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Structural Dynamics
Week 4: Module 02

Special Cases in Arbitrary Force

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Welcome to the structural dynamics class. In this class we will study special cases in arbitrary force. So in the previous class we have studied about the arbitrary force and how to find the solution for the arbitrary force, how to find the response of a single degree of hidden system for any arbitrary force.

So let us discuss the outline of this class, so we will first discuss about the summary of how to find the response due to arbitrary force acting on a single degree of hidden system.

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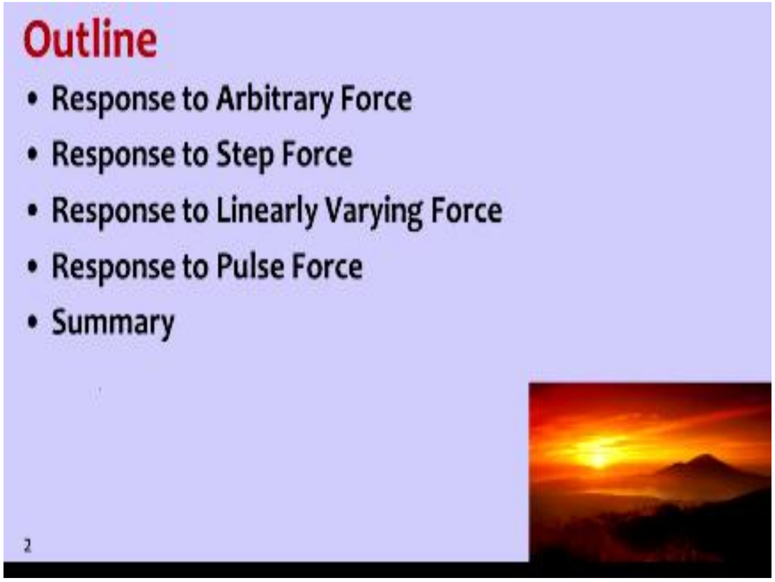

And then next we will find out the response to step force, step force means if a force is applied suddenly how the response will be, and then if the force is linearly varying how the response will be and the force is pulse type so that means I do not really applied force and then after sometime that force is removed. So these three special cases we will discuss in this class.

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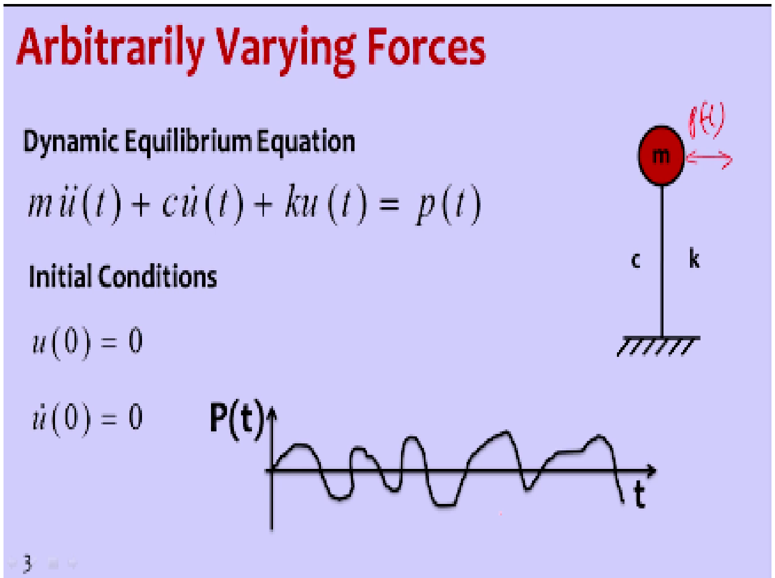

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The first as you know this is a dynamic equilibrium equation mu..+cu.+ku=p, so the dynamic equilibrium equation is equation of motion for the single degree of hidden system. So if P(t) is sinusoidal we have discussed this one and if it is arbitrary so that also we have discussed. So how the, how can we calculate the response due to arbitrary force.

