NPTEL

NPTEL ONLINE COURSE

Structural Dynamics
Week 4: Module 03

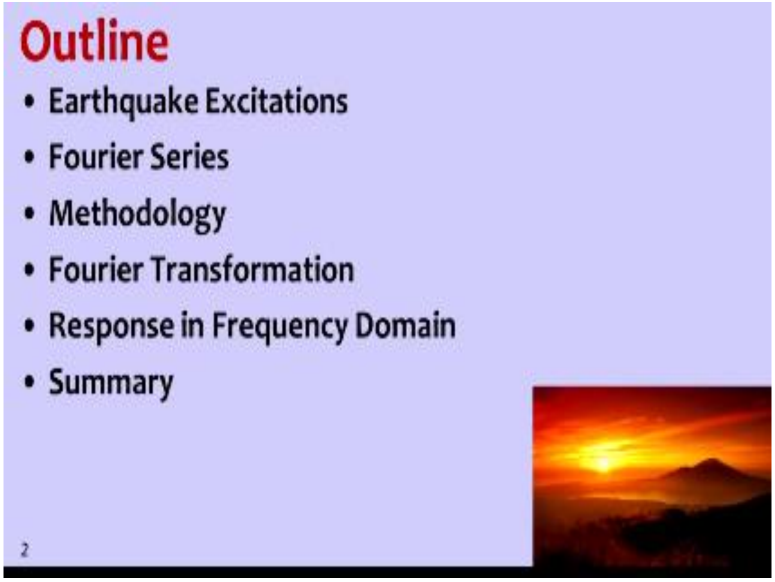
Fourier Transformation

Ramancharla Pradeep Kumar
Earthquake Engineering Research Centre
IIT Hyderabad

Welcome to structural dynamics class. So, so far we have studied the solution of dynamic equilibrium equation first in classical manner and second we have studied that in the Duhamel's integral of solving it. So if any arbitrary force is acting on the structure then we have studied the response, so integral one is classical by decomposing that into sin and then Duhamel's integral by decomposing that into a series of impulses.

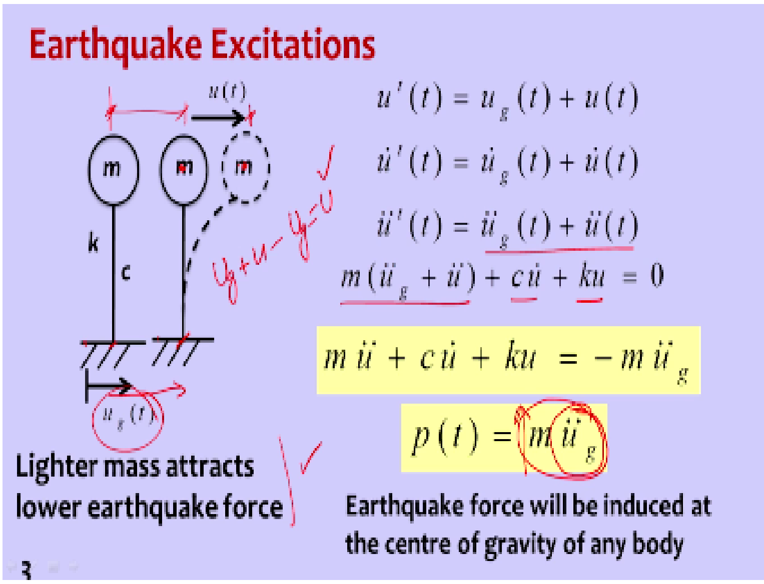
So now we are going to study Fourier transform method. So how do we use transforms and convert differential equation into algebraic equation and solve it.

(Refer Slide Time: 00:59)



So before going into the transform method let us first understand what is a Fourier series and how earthquake excitations act, how earthquake is exciting the building and then how do we write the equation of motion for that and then finally the solution of this dynamic equilibrium equation in frequency domain.

(Refer Slide Time: 01:15)



First let us discuss earthquake excitation. So earthquake excitation is let us assume that this is a structure, earthquake excitation means force is acting that is excitation ground is exciting. So ground is vibrating like this. So when ground is vibrating so you can see that because of this ground, ground motion, so ground is moving and structure is vibrating here okay.

