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Structural Dynamics
Week 1: Module 04

Vibrations Of SDOF System

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Welcome to the module of vibration of single degree of freedom systems. So the outline of this module is will introduce.

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I will introduce you to this module and then Vibration of buildings, Single Degree of Freedom Systems, External force and will formulate the Dynamic Equilibrium equation using Newton's Second law of motion and also using Force Approach.

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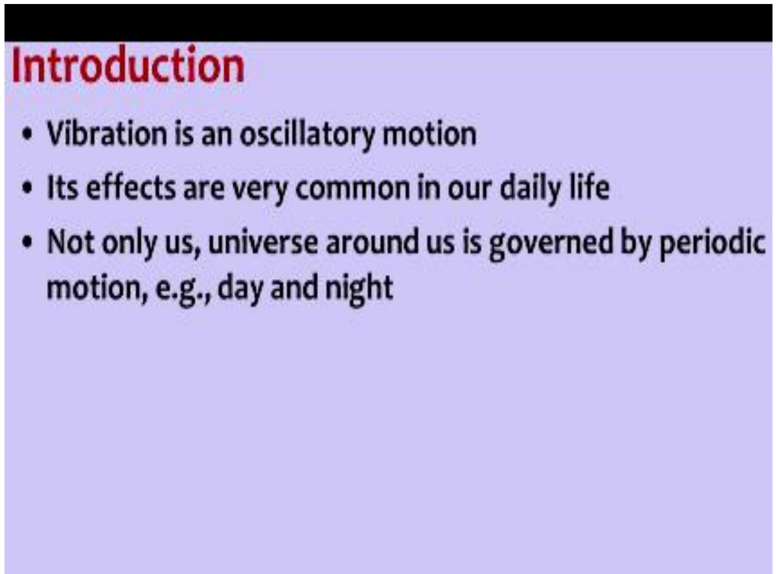

Introduction usually vibration is oscillatory so oscillatory means any to and fro motions so that simple harmonic motion is called oscillatory motion. And its effects are very common in our daily life so for example not only as universe around us is governed by periodic motion that's that is day and night. So this cyclic motion is continusly taking place you see any phenomenal in that oscillatory motion is already present.

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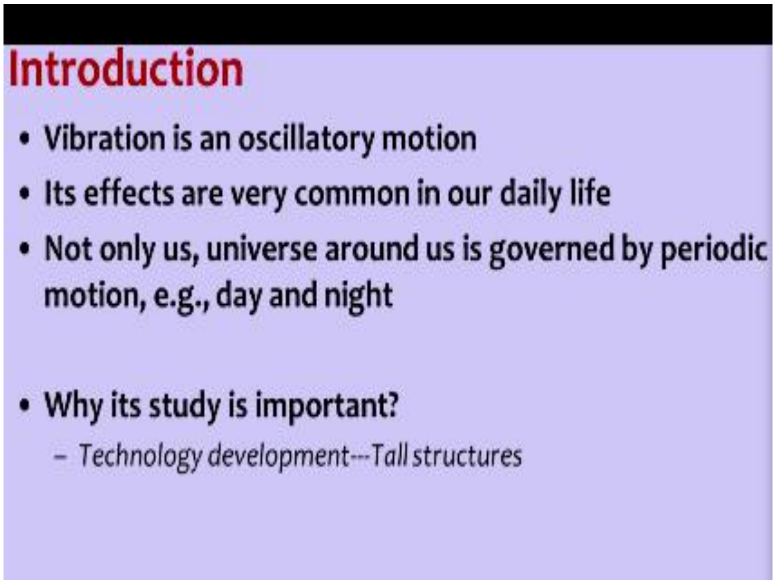

So why we need to study this vibration or why its study is important its study is important because of the construction these days is going towards tall structures so that's is because of the technology development so these tall structures are settable for vibrations from earth quakes or as well as wind and then machines which are installed on floors of the building and these are the forces which.

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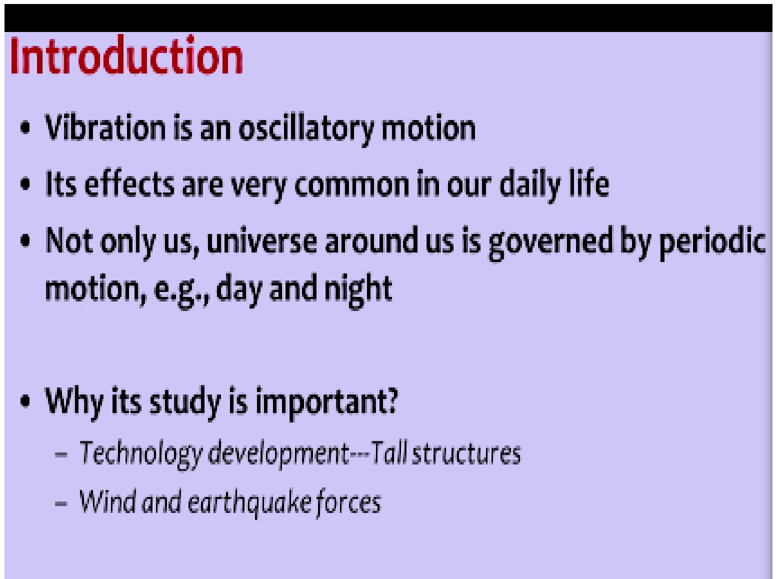

Might come onto the structure.

