass 2 and stiffness 2 T2 is a natural period of this structure.

So natural period of this structure is calculated by T2=2л root m/m2/k2 and third structure M3 mass 3 stiffness 3 in a similar manner we calculate natural freedom of this structure. Now what response spectrum gives is, if natural period of T1, T2, T3 all are same then for given component of earthquake ground motion maximum response is same for all the oscillators. So that means no matter whatever the structure is mass and stiffness be, if natural period is matching then response spectrum says that maximum response is same for all three oscillators.

So the maximum response of linear single degree of freedom system to any given component of earthquake motion depends only on natural frequency or natural period and a damping. So only this two quantities it is dependent. So now I would like to show you a small video clip okay. So explaining the concept of response spectrum let us look at that.

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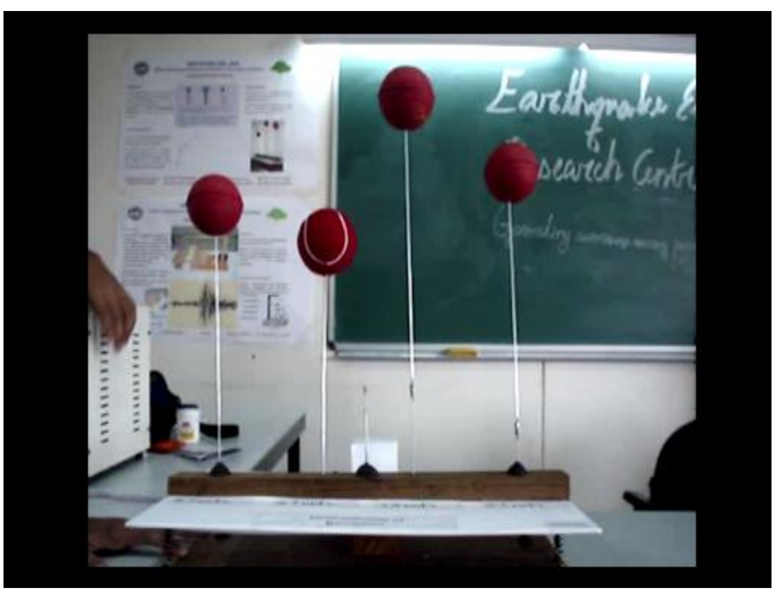

So if we look at this small experiment okay. So in this one there are four oscillators, four idealized single degree of hidden system or we can call it as four buildings. So one from left side the third building is the tallest building and then on the right there is lesser than that okay. So let us take this as like period of this building is some value and then this has lesser period, so tall buildings will have more natural period, short buildings will have less natural period.

So the second one is having slightly lesser natural period and then this first structure the left side structure and the second structure both are matched such that their period is same. So that means what lengths of the buildings are slightly different, but mass is adjusted such that both natural periods are same. So now if we excite all these buildings to some frequency what is going to happen let us look at it.

Now as you can see slowing varying acceleration that means slowly if varying force is applied so because of which that natural frequency is matching with the natural frequency of the tall building. So that is why tall building is vibrating violently similar motion, similar displacements. And then on the same ground like if we increase the frequency of the down motion then this tall building is not having that much amplitude, but the second building is having that is the last one right side building is having more.

And then if we increase the frequency of this one, excitation so this first and second buildings are having more frequency sorry, more displacement. In the video what we have seen first tall building so for slow vibrations tall building got affected and when we increase the oscillation that is the period of oscillation to the increase in the frequency then second building started vibrated.

And after that first and second that is the first one left side one and the second from the left building is vibrated, so from this we can understand that not all buildings will fall down due to earth quakes so the earth quake the frequency of earthquake and in the buildings frequency if they match then peak that is resonating condition might occur.

So like that so the concept of response spectrum says that if we know the natural period of the building what will be the maximum response of that building to a given component of earthquake can be known from responses spectrum concept.

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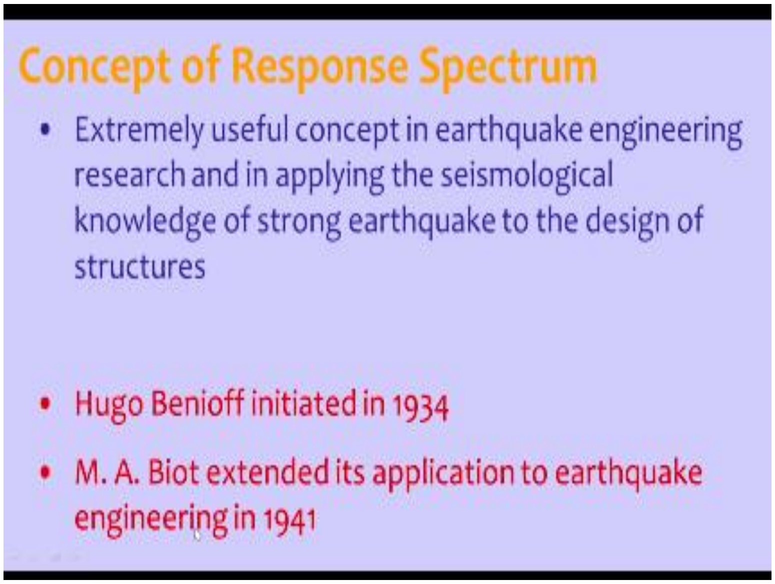