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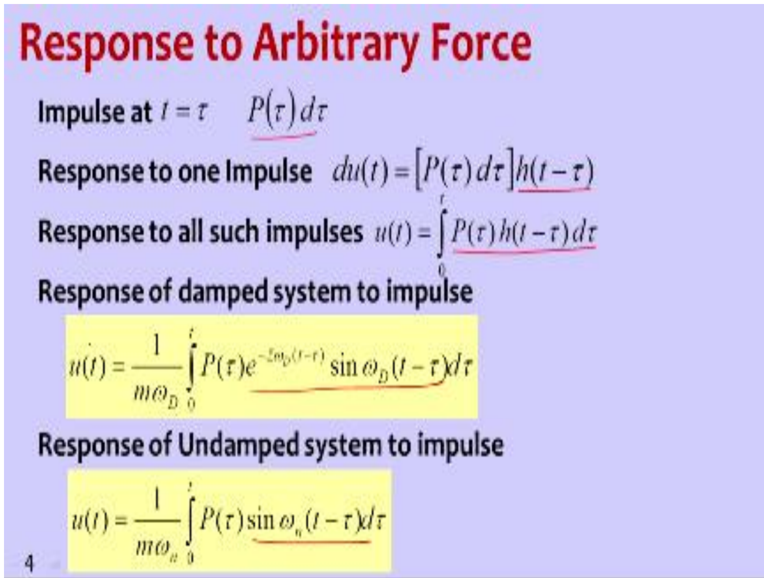

The procedure is that we will first try to find out the impulse at every point and then response due to each impulse before that we find the response due to unit impulse. So h(t-τ) is the response due to unit impulse. And then we will multiply that with the impulse value at any point and that will give us the response to one impulse. So response to all such impulses is the integral of 0 to time t up to time t.

So tτ h(t-τ)dτ so this is the convolutional integral. So if we substitute h(t-τ) in this we get for damped free vibrations and fort undamped free vibrations we will get the complete response. So now we will make use of this one in some special cases.

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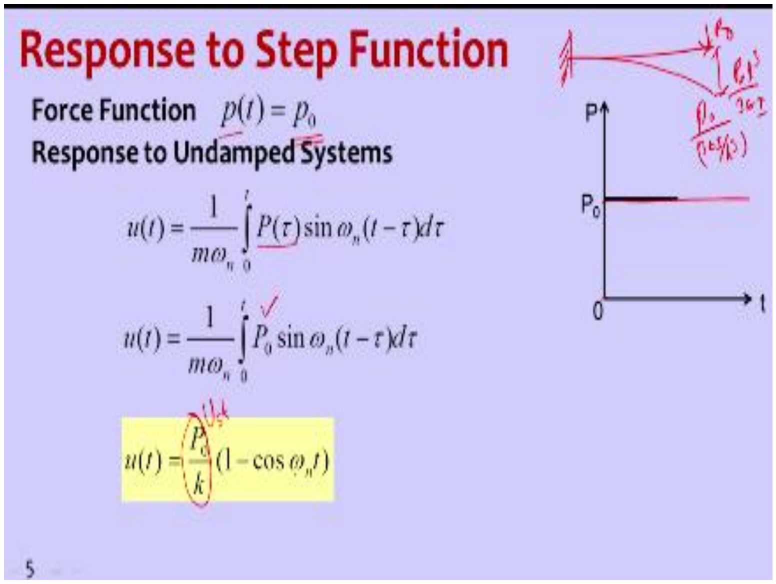

So here we are discussing three special cases. So the first special case is if a force is applied suddenly, so let me explain you what is the meaning of that if the force is applied suddenly. So let us take this cantilever beam and a force is suddenly released and then it is falling suddenly on this beam at one location or as a simply supported beam and force is suddenly applied on this one or on slab force is suddenly applied.

So now how do we calculate this one, so as you can see in this graph at t=0 force is suddenly applied and this force is continuously there on the system. So how the system is going to respond okay. So intuitively if you look at this one, if a force is suddenly applied what happens is because of this it will come and vibrate, oscillate.

And then after that what is going to happen, because force is not going to be removed, I mean if you are not removing that force, force is still there. So how the dynamics will be obvious member or simply supported beam and force is suddenly applied how it will be or on a slab if we suddenly apply a force how slab will vibrate.

So that is what we are trying to understand, so force function p(t) which is constant, so continuously at any point of time P_0 is there. So constant that is fixed force is there on the system. So then let us find out response to un-damped system, so if it is un-damped system we know that u(t) = 1/m ω_n integral 0 to t P(τ) sin $\omega_n(t - \tau)$ d τ so for that P(t) p(τ) so you can see this at any point of time p(τ) value is P(0) so if you replace that p(τ) value with P(0).