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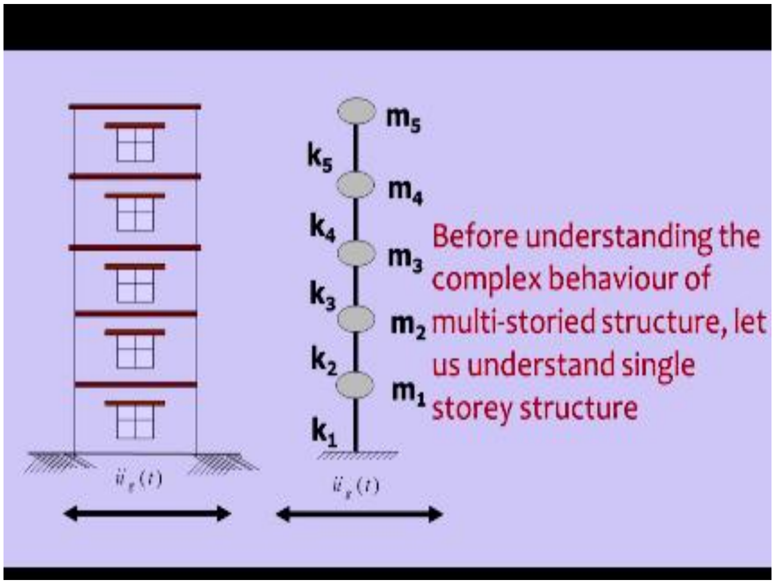

Now if you take a typically building which subjected to earth quake ground motions so as you can see here this

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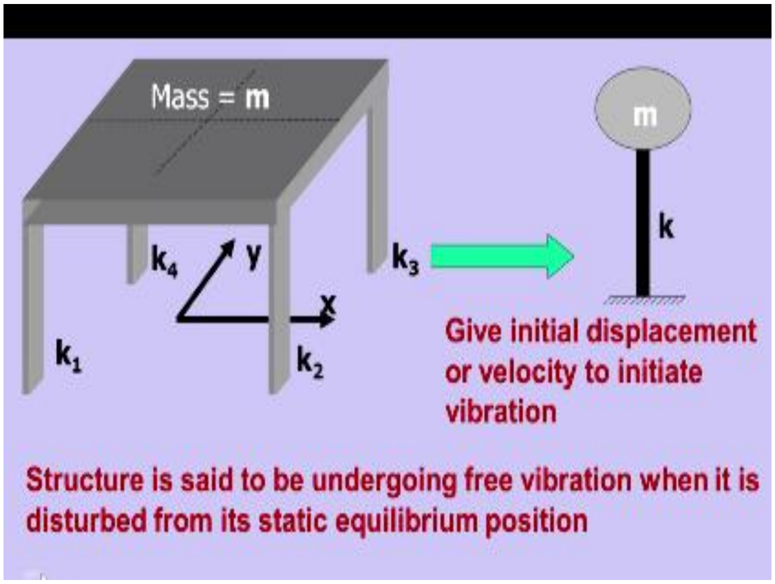

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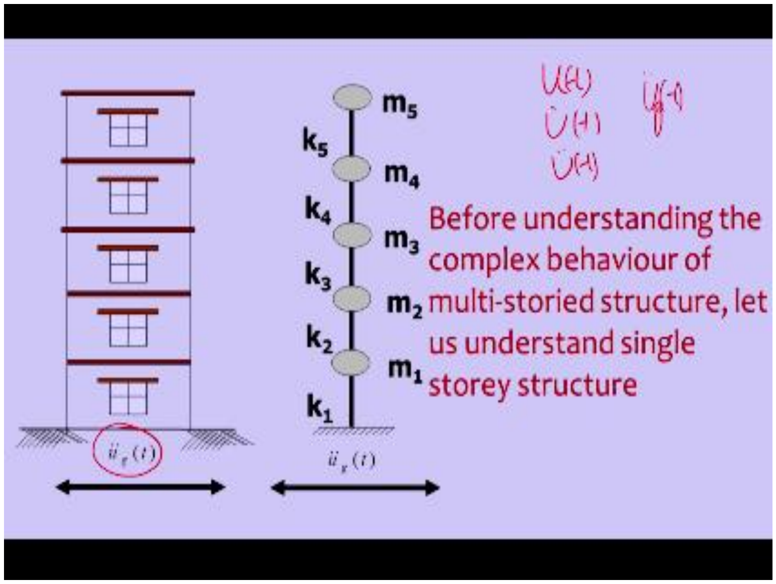

The U double dot G so we, call U as displacement U. which is a deviatory of displacement as velocity and U.. Is a acceleration of the structure and when we use suffix G we call that as acceleration of the ground. So this building is subjected to earth quake ground shaking so a multi-story building. So this multi-storied building can be idealized as many degrees of freedom.

So like mass 1, mass 2, mass 3, mass 4 and mass 5 so five degrees of freedom system like supported by columns which are having stiffness from k1 up to K5. So this is a complex system before understanding the complex dynamic behavior of multi-degree of freedom system. Let us discuss single degree of freedom system.

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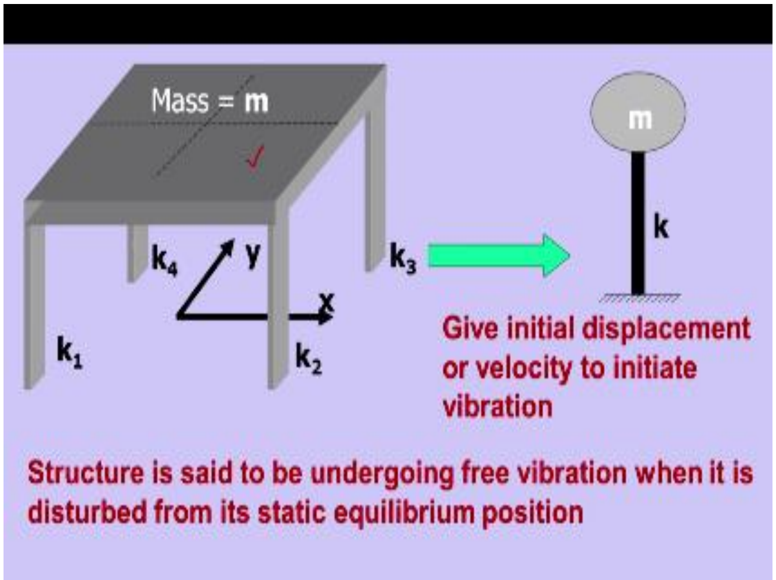