So now the response spectrum is extremely useful concept in earthquake engineering research and in applying seismological knowledge of strong earthquake ground version to the design of structures, first proposed by Hugo Benioff in 1934 and Biot extended its application to earthquake engineering in 41.

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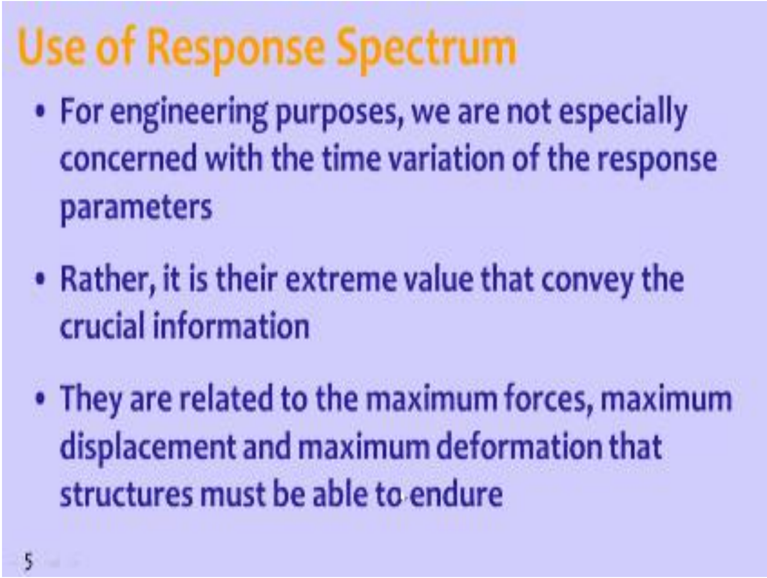

Now uses of response spectrum, where we use for engineering purposes we are not especially concerned with the time variation of the response parameters rather what we need is maximum responses so that is extreme value that convince the crucial information, now they are related to maximum force maximum displacement and maximum deformation that structure may be able to endure, so we need maximum values.

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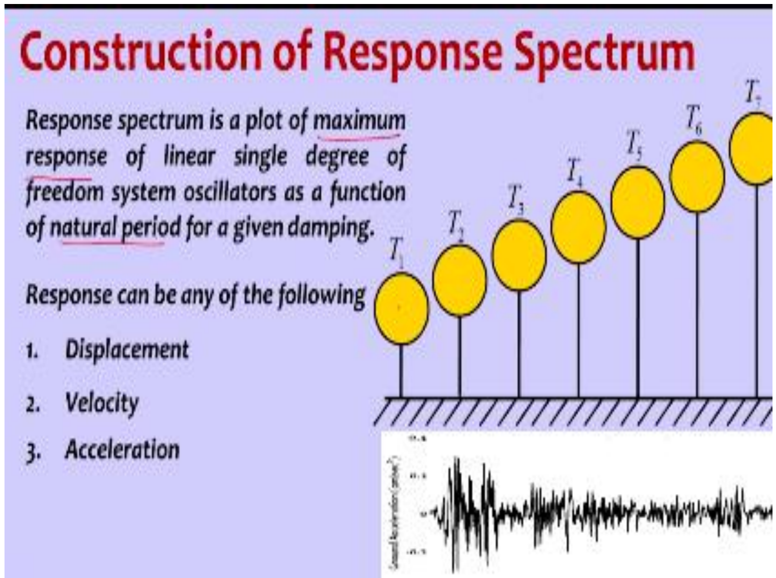

So hence responses spectrum is needed, so now how do we construct this response system, so what we do is like we first take a ground version as you can see here this acceleration is given ground version acceleration and then we put a soot of oscillators linear single degree of freedom oscillators with some fixed damping values so let us say 5% so t1, t2, t3, t4, t5 like that so on so usually oscillators will range up to a period of say 0.1 seconds to say 4 seconds or sometimes 6 seconds oscillators.

