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Structural Dynamics
Week 5: Module 03

Central Difference Method

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Welcome to the structural dynamics class.

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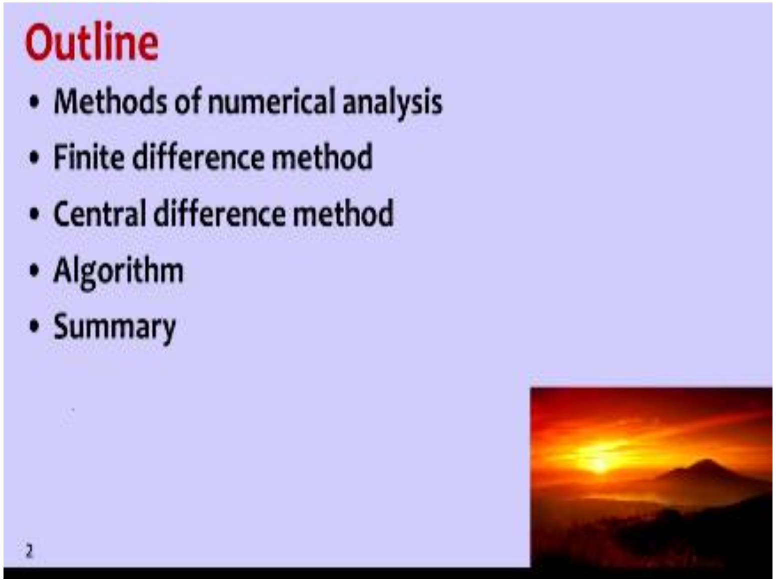

In this class we will study central difference method to the outline of this lecture is we will discuss about methods of numerical analysis, finite difference technique and in that central difference method we will discuss and the algorithm for getting the time history of time history response of a building that we will discuss today.

The methods of numerical analysis, so we have discussed if a structure is given a input motion either round motion or say some motion force applied at the center of the mass how do we get the solution of dynamic differential equation, that is dynamic equilibrium equation or equation of motion. So we need to find out U which is the response of the structure, so for that various techniques are available.

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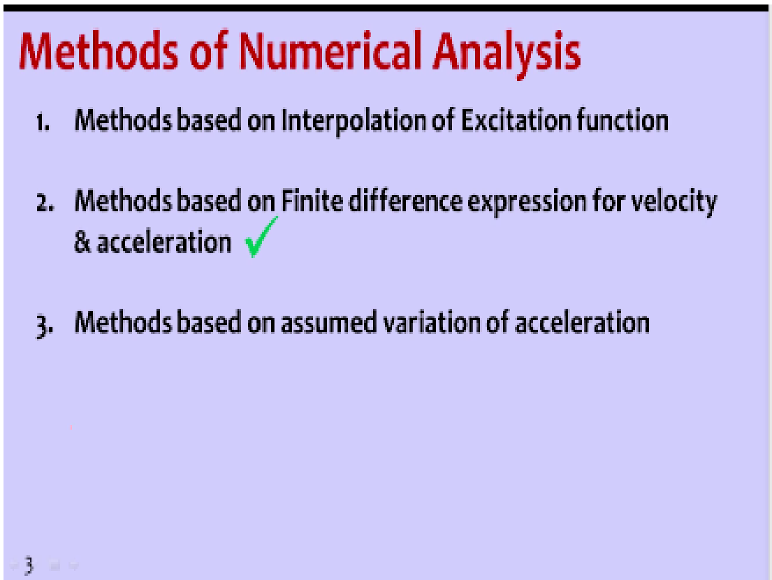

So one method is method based on the interpolation of excitation function. In interpolation of excitation function we divide the excitation into several parts and find the solution. So that we have discussed in the previous classes, and in this class we will discuss about finite difference expression for velocity and acceleration, and the other techniques we are going to discuss in the subsequent lectures that is methods based on assumed variation of acceleration.

So in this class we will discuss about the finite difference expression for velocity and acceleration. And how we make use of that and find the solution of governing differential equation.

