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Structural Dynamics
Week 5: Module 04

Numerical Methods based on Variation of Acceleration: Newmark's Method

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Welcome to structural dynamics class. In this class we will study about numerical methods based on variation of acceleration. So and special application in that we study about Newmark's methods. As you have seen in the previous class we had discussed about central difference method. So in central difference method what we were doing is like we are calculating at displacements at each time instant and we converted acceleration and velocities in terms of displacements given displacements okay.

So in Newmark's method what we do is forcing function which is in the form of acceleration or force we will in between two time instance we will calculate other linear variation in between the two time instance or average variation in between the two time instance.

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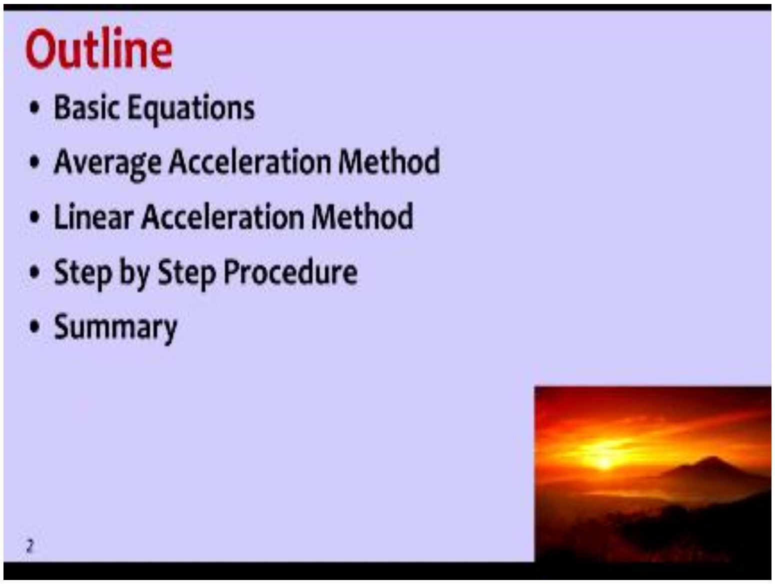

So those two topics we will discuss in this class.

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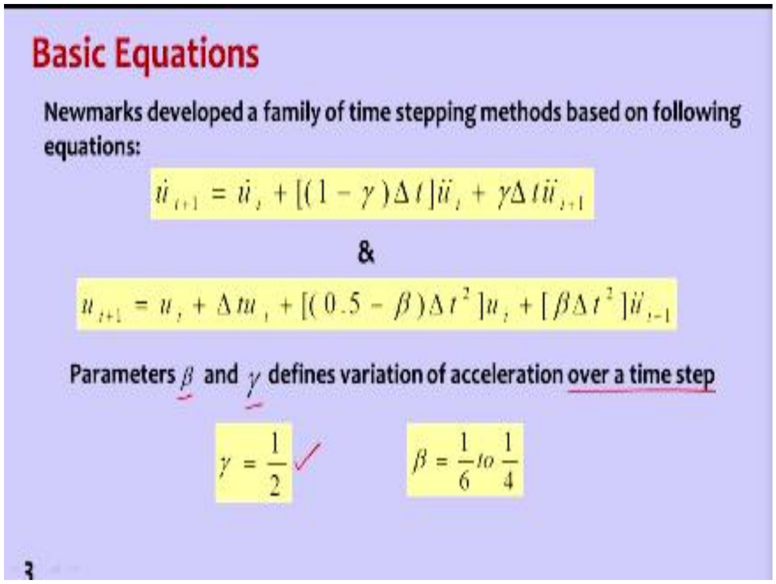

So Newmark has developed a family of time stepping methods based on following two equations, these two equations have been developed that is velocity at I+1 the time instant is dependent on velocity at that time instant ith time instant plus 1-γ into Δt x acceleration at the time instant plus again γΔtu..i+1 response acceleration is needed.

So this is the equation and the displacement equation, so these two equations have been developed for time stepping methods. So this parameters β and γ defines the variation of acceleration over time step. So usual values of γ½ and β value can range from 1/6 to ¼ depending on cases we will discuss these and we will use this β value.