And then we had already study the how to find a response of structure to give a random motion by using numerical techniques so we can use either new mark techniques or this central difference method and then find out the response of this linear single degree of freedom oscillator, so second one, third one like that all oscillators response we find out, so the response spectrum is a plot of maximum response of linear single degree of freedoms system oscillators.

As a function of natural period, so if we put that okay response quantity can be either displacement, velocity, acceleration.

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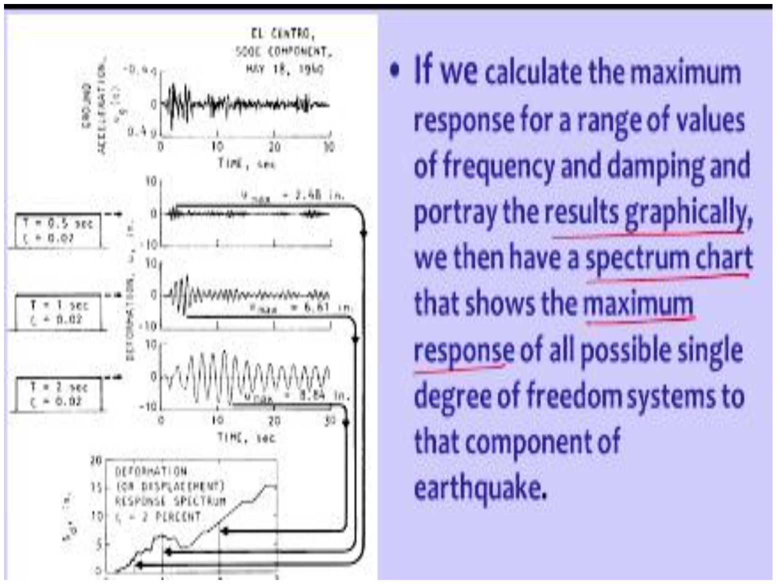

Now you look at this one okay so this has three structures so structure of natural period 0.5 seconds and damping 2% and 3rd second structure is structure of 1 second natural period and damping value as again 2% and 3rd structure is natural period as 2 seconds and damping percentage is also 2, so well this three structures we are giving L center into long motion as input, so when we are giving L center into long motion as input.

And using some numerical technique either central difference method or new marks method we get the time history of this structure this structure time history and then similarly is a second structure we get time history and third structure we get time history so in the first structure maximum displacement as u max = 4.48 inch and in the second structure maximum displacement is 6.61 inch.

And in 3rd structure for the same earthquake the maximum displacement is u max 8.84 inch so this maximum displacement is maximum value at any point of time in this time history response, so here also maximum value at any point of time so one should observe that this maximum values are not occurring either same time in all the oscillators all the structures so they can occur at any different points or point at that point so now what we do is, this natural period of this structure is 0.5 we put 0.5 on the x axis natural period of this structure is 1 second we put 1 second on the x axis and natural period of this structure is 2 seconds we put 2 seconds on so corresponding displacement values we plot of y axis so this is the point the another point so now what we do is, we take many more structures like this.

And then plot this curve so this curve is called responses spectrum of displacement for L centre down motion and damping assumed as 2%, so this is the concept of response spectrum, this is how we construct response spectrum we can say here if we calculate the maximum response of the range of values of frequency and a damping and portray the results graphically with them have a spectrum chart that shows maximum response of all possible single degree of freedom systems to that component of earthquake.

Now once we have this kind of responses spectrum developed for an area, then in that area whatever kind of building is coming say single story building, two story building, three story 20 story building, 30 story building what we need to understand is before design of that building itself we can calculate the natural period of that building and put on the x axis and check the maximum displacement value.

Now that is the maximum displacement with that structure will undergo in the event of this earthquake or similar earthquake, okay. If they are having the same frequency components in it, so this is a concept of responses spectrum it is extremely useful concept in the application of earthquake engineering.

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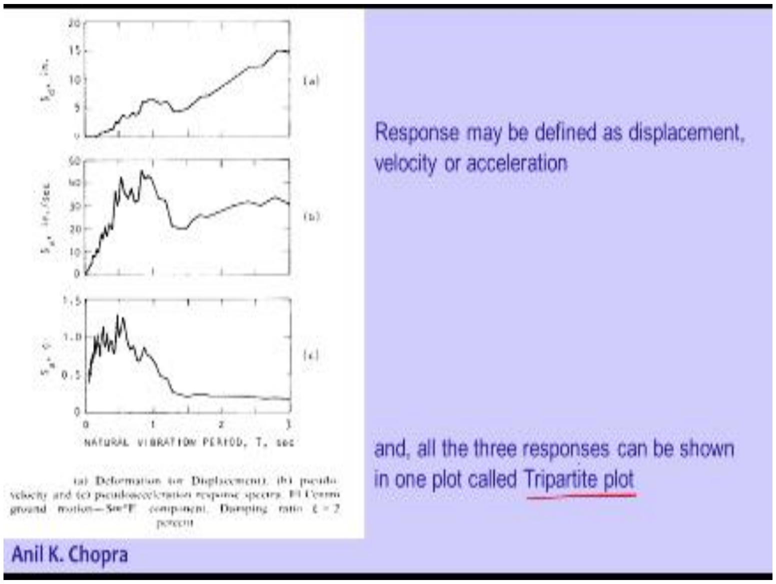

