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Structural Dynamics
Week 5: Tutorial 1

Central difference Method

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welcome to the tutorial on central difference method as discussed already central difference method is one of the techniques which is used for solving the differential equations basically this is used as a method in finite difference techniques where we have forward difference as well as backward difference but we will be discussing about central difference method in this tutorial so the central difference method is basically depends on.

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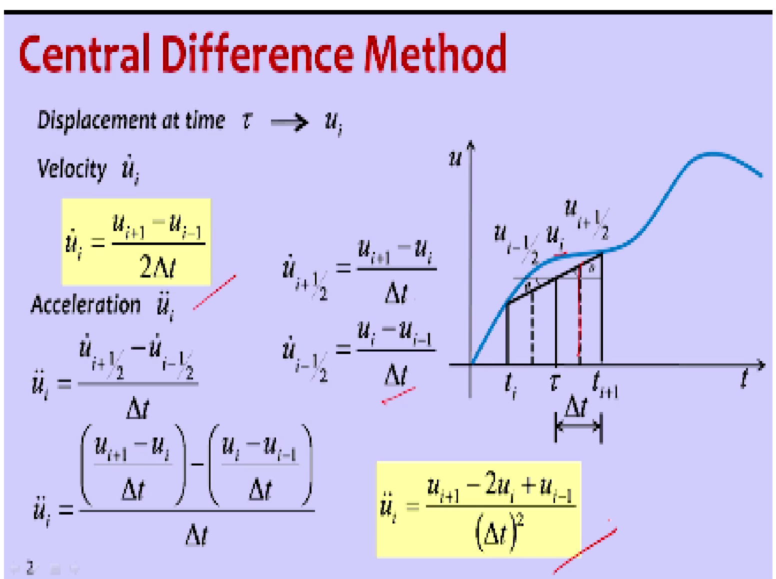

This assumption that a time step T = τ we need to get or we need to know the values of TI + 1 and T P I or TI-1. So u are the displacement at time step I will be calculated by using I + 1 step and I - 1 step similarly for velocity also so velocity as we know rate of change of displacement can be represented in this manner UI +1 - UI - 1 by 2λ T and acceleration is represented in this manner u. I + 1 / 2 that means1/2 I + 1 / 2 which is this half or it can also be represented in terms of I also where we have derivation like this so I + half will give you I + 1 - UI I /λt and u dot i - 1/2 = UI - UI - 1 /λ T.

So using these two we have u.. to be represented like this u..i = ui + 1 – 2ui + ui- 1 /λt^2 so this is how we represented u.i and u..I this is already been discussed in the lecture.

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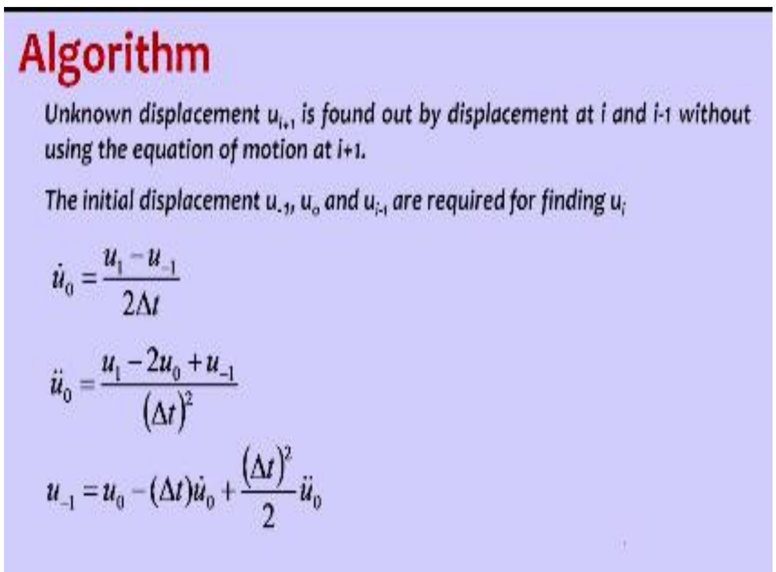

So we go with the algorithm which is used for calculating the displacement of a particular structure or the single degree of freedom system this is the summary what we have to do in this algorithm u . 0 or I = to 0 the initial condition can be obtained from using UI u 1 and u -1 / 2 λ t. And similarly u..0 will be calculated using u1-2 u. + u - 1 /λt^2 and in this calculation one more important parameter or one more important step we need to calculate as you minus 1 because usually this algorithm will be used in calculating the displacement using any program computer program so in computer programs we do not have I -1 step so usually indexing will be starting either from zero or -1.