So were one story structure is a typical example of that so as you can see here like this single story building or say like single story structure where slap is supported by four columns slap and four columns so column1,column 2, column 3 and column 4. Four columns are there so this structure can be given initial vibration or initial displacement or initial velocity. So said the oscillation, so said the oscillation so it will undergo a free vibration if we just give initial condition or initial displacement or initial velocity and leave it.

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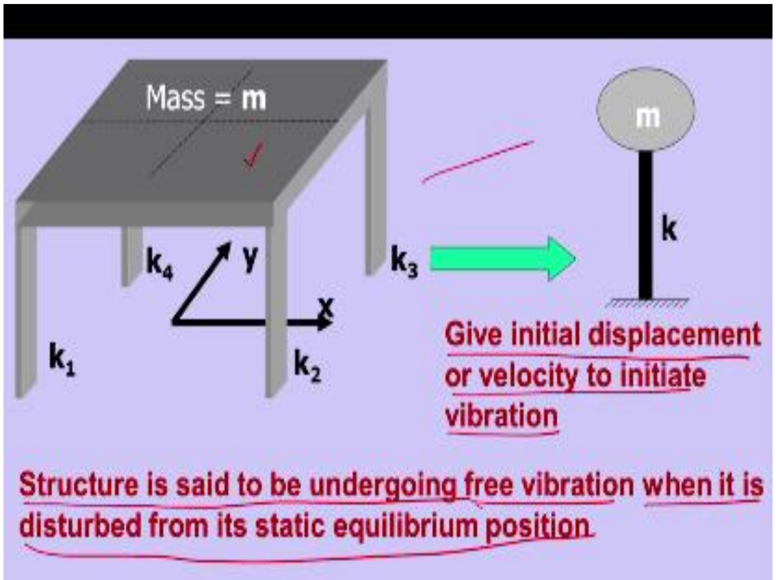

So give an initial condition that is either velocity or displacement to initiate vibration. And the structure is said to undergo free vibration when it is disturbed from its static equilibrium positions so this idealized system can be represented by mass and stiffness mass and spring system.

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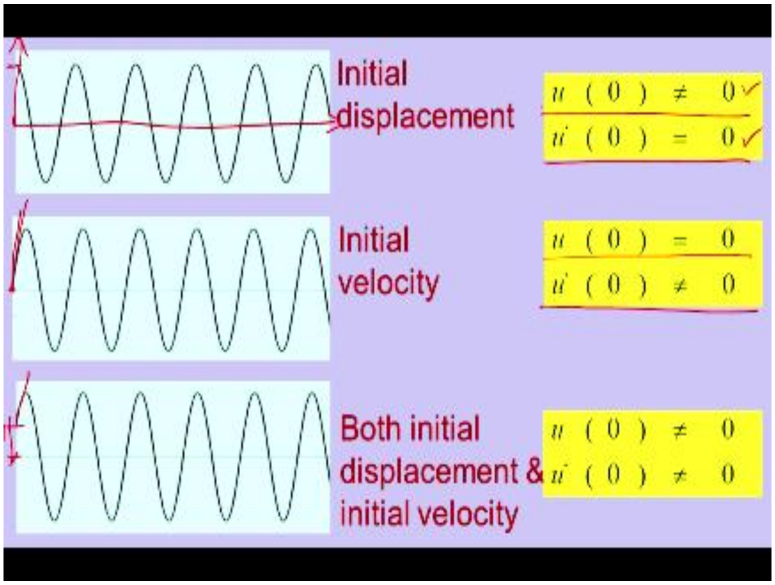

Now initial displacement, initial velocity or both so if we give initial displacement here and as you can see in the horizontal axis this is time scale and when we are giving a time T=0. Some initial displacement so which we are calling it as initial displacement. Here U is 0 U at time T=0 ≠ 0. So that means some non zero displacement is given and U. Which is velocity at the initial time.

The time T=0 is 0 so initial displacement is given and the system oscillates according to its frequency around time scale and then when it comes to initial velocity here initial displacement is zero and we are giving initial velocity initial velocity means you can see tangent at this point and at the time T=0 is not 90°. So it is and you can clearly see here that well displacement is 0 at this point so in the first case we are giving initial displacement but initial velocity is zero in second case initial velocity is there but initial displacement is zero.

Then third case can be like both initial displacement has some value for here to here and then initial velocity also has some value so initial displacement, initial velocity and initial velocity along with initial displacement both.