Okay, now you can see this one this response quantity can be either displacement or velocity or acceleration so this A chart is telling displacement B is telling velocity and C is acceleration, actually these three things are linked also, so all three responses can be shown in one plot which is called tripartite plot, so we will discuss in a subsequent lectures about how to construct this tripartite plot.

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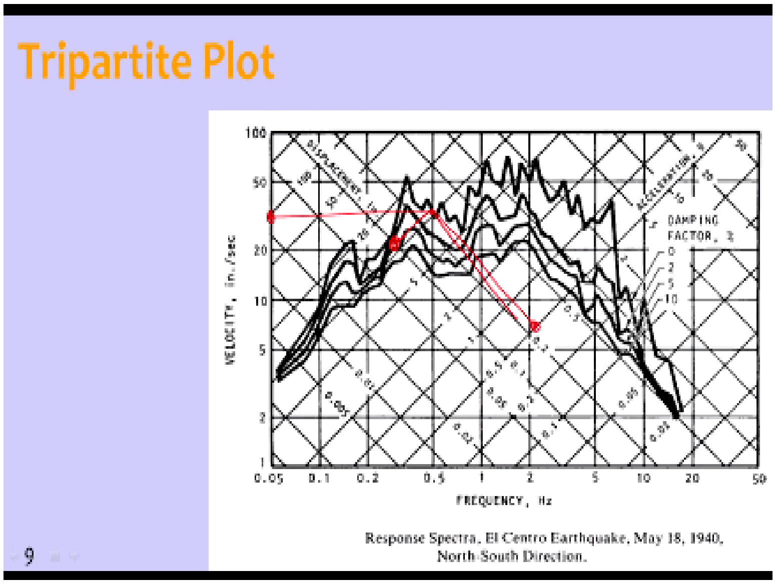

Now let me show you what is tripartite plot, so this tripartite plot is on the x axis we have either natural period or natural frequency on y axis you can see velocity, so that means what velocity have this structure can be measured velocity response of the structure can be measured if we know the natural period or natural frequencies, so we know natural frequency say 0.5 Hz so then we directly go to that and this here we have four plots so each one corresponding to one damping value, so damping 0 is there damping 2%, damping 5% and damping 10%.

So let us take as 0 damping, so 0 damping is a last curve so we go there and hit that curve and then directly come here and read this one, so please note that this plot is in log, log scale so we need to read that like that only read it like that only. And now how to get displacement from this one, from the same point you can see on the like 45˚ line here, we can directly draw like a point or a line perpendicular to this 45˚ line.

Then you measure displacement we can see values here point, 0.1, 0.2, 5,1,2, 5 like that so we can read this is the displacement value, and similarly on the other direction again 45˚ okay, +45˚ and -45˚ lines we can go and drop sorry, we can go and drop another perpendicular line and then read acceleration value, so this axis is telling acceleration 0.2, 0.51, 0.1, 0.2, 0.5 like that so we can read the acceleration here, we can read displacement here and we can read velocity here.

So that is how we can read in one plot all three values, so that is called tripartite plot, so how to construct this tripartite plot we will discuss in next, next lecture.

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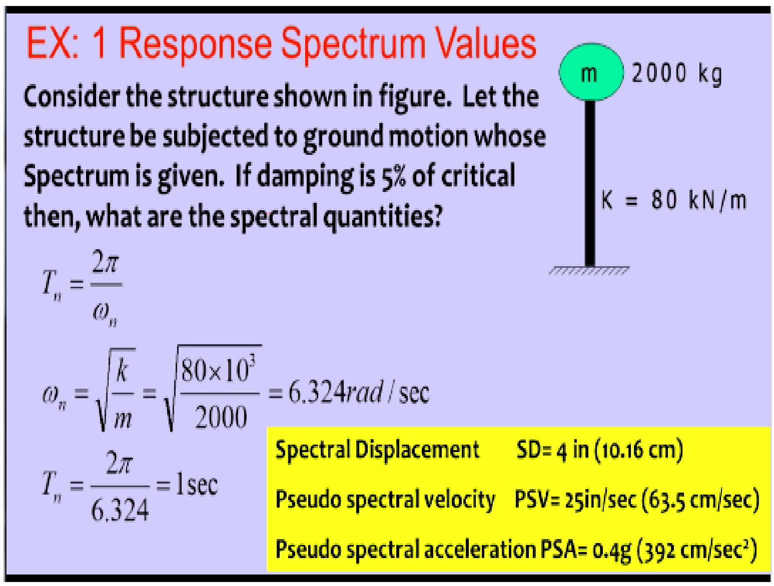