So in order to calculate that we need to initially calculate u minus one this is the condition where we need to calculate already much before so u -1 is calculated like this and initial displacement and initial velocity are calculated in this manner at I = 0 zero that means the first step you have initial displacement and initial velocity equal to zero that means this is completely a forced system I mean to say forced undamped system where the equation of motion is given by this mu..0 or mu..+ u. + ku= p where P is the external force. So from this equation at time step I = 0 .

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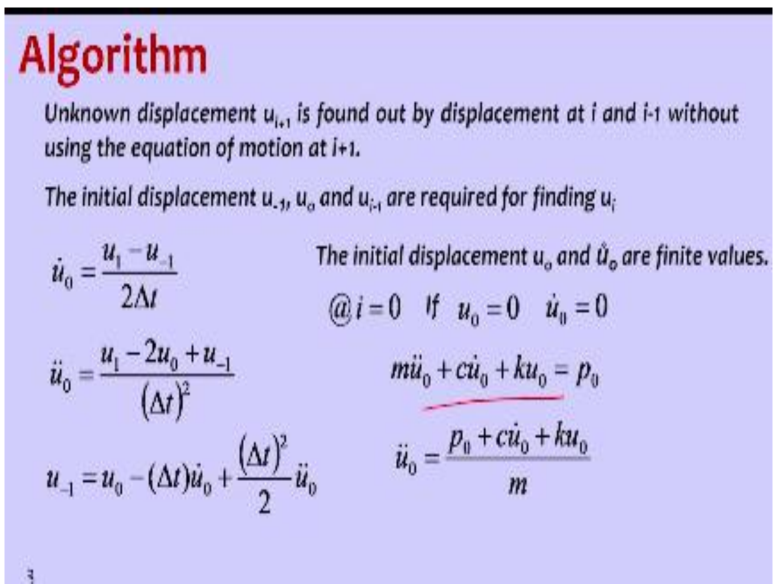

U.. 0 is given as P .+ P_0 +Cu .. + K u_0 / M so this is how we calculate the initial values.

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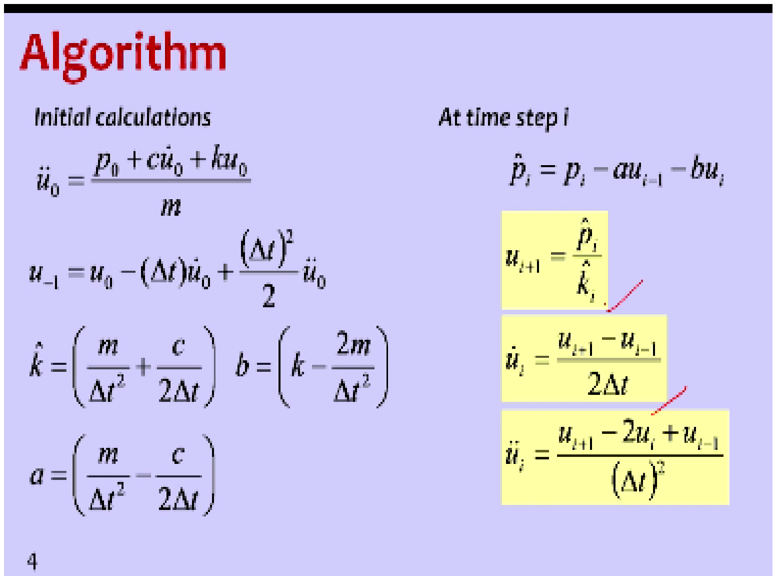

So we calculate or will discuss about the algorithm which is required for calculating the response of the structure so as initial calculations are required so initial calculations for acceleration u..0 at I = 0 we have P_0 + Cu .0 + k u dot u 0. So this is acceleration at time step I = 0 and u - 1 which is the previous value from 0 u 0 -λ T x u.0 + λ t 2/2 u..0 and K^ is an initial value which is almost constant for the particular problem for the given problem M is constant for a particular structure and c is constant and delta t is constant but I'm step for the force applied.

And a is M /λt2 –c/2λt and b is calculated as k -2m x λ T Square so with this initial values at time step I p^ I is calculated as bi minus a into u I - 1 - b UI and u i+ 1 is calculated as p^ I / K^ so here we the one important thing that we need to keep in mind is we are getting I +1 value at the time step I so in indexing the time steps or the forcing frequencies we need to take care of the index that has to be given in the computer program.

