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Structural Dynamics
Week 3: Module 03

Relationship between R_d, T_v & R_a

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Welcome back to structural dynamics class. So in this module we will study about dynamic amplification factors. So last class we have studied about damped forced vibration and undamped forced vibrations. So as we have noticed in that is, when forcing frequency and the natural frequency of the system matches a resonating condition is taking place. So in this resonating condition amplitudes have displacement, amplitude of velocity and amplitude of acceleration will go to very high values.

So sometimes up to infinity or very large values, so because of the presence of damping we can reduce this amplitude to a greater extent, but let us understand the relationship between these response dynamic amplification factor for displacement, velocity and acceleration in this class. So how we are sensitive to frequency ration and like where do we apply all those things we will study in this class.

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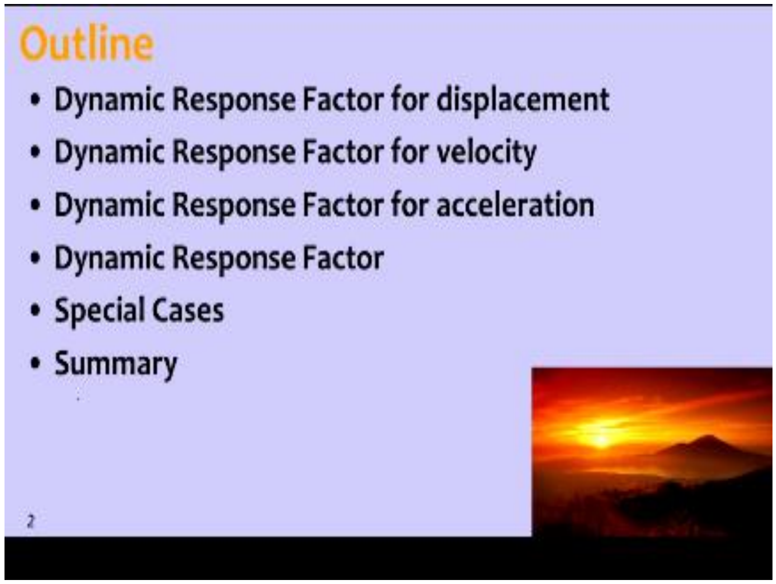

So the outline of this class is dynamic response factor for displacement we will understand, response factor for velocity we will understand, dynamic response factor for acceleration we will understand. We will try to derive the relationship between these three factors displacement, response as well as acceleration. Now if we look at the equation of motion.

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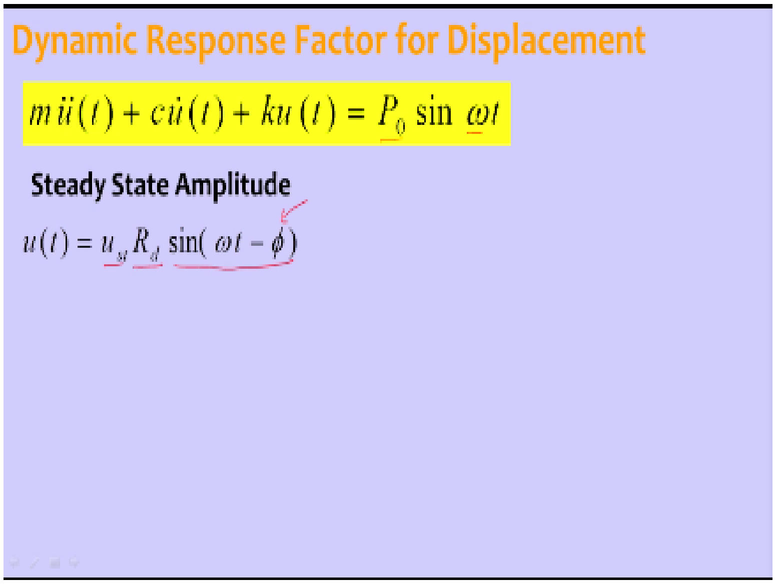

So which consists of three terms that is inertia term, damping term and stiffness term. So all these are not together is equal to the external force which is dynamic in nature. P0 is the amplitude of the force and ω is the forcing frequency. Now if we look at the study state amplitude which we have studied, which we have understood in the last class, so study state amplitude is the product of static displacement and dynamic amplification factor for displacement as well as the sinusoidal component.

So in this we need to observe the presence of this ϕ, this ϕ is a phase angle. So its phase angle means like the way the force is getting applied and the way displacement is taking place, so are they in phase or are they out of phase. So in phase means when we apply force displacement also will be in the same direction, but out of phase means when we apply force displacement will be in the opposite direction. So these conditions also we will try to understand in this.