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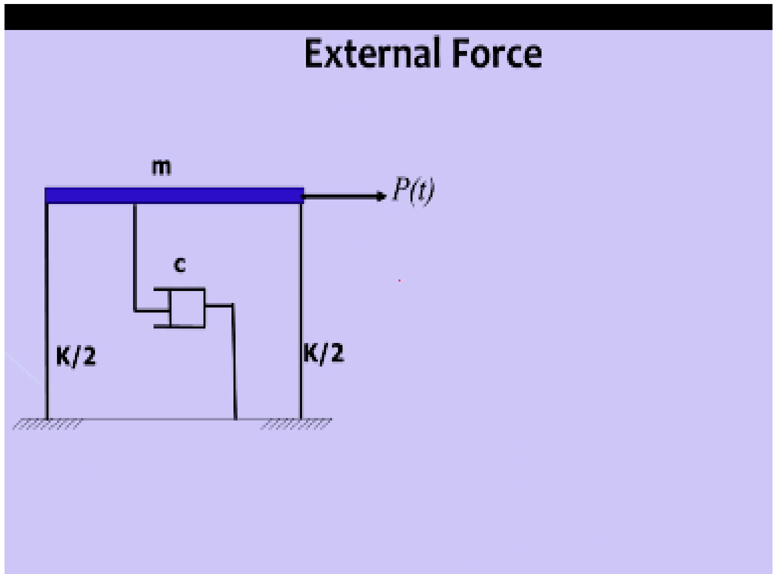

So let's look at how the external force is applied on the structure as you can see the system are the single degree of freedom system can

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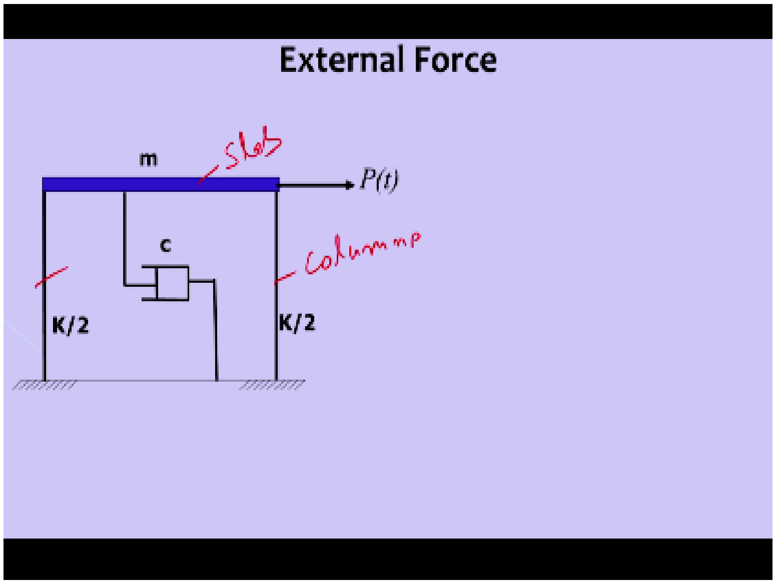

Be represented in the form of say slap and this , this one and this one you can call it as columns and any system will have internal frictions so that means free vibrations is usually not always continues so if you set free vibration after some time vibration dies out that is due to many reasons one important reason is internal friction between the , the material of the column so that will reduce the energy of vibrations so we will discuss this damping phenomenal in detail but to start with we indicate

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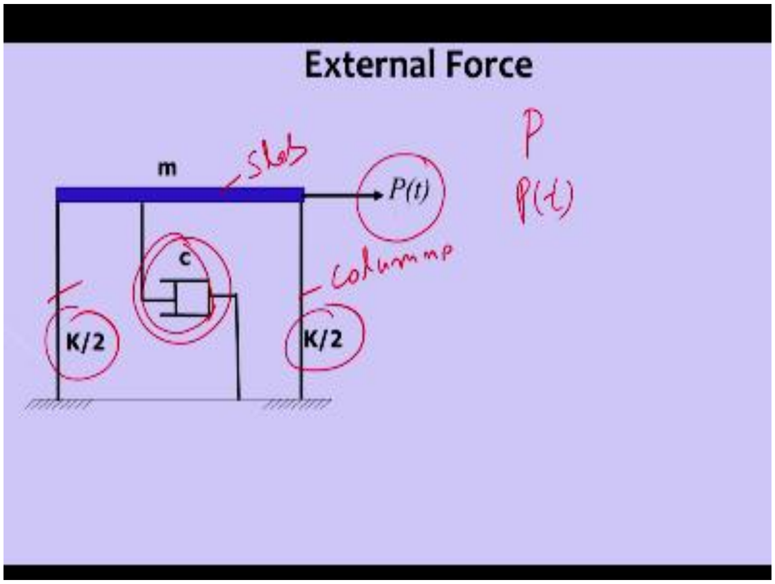

Damping as a dash spot, damping as dash spot. So mass of the structure and then resistance comes by the column which we are representing as stiffness and damping reduces the energy in vibration. In this P is the external force which is being applied now this P as an external force which this is not a static force. A static force is always constant were as dynamic force always changes its value according to time.

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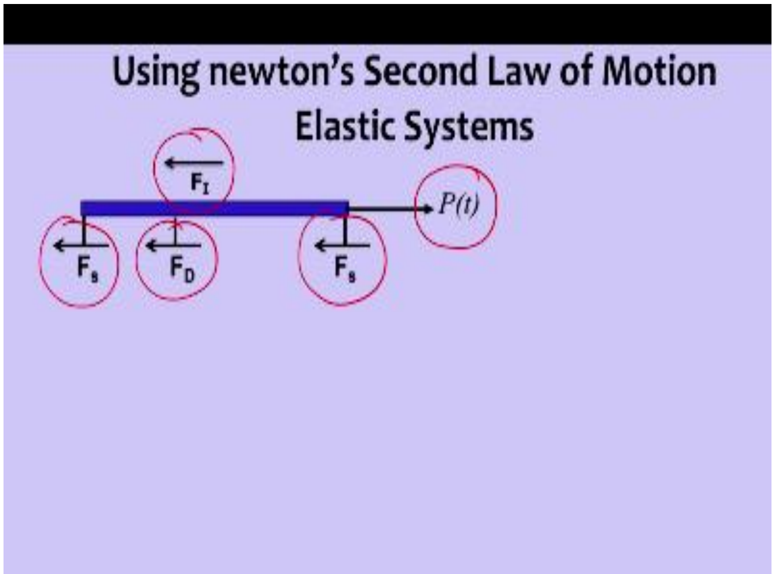

Now we derive dynamic equilibrium equation using Newton's second law. So if we write say a cut the system and write free body diagram this external force is applied here and then inertia of force is opposing the motion and then this is stiffness force, this is stiffness force but which is coming from k/2 and this is damping force.

