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Structural Dynamics
Week 4: Module 01

Response to Arbitrary Force

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Welcome to structural dynamics class. In this class we will discuss about response to arbitrary force.

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So outline of this class is arbitrary varying force response due to unit impulse, response to arbitrary force and then we will discuss some problems.

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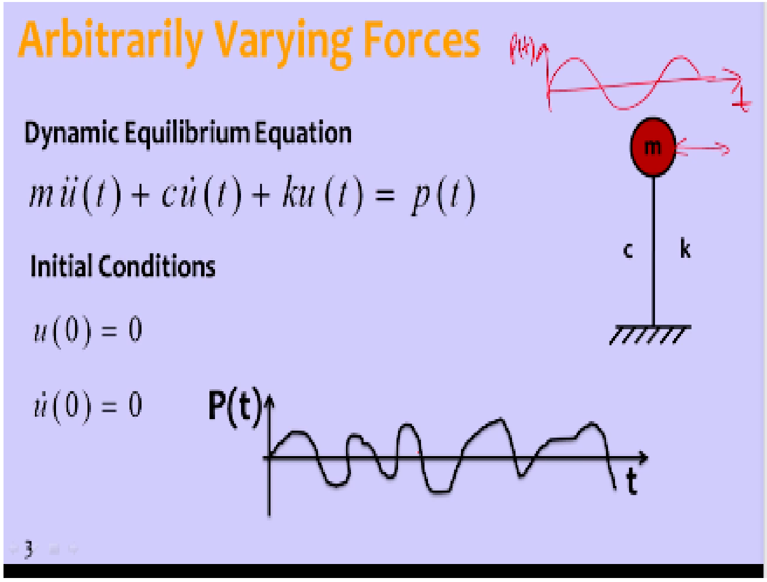

Now if you look at this arbitrarily varying force acting on the structure. So arbitrary varying force means like so far what we have discussed a structure is there and on that we have applied sinusoidal force or a harmonic force we have applied. And now instead of harmonic force if some random force is getting applied on that say for example, some wind force is acting, the earthquake force is acting how do we evaluate the response using the principles of a structural dynamic.

So that is what we are going to discuss in this class. So if you look at this equation, this is the equation of equilibrium mu..+cu.+ku=p, so this P usually is sinusoidal. So, so far what we have discussed was sinusoidal force, sinusoidal force is acting, so we could able to find out the response. But instead of this sinusoidal force if arbitrary force like this is acting which is random in nature.

So if such kind of force is acting how do we find the response from the principles which we have already studied okay in single degree of random system, undamped free vibration, damped free vibration. So using those principles how do we understand this one, so let me explain in the concept of this one.

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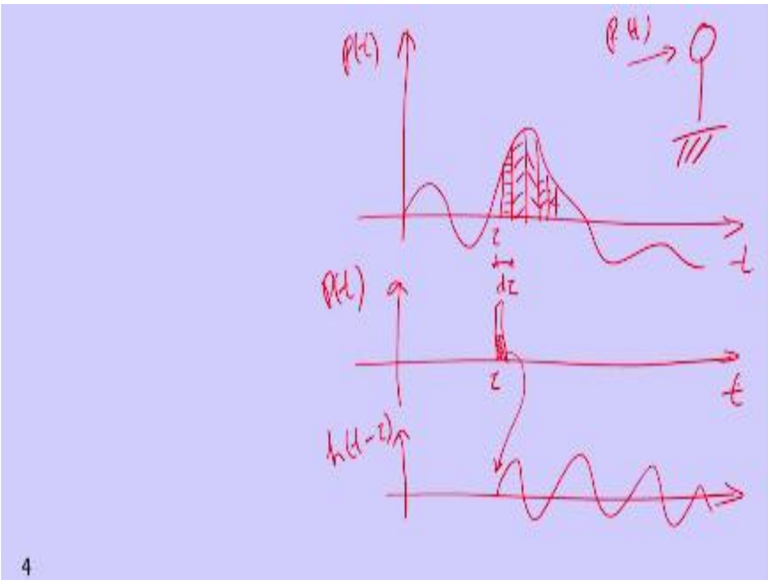

So the concept of this is we will first try to divide this arbitrary force, so whatever manner it is something like this arbitrary force P(t) and T so we will take this force at sometime, so this time we call it as Fτ at τ we will take that and the distance gap between these two is dτ. So τ and dτ so this becomes a impulse, impulse is nothing but force multiplied by time that is called impulse.