And then rest of the thing remains same so now we can evaluate this term so if you integrate this term so P0 will be constant it comes out Sin $\omega_n(t - \tau)$ into multiplied by theta if you integrate it and expand and solve than apply the limits then we get 1- Cos ω_n (P-k) so what is this P- k, p - k is the static deflection $U_s(t)$ something like at rest condition, when force is acting on the body at rest then P0/k that is the deflection.

So for cantilever beam if I take the cantilever beam if P0 force is acting P force is acting so what is the deflection of that cantilever beam, we already know that $P0l^3/3Ei$ So when force up is applied at the free end P0 so deflection will be P0 $l^3/3Ei$ this is the defection so exactly this like $P0/3Ei/l^3$ so this k means this value so k is the stiffness of the cantilever beam so that is the static deflection part. Multiplied by 1- Cos ω_nt this is the dynamic part.

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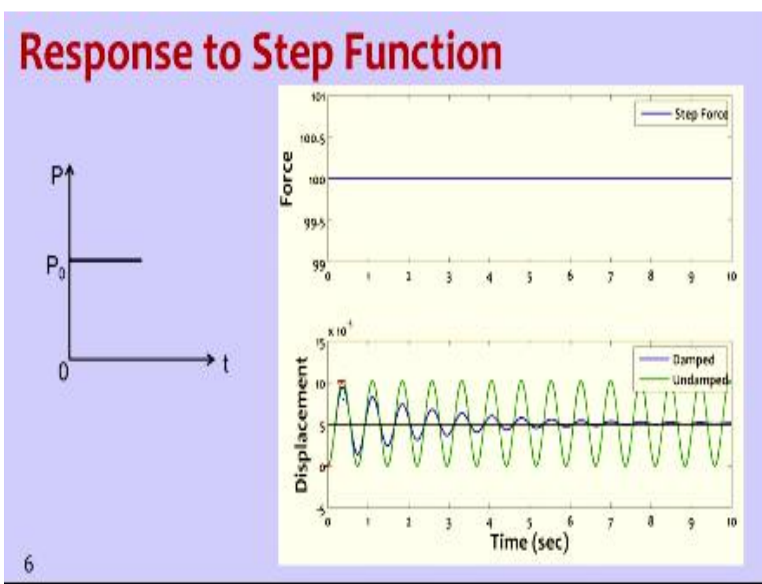

Now if we plot this response it looks like this so force like as an example problem a force of 100km is acting so 100 is continuously there then what is happening is a displacement value which is 0 at the time t = 0 because of the application of the force suddenly it has increased okay, so if you take say un-damped vibration suddenly it has increased and the continuously it is oscillating there.

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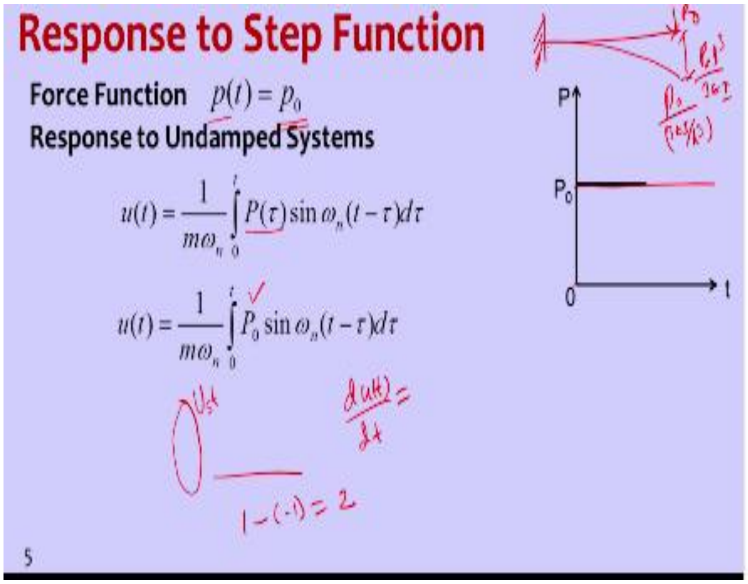

Oscillation part is coming from this one, as you can see this one if you look at this so the maximum value of this one maximum value of this function so of we take one differentiation say du(t)/dt if we take so the maximum value of this so if we make this equal to 0 so then what we get from this is at a regular intervals.