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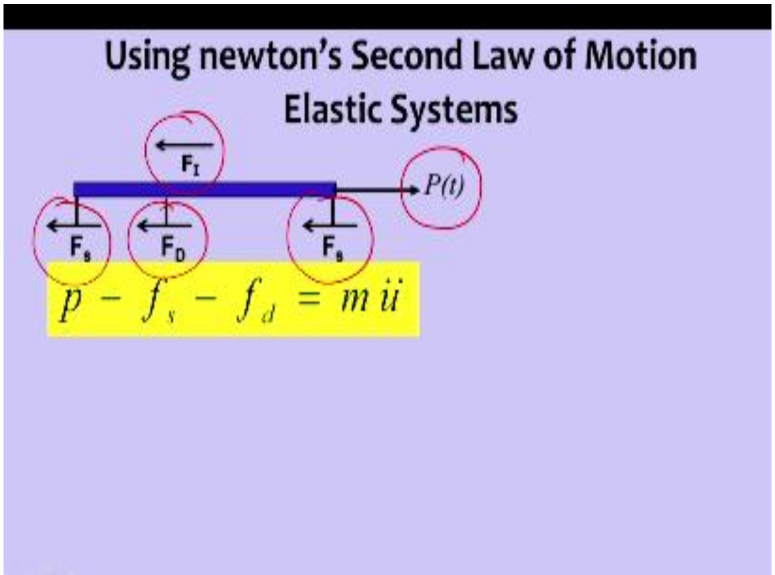

So now if we write equation of equilibrium so this external force, stiffness force, damping force inertia force. If you look at the nature of the arrows.

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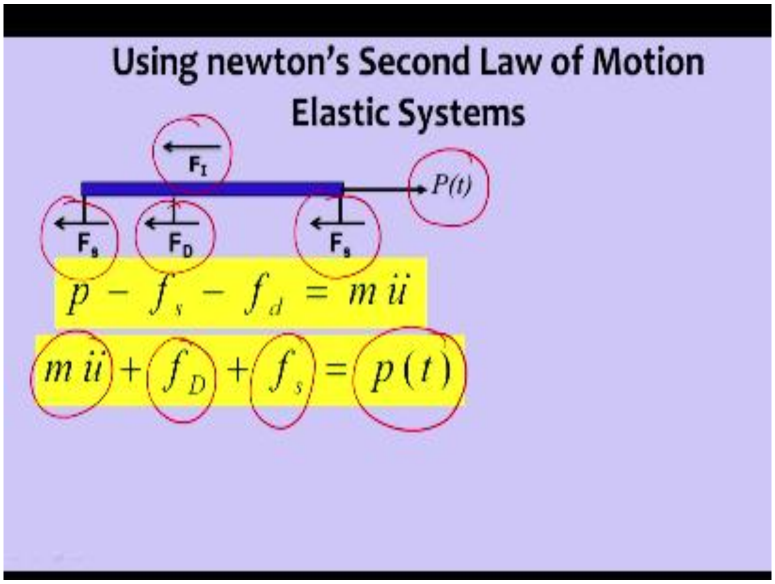

Then MÜ then damping force then stiffness force which is equal to the external force.

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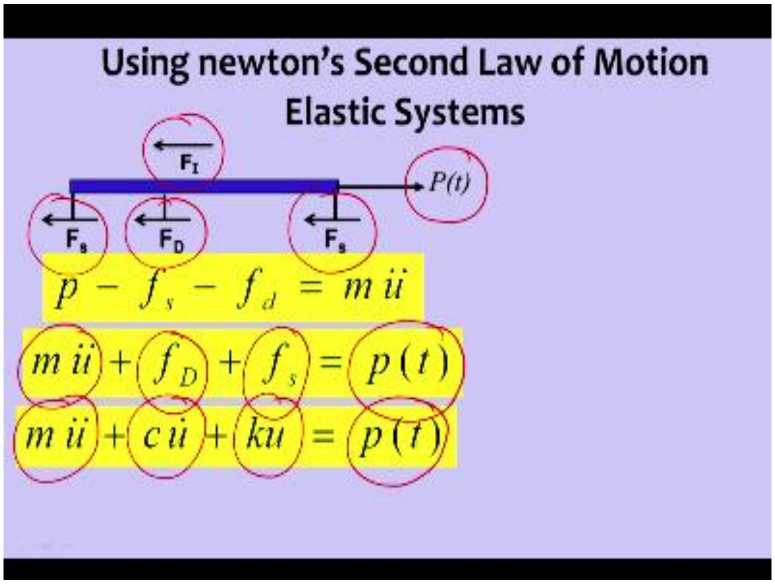

So the same thing can be written like inertia force, damping force stiffness force which is elastic resisting force applied force.

