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Structural Dynamics
Week 5: Module 01

Numerical Methods

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Welcome to structural dynamics class, in this class we will study numerical methods.

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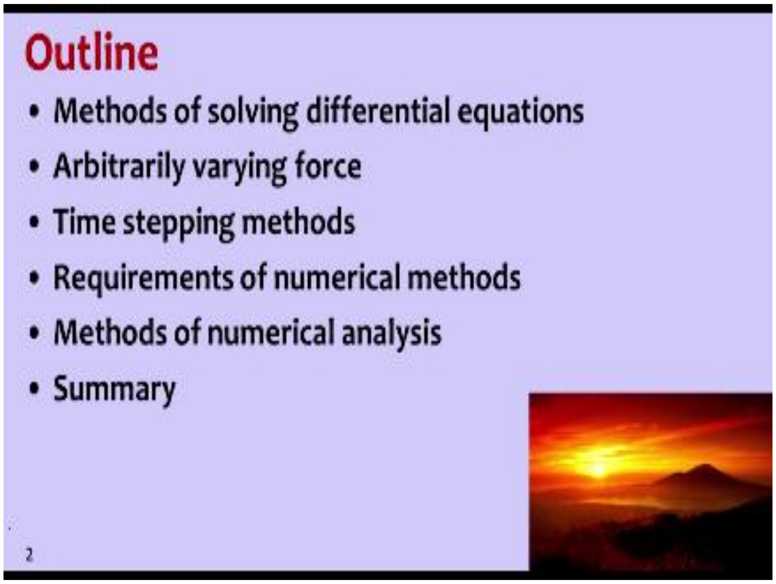

Let us go to the outline of the class, so methods of solving differential equations so we will discuss about that and arbitrarily varying force, time stepping methods and what are the requirements for numerical methods and methods of numerical analysis. So these things we are going to discuss in this class so methods for solving differential equations.

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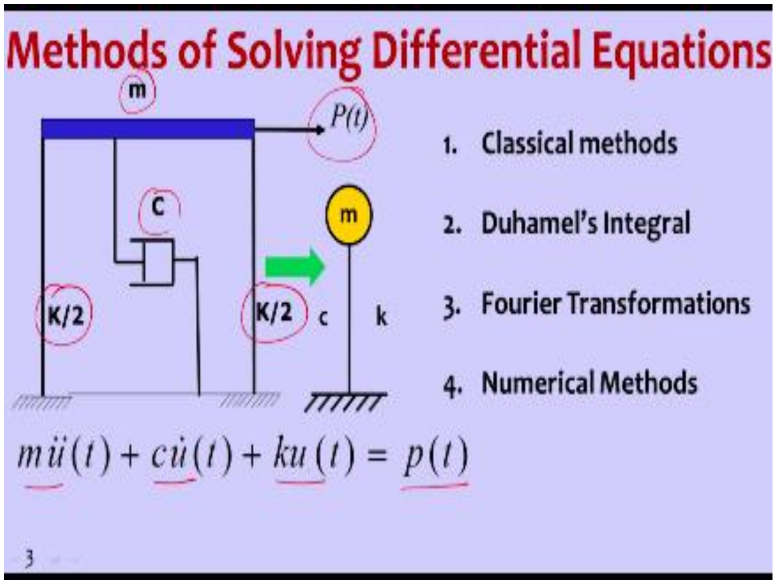

So this is a system a single degree of freedom system where mass of slab is present it is supported by two columns and damping as you can see here mass is present here stiffness of supporting columns and damping to present in the system and then excitation, okay. Then we can idealize this system as a single degree of freedom system, so the equation of motion or governing differential equation will be of the form $m\ddot{u}+c\dot{u}+ku=p$ so p is a force and this $m\ddot{u}$ $c\dot{u}$ and ku are inertia force damping force and stiffness force respectively.

Now when p acting on the body we know how to solve this one, so these are the methods so there are four methods for solving and then find the solution, solution got me what we want is , u(t) we want to find solution. So first one is classical method, second one is Duhamel's integral method we have discussed that and the third one is transform method so we have discussed Fourier transform.

How we convert time domain into frequency domain that we discussed in Fourier transform method and Duhamel's integral method what we did was we took the random forcing function and then divided that into a series of impulses and for one unit impulse we found the response

and then from there we found out the response to impulse and then integrated over a period so wherever we want can get the response so that is what we did in the Duhamel's integral.
And classical method of solution also we have discussed where forcing function is sin our cos, so now in this class we will discuss numerical methods.