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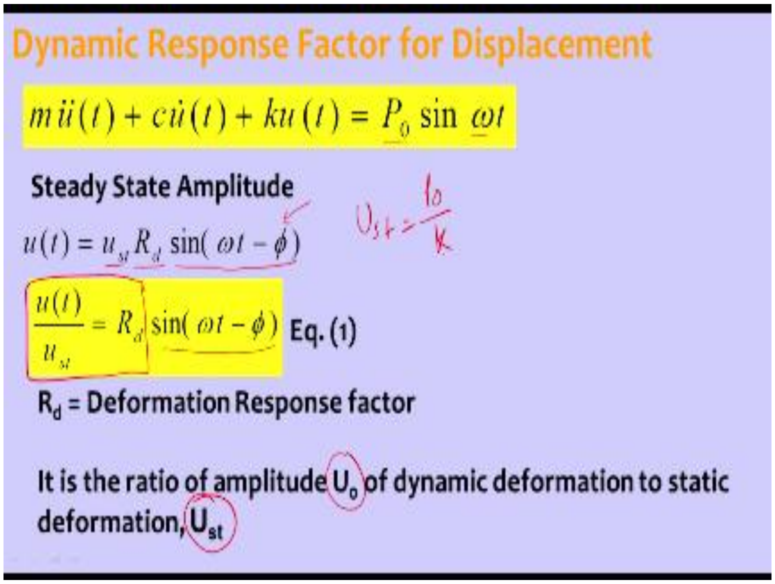

Now if we look at this one so dynamic displacement divided by static displacement is equal to R_d x sinωt-ϕ, but if you take only amplitude it becomes this, this much portion amplitude. So this R_d is called deformation response factor or dynamic response factor for displacement. So it is a ratio of Uo that is amplitude of dynamic deformation to static deformation.

So we know that static deformation can be found out by amplitude of the load divided by stiffness, this is static deformation. So dynamic deformation is it more than the static deformation, less than static deformation, so what are the cases which increases, decreases this dynamic deformation compared to static deformation we will try to understand.

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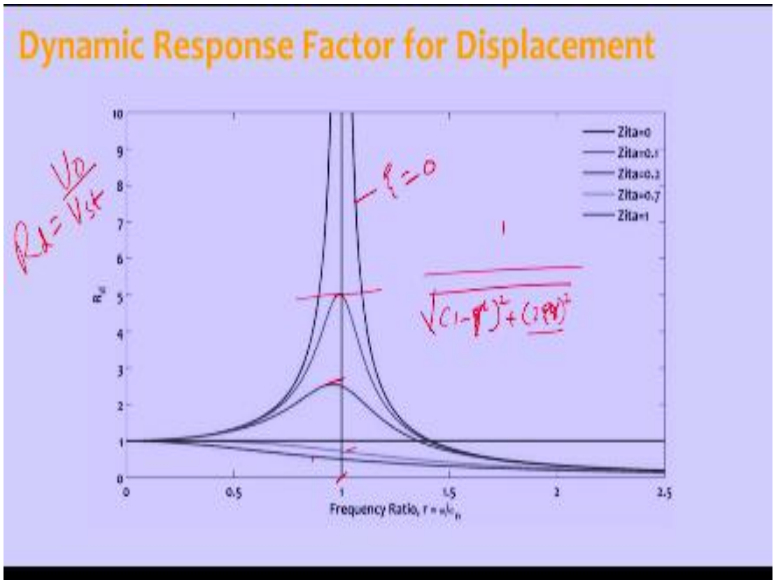

Now if we plot this curve R_d, so that Rd is U_0/U_{st} and this R_d is the equation is 1/(1- ζ^2+2 ζ2 ζr) sorry 1-r^2+(1-r^2)^2+2 ζr^2. So if you substitute R in this one r value from 0 up to say 2.5 will get this response so this response is for different damping value so this the first this one is ζ value is equal to 0 and here ζ value is 0.1, 0.2, 0.7 and ζ 100% damping and you can see in this curve we can see that at when r = 1 in this if I substitute r = 1 and when ζ = 0.

So this term will disappear and it we will be left with only 1 – r^2 when r is one this will become 0 so that means the amplitude Rd value will be infinity and when we introduce say 10 % damping, 20% damping something like that so suddenly this amplification that peak value will be it is getting reduced this is dynamic amplification factor for displacement.

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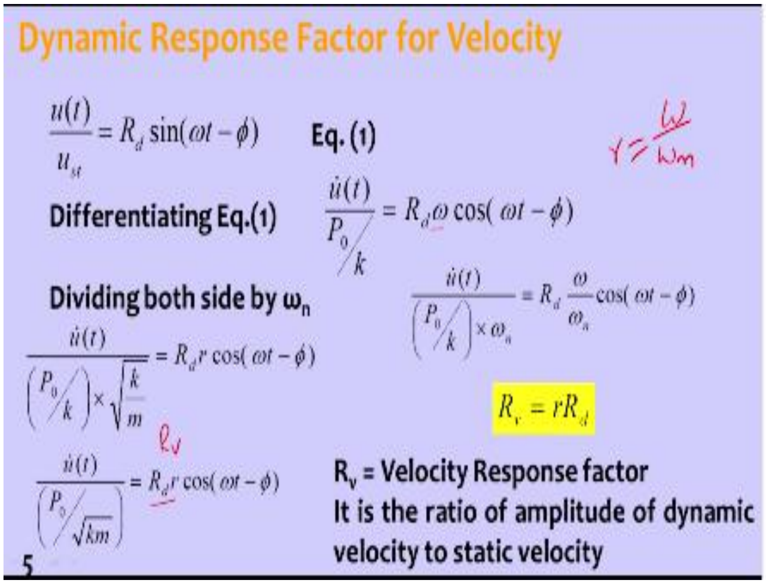

Now dynamic amplification factor for velocity so as we know this is a equation so u dynamic displacement divided by static displacement is equal to Rd multiplied by sinusoidal path that is ωt – 5 so if we differentiate this equation then what we get is so on a numerator we get velocity and here if we differentiate one ω comes out of that sin ωt – Ø so ω and the Cos t it will remain so Us(t) is nothing but P0 by amplitude by stiffness.