So we get only i -1 values when we are calculating for I values so u .i which is velocity is calculated as UI + 1 - UI – 1/ 2 λ where velocity is calculated at time step I only displacement will get at i + 1 step so this has to be taken care when we are writing any computer program and acceleration similar to displacement we have acceleration which is at time step I only u.. i is equal to u of I + 1 - 2 UI + UI – 1/λT whole square.

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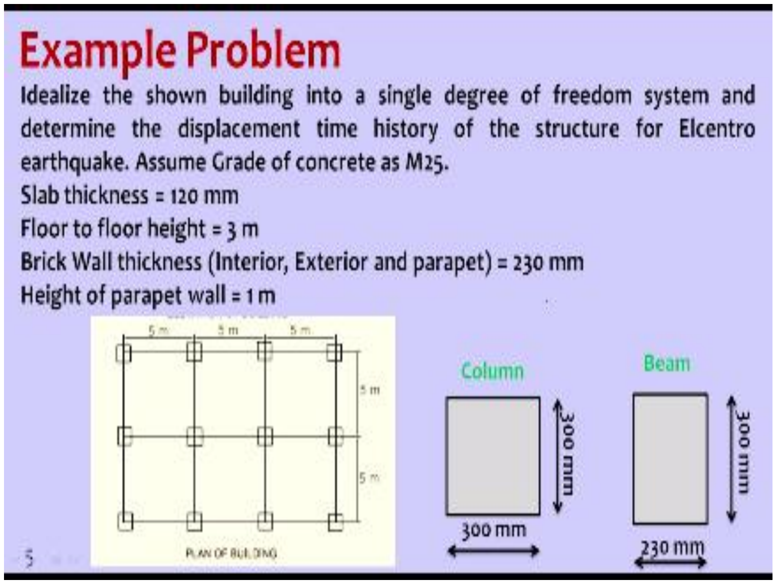

So this is algorithm that we have to follow for this problem we will be going through the different steps of the algorithm which we have discussed now so I realize the so Shawn building into a single degree of freedom system and determine the displacement time history of the structure for El Centro earthquake so assume grade of concrete as m25 and the given details are slab thickness is120mm floor to floor height is 3 meters brick wall thickness internal external and parapet all are equal which is equal to 2 30 mm.

And height of parapet wall is1 meter so in this plan we have equal size of columns which is equal to 2 30mm this is the size of column and beam so 300 by 300 is the column size and beam sizes to 30 x 300 so each bay length is 5 meters.

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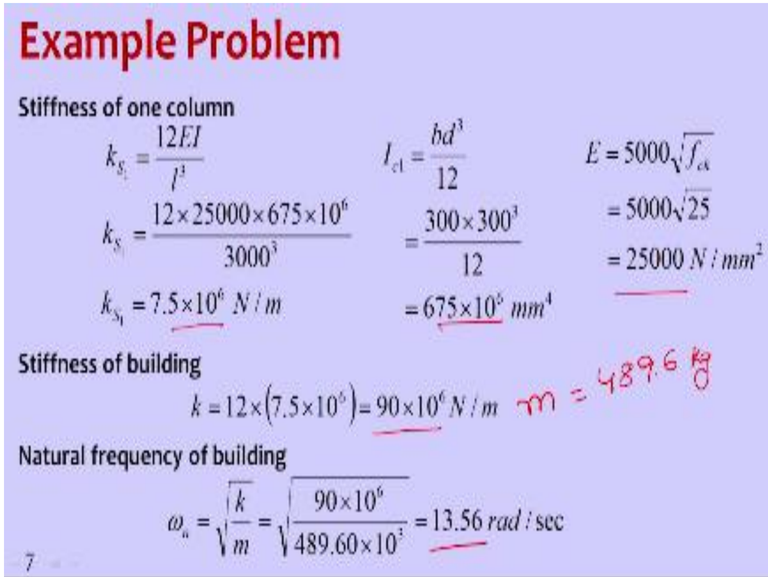

So as we have already discussed in week 7 tutorial mass of each floor is calculated and mass of first and second floors this one 76.6 9 into 10 to the power of 3 kg and mass of the top floor alone is around 130 6.22 into 10 to the power of6 kg so all put together what we are trying to do is we are calculating the mass of total structure wherever even though it is a three-story structure even though this can be lumped as a three-story structure as a multi degree of freedom system.