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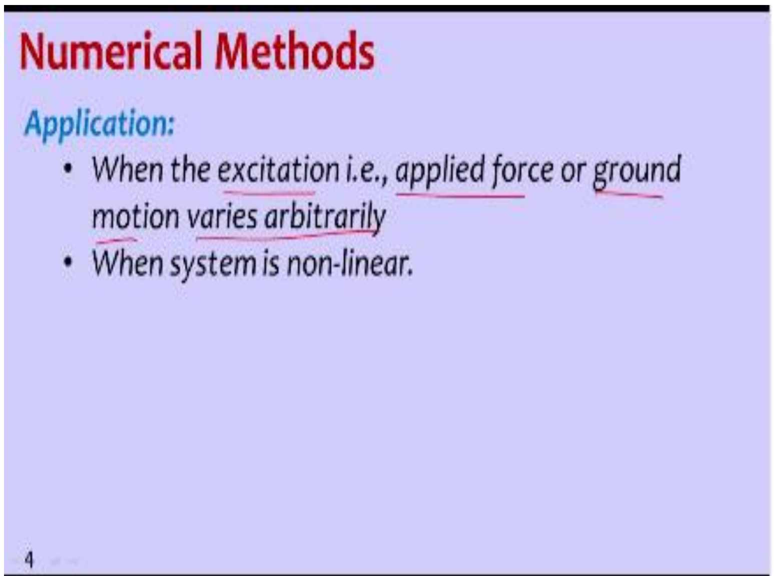

So what are the applications of numerical methods, so when do we apply numerical methods. So numerical methods we apply when excitation so that excitation are the applied force or ground motion varies arbitrarily, when it is varying arbitrarily so we cannot go for classical method or other methods then the second one is when the system is nonlinear so all the other three methods can be more or less can be used to find out the response of arbitrary force also but when system becomes nonlinear then we have to rely on numerical methods.

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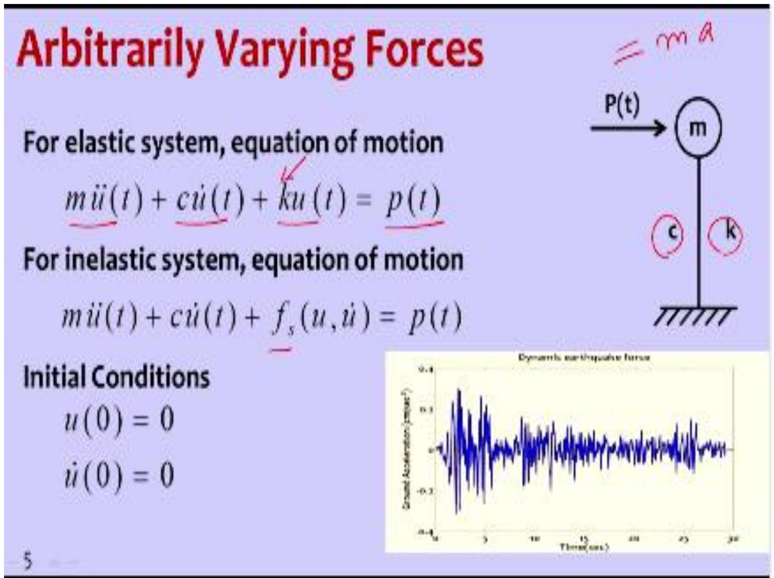

Now utterly varying forces so let us discuss that so this is a single degree of freedom system and idealized a single degree of freedom system where mass is there, stiffness is there and damping is there so as a structural engineering application if you take single storey building so all the mass we assume it to be concentrated at the roof level so that is at the center of mass because more mass is there at that location and the all the supporting say columns and walls put together or offering stiffness k and the damping which reduces the energy of vibration so we call that as a damping factor that is dumping c, and then force P is acting on the system. So if force is so this one in terms of ground acceleration it is acting so that force can be calculated as P is equal to mass times acceleration so that is force.

A force is this random force is acting how do we find out, so for elastic systems equation of motion will be we know this one for elastic systems equation of motion will be $m\ddot{u}$ $c\dot{u}+ku=p(t)$ so in this one the stiffness is elastic forces random. Whereas for inelastic system force that is a stiffness force is dependent on displacement as well as velocity here it is dependent only on displacement, so we can solve this one by using initial conditions that is $u(0)=0$ and then $\dot{u}(0)=0$ by using initial conditions we can solve this one.

