

**NPTEL  
NPTEL ONLINE COURSE**

**Structural Dynamics  
Week3: Module 05**

**Transmissibility**

**Ramancharla Pradeep Kumar  
Earthquake Engineering Research Center  
IIT Hyderabad**

Welcome, back to structural dynamics class in this module we will study transmissibility so what is transmissibility.

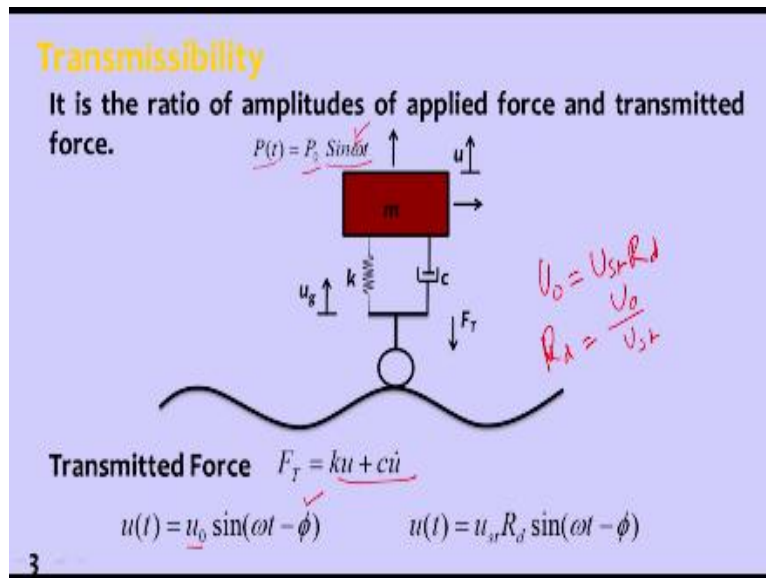
(Refer Slide Time: 00:22)



So transmissibility is when a force is applied on to a body how much forces is getting transmitted so that is transmissibility so as an example, if we look at say a vehicle is passing on the road, vehicle is on the road. So vehicle has a certain weight and then it is transferring the force to the supporting road so how much forces is getting transferred to that, that is called transmissibility. So we will discuss that, in static condition all load will be transferred but in dynamic condition it

depends on the speed of the vehicle it depends on the natural frequency of the vehicle and then a damping present in the system, so all these three things. So how we work out the force which is getting transmitted that we will discuss in this class.

(Refer Slide Time: 01:16)



So transmissibility is it a ration of amplitudes of the applied force and the transmitted force. So if you look at this figure this is the idealized masses spring and damper system, so mass is representing the entire mass of the vehicle and then stiffness and damping together they are representing the suspension of the vehicle so usually we have shock absorber, tiers and all those things that is the suspension of the system and you can see here this P(t) is a force applied so this has two components in it.

One is the amplitude part of it that is P<sub>0</sub> which is amplitude and then second one is the sinusoidal part of it so this ω is a dynamic part of it, so this is dynamic part and this is starting part and because of which this mass is vibrating so this u which is measured at the center of mass is a displacement of the system, and when it comes when we will look at the bottom here like the tier is in contact with the road. So in a way this P sin ω t this sin ω t is coming because of the movement of the vehicle on the road.

If it is flat then this  $\omega$  will become very less value if it is say pumping kind of road this value increases and that also depends on the speed of the vehicle. So now vehicle is moving in the towards the right side and then if it is very high speed then  $\omega$  will increase, low speed  $\omega$  will decrease like that. Then  $F_T$  is the force which is getting transmitted to the base, okay and then  $u_g$  is the ground displacement so the displacement which is imparted because of the road.

Now if we calculate this  $F_T$  so  $F_T$  has two components in it, one is stiffness multiplied by the displacement and damping multiplied by velocity two terms stiffness multiplied by displacement and damping multiplied by velocity. Now let us find out the solution of this like equation, equation of motion if this is a proposal solution as a solution that is  $u(t)=u_0$  which is amplitude  $\sin\omega t - \phi$  so  $\phi$  is the phase angle,  $u(t)=u_0$  is nothing but  $u_0=u_{st} \cdot R_d$   $rR_d$  we know is a ratio of dynamic displacement to static displacement amplitude, so this is a equation.

(Refer Slide Time: 04:11)

**Transmissibility**

Substituting  $U(t)$  in  $F_T$

$F_T = k u_{st} R_d \sin(\omega t - \phi) + c u_{st} R_d \omega \cos(\omega t - \phi)$

$F_T = u_{st} R_d (k \sin(\omega t - \phi) + c \omega \cos(\omega t - \phi))$   $F_{T0} = P_0 R_d \sqrt{1 + (2\xi r)^2}$

$F_{T0} = u_{st} R_d \sqrt{k^2 + (c\omega)^2}$   $\frac{F_{T0}}{P_0} = R_d \sqrt{1 + (2\xi r)^2}$

$F_{T0} = \frac{P_0}{k} R_d k \sqrt{1 + \left(\frac{c\omega}{k}\right)^2}$  **Since**  $R_d = \frac{1}{\sqrt{(1-r^2)^2 + (2\xi r)^2}}$

$F_{T0} = P_0 R_d \sqrt{1 + \left[\frac{2m\xi\omega_v\omega}{k}\right]^2}$   $\frac{F_{T0}}{P_0} = \frac{\sqrt{1 + (2\xi r)^2}}{\sqrt{(2\xi r)^2 + (1-r^2)^2}} = \frac{\ddot{u}_0}{\ddot{u}_{g0}}$

4

Now if you substitute this in the above equation of  $F_T$  so what we will get let us see that, so  $ku_{ut}$  and then  $c \cdot \dot{u}$  so if we differentiate that  $u$  once so what we will get is from this one, so if we differentiate this one so what we will get is  $\omega$  comes out, okay and then that becomes  $\cos$  as you

can see, okay. So  $u_{st} \cdot R_d \omega \cos \omega t - \phi$  so then up on further simplification  $u_{st} R_d$  can be taken out  $k \sin \omega t - \phi$   $c \omega \cos \omega t - \phi$  so what is important to as this is the amplitude, so amplitude of this sin and cos is if we have  $A \sin \Theta + B \cos \Theta$  amplitude of vibration is square root of  $A^2 + B^2$ .