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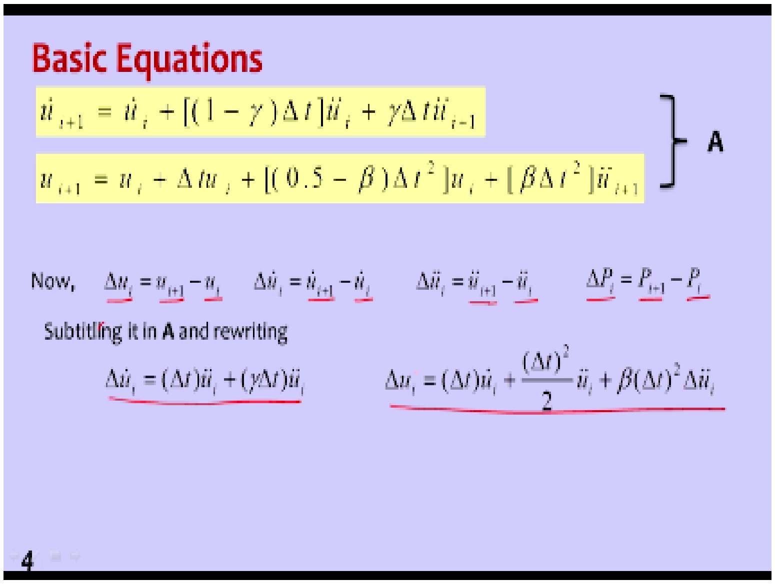

Now these are basic equations as shown you in the earlier slide now if Δ ui is ui+1 –ui and Δ ui. That is a increment in displacement is equal to displacement at a time t_i+1 – displacement at time t_i velocity also increment in velocity is velocity at i+1th time instant minus velocity at ith time instant acceleration increment is i+1 time instant minus acceleration at ith time instant.

And Δ 4 that is increment force increment is force at i+1 time instant minus force at I, now substituting it in the A, so and rewriting the equation so what we get is this for velocity term and this for the acceleration term so first and second so if we substitute these things in the above equation we get Δ u_i, Δ t u..i + γ Δ t u.., and similarly Δ u increment in the displacement increment in the velocity.

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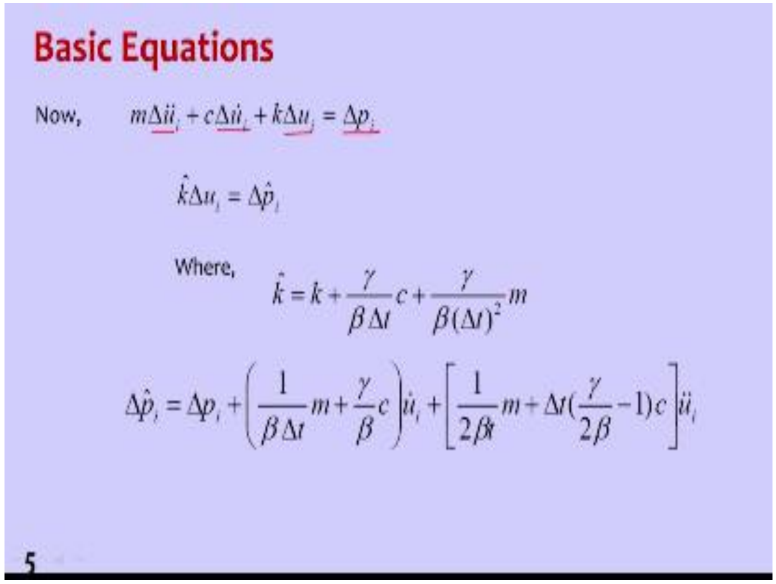

Now basic equation is Mu.. + u. + ku = P we are rewriting that equation in the form of incremental acceleration incremental velocity and incremental displacement = incremental force so converting this equation into this type k^ Δ u/Δ i Δ p^ so whereas k^ is k + γ/β Δ n to c, γ/ β Δ t^2 m and then Δ pi^ is given in this form.