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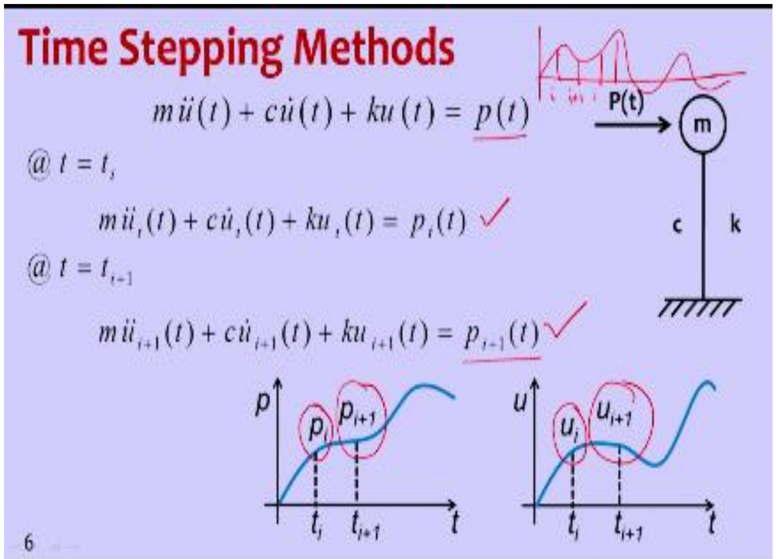

Now time stepping methods, so in time stepping methods like we discretized the force forcing function that is force into different, different time intervals so like this we discretized that and then give i, i+1, i+2 that is t at it, t at i+1, t at i+2 like that. So forcing function and then you can see this is the response at time $t=t_1$ this entire equation will become m into acceleration at $\ddot{u}_1$ and damping into velocity at time $t=t_1$ that is $\dot{u}_i$ and displacement at that time $t=t_i$ that is u_i multiplied by k that should be equal to force at that time instant.

And then if say time $t=t_{i+1}$ then this equation should hold good, that is acceleration at time t_{i+1} velocity at t time i+1 and displacement at time i+1 so that should be equal to all these products should be equal to four set time i+1, as you can see in this one so force at this point okay, should correspond to the displacement that i and force at p_{i+1} is corresponding to or displacement at i+1 is corresponding to for size i+1.

So here you can see this equation of motion for time $t=t_i$ and here the equation of motion for time $t=t_{i+1}$, now how do we solve this one.

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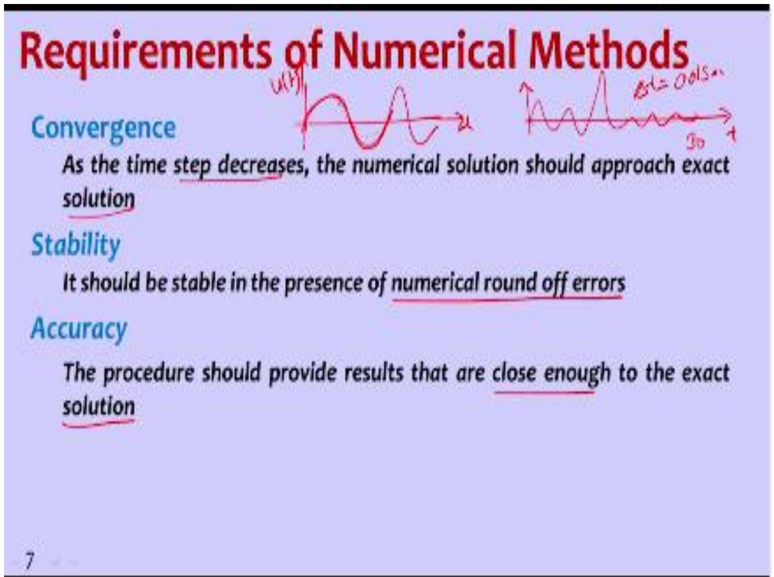

We have written equation, but how do we solve this one and now when we are solving like dynamic equilibrium equation using numerical methods what are the conditions. So first one the solution should converge so what is the meaning of convergence as the time step decreases the numerical solution should approach exact solution, so that means what if say excitation is for example say for 30 seconds, for 30seconds if my say time step Δt is say 0.1 second so that means out how many times it is 30 /0.1 second so that many times I am solving this equation.