Now usually what we will do is take the response due to that impulse only that impulse, find out the response. Now where this impulse is acting, this impulse is acting at τ, so that response due to impulse which is acting at that point we call it as ht-τ. So that means what, if a structure is there and this force P(t) is acting on this one is a full force.

But this impulse is acting at τ, so that means before this τ value there will not be any response. So if I plot the response of this system, so the response of that system this is response h(t-τ) so response will start at this point only, only because of the impulse. So before understanding the response due to impulse let us try to understand the impulse which is of one unit value yeah, this is response.

So this is impulse, so in the larger arbitrary force we have subtracted, we have taken some small bit of force that is impulse and this one we are calling it as response due to unit impulse only this much value okay. Then it is acting here, so how the response will look like. And then after this one we will multiply it with the total impulse value that becomes the response due to this impulse.

And then we will integrate all the responses like similar responses due to similar such kind of impulses at the different, different location. So that is what we are going to study in this class.

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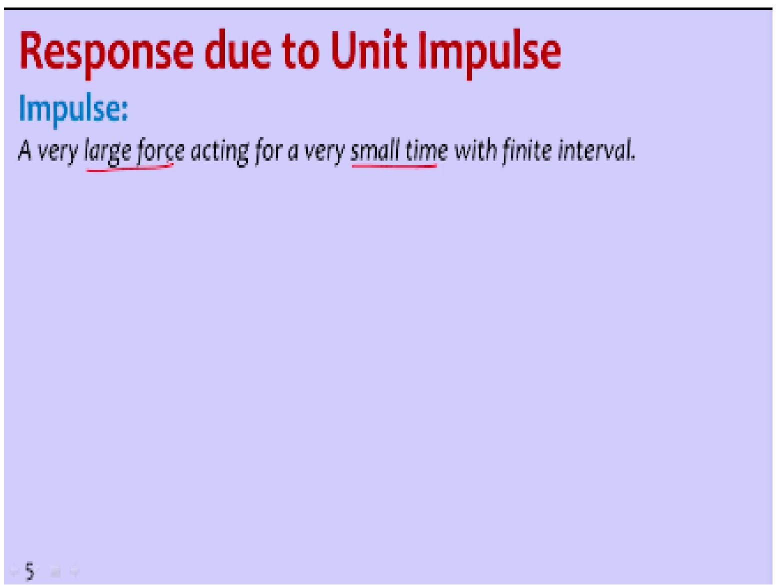

Now let us continue. So response due to unit impulse, so very large force acting for a very small time with final time interval.

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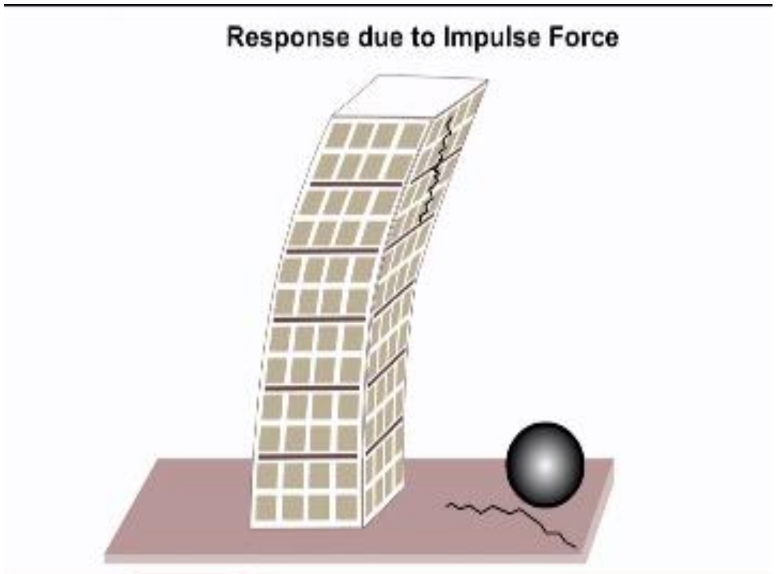