So that is what is worked out, so this is a coefficient and  $C \omega$  is a coefficient so  $u_{st} \times R_d$  under root  $k^2 + C \omega^2$ . And here you look at it, this is a amplitude, this is a variation with time function and this is only amplitude. So we are taking out amplitude from it and then UST is nothing but  $P_0/k$   $R_d$  value remains  $k$  if we take out it is  $(1 - C \omega/k)^2$ .

So upon further simplification of expanding of the values okay of  $C$  so  $2m\zeta\omega_n$  is a  $C$  value, so if we expand it, it looks like this and upon further simplification we get, so if we substitute  $R_d$  also in this one and then take the ratio okay, this  $PFTE/P_0$  so  $FT$  is a force transmitted and  $P_0$  is the amplitude of the applied force.

So amplitude of transmitted force by amplitude of applied force is equal to like  $R_d \times$  under root  $(1 - 2\zeta^2)^2$ , if we put  $R_d$  value in this one, so what we will get is this. So amplitude of applied force transmitted force by amplitude of applied force is equal to this whole big equation, that is actually equal to amplitudes of this one  $U..0/U..G0$  this is a applied force and that is a transmitted force.

(Refer Slide Time: 06:39)

**Example Problem**

A weight of 150 kg is acted upon by vehicle of force  $F = 300\sin 6t$ . Assume  $\xi = 10\%$ . If stiffness of surface on which vehicle is resting is  $50\text{kn/m}$ . Find the amplitude of force transmitted.

**Solutions**

**Natural Frequency**

$$\omega_n = \sqrt{\frac{k}{m}} = \sqrt{\frac{50000}{150}} = 18.26\text{rad/sec}$$

**Static deflection**

$$u_s = \frac{P_0}{k} = \frac{300 \times 10^3}{50 \times 10^3} = 6$$

**Frequency Ratio**

$$r = \frac{\omega}{\omega_n} = \frac{60}{18.27} = 3.26$$

5

Now let us solve one problem using this principle okay to understand this phenomenon let we will learn more. So a weight of 150kg is acted upon by vehicle of force  $F=300\sin 6t$ , so this 6 is a forcing frequency and 300 is a amplitude of that force, and the damping assumed in the system is 10% and if stiffness of surface on which vehicle is resting is 50kn/m. Find the amplitude of force transmitted.

So let us first find out natural frequency which is under root  $k/m$  so  $k$  is given as 50,000 and mass is given as 150kg so with that we will get 18.26rad/sec so that is natural frequency. Then frequency ratio forcing frequency is 60 oh! so this is 60 not 6 this is  $60 \sin 60t$ . So forcing frequency is  $60/18.27$  so that is 3.26 is  $R$ , so frequency ratio  $R$ . So if we find out static deflection that is  $P_0/k$  so force amplitude is 300 and  $k$  is 50 so if we detect the ratio we get 6.

(Refer Slide Time: 08:00)

### Example Problem (Cont...)

Steady state amplitude  $u(t) = u_0 \sin(\omega t - \phi)$

Displacement response factor

$$u_0 = \frac{u_{st}}{\sqrt{(1-r^2)^2 + (2\zeta r)^2}} = \frac{6}{\sqrt{(1-3.26^2)^2 + (2 \times 0.1 \times 3.26)^2}}$$
$$\frac{F_{T0}}{P_0} = \frac{1 + (2\zeta r)^2}{\sqrt{(2\zeta r)^2 + (1-r^2)^2}} \quad \frac{F_{T0}}{P_0} = \frac{1 + (2 \times 0.1 \times 3.26)^2}{\sqrt{(2 \times 0.1 \times 3.26)^2 + (1-3.26^2)^2}} = 0.124$$

$$A_T = F_{T0} \times 0.124 = 0.124 \times 300 = 37.11 \text{ kN}$$

6

Then steady state amplitude is  $u = u_0 \sin \omega t - 5$ , so displacement response factor, so with this one  $u/u_{st} = R_d$  that is  $1/\sqrt{(1-r^2)^2 + (2\zeta r)^2}$  all this value. So  $u_{st}$  we know  $u_{st}$  value we have put there and  $F_T/P_0$  is  $1 + 2\zeta^2 r^2$  or all this value, so we substitute appropriate values in that. So we get 0.124 as the ratio. So if we want acceleration  $t$  which is transmitted value is  $F_T \times 0.124 = 0.124 \times 300$  given, so 37.11 kN. So when the force applied is 300 kN the 37.11 kN is the force transmitted.

(Refer Slide Time: 08:51)



So in this class we have studied how the force is transmitted from the moving vehicle on to the road like it depends on like three factor one is the weight of the vehicle and second one is the speed of the vehicle which is deciding the frequency of application of force and then third one of the damping. So with which the load is getting transmitted.

### **IIT Madras Production**

Funded by

Department of Higher Education

Ministry of Human Resource Development

Government of India

[www.nptel.ac.in](http://www.nptel.ac.in)

Copyrights Reserved