It will become maximum minimum so if we substitute this value in the time at this location so we get this as –(-1) so that means 2 so static deflection twice the static deflection is the maximum value of displacement.

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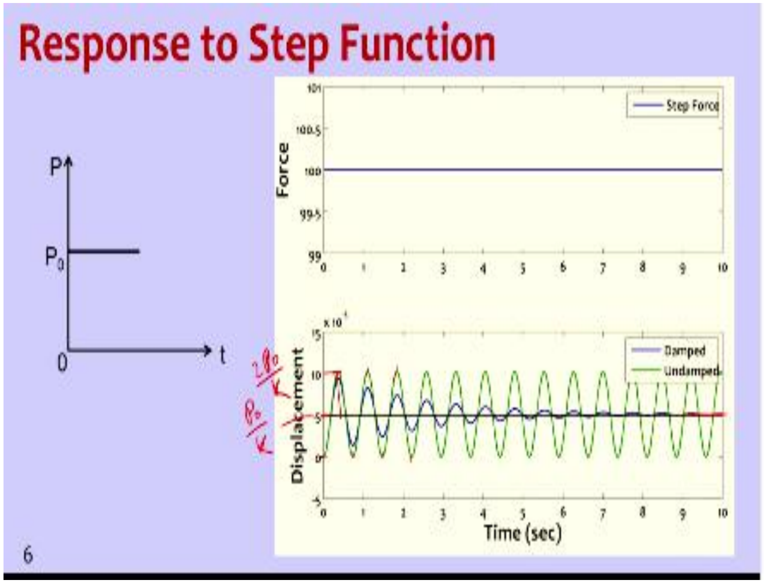

And then if I look at this one you can see this one this 5 is a P0/k that is static deflection value so as you we can write like that also, this is P0/k and this value will be 2P0/k so this is oscillating of green one.

Un damped system continuously it oscillates about the mean position, mean position is static deflection now if we provide damping in the system which is blue color so you can see this one after few cycles vibration amplitude reduce and it merge with the it comes to rest at the static defection value.

So if we understand this phenomenon okay, is a cantilever beam and then load is applied here so because of the application of the load it comes here and vibrates and then finally becomes rest at rest condition, so at rest condition this continuously it goes as a static deflection before that it is a amplitude is the double amplitude is that then slowly it vibrates few cycles and then comes to rest.

So at rest condition in case of suddenly applied force is P0/k and the minimum and maximum values will be minimum value will be 0 maximum value will be double of P0/k so that is the response of a step function. Now let us consider another special case.

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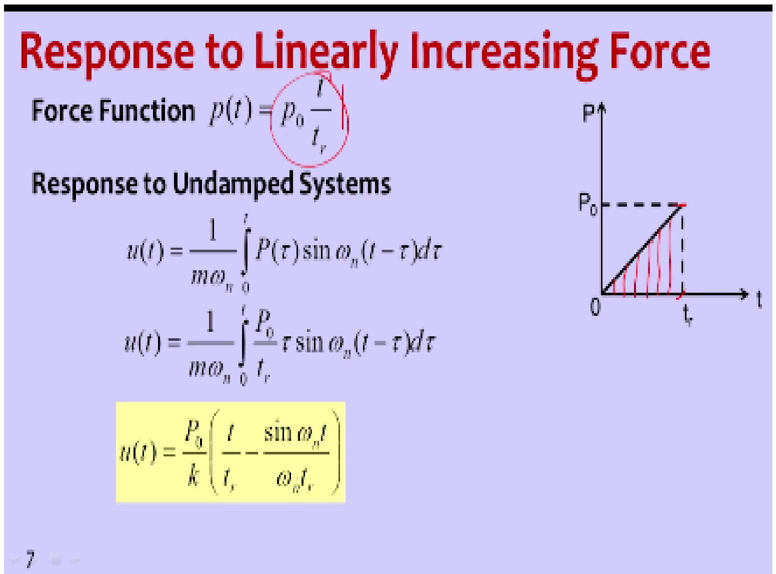

Which is linearly varying force, so here force is not all of a sudden here force is acting gradually force is being increased gradually step by step, step by step like that, step by step. So it is something like so there is a member on which say I am putting 1kN load and then another kN load and then another kN load another kN load so it is increasing gradually. So when forces id increasing gradually how do we find the response to that, so again using the same principles.