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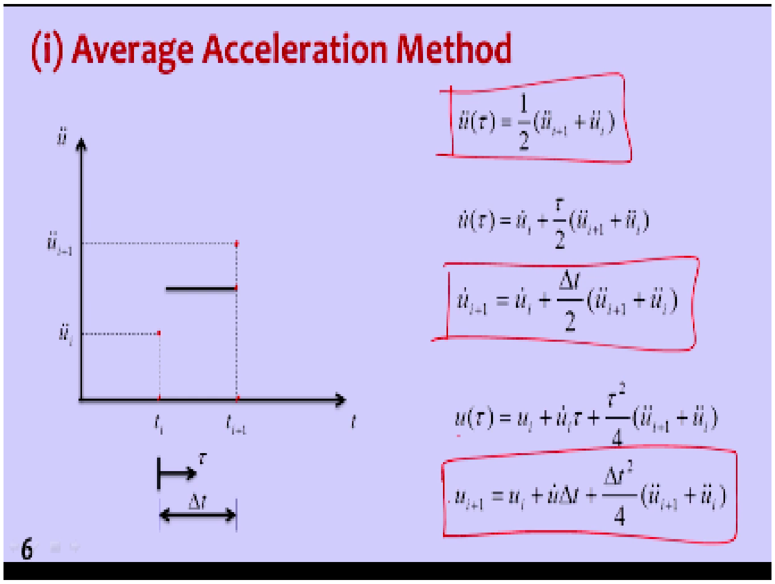

Now let us discuss the average acceleration methods, so what is average acceleration, t = ti this is a acceleration value that is üi acceleration value $t=t_i+1$ this is ü1, sorry, $\ddot{u}_i$ is here $\ddot{u}_i+1$ is here, $\ddot{u}_i$ is here, so the average acceleration means you can see at any time instant in between these two t_i and t_i+1 üτ=1/2 that is average of acceleration at i+1th time instant plus acceleration rate i^{th} time instant as you can see here half of that, so average of that.

Then u̇τ is equal to how do we get that, so you integrated you will get it so $\dot{u}_i$ you integrated and at u velocity at $i+1^{th}$ time instant is $\dot{u}_i+\Delta t/2$ $\ddot{u}_i+1+\ddot{u}_i$, so this is how we get displacement value. And displacement τ sorry, velocity value earlier one and displacement value you can get from this equation again integrating it once and then substituting this time instant, so this is acceleration term this is velocity term and this is displacement term we make use of these things in the further equations.

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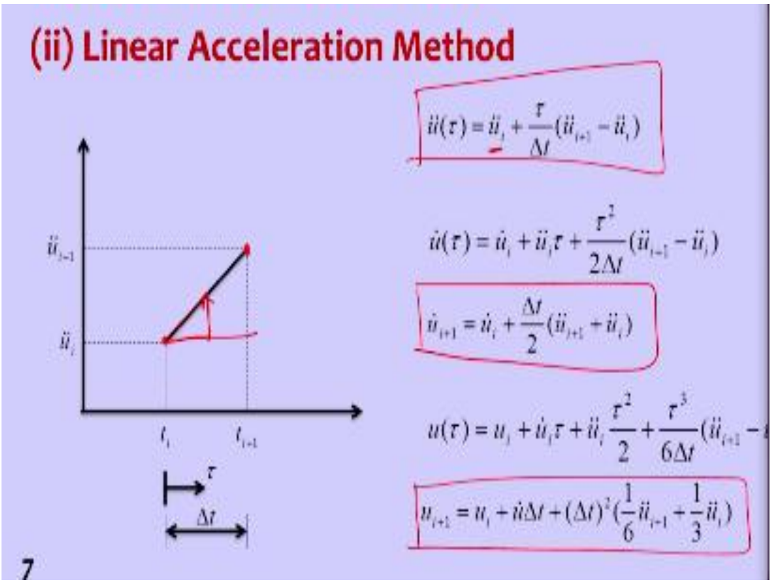

And then linear acceleration method, in linear acceleration method as you can see acceleration at this time instant t_i acceleration at next instant t_i+1 are known to us then how do we define acceleration in between, so in between means what so i^{th} term acceleration will remain as you can see in between this we need this value, so that value can be found out by $\ddot{u}_i$+1-$\ddot{u}_i$ this value divided by Δt into whatever time is there, so this is from similar triangles we can get it, okay.

And then again if you differentiate it you get velocity term so velocity term you are present in terms I and displacement term again if you double differentiate it then you get again displacement term at this time instant. So we need this 3 so linear acceleration method either acceleration we founded.