So ground is moving and structure is vibrating here, so if you look at this closely what is actually happening is in this one you can see Ug is the ground displacement, so this part is moving up to this okay. So this ground is displacing from here to here, this is Ug ground displacement. And then when the structure base of the structure is stopping here mass is not stopping there it is going further.

So that is Ut that is the displacement of mass. So the total displacement measured from this part is Ug+Ut okay. So it is total displacement if Ug+U so Ug(t)+U(t) so velocity if you differentiate it once we get u.g and u.g and u. and acceleration is u.., u..g and u.., so now let us write the equation of motion in this condition. So equation of motion is the acceleration mass is related to the acceleration, so inertia force is related to movement of the mass.

So mass has moved from this point to this point, so that means inertia force should contain this two term that is u..g as well as u.., whereas the stiffness force if I explain you like if base has moved 3 centimeters and top as tip has also moved 3 centimeters so the force in this one is 0, so because there is no relative displacement between these two points. So it is 0, so that means what so far us to calculate the force in the column we need relative displacement.

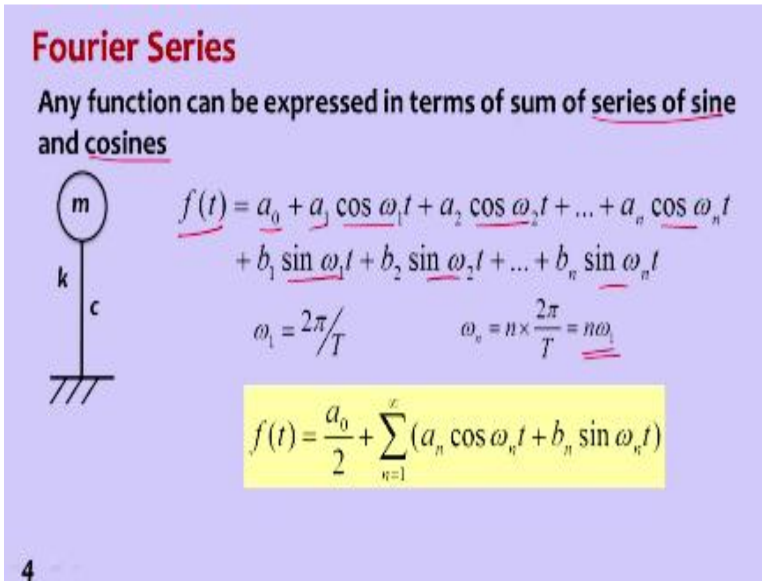
So that means what is relative displacement, relative displacement is the displacement between base and the centre of mass. So that is only even though the mass has moved from this location to this location new location, that mass has moved from here to here and base has moved from here to here. So that means relative displacement can be calculated something like Ug+U as the tip of the mass, motion of the mass minus Ug so which is equal to U.

So this is U, U is the relative displacement okay, so stiffness force is related to relative displacement inertia force is related to absolute acceleration, damping force is related to relative velocity, so if we write the equation of motion it looks like this inertia force, damping force and stiffness force.

So then if we re-write this equation $mu.. + cu. + ku = -mu_g$, so that means what earthquake force is nothing but mass times ground acceleration so like this Ug is actual main component of earthquake force U..g is main component which is earthquake excitation itself and mass is the component which is increase which may increase or decrease so if mass is higher force is higher if mass is lower, force is lower.

So that is why it is said in earthquake inhering that lighter mass attracts lower earthquake forces heavy mass attracts higher earthquake forces, so earthquake force will be induced at the centre of gravity of any body, so if the induces forces that is this, inertia force is a induced force or earthquake force is a individual force.