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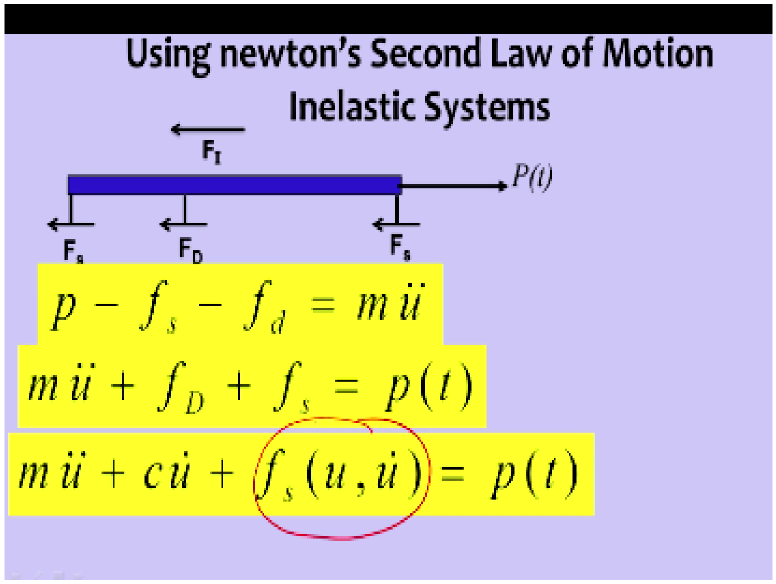

So if the same thing if we write in elastic systems which will become like this here resistance resisting force is not only function of displacement but also it is a function of velocity.

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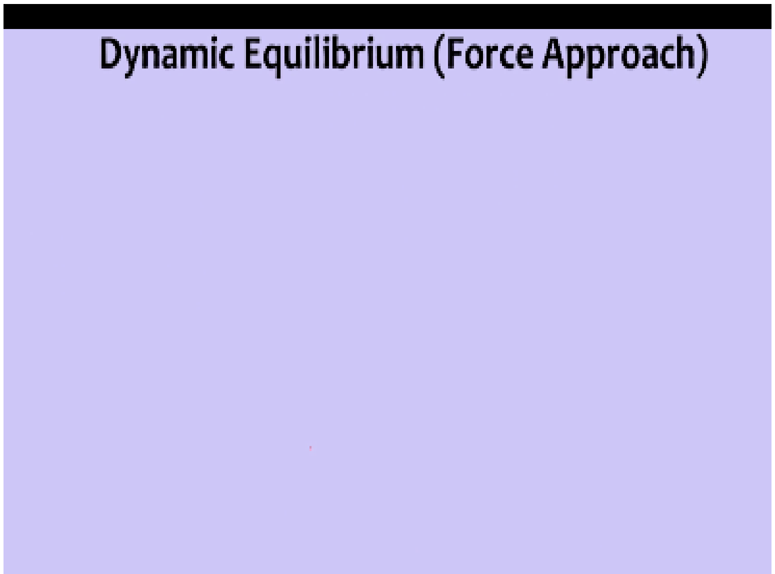

Now the same dynamic equilibrium equation if we get it through force approach how does it look like.

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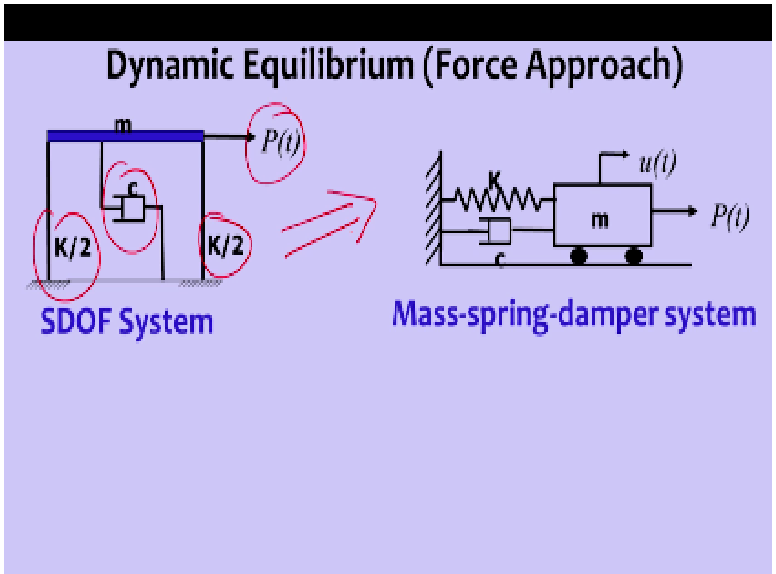

So this is like single degree of freedom system mass is the there springs are there and then damping is there and external force is applied on the system now if we idealized these system as mass is spring damper system so mass means damper system means say if I am vibrating.

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The structure if I give initial displacement or initial velocity structure this structure vibrates so that means elastic force that is the a resisting elements which is columns are trying to pull the mass back so that is represented by.

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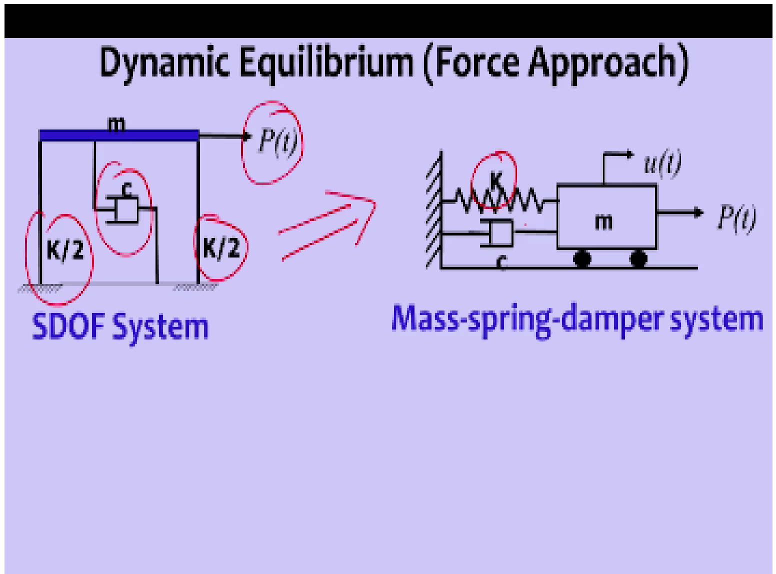

The spring so spring is having stiffness and then after some time after few cycles it will come to rest so that is represented by.