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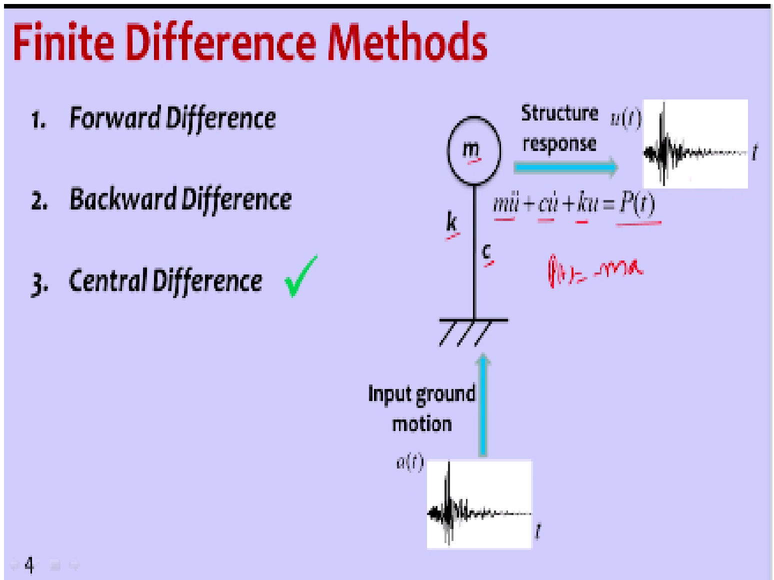

The finite difference methods, so in finite difference method there are three kinds of finite difference methods one is the forward difference, so then the forward difference means what we make use of the function value at I+1, so I is the time step we make use of I+1 and then it in backward difference we make use of the one step before the time step given. And in central difference method we make use of both forward and backward, so we evaluate the velocity and acceleration at a given point by making use of one step forward and one step backward.

So how does it work, let is discuss this. As you can see the structure idealized structure single degree of hidden system is here, mass is given, damping is given and stiffness is given. So acts at inertia force, damping force and stiffness force that combination is equal to the applied force. So this is applied force, so how do we get P, P(t)=-ma, acceleration value is given. So now we need to find out the response of the structure.

So how do we solve this one using numerical methods. Earlier we had discussed using classical method and then Duhamel's integral upwards or convolutional integral upwards and then Fourier transform methods. So these three things we have discussed and now we are discussing the process of discussing this numerical method.

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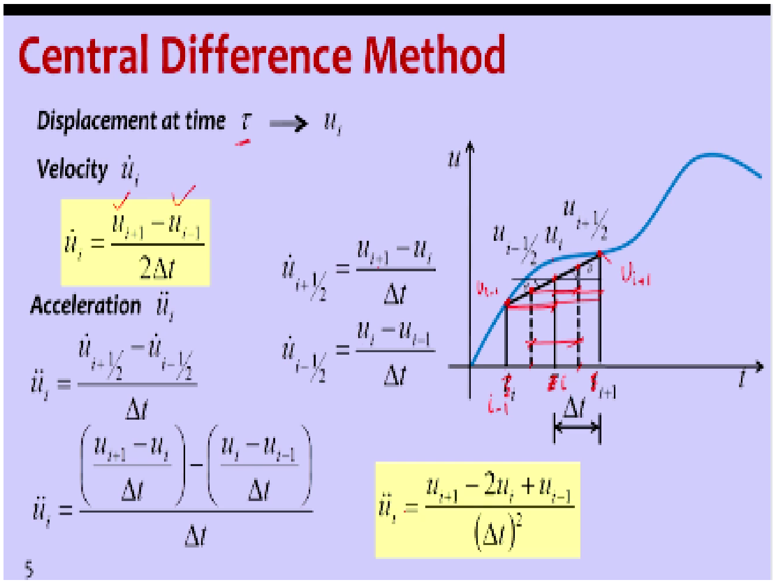

In numerical methods we will be discussing central difference method. So before going to the technique of central difference method how does it work let me explain. So if we take say a time instance which is you can see this is in between ti and ti+1 so if we want to get velocity or displacement at this point, displacement at this point, displacement at this time. So you can see this one, so actually displacement is here and displacement is here these are the displacement value this is displacement function.

So at ti it is t=ti displacement value is i, actually this is i okay. So this one is i-1 this is i+1 so i at i instant U=Ui at i-1 instant U=Ui-1 and i+1 it is u = ui+1 here in middle of this that means ui-1/2 and here ui+1/2 so this is i-1 time instant I time instant i+1 time instant and this is i-1/2 time instant i+1/2 time instant so this five displacement values are available here so now velocity that is u. at I that is equal to.