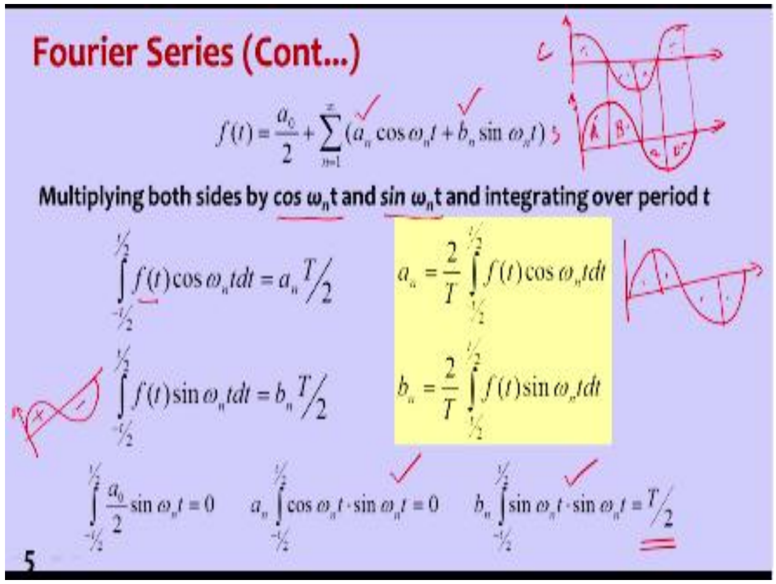
(Refer Slide Time: 05:23)



Now let us discuss a Fourier series so how we make use of this Fourier series in Fourier transform and then how we make use of Fourier transform for finding the solution in frequency domain so first let us discuss briefly a Fourier series, so Fourier series is any function can be expressed in terms of some of series of sin functions and Cosine function, so we can see this a force or displacement or any signal f(t) can be represented like this so A0 is constant so A1 Cos ω 1(t) A2 cos ω 2(t) like that series and fine so this up to n so n can be any number I can go up to infinity also.

So whereas ω1 is as small frequency as possible that is 2π by say t, so t is very small component okay and ωn is n times 2 π/t that is nω1 is a frequency, so this ω, ω2, ω3, ω4 can be represented in terms of this, so they are all linked with ω1, ω1 is a small quantity as small frequency as possible, so in that case we can represent the entire function something like f(t) A0/2 + Σ(n) from 1 to infinity Am Cos ωnt + Bn Sin ωnt. So the any function can be represented in the form of series of Sin and Cosine with this summation form.

(Refer Slide Time: 07:06)



So now so let us go further so any function can be represented like that so let us understand the properties, okay and then we need to find out what is Am and what is Bm which is co-efficient so if we multiply both sides left side and right side by Cos ωnt and also Sin ωnt and integrating over one period so one cycle so what we are going to get, so this is half cycle so that is t/2 and this is either half cycle so -1/2 to +1/2 this is one full cycle, so if we integrate it over one full cycle that co-efficient value is that product value is Am t/2 similarly in sin also Bm t/2 becuas4e it is multiplying with this function.

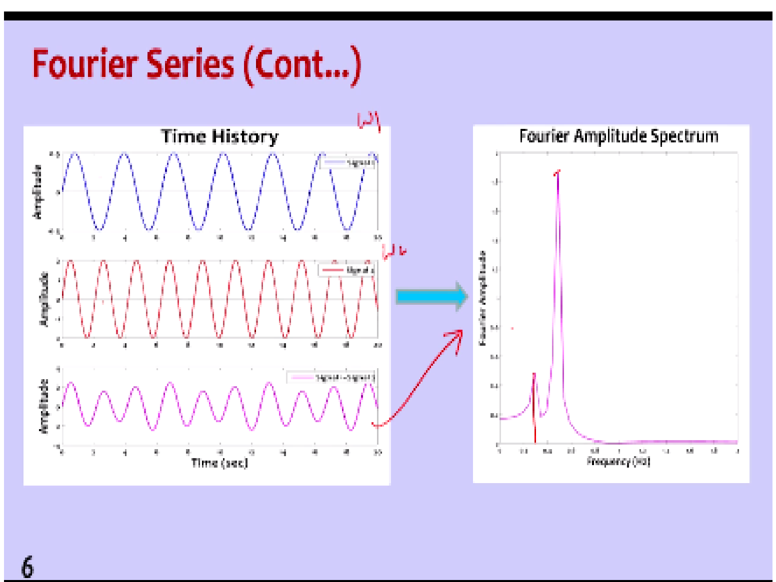
Now An will be 2/t into this one Bn is 2/t into this function, so in if we sum the cos, sin multiplications of cos, multiplications of sin over one period what values will get. If you look at this one a/2 $\sin\omega_n t=0$ so how it is 0 so if you take say one sin function, if you add up all values so this total value is positive this total value is negative so both if we you add ∑ will be 0.