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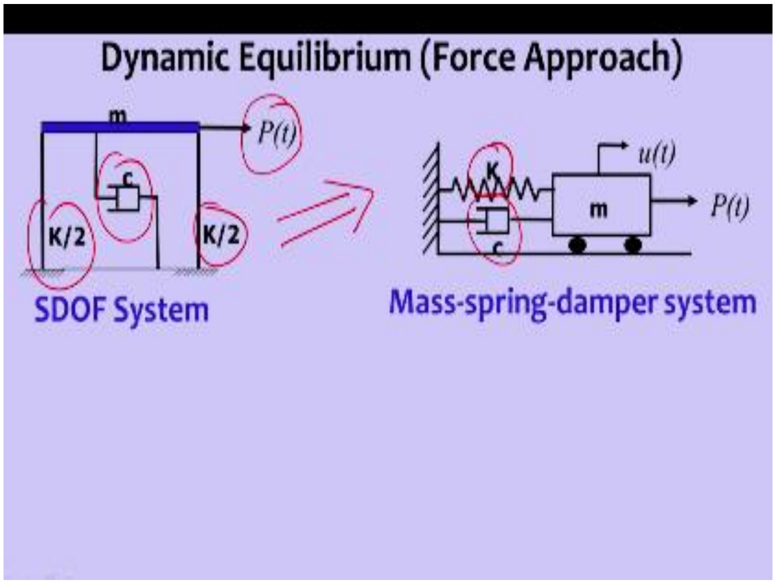

A damping present in the structure and mass when it comes back it over shoots so that means mass is oscillating to and fro at the mean position and then this is done by inertia force.

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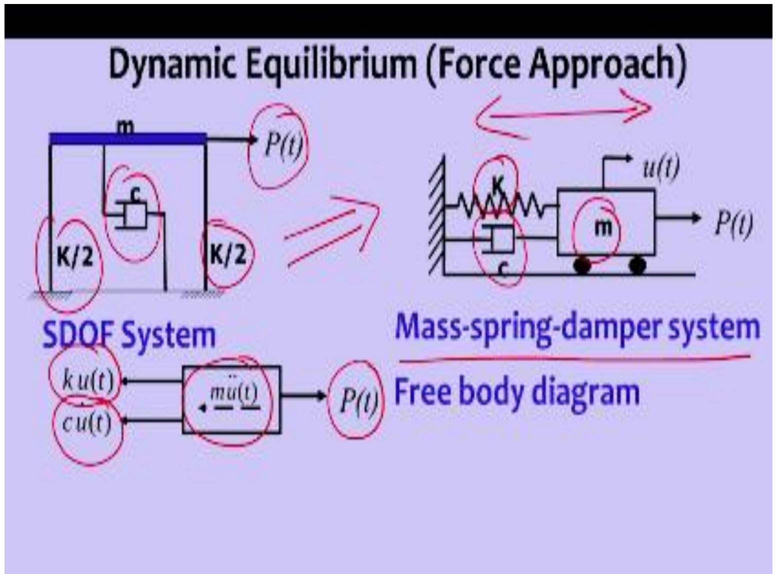

So in this one if I'm idealizing it as mass spring damper system and give initial displacement to this one so this will set to and fro motion now for dynamic equilibrium equation. So what I am doing is I am drawing a free body diagram here so this free body diagram is mass that inertia force because the mass is in the direction opposite to the application of the force that you can see in terms of MÜ.

So we call this as inertia force and then if I apply this force in the forward direction then the resistance of the spring comes in the backward directions so you can see this one K into Y and then damping force also in the form of KÜ. And then external force in the towards the right direction now if we write

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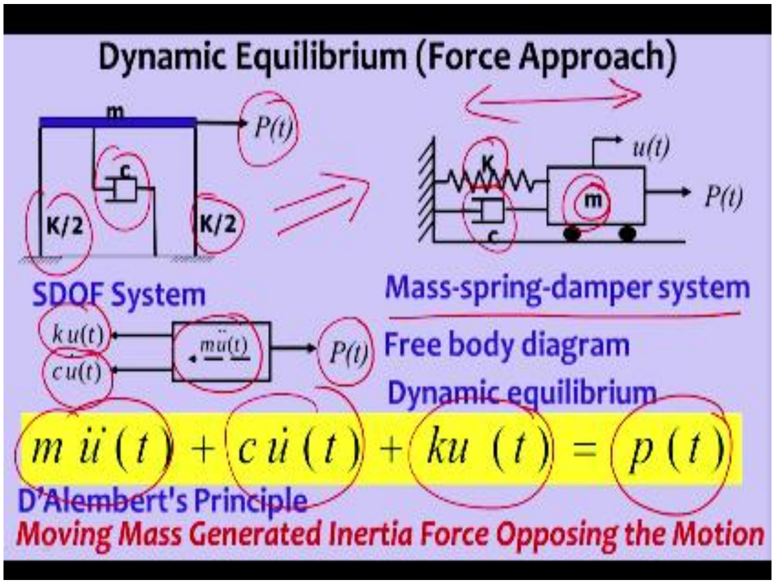

Is equilibrium equation so we get MÜ and CU + KU all these are U U dot and Ü are respectively displacement of the system velocity if the system and acceleration of the system measured at the centre of mass of the structure okay all these things they are respectively displacement velocity and acceleration so this first term we call it as inertia force second term we call it as damping force and third term we call it as elastic force so all these forces are added together will become equal to externally applied force.