Now let us solve one problem okay, let us you this response spectrum and find out the response quantities so this example says that consider a structure shown in figure so this figure where mass of the system is 2000 kg and stiffness value is 80 kN/m. Let the structure be subjected to ground motion whose spectrum is given, so spectrum is known to us you damping is 5% of critical then what are the spectral quantities.

So spectral quantities means we need spectral displacement, spectral velocity and spectral acceleration we will discuss about why we are calling pseudo spectral acceleration, a pseudo spectral velocity in the next class. So this is spectral displacement, this is pseudo spectral velocity, pseudo spectral acceleration. Now for that we need natural period of the building, so natural period of the building that is $T_n=2\pi/\omega_n$ so where ω_n is a natural frequency which can be calculated using stiffness and mass parameters.

So stiffness is given to us that is 80 kN/m so I substitute that 80 kN/m here and then mass is also given 2000 kg. So the result is 6.324 rad/sec is a natural frequency of the system. Now I substitute that 6.328 here $2\pi/6.328$ that is one second, so natural period of this system is one second. So now for one second these are the corresponding displacement values velocity value and acceleration value how did we get it, now let us looks at this chart.

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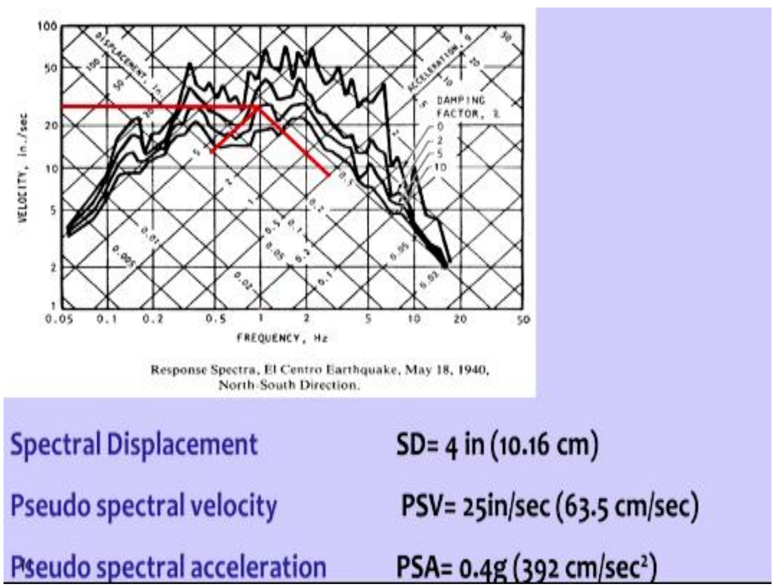

So what I did is so I took that frequency natural period is one second so that is frequency is equal to 1/ t so 1/1/= 1 only one hart this is so this one had frequency with this using this now we are given 5% damping. So this one hart frequency is equal to one only one hart this is so this one hart frequency using this one we are given 5% damping so second line second plot so from there I can directly go to like y axis and measure the velocity so velocity 25inch per second per and you convert in this system that is 63.5 cm per second this is velocity this is Pseudo spectral velocity and then from the same point on the curve we drop like on $+45^0$ axis so we get we hit this point so this displacement.

So that is 4 inch displacement that is equal to 10.16 cm and then in -45^0 axis if we drop that is perpendicular line we get acceleration that acceleration is 0.4g that 392 cm per second square so that is how we get a spectral displacement spectral velocity and spectral acceleration. Three values so we can get these spectral values, then it is very important for us to calculate the maximum forces.

So what we got from response spectrum chart is spectral displacement spectral velocity and spectral acceleration so actually what we want is for designing of a building we need forces so we need basher and then the distribution of basher over the height of the building so how to calculate basher and how to calculate the base movement, so let us discuss that.

