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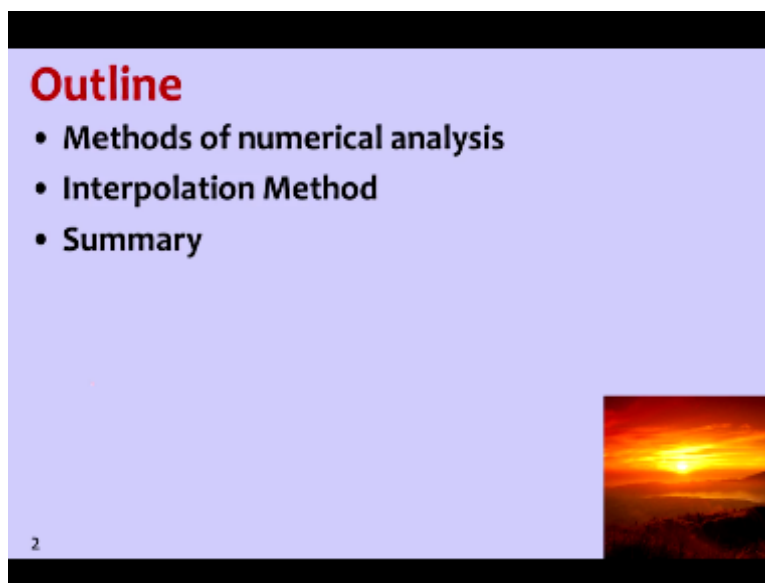
**Structural Dynamics
Week 5: Module 02**

**Methods Based on Interpolation of
Excitation**

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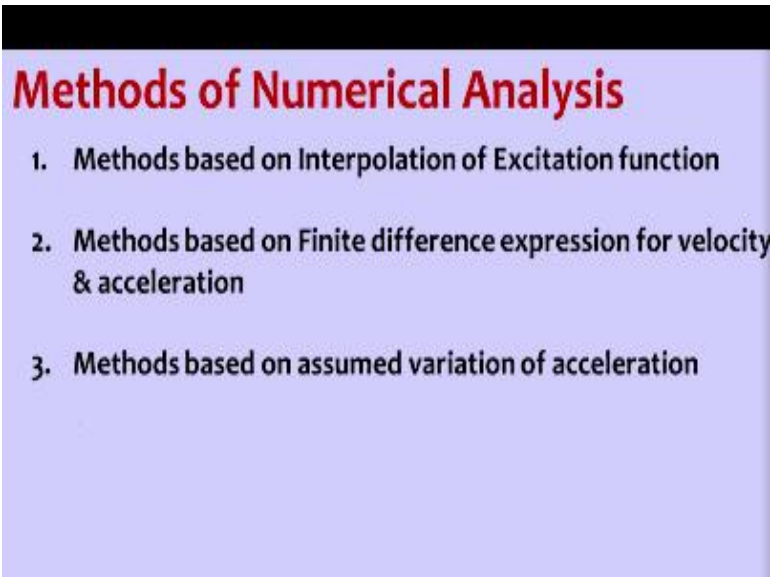
Welcome to structural dynamics class in this class we will study numerical methods so that method based on interpolation of excitation function

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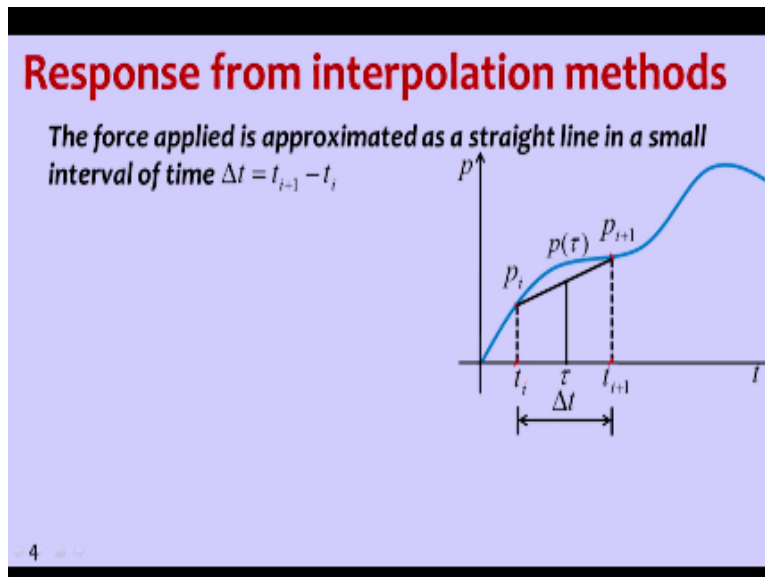
The outline of this lecture is methods of numerical analysis details of interpolation method and then we'll discuss summary.

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So methods of numerical analysis number one methods based on interpolation of excitation functions so kind methods based on finite difference expressions for velocity and acceleration and third methods based on assume variation of acceleration so we will discuss first method in detail in this class so method based on interpolation of excitation function.