As a time step is decreasing solution should approach to the exact solution whatever classical solution gives it should be near to that solution. But as time step increases so what happens so we will get approximate solution not the accurate solution so convergence means as time step decreases it should converge the solution should converge, that means what solution should be so for example if displacement function the original displacement function is this, the final solution is that is t and u(t).

Then approximate solution should be near to this one and as the time step is decreasing then it should match the exact solution that is what is said in the convergence. Then the second one is stability, so stability means what it should be the solution should be stable in the presence of numerical round off errors. So this numerical round off errors will occur but the solution should be stable enough, so that means what wherever we are getting solution that should be near to the accurate one, it should give approximate value but it should be near to the accurate results.

Then the accuracy third one, so the procedure should provide the results close enough to the exact solution. So these three things are there as a criteria for numerical methods as a requirement for numerical methods one is convergence, second one is stability and third one is accuracy. So solution should converge to exact solution as the time step decreases that is convergence and it should be stable in the vicinity of the solution that is a stability and the accuracy is the procedure should produce a close enough results that is accuracy.

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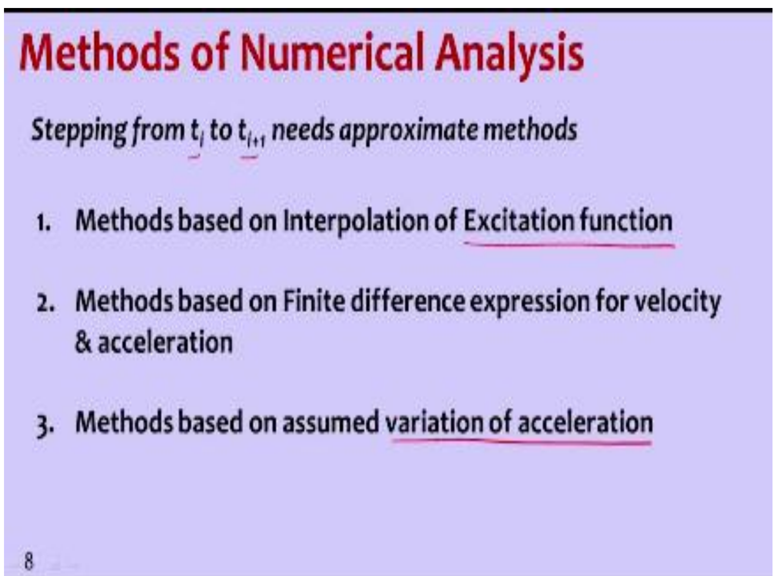

Now what types of numerical methods are available for solving dynamic equilibrium equation, so first one is like stepping from time t_i to t_{i+1} needs approximate methods so what are the approximate methods. So there are methods which are based on interpolation of excitation function, so as force is an excitation function so if we start interpolating this excitation function, so that means what we break down into small, small components and try to find out solution for each component and later add them.

So that is the first one and the second one is methods based on finite differences of velocity and acceleration, velocity and acceleration. So we have this quantity displacement which we are trying to find out as a solution of the dynamic equilibrium equation, okay. So then the derived quantities are velocity as well as acceleration, so methods based on finite difference expression for velocity and acceleration this is second type of methods.

So in this we discuss about three methods that is forward difference method, backward difference method and central difference method so for our dynamic equilibrium equation solution for finding solution to dynamic equilibrium equation we use central difference method in this case, and the third one is methods based on assumed variation of acceleration. So in the forcing function acceleration is there, so force is equal to mass into acceleration this acceleration is the actual input.

So now this acceleration how it is varying, so in between two time intervals say time t_i and time t_{i+1} how the acceleration is vary it to vary linearly are we assume average between these two points, so whatever acceleration is there at t_i and whatever acceleration is there a t_{i+1} we average it and take that average in between these two time interval or there is another way is we linearly vary that, so that means whatever is acceleration at t_i and whatever is acceleration is t_{i+1} so we will not use the forcing function but we will take a linear variation in that so care should be taken such that the time interval is very small so as we are not losing the accuracy of the problem.

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So in this class we have discussed the necessity of the numerical methods okay, so what is the necessity of numerical methods. Numerical methods are needed in two conditions okay, number one when force is random in nature, number two when the system is nonlinear so in these two cases we will use numerical methods and then the types of numerical methods are the essentials of numerical method we have discussed one is a convergence, second one is stability and third one is accuracy and then later we have discussed the types of numerical methods available.

So first one is interpolation of excitation function, second one is finite differences between the velocity and acceleration and third one is assumed a variation of acceleration.

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