So this function p(t) is p_0 which is amplitude that final value into t/t_r t_r is a time up to which this ramping up to that point t_r. So even time t=0, p=0 from this function when time $t=t_r$ it is attaining it is value, so if we take this as a force and apply into the out Duhamel's integral or the conventional integral how this solution will be, let us look at that response of undamped systems. So $1/m\omega_n \int p(\tau) \sin\omega_n (t-\tau)d\tau$ if you look at this one response and then substitute in the place of p force value so this function. So p/t_r t is now τ here $\sin\omega t(t-\tau)d\tau$, so the response of that will be $P_0/k(t/t_r - \sin\omega_n t/\omega_n t_r)$.

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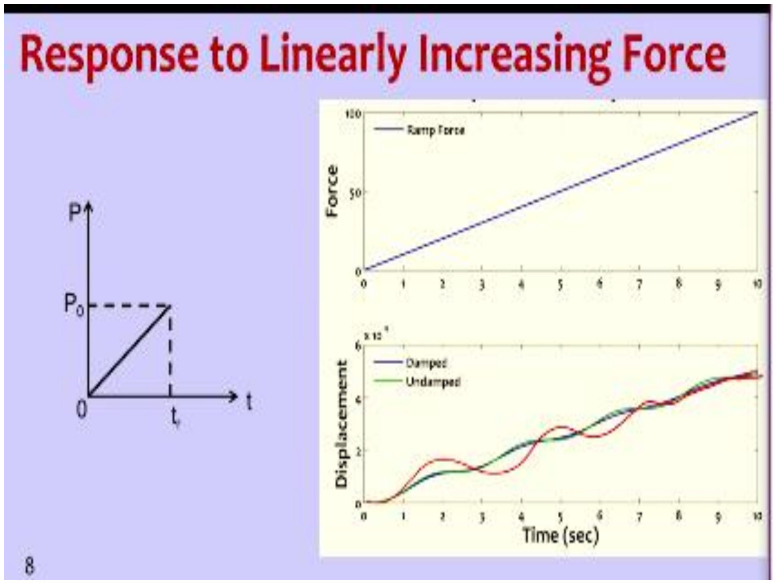

So the function the response or displacement value will be something like this, okay undamped system is a blue line damped system is a green line actually if we use another value of as an example, so how it looks like is so it is something like this it merges in that so this is response to linearly increasing force.

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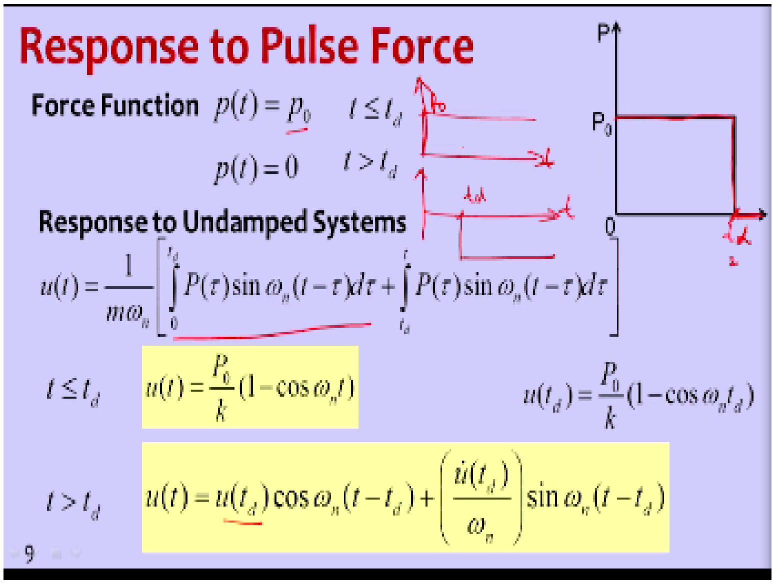

Now let us discuss another special case that is of pulse force so pulse force is this one so as you Can see in this plot so this force is applied to some extent till some extent select to this is t r and after that it is stops so force is for example suddenly force dropped a time t = 0 attitude of force is p 0 and it applied for a few seconds and then removed again so what will happen due to the application of this force.