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So now what we do in this is the applied force is approximated as a straight line in a small interval of time so that is Δt is equal to $t_{i+1} - t_i$ so this is a force any arbitrary force we are taking as you can see in this figure this is arbitrary force blue line and say we are interested in say P_τ so we have two time intervals that is time marks that is at t_i and then t_{i+1} that is $t_i + 1$ so force at t_i is P_i and force at t_{i+1} is P_{i+1} and the distance that is time interval between these two points is Δt is equal to $t_{i+1} - t_i$.

So we need to understand one thing we are taking this t_i and t_{i+1} from a force okay so that means before that also force is acting and after this also force will act so we are taking one time interval between t_i and t_{i+1} so in between that we need to find out what will be P at any point in between at any point in between so that will work it out.

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Response from interpolation methods

The force applied is approximated as a straight line in a small interval of time $\Delta t = t_{i+1} - t_i$

$$\therefore p(\tau) = p_i + \left(\frac{p_{i+1} - p_i}{t_{i+1} - t_i} \times \tau \right)$$

$$p(\tau) = p_i + \frac{\Delta p_i}{\Delta t_i} \times \tau$$

For undamped system

$$m\ddot{u}_i + k u_i = p(\tau)$$

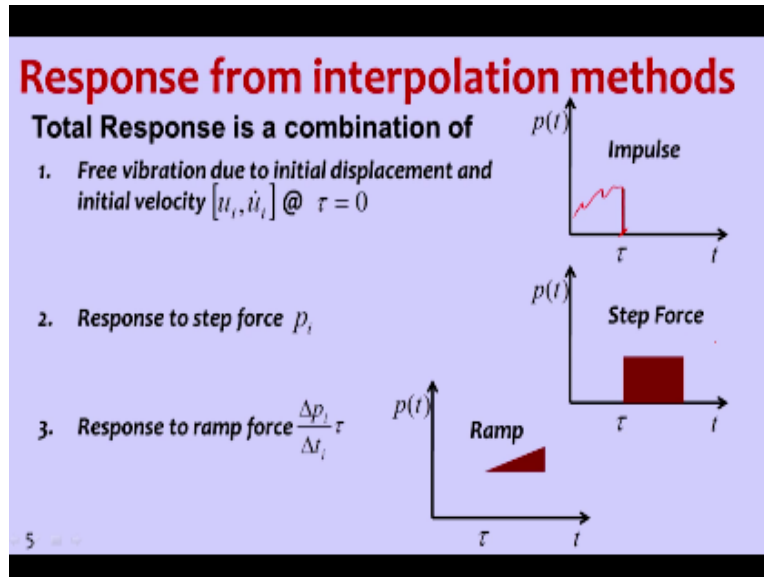
$$m\ddot{u}_i + k u_i = p_i + \frac{\Delta p_i}{\Delta t_i} \tau$$

So as you can ΔT this, this one is drawn separated separately here so ΔT and this is ΔP so ΔP is nothing but $P_{i+1} - P_i$ so is this much quantity and then p_{i+1} is full quantity so we want this so that is Δp and this is ΔT now how do we get force at any time in standing between T_i and T_{i+1} so first of all P_i will anyway be there and then after that we need to find out the slope of this ramp so slope of the ramp is Δp by ΔT so ΔP is $P_{i+1} - P_i$ divided by ΔT that is $T_{i+1} - T_i \times \tau$, τ is measured from here to here.

So τ value can be 0 and time value can be ΔT so time range is 0 to ΔT so if τ is 0 then our P τ will be equal to P_i something like if they say this is 0 then this whole thing will become 0 the p that τ is equal to 0 means p at T_i that will be equal to P_i so exactly same now if τ becomes ΔT if this is a ΔT so Δt and ΔT will get cancelled p_i and p_i will get canceled.

It will be we will be left with P_{i+1} so that means this function holds good for defining force at any τ in between T_i and T_{i+1} here so P_i is equal to ΔP_i by ΔT_i into τ . so now for undamped system if this is a force so how do we apply $m\ddot{u}_i + k u_i = p(\tau)$. So this is a τ in between then so $m\ddot{u}_i + k u_i$ at i plus k you at i that should be equal to $P \tau$. So this is a τ in between then so $m\ddot{u}_i + k u_i$ at i is equal to $p \tau$ function that value we are substituting here $p_i + \Delta P_i$ by ΔT_i into τ .

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Now how do you find the response to this system now a total response is a combination of you can understand three things are there in this one, one is the impulse impulsive at t is equal to τ there is an impulse suddenly there is a jump okay so that means what up to that point free vibration due to initial condition so at this point so whatever our free vibrations are there because of the chopping of the force at that point we need free vibration due to initial conditions and then the second one is response to a step forces this is a step force.

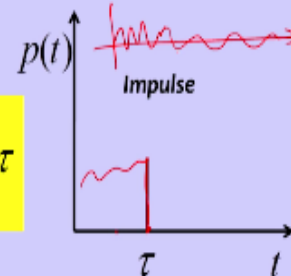
And the third one is response to ramp force ramp Forces ΔP I by ΔT a into τ these a ramped force so three things are their response to free vibration at this location okay because actually the force is coming and stopping at this point so from we are interested at this point so from here there is free vibration that component we want and then suddenly there is a step force at that point and then ramp is also there in that so these three quantities we need to find out.