We are not concerned with a multi degree of freedom system we are converting it to a single degree of freedom system so all the mass is lumped at the top floor only so whatever force that is acting on the structure will be acted upon assume it to be acting on the top floor so total mass is lumped here and stiffness is lumped only in this section whereas damping is also present here and mass is lumped here.

So total mass which is edition of 176 x 6 9x 10^{3} kg and 136. 22 x 10^6 kg is added up and finally the mass is calculated as 489 . 6 x10^3 kg so similarly we have stiffness of one column which is equal to K s 1 is equal to 12 EA / L cube for which E is calculated as 5000 √ fck5000√ 25 which is already given value which is obtained as to 25,000newton per mm square I is $BD^{3/}$/ 12= 300 /300 column so300 x 300^3 / 12 which is coming around 675x $10^{6.}$

When you substitute E and I in calculating stiffness we get around 7. 5x 10^6 Newton per meter so stiffness at each floor is similar and we have 12 columns in the complete structure so we are idolizing it as a single degree of freedom system so we are calculating 12 x 7.5x10^6 which is coming around 90 x 10^6newton per meter so from we got stiffness value and the mass value which is already coming around m = 489.6 kg.

So using this mass and stiffness we calculate natural frequency of the building which is equal to root over K/ M which is calculated to be equal to13.5 6 radians per second.

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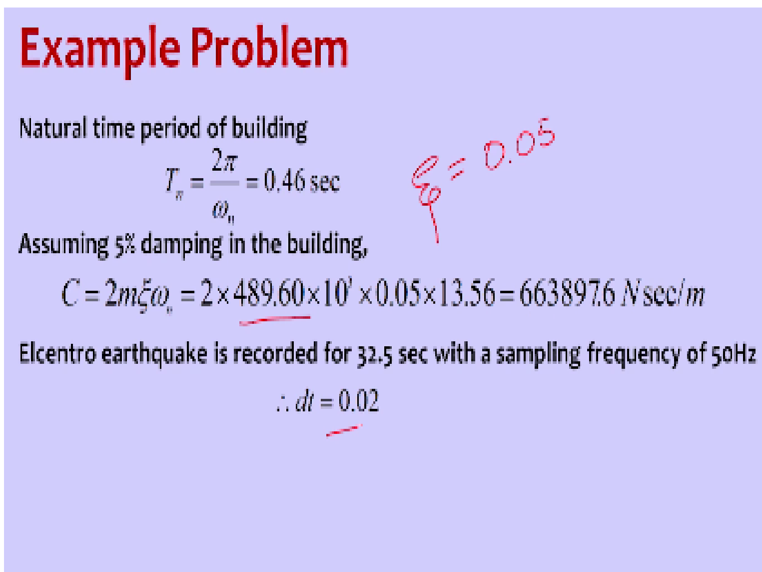

So after completing calculation of natural frequency we go up on calculating natural time period of the building which is equal to 2π / ω n which is coming around 0.4 6 seconds assuming 5% dumping in the building C is calculated which is called damping is calculated as 2 m ζωn where m is 4.89 0.96 x 10^3 and ζ is calculate ζ is 5% so it is around 0.05 so 2 x 489 . 6x 10^3x 0.05 x13.56 = 6 638 97.6 Newton second per meter.

So Alessandro earthquake is usually recorded for 32.5 seconds with a sampling frequency of 50 Hertz that is λ T that is time stepping time step is coming around 0.02 seconds.

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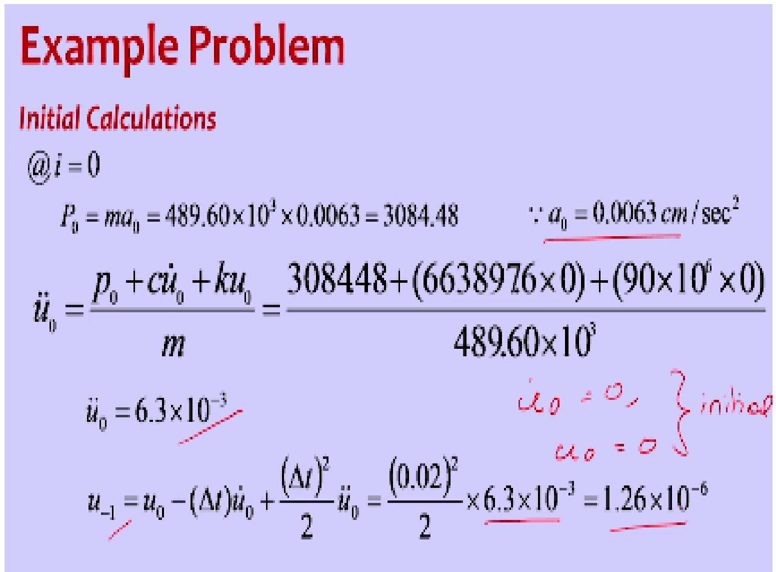