Now in this case, so cos and sin both multiplied together and then adding, if I draw this say cos function this is cos function and then if I draw say sin function is something like this, so this peak is here, here this is here and this peak is here like this, so if we look at this one okay, so if I say call this as A, this as B, this as C, this as D okay, ABCD kind of thing so you can see this values, so they are values are same but so signs are different.

So you can see this one this is positive this is positive so both will become positive and this is positive this is negative, negative so this is negative and this is negative this will become positive so this is negative and this is positive will become negative, so if we add up this multiply by this, this multiply by this, this multiply by this, this multiply by this and add it up we will get 0. But whereas in this case if you take another sin function so you can see this one, this multiply by this positive value, this multiply by this again positive value this multiply by this negative, negative again positive value, this multiply by this again positive value.

So that means sin function multiply by sin function integrated over the period is non-zero and sin function and cos function integrated over the period is 0, and only sin function integrated over the period is 0, only cos function integrated over the period is 0, so that means so this has value b_n has value a_n will be coefficient will be 0, so this value is 0 and this value has some, this has non-zero.

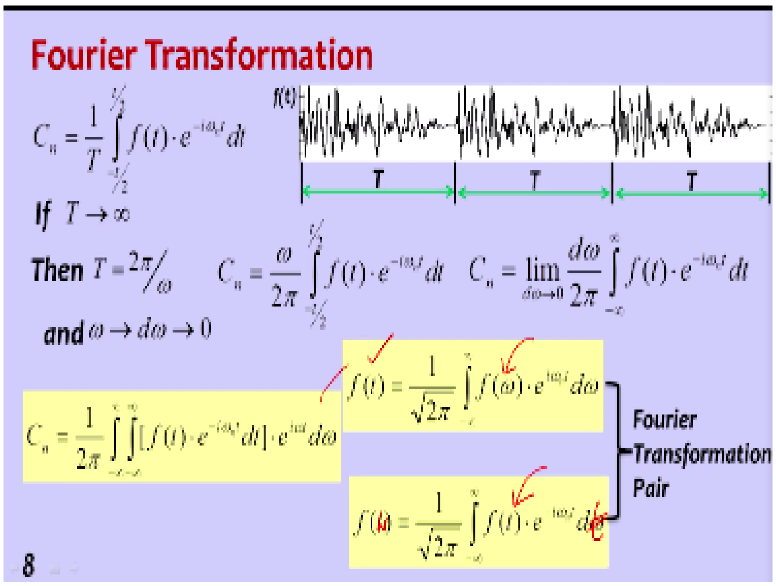
(Refer Slide Time: 10:54)



Okay, now let us look at the diagrammatic view of this frequencies okay, so if you can look at this signal so it is a sinusoidal series signal 1 this is sinusoidal series signal 2, so in this one you can clearly observe that this signal and this signal has different frequencies so this is signal 1 having ω1 has frequency this is signal 2 having ω2 has frequency. Now if we add these ω1 signal to ω2 signal both signals will be there as you can see this one two signals are included in this signal.

Now if we take the Fourier transform or Fourier amplitude spectrum of this one we get peak set one frequency peak at this frequency that means what, Fourier transform is telling us that what kind of frequencies are present in many signal, so now how do we get from here to here so this is a time series and this is a frequency domain function or frequency domain chart or the spectrum. So how do we get from here to here let us discuss that?

(Refer Slide Time: 12:01)



So like a0 / 0 that is a signal which is represented in the form of series of sine and cosign summation of sin and cosigns so this cos ω nt sin ω nt can be represented in the form of exponential functions so lou is if we substitute cos and sin in the form of exponential functions in this one so and rewrite it so we get cn which is a constant term co efficient term and cn conjugate. So this is a co efficient term in the conjugate side.