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Response from interpolation methods

1. Free vibration due to initial displacement and initial velocity

$[u, \dot{u}] @ \tau = 0$

$$u(\tau) = u_i \cos \omega_n \tau + \frac{\dot{u}_i}{\omega_n} \sin \omega_n \tau$$


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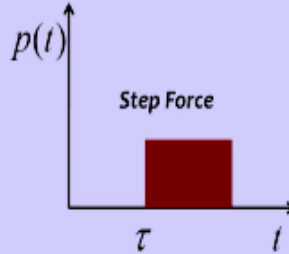
Now first one is response due to initial conditions so initial condition so that means what some force is acting up to this point what are the initial conditions at this point so we need U at I u dotted I at τ is equal to 0 so at how is equal to 0 so we are putting the τ at some point that is at I okay that how is zero at I τ is equal to ΔT at $TI + 1$ so for that we already know from impulse response function that you τ is equal to UI that is initial condition $\cos \Omega n \tau$ plus u dot by you $\Omega n \sin O\Omega$ into τ .

So we need to understand this one this you I and u dot I these are initial conditions due to the application of force up to this point if we suddenly stop force at this point how the system will vibrate so let me explain you that say if this is forcing function T and P of T and say we are suddenly stopping force at this point okay so if we suddenly stop force at that point something like this so that means what response also should follow like that so what will happen is refer response for this. One maybe something like this and then after this it is just free vibration like this so that this component is representing this free vibration.

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Response from interpolation methods

2. Response to step force P_i

$$u(\tau) = \frac{P_i}{k} [1 - \cos \omega_n \tau]$$


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And the second component second component is response to step force so suddenly there is a step force so first one is free vibration component due to initial conditions at τ and then second one is the step force so we already studied from the application of dermal integral or convolution integral that what is the response to step function so you may refer to that lecture for the derivation part of this one okay so p I that is force at I by K so this will use a static response so $1 - \cos \Omega \tau$ is suddenly this force is applied so $1 - \cos \Omega \tau$ will give the dynamic component of that.

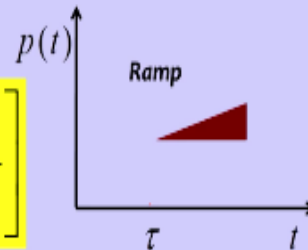
And this is a static component so when sudden force is applied suddenly amplitude will be double usually so this is the step force response due to step force and the third one is the response due to.

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Response from interpolation methods

3. Response to ramp force $\frac{\Delta p_i}{\Delta t_i} \tau$

$$u(\tau) = \frac{\Delta p_i}{k} \left[\frac{\tau}{\Delta t_i} - \frac{\sin \omega_n \tau}{\omega_n \Delta t_i} \right]$$



Ramp force so this also we have studied in the dermals integral you may refer to that lecture so $U \tau$ is equal to $\Delta P I$ by $k \times \tau$ by $\Delta T I$ - $\sin \Omega a n \tau$ by Ωn into $T I$ so this $\Delta T I$ is the time interval which is usually constant and Δp_i also tilted up a i need not be constant but $\Delta T I$ will be constants of that place same ΔT we can use throughout so these three responses we need to add for getting.

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Response from interpolation methods

Total Response is a combination of

1. Free vibration due to initial displacement and initial velocity $[u_i, \dot{u}_i]$ @ $\tau = 0$

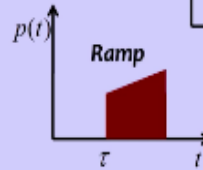
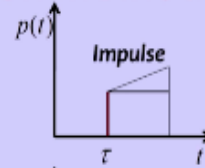
$$u(\tau) = u_i \cos \omega_n \tau + \frac{\dot{u}_i}{\omega_n} \sin \omega_n \tau$$

2. Response to step force p_i

$$u(\tau) = \frac{p_i}{k} [1 - \cos \omega_n \tau]$$

3. Response to ramp force $\frac{\Delta p_i}{\Delta t_i} \tau$

$$u(\tau) = \frac{\Delta p_i}{k} \left[\frac{\tau}{\Delta t_i} - \frac{\sin \omega_n \tau}{\omega_n \Delta t_i} \right]$$



The response using interpolation methods so first one is what free vibration due to initial displacement and initial velocity at time t is equal to τ or at that that point and then second one is at the response to step force P I this is a sudden jump and third one is response to ramp so actually total forces this as represented in the third diagram total forces this so we need to add these three things to get the total response at using interpolation methods.



In summary what we have discussed in this class so response using interpolation method so in that three steps are the number one at any point of time we need to for finding response at any point of time so we need to first find the response due to initial conditions at that point that is one second one is response due to step force in the time interval and then third response due to ramp force in the time interval and then add these three things so we will get the response between time T_i and $T_i + 1$ so like that we can go from zero seconds up to the up to the time our till excitation loss.

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