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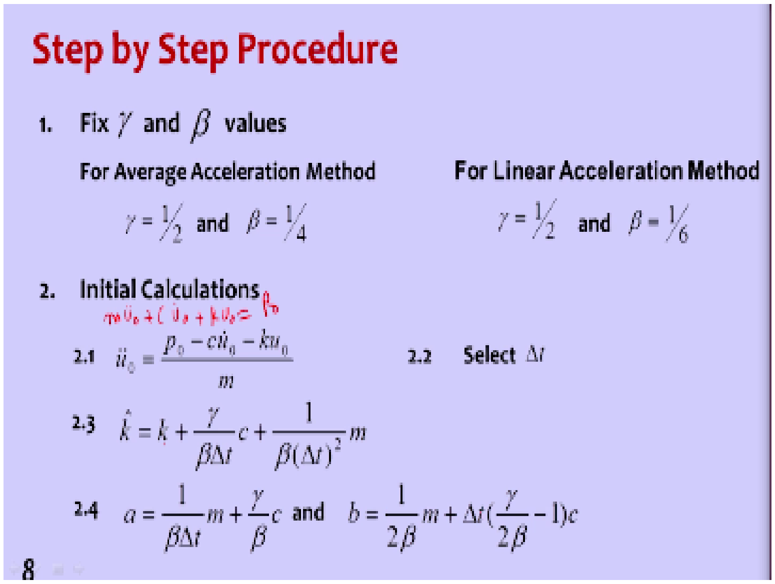

So now let us discuss step by step procedure for this one so first of all we need to fix γ value and β value okay so for average acceleration method γ value is ½ beta value is ¼ for linear acceleration method γ value is ½ β value is 1/ 6 so this methods are very sensitive to this β values so fir initial calculation we need first initial acceleration which we have earlier discussed this is coming from the equation of motion that is mu..0 at time a 0 time instant cu.0 + ku0 = p0.

So from that we get initial acceleration now select Δt and then find out k^ okay k^ = k known stiffness of the system + γ as known here and damping value is known β value is known and time step is known so 1/ β x λ t2 x m so all these ones so this value so this is the constant value k^ and then we find out a and b so a is dependent on like all this parameters mass and damping parameter γ and β value and b also same thing mass and damping parameter γ and β are the time instant. So a b for values we formed out.

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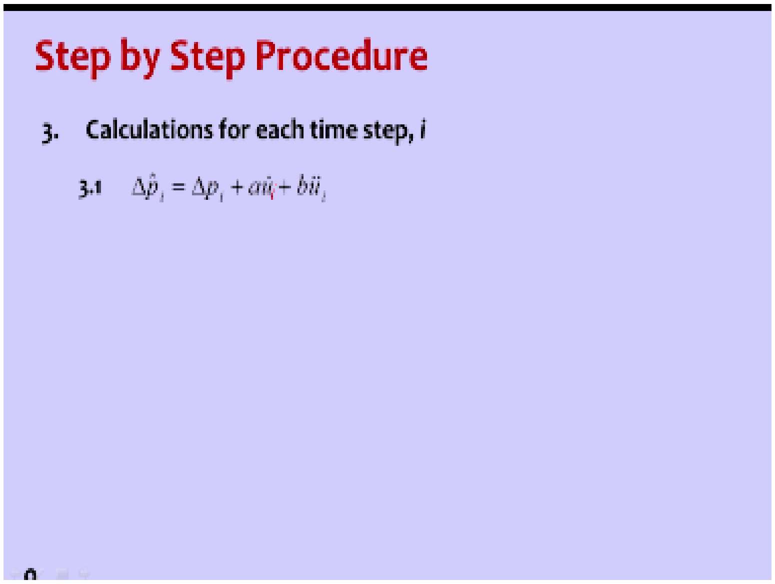

Then after that calculation for each time step so calculation for each time step is λ pi=λpi^= λpi+au.+bu. with this I only okay bu.. so we know what is this a b values.