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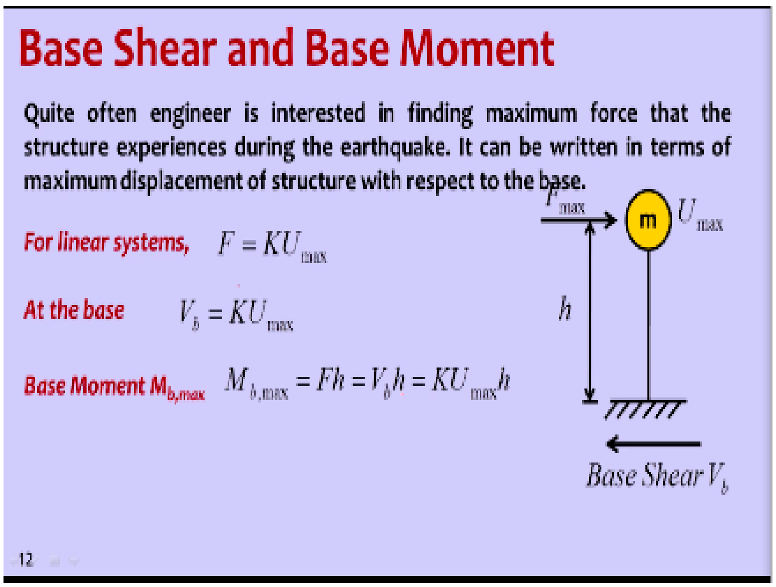

So quite often engineer is interested in finding maximum force that the structure experience during earth quake it can be written in terms of maximum displacement of stricture with respect to base, now if you look at this force is mass in to acceleration and the same force as are action here that is called basher so w can calculate basher and also base movement because force is acting at x from the base so how do we get that force so force is simply f= k stiffness multiplied by maximum displacement.

So this maximum displacement is what this is the spectral displacement what we get from response spectrum plot. So at base we can that same thing it is v b that is basher = k x u max so then base moment is m v = f force in to live on that is h v b x h.

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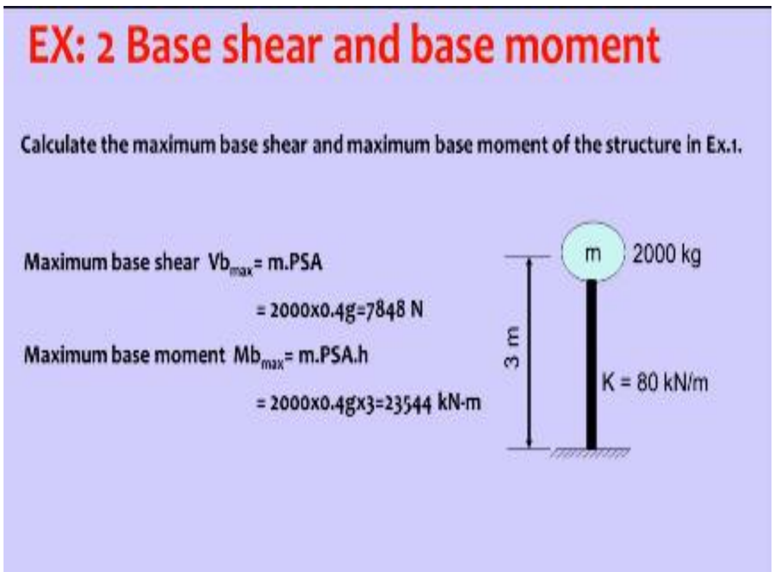

So once we can call this directly as k x spectral displacement multiplied by h then calculate maximum basher and maximum base movement of structure in given example as example one so basher can also be calculated in another manner that is mass x acceleration sop what is that acceleration Pseudo spectral acceleration from the dry plotted part or response spectrum plot for acceleration so mass given is 2000 and the acceleration value is 0.4g is a acceleration value here you can see 0. 4g then if I multiply this two I get like 7.48kn is a basher maximum basher.

And similarly if I multiply it with height of the structure then I get 23544kn m as base moment of the structure so these basher value and base moment value are required for the design of this structure.

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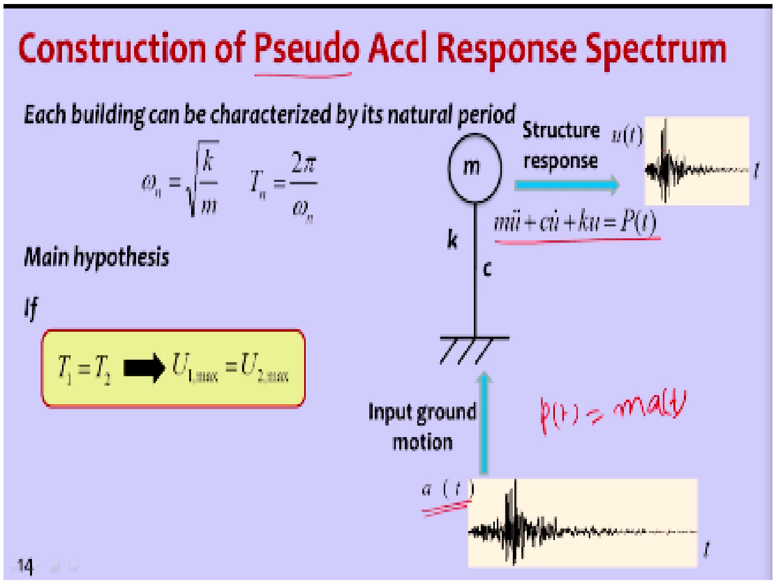