So how do you get that you can see this one,, velocity at the displacement at this point displacement at this point divided by fold at that t this is delta t this is delta t so this two delta t, so displacement a velocity is nothing but rate of change of displacement, so this is change of displacement so this value is ui+1 this value ui-1 so this change of displacement so ui+1 is here ui-1 is here divide by the time interval between these two points.

That is 2 delta t so we can write the expression of velocity in terms of displacements you can see this one displacement so where we are calculating velocity, we are calculating at a I and we are making use of one step ahead that is ui+1 here and next step one step backward that is ui-1, similarly we get acceleration, acceleration is rate of change of velocity, so we first calculate velocity at this point and the change in velocity divided by the time interval between them.

If you can see here velocity at a ui., u.i+1/2 here velocity at this point so we need velocity at this point divided by this time interval so that is delta t only and now we can we already know how to find out the velocity at any time instant so we substitute that in this one, so that is you can see velocity at I + ½ is this point if you want velocity at this point displacement of this point minus displacement of this point divided by delta.

So you can see this one displacement this point is, ui+1 displacement at tis point is ui and divided by delta t, similarly at this point if you want to calculate velocity we have to subtract this displacement minus this displacement divided by delta t, so you can clearly see this one u. is here ui-1 is here and delta t is this so this give velocity at this point earlier one is velocity at this point.

Velocity at this point minus velocity at this point divide by time interval between them, so that is what is here now if you substitute these u.i+1/2 – u.i-1/2 these two expression in this and the simplify it, how does it how it looks like let us see, so I am substituting this here in place of this and I'm substituting this in place of this divided by, so as you can see in this one ui+1 remains, so now here u_i- u_i so here minus sign here minus sign.

So 2ui and this minus of minus, plus, +ui-1 and then delta t, delta t is there common so this will become delta t^2 so as you can see here u..i = ui+1 – 2ui + ui-1 so acceleration term can be represented by displacement at three points, velocity term can be represented by displacement at two points we wrote expression for velocity, expression for acceleration so we have two expressions in terms of displacement.

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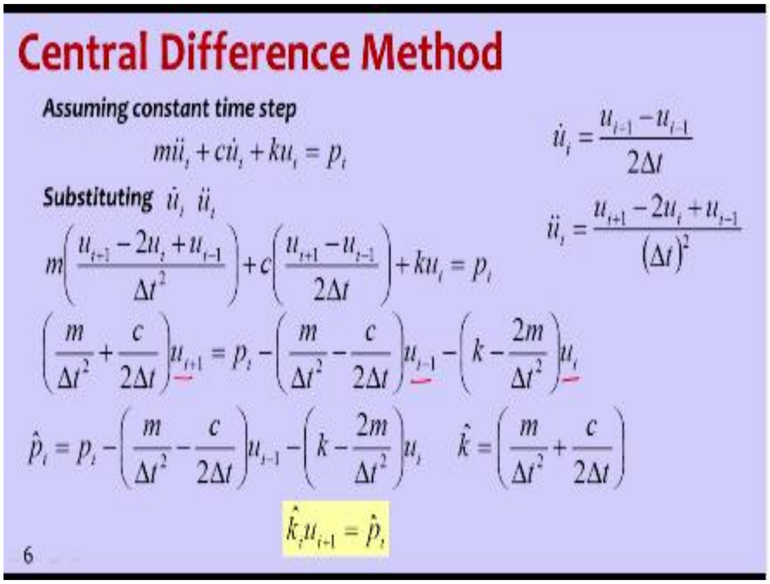

Now we will use of this one in our governing differential equation, so assuming a time step at any time, time p=ti at i[th] time step $m\ddot{u}_i + c\dot{u}_i + ku_i$ is equal to applied force at that time instant. So if we substitute we already know that velocity can be represented in terms of displacement, acceleration also can be represented in terms of displacement we substitute velocity value and acceleration value in this equation and simplify it further.

So substituting that as you can see this one $m(u_i+1-2u_i+u_i-1)/\Delta t^2$ acceleration term we substituted velocity term we substituted and displacement term is $u_i=p_i$. Now in this one how many different terms are there, different terms are there, different displacement values are there, so u_i+1 is one displacement u_i is another displacement u_i-1 is another displacement, so we have three displacements in this, so we have three displacement in this one. So if you simplify that in terms of the displacements so what we get is you can see u_i+1 is here and u_i+1 is here, so that means mass/Δt^2 will come here, and then c/2Δt will come here, so that will give us the coefficient for u_i+1.