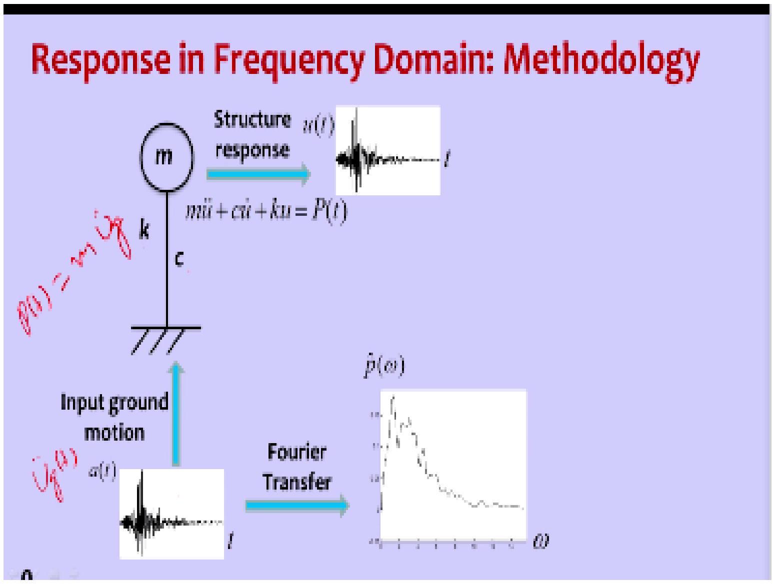
So like the both things can be added and can be represented in the form of cn 1/ 2 an – I bn okay, so in is a imaginary part so an and bn we already know so if we substitute an and bn in this equation and evaluate it so we get cn value so we can write f (t) = Σ - ∞ t0 + ∞, so this is like -∞ to 0 and this is 0 to + ∞ so 0 to + ∞ - ∞ to 0 so both added together we get one constant value one single term furrier series.

And then cn can be evaluated like this so if you back substitute this okay then what we get is something like this one so in earthquake signal which is repeating after say the entire signal period so something like a this is a random vibration and this is repeating after every cycle so when the total time 10 to ∞ so we can assume that each earth quake signal is continuously repeating.

So then t is 2π/ωn so if w substitute this and ω tens to ω w and d ω tends to very small value so then if we find the limits for that for evaluating cn so cn will be 1/ 2π - ∞ to +∞ double integral of this function. Now from this we get this f t if we substitute this one so 1 / 2π - ∞ to +∞ f of ω e $^{i\omega nt}$ d ω and then next one is f(t) 1/ 2 π - ∞ to +∞ ft x e –$^{I\ \omega nt}$ this is dt, so in this one so this is a furrier transformation pair so furrier transformation states that any time signal is available to us.

So we can convert that in to frequency demine by putting here time signal and any frequency doming signal is there you put that in this place and convert that in to time doming signal so frequency doming signal can be convert in to time doming signal and time doming signal can be converted in to frequency doming signal.

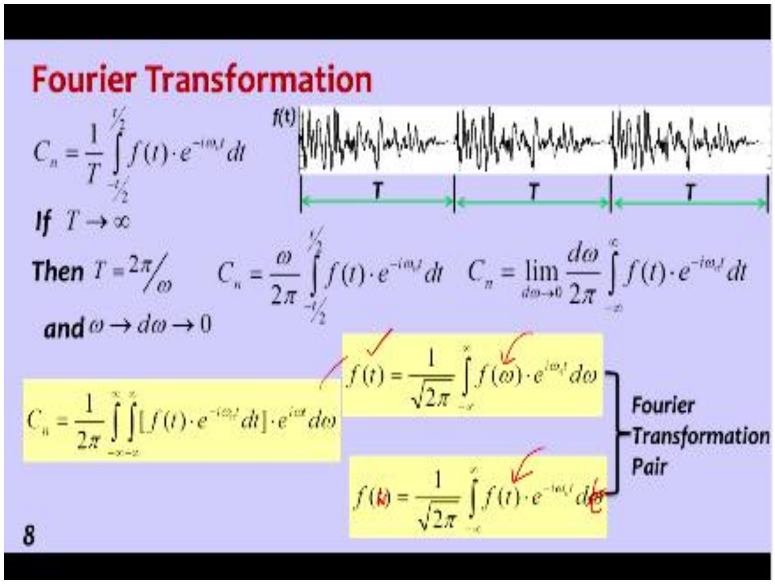
(Refer Slide Time: 15:06)