So the initial calculations at I = 0 p_0 is calculated as M x a_0 which is the acceleration that is coming from this earthquake in a 0 is the first value which is multiplied by the mass of the structure is the induced force that is acting on the single degree of freedom system p naught is calculated as 489 . 6 x 10^3 x 0.0063 which is the value which was recorded the first value of the record so 308 4.48 is p_0 where a_0t is 0.0063centimeter per second square.

So this is the first value of P_0 naught and u..0 this is the initial acceleration which is equal to P_0 Cu . 0 + k u 0 / M which is already discussed so calculating this1308 4.48 + 663897.6 x0 where you we assume u. zero as zero and also u of 0 is 0 so these two values initial conditions are 0so initial conditions are calculated the initial conditions are assumed before we start the problem so calculating this acceleration initial acceleration we got around 6.3 x $10^{-3.}$

So in calculating as already discussed in calculating u at zero we need to calculate u - 1 which is the imaginary value using u 0 - λ T x u . 0 + λ T^2 / 2 u..0, now λ T is 0.02the sampling frequency and u. 0 is 6.3 x 10^{-3}which you which we got before so calculating u -1 we get around1.2 6 x 10^{-6}this is displacement at minus one step.

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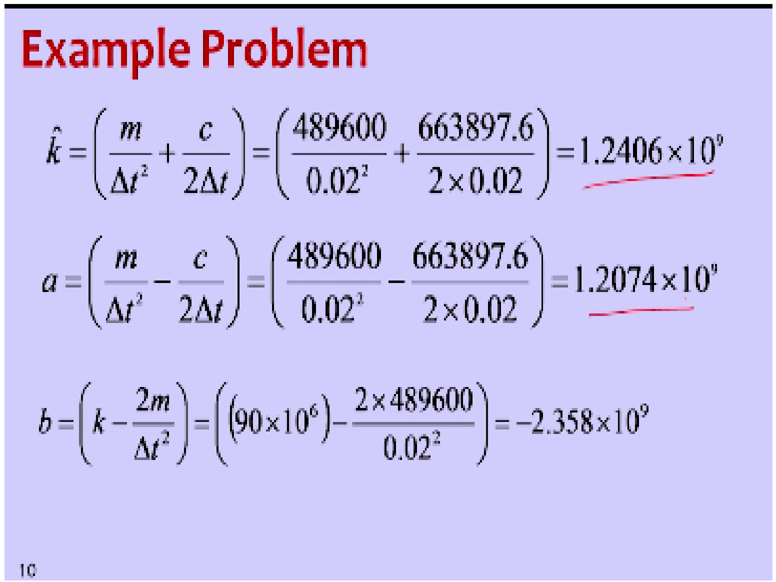

Imaginary step so k^ which is a constant is calculated as m /λ T square + c / 2 λ T so M we have calculated as 4489 600/ 0.02square + double 638 97.6 / 2 x0.02 which is coming around 1.2 4 x10^9 similarly we have another constant which is a which is calculated as m by delta t square - c by 2 x λ T which is coming around 1.20 7 4 x 10^9 similarly B is calculated as K -2m x ω T where K is 90x 10^6 - 2 x 489600 which is a mass of the structure by 0.02 square which is coming around minus 2 .3 5 8 x 10^9. These three values are constant throughout the program or throughout the problem for specific to that problem.

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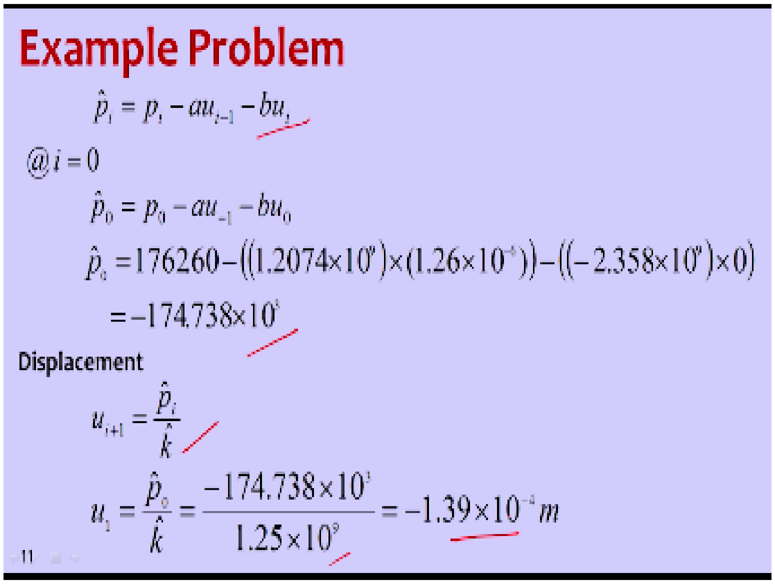