So this equation can be understood as a appearing from the D'Alembert's principle which, which says that moving mass generates inertia force opposing the motion so as you can see here mass is moving in the forward direction inertia force is generated in the backward direction.

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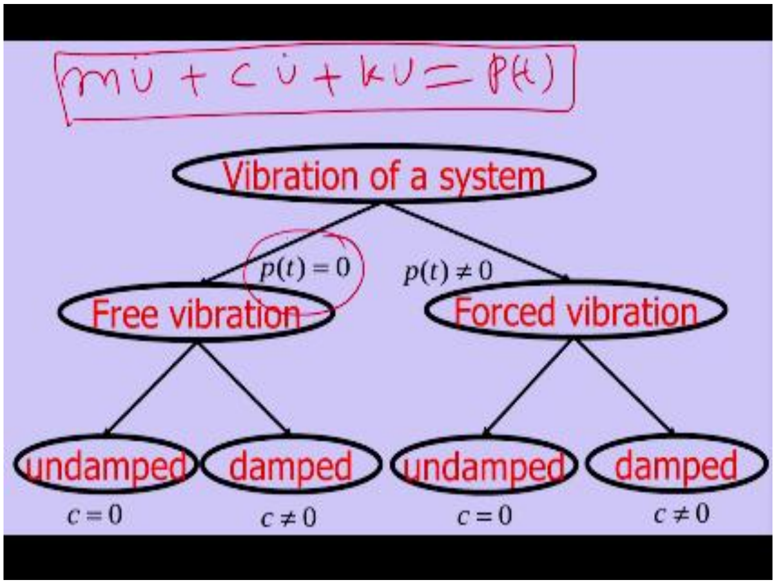

Now this vibration of a system let me write MÜ + CU dot + KU= P (t). So in this we can understand this equation of motion as a general equation of motion but from this varies conditions can be understood so that are free vibrations so free vibrations means when a external force is not acting on the system we call it as free vibration so when external force is not acting how the vibration is taking place so vibration takes place because of the initial disturbance to the system.

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So initial disturbance to the systems system can be so if I give initial displacement it sets to oscillation and initial velocity means I'm giving impulse to this one initial velocity and this initial velocity or I pull this one hit it initial displacement along with initial velocity so.

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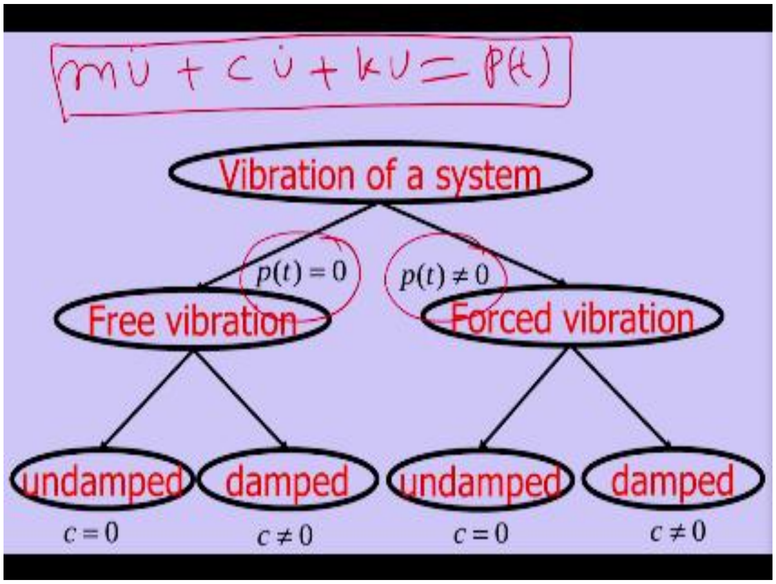

This is free vibration and forced vibration means like when externally applied force is there which is dynamic in nature so that is D'Alembert's principle thank you .

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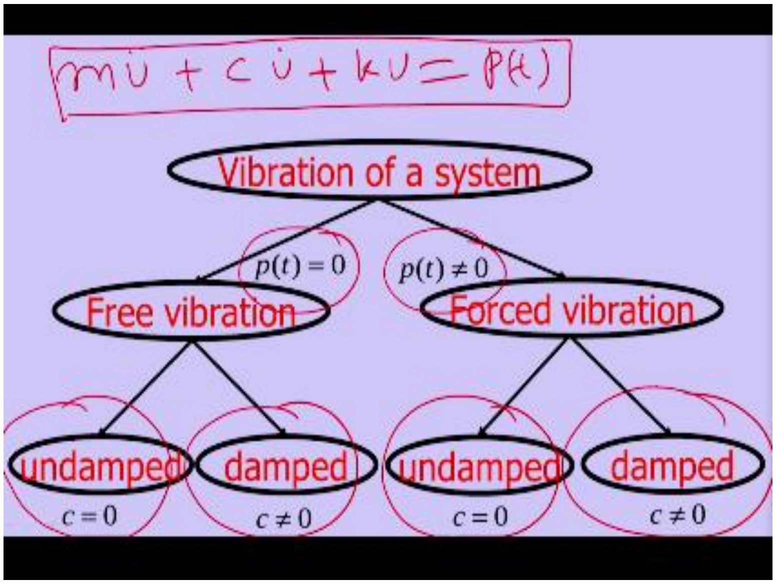

Forced vibration and the another case is un damped free vibration damped free vibration and then un damped forced vibration damped forced vibration so these are the four cases so which will discuss in subsequent modules.

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So in summary what we have studied in this module is vibrations of building and what is a single degree of freedom system how external force is applied and how to formulate dynamic equilibrium equation in the formatted into two methods using Newton's second law of motion and second one is force approach that is force approach is D'Alembert's principle Thank you.

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