We are also sending all these things to right hand side that is p_i-u_i, ui term if I look at it ku_i is here and in this one we have another u_i term, so when it goes that side it becomes –k minus that

$(2m/\Delta t^2)u_i$. and then similarly u_i-1 term will come from this one term will come from this one, another term will come from this one so that is in this one $m/\Delta t^2$ is here and here $c/2\Delta t$ term is here.

So now we have expression rearranged in the form of u_i+1, u_i-1, u_i so again this expression can be simplified and if we rewrite it, all this one is a force term you can see this one so p_i-$m/\Delta t^2$-$c/2\Delta t$(u_i-1 -2k$m/\Delta t^2$)u this is force term and then k value, so k value is this one you can see $m/\Delta t^2$ $c/2\Delta t$ this is, so this entire expression can be written in simple form that is $\hat{k}.u_i+1=\hat{p}_i$ so this entire expression is $\hat{p}_i$ and this expression is u, so in this one we can clearly observe that for getting displacement at i+1th step that is forward step.

Next step, we need p_i of that term displacement is dependent on displacement of that term and the previous term you can see this.

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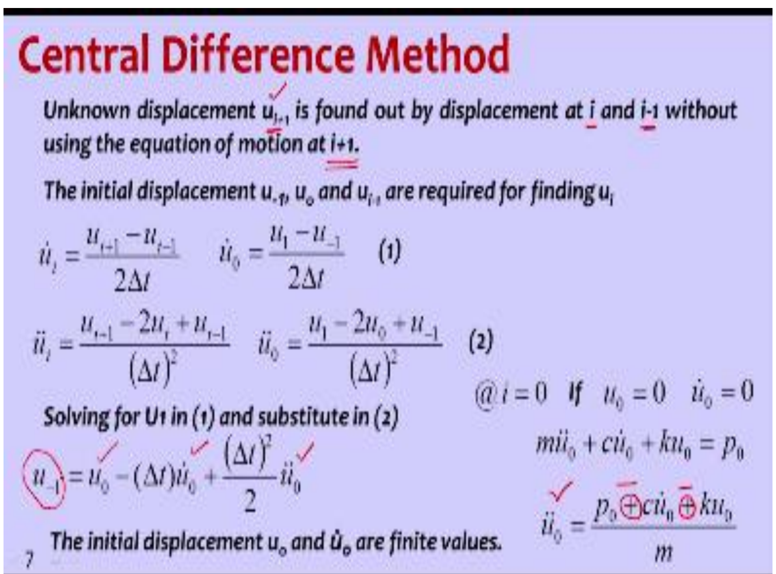

So unknown displacement u_i+1 is found out by displacement at i and displacement at i-1 without using equation of motion at i+1th time step, so that is the beauty of this central difference method, without using expression equilibrium equation expression for a time instant i+1 we are getting this. Now how does it happen, it happens by initial value.

So the initial displacement u i -1 so this is a fix tedious time because time starts at 0 but we need one displacement it negative time and then i-1 required this things are finding ui so if we look at ui u.i velocity ui+ 1 – ui-1/2 Δt. Then if we wanted 0 u0 so I is if you put io = 0 u.0 so if i=0 that will become u1 if i=0 that will become -1 so that means what we need dist\placement at –time so that means before 0 term so this is fixation velocity displacement term we need to find out we will discuss about how to find it out. Then acceleration value is ui+1 – 2 ui + ui-1/ Δ t^2 this acceleration it on in terms of displacements.

Then that one if I want initial acceleration you put u. 0 a time instant 0 so you will get u1 you will get here u0 you will get here u-1, so u0 can be initial condition that is initial displacement if you defined initial displacement u0 can be there and the u1 we need to find out so what we need to find it out solving for u1 u1 is unknown here, solving u1 in this equation and substitute this u1 value in the second equation so if we put if you substitute that solve for this one.