So force function is p= p_o at time t ≤ td so this is td and pt= 0 when t > td so that means after this one it is 0 so it is a force is something like this as suddenly it is increasing to p0 and then td and then it will comes down and after that this 0 this red one is force. So again we come to response to undamped force systems as you can see in this one there are two things one is up to td and then the second component is starts from td.

So p τ sin ω n t- τ d τ and p τ sin ω and t- τ dτ, so this is from td up to any time and here the time is between 0 to td, so 2 functions are there so in between t and td so any time value less than say if this is 2 seconds less than 2 seconds this value is holds good and then beyond that this value holds good so we need to consider one important thing in this one that is at t= td so this second function starts.

So in that the initial displacement term also will be there that is utd will be there cos ω n t – td u dot td / ω n sin ω n t – td so that will be there so this is after td, so this one is before td so before td is this function after td is the second function the same problem can be understood ion a different manner so that is the same thing we can continue it like this say force is acting continuously from here forever.

And then the second one is force is acting in the negative direction from td like this so this is td, so what this gives is the constant impulse value so p not okay. A step force response due to step force from time t = 0 continuously up to any point of time and this function gives response to step force from time T=Td if you subtract the this from this one then also we will get the same thing value.

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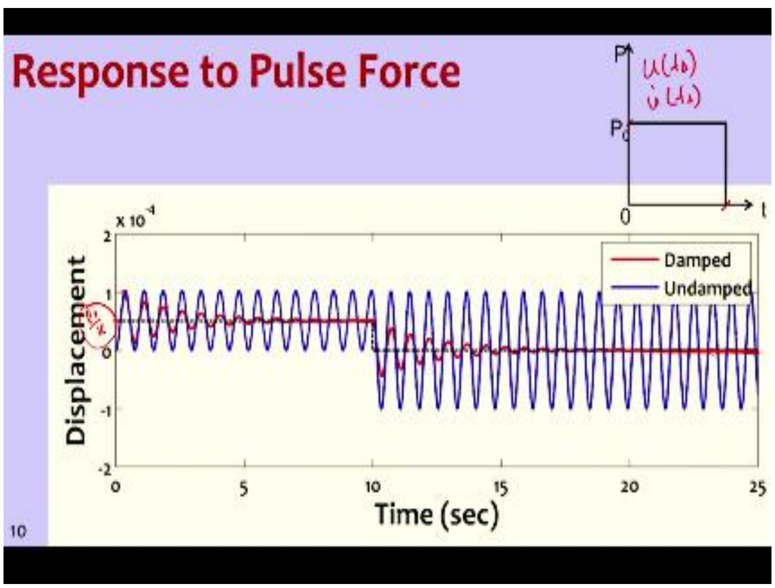

So how does this response look like so you can see this one suddenly when force is applied so vibration amplitude which starts from 0and this is P0/k starting deflection part suddenly it increases and then it is oscillating so let's look at the blue line and at this point suddenly force is removed when force is removed so whatever is initial displacement that is U(Td) and then U dot

(Td) will become initial conditions for free vibrations starting from this point so that's how you can see this that is initial conditions.

Come from free vibration okay and then if we look at say damped free vibrations so we can see that the damped the red line so because of the damping it came to rest almost the total energy is removed from the system and suddenly after this one we are removing the load and when we remove the load again because of that oscillations will take place and then finally comes to 0 so here it is not coming to 0 line it is coming to static equilibrium position.

And here it is coming to a equilibrium position were force is not there so 0 displacement condition here static equilibrium but equilibrium condition is met with at P0/k so thet is static deflection here it is 0 because force completely removed here it sis 0 so these three important cases.

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We have studied so in summary in arbitrary force gives so first we have understood how the response to arbitrary force can be computed by dividing the force into series of impulses and find the response due to unit impulse multiply by the impulse and then what sum of all the responses to get the total response and then. We have applied to three special cases that is step force sec and one is linearly varying force and third one is a pulse force so these three forces special cases we have studied in this class.

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