Now let us discuss response of structure in frequency doming so the methodology is as follows so this is the structure which is have been mass damping and stiffness so three parameters mass of the structure damping of the structure and stiffness of the structure if we give input ground motion to this structure then we get response of the structure in time doming so how we get that is mu double dot + cu dot + ku is = p(t).

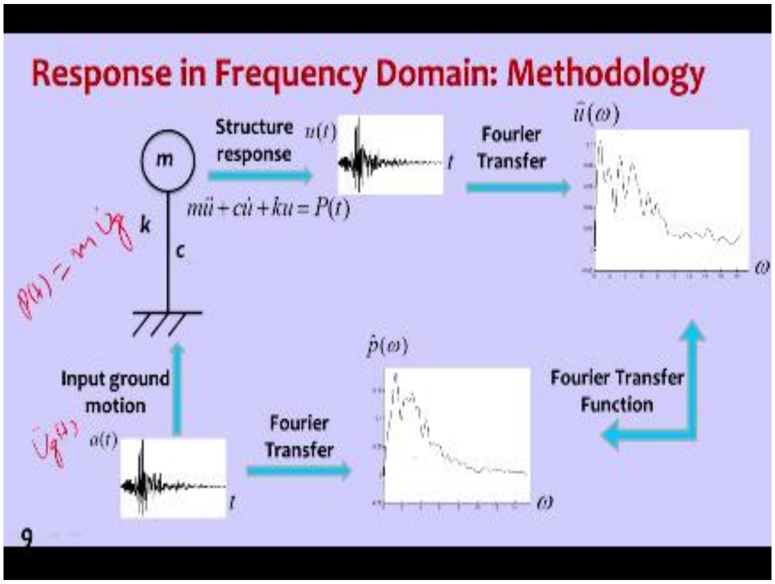
So input motion is given here at pt so this is a acceleration can also be called as u double dot g and this pt is p (t) = m xu double dot g, so mu double dot + u dot + ku = p(t) so we have discussed and understood how to get the response okay to this one so response is ut. UT is a displacement response U dot T is a velocity response and ÜT is a acceleration response we can solve this one and we can get UT. But now how to get this in frequency domain so that's what we are discussing so if we convert this time history input ground motion using Fourier transformation payer into frequency domain.

(Refer Slide Time: 16:31)



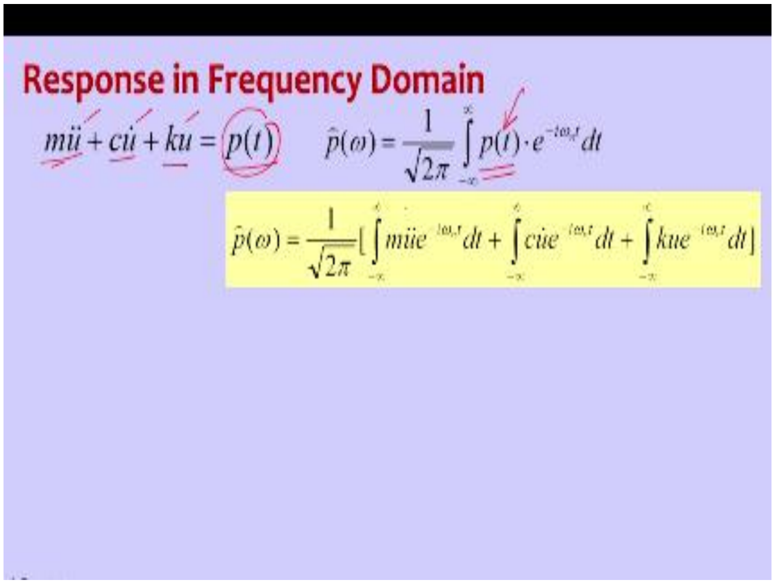
So if I show you the previous this one so that time signs al force if we input here then we get force in frequency domain that's what is required here.