And substitute in the second equation we get u-1 term that is u0 that is initial displacement initial velocity and Δt^2 /2 x initial acceleration. So know initial displacement and initial velocity but how about initial acceleration so for finding initial acceleration a time i=0 if ui0 is 0 and u.0 is 0 before the start of vibration initial displacement is also 0 initial velocity is also 0 so both are 0 .

So if you take the equation of motion at 0 at the time so that is mu..0 +cu.0 +ku0 = p0 so from this we can find out initial acceleration, so that initial acceleration is like this t. this is – sin – c0 this is – sine p0 = cu.-ku/m so we get u..0 from this. So once we get u..0 from this you substitute here then initial displacement you substitute initial velocity substitute then you get a fit tedious displacement term. So now to start the process of getting because this pi is known to us so how to start the process.

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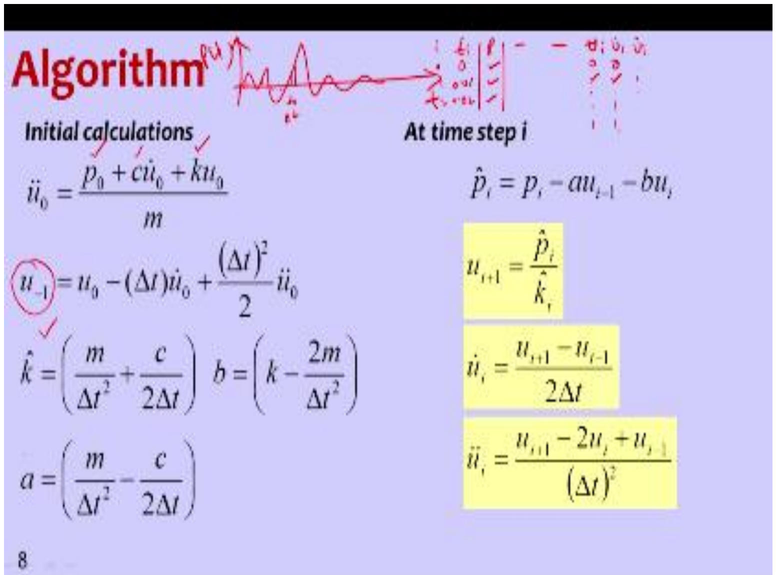

Let us see the algorithm for that is initial calculation we do actually let me explain first this is the force given to us so this can be written in terms of the table p at I p so i=0 at force p =0 this one I = I means time instant this is the time instant different time instant we have all force values defined say 0.01 0.02 the ti I is the time instant so 0 1 2 like that so now we need to find out the parameters and then finally displacement velocity at that acceleration at i.

So this things we need to find out. So now how do we go about initial calculations from the given system so we have we know already p value so p0 value we know damping value is given in the system an stiffness value is given in the system initial velocity we assume initial displacement we assume we get initial acceleration from initial acceleration we get the factitious displacement of term U – one term then once we have the that we find out K M/Δt^2 + c/2Δt.Δt Is the this time instant this is Δt okay It can be fixed though it need not be fixed value at the every time.

But for convince sake we fix it and then a is this b is this value this is for simplification in terms of P. and then P^ we get for that time instance so PI –au i-1 bu$_{i.}$ Pi P^ I =pi –a ui – 1 bui so this Pi will come from this column okay whatever is I step that P we will substitute here and then UI

-1 we will calculate then U i+1= P^i/k^ so K^ we will calculated P^ we calculated we get displacement so when once we get this initial displacement initial velocity is 0.

Once we get displacement we write this value here and then through that we calculate velocity and calculate velocity we write here so this column will be filled and then we write derive acceleration we write acceleration here so like that these columns are populated.

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In summary we have discussed in this class in this class we have discussed about central difference method so equation of motion which is MÜ cu dot +ku =p so in numerical methods we calculate this at ever time instance so MÜ of I step + Cu dot of ith step and KU of ith step = P(I th) step velocity and acceleration we represent in terms of displacement using central difference method.

So these two expressions of the velocity and acceleration we substituted in the governing substitute equation to get a equation so then we could see in that equation that Ui+1 Is displacement time i+1 is dependent on the previous displacement value on the not the forward or the time instance so n tat manner we can calculate displacement time history so I can vary from 0 to n times as long as the forcing function is there so as long as that I can vary say 0 to 1 2 3 4 like that so we can calculate displacement velocity and acceleration at all those time instance and an example problem will be shown in tutorial of this class.

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