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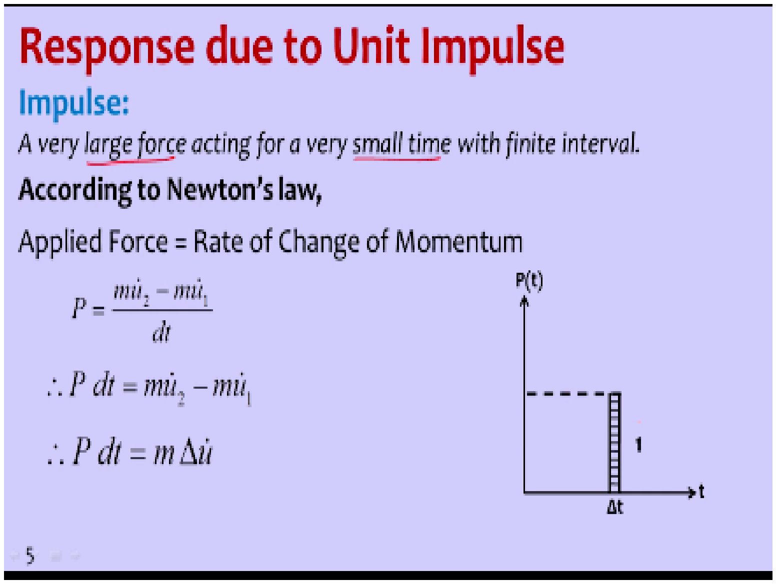

So according to Newton's Law applied force = Rate of change of momentum, so applied force P = momentum at a time 2 at a time t2 and momentum at time t1 so mass into velocity at time instant 2 mass x velocity at time instant 1 divided by dt so the time interval, so that is force and momentum principle now if I multiply this P into dt so which is impulse so change of momentum is called impulse or force into dt is impulse, now here if you can take this m as common u2. –u1.

So that means velocity at a time 2 – velocity at time 1 so that is Δ velocity so that is change of velocity, so change of velocity multiplied by mass is impulse and force into time is impulse that is equal to force into time is impulse, now as you can see here Δ t and this so this is unit impulse 1 unit.

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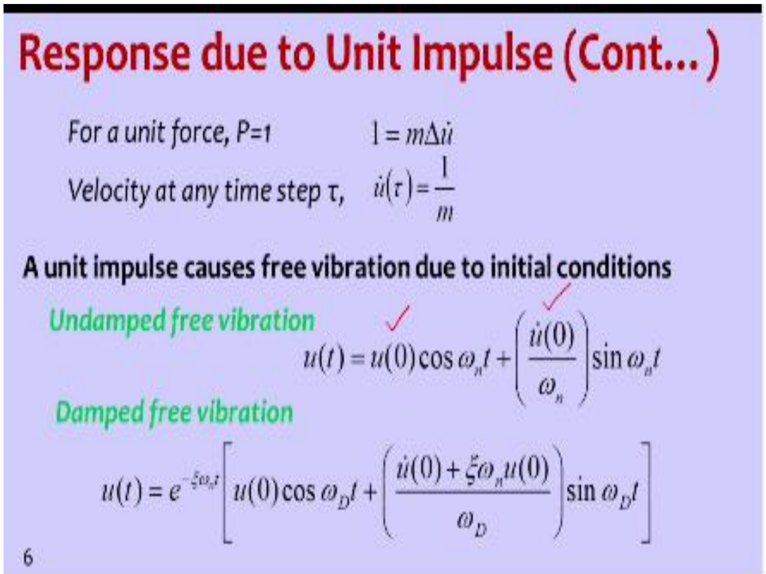

So we are making it 1 unit, so if force = 1, P =1 in the above equation if that is 1 pi x Δ t value is 1, so p to Δ t value into multiplied by Δ t is 1 so that means 1 = m x Δ U. so from this we can get that u. τ = 1/m assuming that initial condition that is initial velocity is 0 so u. at τ value is 1/ τ so if this is 1 so Δ u. which is u2 – u1 so your u2. –u1. So u1. Is 0 so u. τ = 1/m.

So a unit impulse causes free vibration due to initial conditions, so what are the initial conditions in this one, so if it is un-amplified free vibration equation we have displacement at a time t = 0 and velocity at time t = 0 so this and damped free vibration that is $e^{-\zeta\,\omega}$ m(t) multiplied by u0 that is initial displacement Cos ω dt this is damped natural frequency and u.0 + ζ ωn u0/ ωdt into Sin ωdt so this is unamplified vibration response they amplified vibration response , so now in this one we need to use initial conditions.