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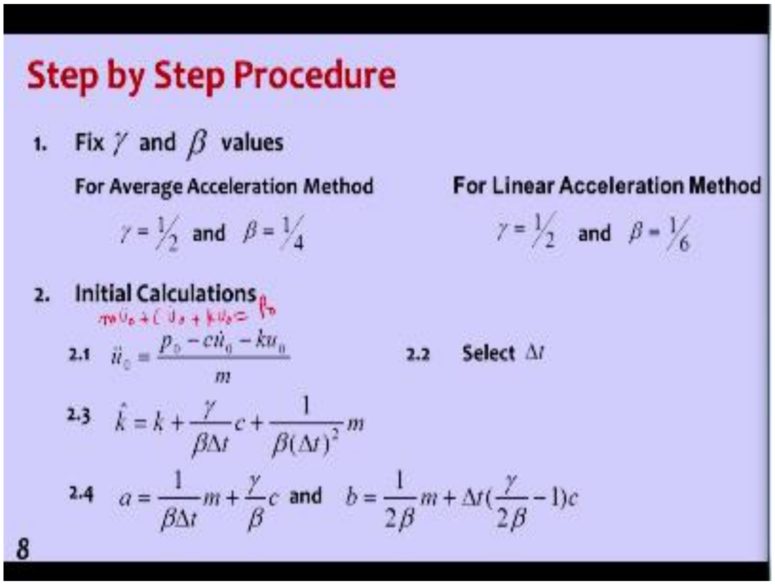

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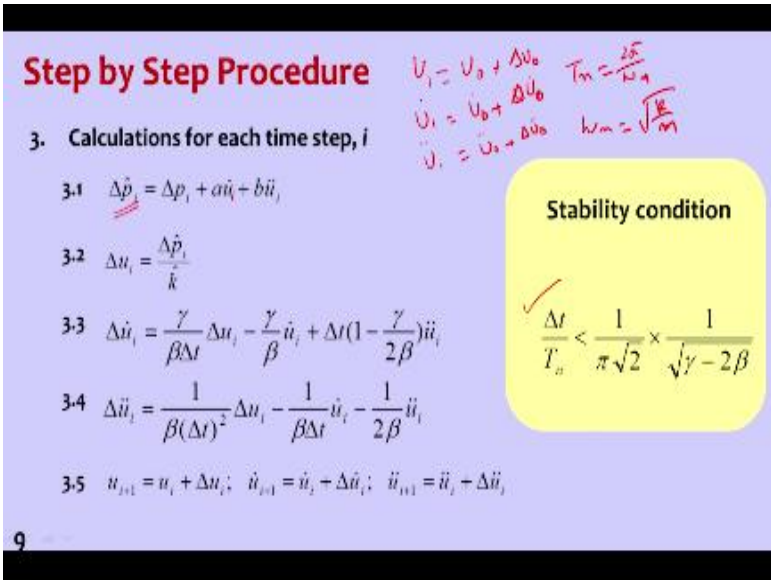

So substitute that ab values and then Δu_i we get =ΔP^i/K^. remember this one Δp is calculated at every time instance were vas k is a constant value which is fixed so it will not change in the analysis then similarly we find out velocity increment then we find out acceleration increment so like this we populated so we want to find out displacement so displacement at U1 = displacement 0+ displacement ΔU at 0 so like this velocity Ui 1 = velocity 1+Δ velocity at 1 acceleration at 1 i= acceleration at sorry 0 acceleration at 0+ Δ acceleration at 0 so that's how we get.

So from the current displacement and the difference in displacement we get the next step displacement difference in the current velocity we get the next step velocity current acceleration and difference in current acceleration we get a acceleration of the next step so that's how we populate the table. So in this one we should remember one thing that ability condition so usually Δt usually is small by Tn.

Tn is a natural period of the system how do we get this Tn. Tn =2Л/Ωn and we get Ωn natural frequency under root k/m so if we get this one Δt/Tm should be less than 1/Л under root 2 1/under root gamma – 2 beta so is the stability conditions so usually this is met but if we are not getting proper result that we can check this condition whether it is mt or not as in discussed in previous class there are three important criteria.

Methods one is convergent the solution should converge to exact solution as a the time step decreases the second one is convergence stability second one numerical should be rounded of otherwise it will blow up so that is a second one and the third one is accuracy so convergence stability and accuracy these three conditions for the numerical method so in summary what we have discussed is the Newmarks method so Newmark has given two equations for velocity and one for acceleration and these thing will be used as a numerical as to find out so acceleration velocity and displacement each time step.

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