Now how do we construct this acceleration response spectrum why we are calling Pseudo I will discuss Interviewer: he next class, construction of acceleration spectrum so we have linear single degree of freedom system and the equation of motion for that linear single degree of freedom system is mu.. +cu.+ ku = applied force. That applied force is coming from acceleration so that p (t) = mass x acceleration and then this is the response now what we do is we get the response for many structure like that okay and then use the concept of this spectrum.

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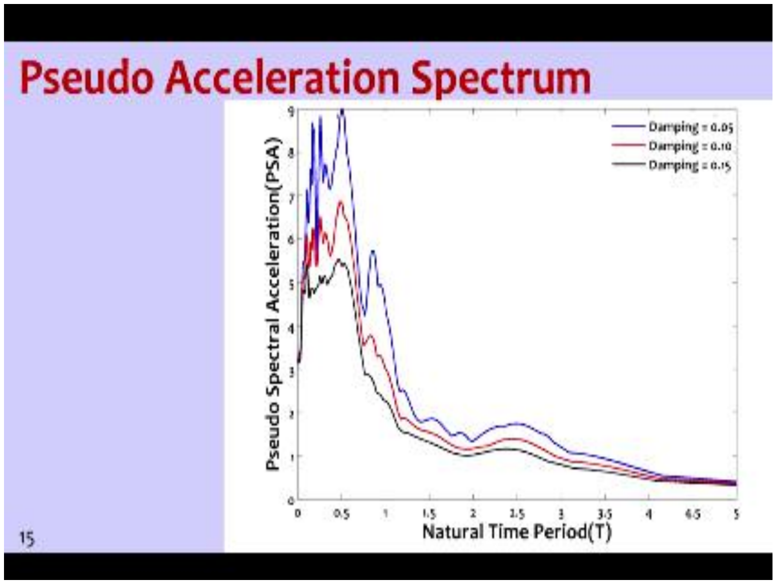

So for given l center ground motion for different damping values we have polluted this response spectrum for acceleration you can see this one black one is for 0.15% damping and red line is for 10% damping and blue line is for 5% damping so you may note this one as damping increases it reduces the amplitude value at the same natural period okay it reduces the amplitude value so this is a responses spectra for varies damping values for given component of earthquake in this example we have used l center ground motion.

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Now how do we construct design spectrum from that and what is the design spectrum so the plot what is given in is 1893 that is criteria for earthquake western design of building so in that the plot given is called design spectrum it is plot responses spectrum so design spectrum is a plot which comes from responses spectrum so what is design spectrum okay so it accounts for issues that have varying on size big safety something like over strength redundancy and ductility.

So these three things are it accounts for these things and the design spectrum must be accompanied by what load factors and permissible stresses damping to be used in the design method of calculation of t and type of detailing for ductility.

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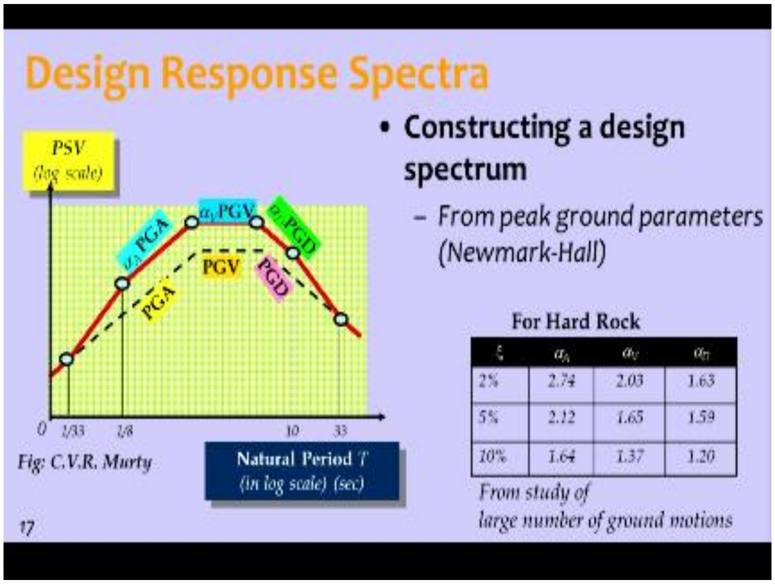

So now how we get this design spectrum so after getting suit of a response spectrum or spectra the different, different ground motion . What are possible are what are likely to occur in the area so we get responses spectra foe different, different ground motions. So what we do is from there okay we do a average kind of things so this average is called as statically averaging statistical averaging so this the procedure is detailed procedure is given in the Newmark and hall text book so what we need to do is, is the design spectrum will be smooth.