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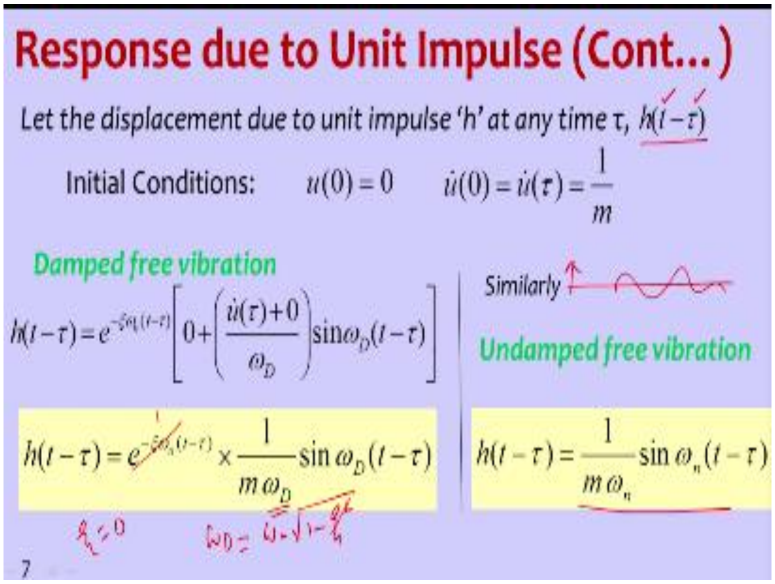

So due to impulse initial condition are like at a till time $t = \tau$ there is no force acting on a system so at time $t= \tau$ force is acting so impulse is acting in the system, so impulse is generating this velocity which is equal to 1/mass, so let the displacement due to unit impulse that is h at any time τ is this value so ht- τ is due to unit impulse so 1 unit impulse is causing ht- τ so it means time that is response at any time okay.

Which is greater than τ value so before it is 0, so initial conditions are u0 = 0, u.0 that means u. τ = 1/m, now if we apply this one and put in damped free vibration so $e^{-\zeta\ \omega}$ m(t) instead of t that is t- τ and this value initial condition u0 is 0 and u.0 becomes u. τ u0 is 0, so u. τ/ ωt Sin ωt . t- τ so this shows that if time scale is given so till time $t = \tau$ there will not be any response after that response starts.

Is going to see this one h that is the response due to unit impulse applied at time $t = \tau$ that is equal to $e^{-\zeta\ \omega}$ m(t- τ) multiplied by 1/ mass ωd sin ωd (t- τ) so similarly for un amplified free vibration so if a damper free vibration is a response here for un amplified free vibration this is a response so difference is this, so if in this equation also if you put damping equal to 0 same thing we will get so only difference is here if say in this case if $\zeta = 0$ so that means what if this is 0 this term will disappear it will become 0ne and ω_d which is = $\omega_n \sqrt{1-\zeta^2}$ so if ζ is 0 this is term disappear ω_d will become is equal to ω_n so this becomes ω_n. So essentially both will become same when ζ value is equal to 0.

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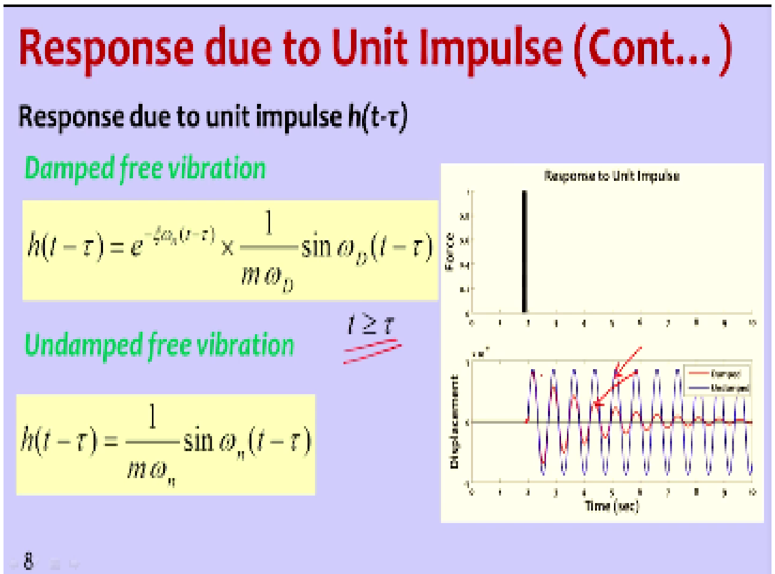