(Refer Slide Time: 16:39)



And then this response of the structure if you convert that into again put this in the Fourier transformation payer we get response here okay now instead of that how to get so this is a input ground motion in frequency domain this is the output response in frequency domain so this is Fourier so how do we get from here to here directly instead of doing this one instead of doing say input time history and get output time history. Instead of that how to do from frequency domain to frequency domain so that is what we are going to calculate.

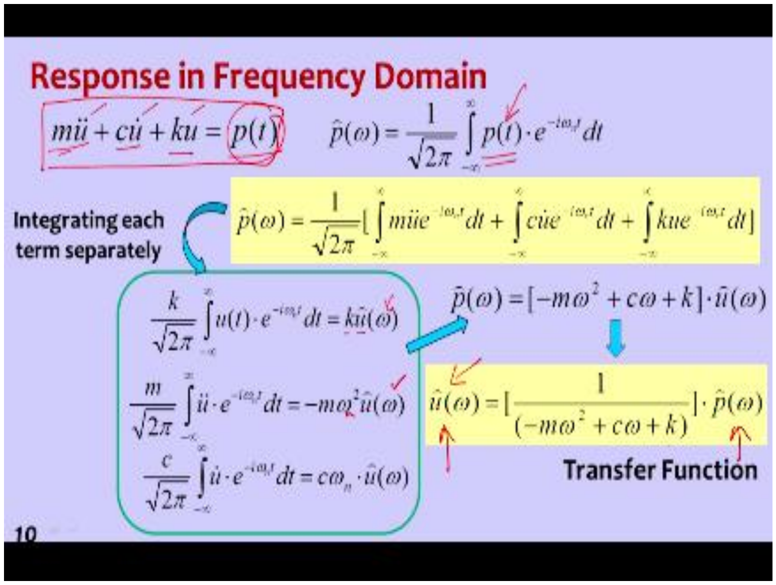
(Refer Slide Time: 17:19)



So this is equation of motion so MÜ+CU dot + KU= P(t). so the dynamic equilibrium equation so first term is inertia force second term is damping force and third term is stiffness force and all put together is equal to the external force. So we need to convert this force this into frequency domain so by using that Fourier transformation payer so this P(t) put at the time function and then convert that into frequency domain so it something like that.

So this is a right hand side and the left hand side all the terms we convert that we apply here so minus infinity to plus infinity MÜ$^{-i\Omega nt}$ dt + cU dot –$^{I\omega nt}$ dt and KU so all these three term so now this UT is a time domain function U dot T that velocity is also in time domain acceleration is also in time domain so we need to convert each function into frequency domain.

(Refer Slide Time: 18:25)



So this is how it is done so the first term that is stiffness terms Is getting converted into frequency domain so we write that as stiffness okay. U^ function of Ω and mass term – MΩ^2 ^{U}^Ω and then damping term is converted like the this CΩN U^Ω. so this is also Ωn and then if we combine all these three things into a equation that's is U^ Ω i= -MΩn^2 CΩn + K * by U^Ω. So from this we can get so the response in frequency domain by giving input in frequency domain so here you can clearly see this is a time history this is a equation of motion.

In time domain and this is a response of the system in frequency domain so here we have converted this differential equation into algebraic equation and it is very easy to find out the solution in this form. Summary what we have discussed in this class so first we have understood how a signal can be decomposed into a series of sin and co sin functions and how a signal can be converted from time domain function into frequency domain function.

And later we have understood how to solve the dynamic equilibrium equation which is in the time domain into a frequency domain so in a Fourier transformation method of a solving this a dynamic equilibrium equation we convert differential equation to algebraic equation and solve it.

IIT Madras Production

Funded by

Department of Higher Education

Ministry of Human Resource Development

Government of India

www.nptel.ac.in