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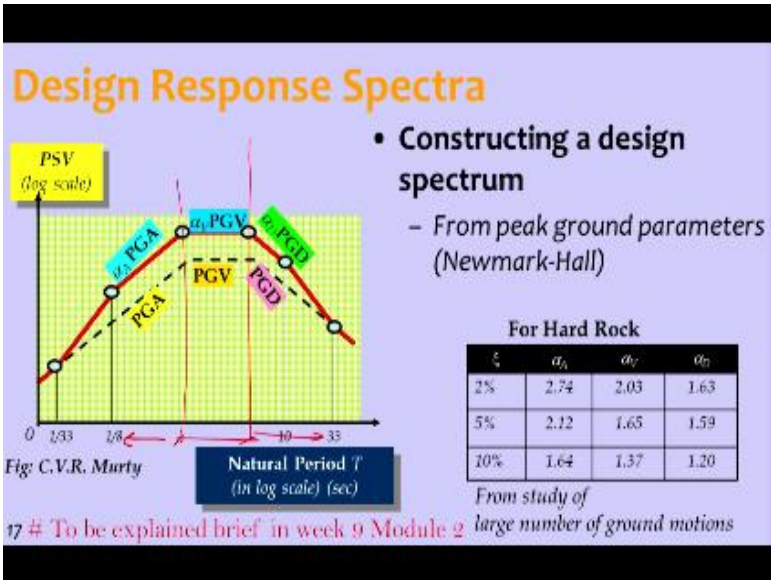

And out so something like we have many spectra say T this that acceleration so we get the average responses spectrum so something like say one average response spectrum something like this so but in design spectrum the issue is what we use this average response spectrum is like there is a mistake in the calculation of natural period for any structure okay instead of this on if it is another one value then suddenly at the point response may be peak so in order to, to remove this kind of describe the plot is smoothen this average spectrum plot is smoothen by using coefficient so if it is for acceleration the2% coefficient is used for velocity and for displacement

These coefficient are used and then this limit are set so this one is natural period on x axis and psv that is velocity rural spectra velocity on y axis this control points are given at 1/33 seconds 1/8 seconds 10 seconds and 33 seconds these points are given and these point the middle point is achieved by using trial and error method so this region this region is acceleration sensitive region this is velocity sensitive, sensitive region and after this is displacement sensitive region in the design spectrum.

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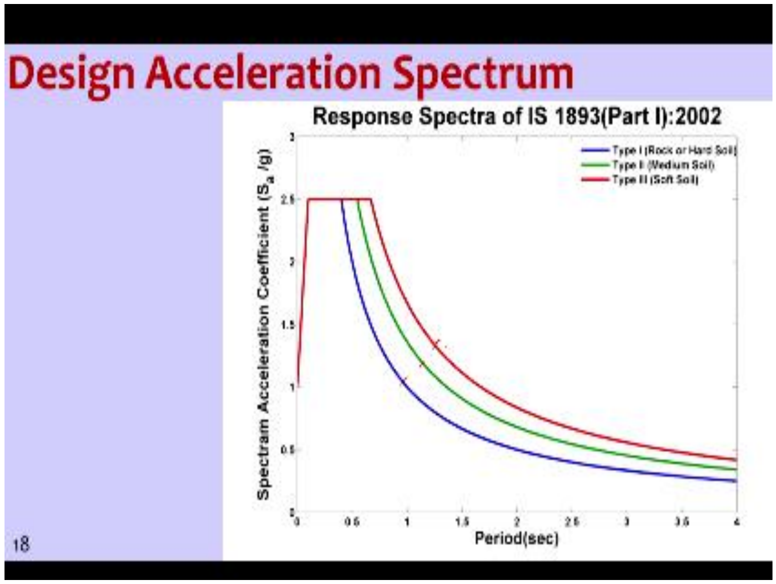

So this is a design spectrum what we use in a is1893 that is a criteria for earthquake resistance of a building so here the spectrum is given for different types of soil conditions one is type one rock or hard soil and the medium soil sorry this is hard soil medium soil and soft spoil so obviously soft soil response is higher as well as the coefficient values are higher.

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So in summary what we have discussed in this class so in this class we discussed the concept of response spectrum what is response spectrum and then what are the advantages of the response spectrum and how do we construct response spectrum and how do we use response spectrum and briefly we have discussed concept of design spectrum okay in tutorial you will be explained about example problem.

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