Now response due to unit impulse that is h t – τ is given like this and undamped free vibration damped free vibration both values yup given so here we need to remember that the t value should always greater than or equal to τ and this is our response will be force of one unit is acting at a this location so at this location exactly like that time as you can see the plot of these two things the blue is indicating the undamped vibrations so we can call it as undamped free vibration but initiate this vibration we need in initial disturbance which is cause it due to impulse.

And here this is damped vibration as you can see damping is taking place continuously damping is taking place that means vibrating energy is removed from the system.

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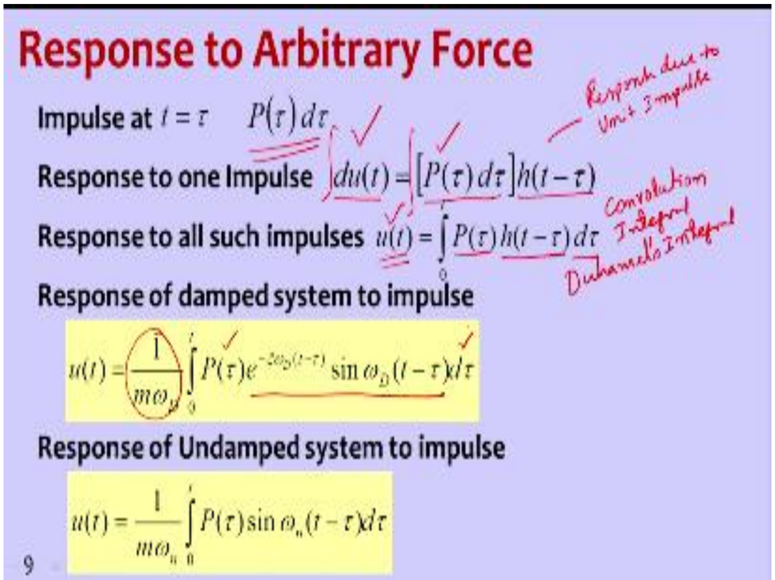

Now impulse so till now what we have discussed is this the response is due to unit impulse so now if it is impulse value some value is there now how the response will look like so if we find out first the response due to unit impulse and multiply with the impulse value at the time t = τ, so then we will get the response due to impulse you can see this one ht – τ is response due to unit impulse.

Now if we want response due to impulse then we need to multiplied with impulse value force in to dt okay the time interval so this is impulse so impulse value multiplied by response due to unit impulse which is will give response due to impulse, so which is this impulse so this is a impulse and this entire thing is response due to impulse, so this du of t is write is because this is only due to one impulse so in any arbitrary force if you take al large arbitrary force in that there can be many impulse serious of impulse will be there.

And this is response due to one such impulse and then response to all such impulses if you want we put together so that is integral of du if you take integral here and integral here as you can see integral of d u t you will become ut = integral 0 to t, t τ h t – τ x dτ, so this is called convulsion integral. Or this can also be called as Duhamel’s integral, so it is a very powerful tool to find out the response due to any arbitrary force just by decomposing that force in to a serious of a small impulse values and then find out the response due to a alit impulse multiply by the impulse value and then integrated over the time.

So then we will get the response at any point of time, okay so you can see this response of damped systems to impulse now if we substitute this h t –τ dτ as you can see this is ht – τ dτ damped systems, and this is for undamped systems so 1/m ω d x e $^{-\zeta \omega d t}$-τ son ω d e – dτ is a response due to unit impulse now multiplied by impulse value. So let us go back and look at it.