So after calculating the constant values for I^{th} step p^ I is calculated in this manner p I - a x u i - 1 - B x u I that means we need u i - 1 through so at i = 0 that is the first step p^ 0 is calculated as p0 minus a- 1 minus b u 0 so p^ 0 is 1762.6 which is P_0 which we have already calculated minus 1.2 07 4 x10^9which is a value which we got from the calculation and1.2 6 into 10 to the power of minus 6which is u minus 1 which we have already calculated minus B we got as be got as minus 2 . 3 5 8 x 10^9 + u 0 which is the initial value which is equal to 0 that we have assumed there.

So finally we are getting a p^0 value as minus 1 74.7 3 8 x 10^3 from this value will be calculating the displacement as u I + 1 is equal to P^ / k^ so u I plus 1 where i is equal to 0 so we are calculating it u 1 that is equal to P ^0 / K^ that is equal to P^ 1 7-1 74.7 3 8 x $10^{3/}$ 1.2 5 x10 ^{9}which we got already that that which we got from the calculation which is made before so u1 we got around minus 1. 3 9 x 10^{-4} meter.

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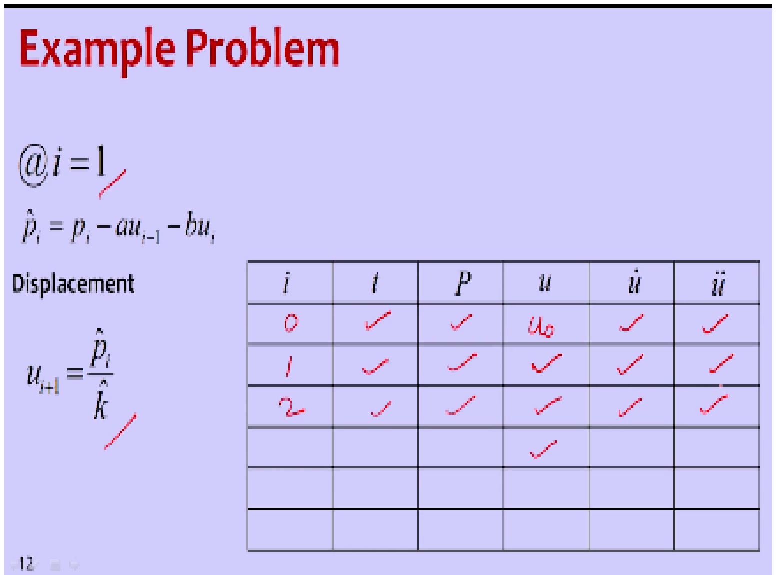

So now this is the one first iteration similarly we can do for some other iterations also we can calculate it i is equal to one where p^ I is defined as P I – ax u I – 1 – b x ui and displacement is calculated I plus 1 step UI + 1 is equal to p^ i / K^ so at time step I is equal to 0 T is given to us and the external force p is given to us and finally displacement with the algorithm which we have discussed now will be getting a getting ui+ 1 will be getting at UI + 1so UI + 1 is equal to P^ I / K^.

So using API that is p_0 and T_0will be getting it u one but velocity will be getting it u u0 itself so these are the initial calculations that we have already mentioned so when ni = 1 T value is given and p value is given and u is obtained it 2, so u2 getting and velocity using this u2 will be getting at velocity that is a tone step and u double dot one similarly for second step we have t 2 p 2 and u 2is up you three is obtained from this equation and u dot 2 and u..2 is obtained here similarly for all other steps.

So as a summary we can discuss this central difference method in case of any random signal which is basically an earthquake force and this has to be kept the most important a point that has to be kept as the indexing of the iterations so when we are going for a computer program we need to take care of them I - 1 and I + 1 value so displacement will be obtained for a one extra value and well aware as velocity and acceleration are obtained at the same value so I I number of steps or T number of steps will be there in velocity and acceleration whereas displacement will have I plus 1steps.

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