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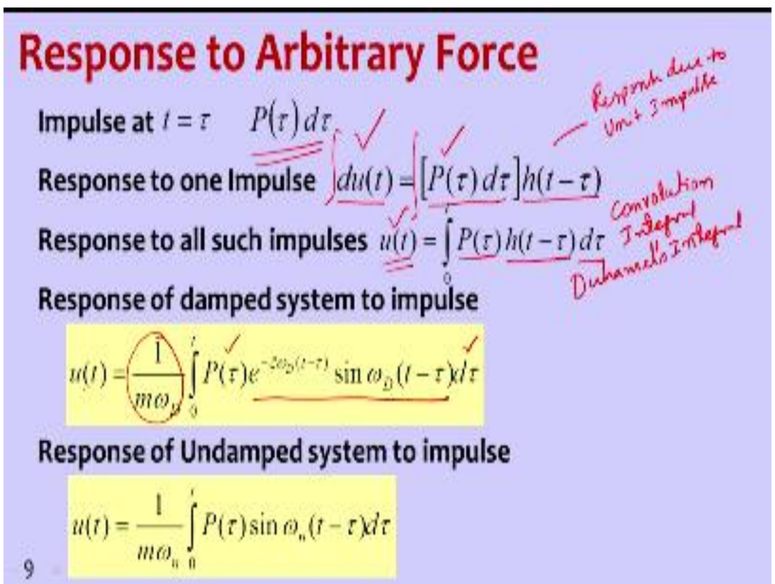

Yeah this one this is damped free vibration response so ht – τ is substituted that value and then we get this entire response.

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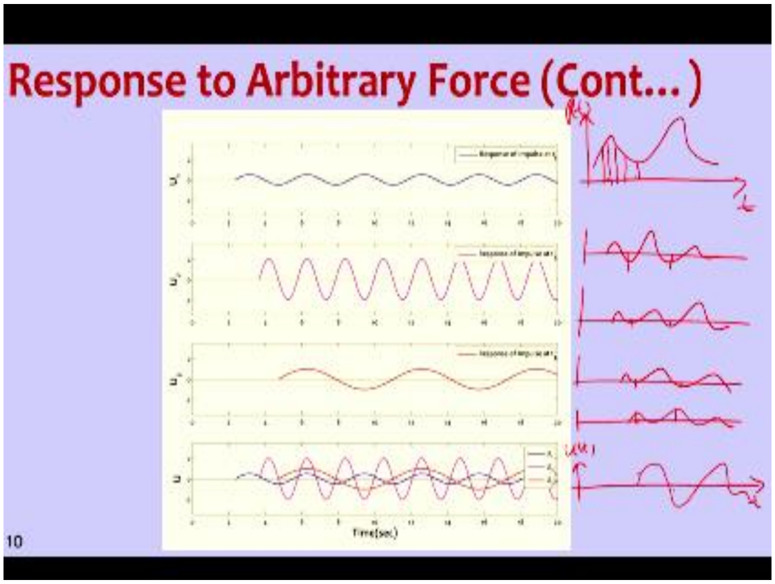

Now how does it look like so it looks like this so this is response due to impulse one response due to impulse two response due to impulse three response to all impulses put together so if we have a force like this something like this a force T, PT so this is first unit impulse this is second unit impulse third unit impulse forth unit impulse so what happens is this response at this occasion so we can get it as response at this point.

Then next one is response at this point third, third response at this point and forth one is forth response at this point if we want to get response at any point that is full U of T so at any point you add this value +this value + this value+ this value all put together we get that value so at any point here we add this value+ this value+ this value+ this value we get response at that point so can be positive or negative so this is a total response we will get. So that is how we add all the responses

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So in this summary what we have studied in this class so in this class we have studied first how a response to arbitrary force can be computed or calculated so first we take arbitrary force and divide that into two series of impulse so impulse value is P at τ time T τ multiplied by data is an impulse value and then we have understood how to get the response due to unit impulse so one unit impulse will generate velocity =1/M so 1 unit impulse then after that after getting that initial velocity so we take undamped free vibration or damped free vibration.

In that we substitute initial displacement value and initial velocity value so in solution initial conditions are impulse inside so we are inputting the input conditions that is initial displacement and initial velocity so we can get un damped free vibration response damped free vibration response for unit impulse so after getting that unit impulse is that is H T – τ value so we will multiply that with impulse value that is P τ D τ the P of τ multiplied by T τ multiplied by H so T- τ so we will multiply so this is a response due to impulse.

So foe getting the response of the total force at any point of time we add all such kinds of impulses in the form of integrals that is called conversional integral or Duhamel's integral so that's the summary of the today's class and we will discuss some applications or some special cases related to conversional integral in the next class.

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