So if you rearrange this terms and by dividing by ωn so what we get is this ω/ωn will remain and this To/K x ωn so if we rearrange this one ωn is under root k/V0 ω/ωn is a ratio of frequencies so R = ω/ωn it is ratio of a frequencies so we can write that u./P0/√ Km = Rd multiplied by R x Cos ωt – 1 so what we can look at is, you can see this one u.t so u.t means velocity okay in that so this term can be treated as Rv so this is called dynamic response factor for velocity it is s ratio of amplitude of dynamic velocity to static velocity is like Rv so dynamic velocity and this is static velocity.

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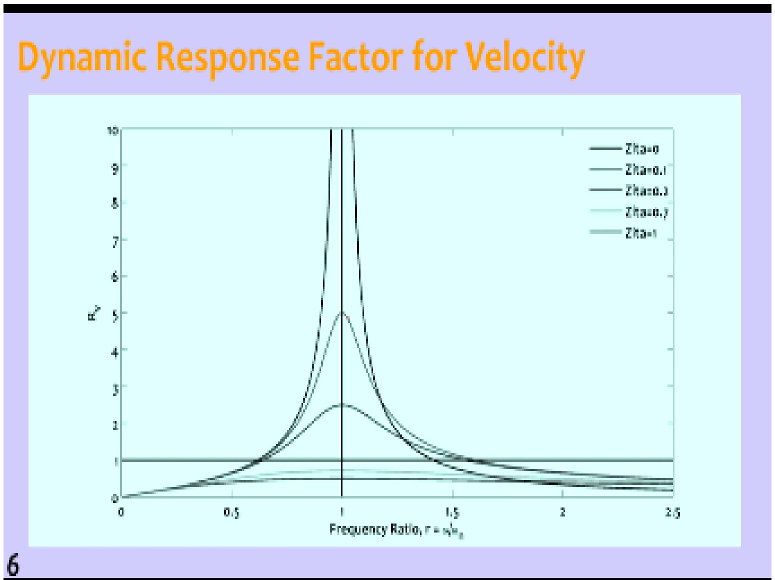

So the graph if we plot this ne the graph what we will get is like this okay again in this one you can see when damping is not there this value is goes to infinity and with the presence of damping that peak amplitude drastically reduces.

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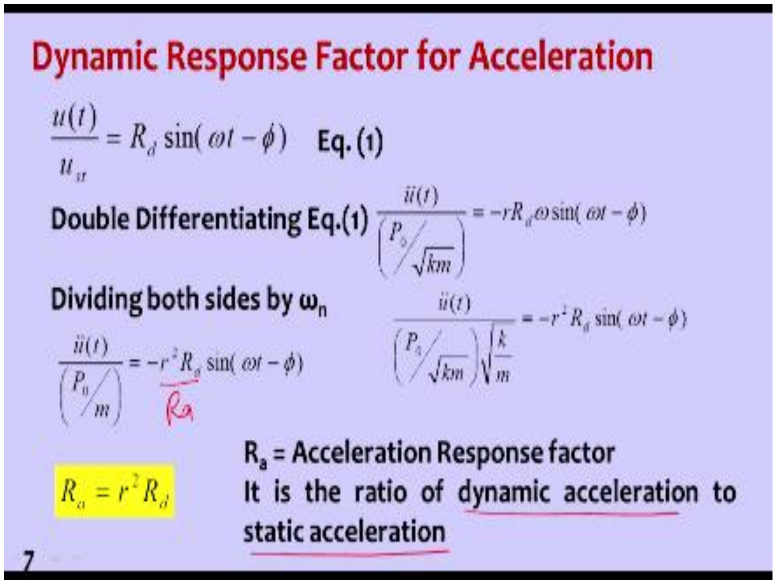

And then in the similar manner let us go further and analyze dynamic amplification factor for acceleration so the same equation if we take but now we will differentiate it twice so if we do it twice what we will get is u.. on the numerator and here if you look at this K filling Rd multiplied by we need to do twice so ω^2 down will come so 1ω came down here so another ω is remains here can see if you divide by ωn so what we get is u.. P0/√ kM and again √ km because of ω natural frequency return by natural frequency.

We get R^2 Rd Sin ωt – 1, so this negative sign can be removed because we are testing the amplitude only so now u., U.. /P0/k we using this equation. If you look at it, then $-r^2 R_d \sin\omega t-\phi$, so in this one so this term can be treated as R_a dynamic amplification factor for acceleration so R_a is acceleration response factor it is a ratio of dynamic acceleration to static acceleration values.

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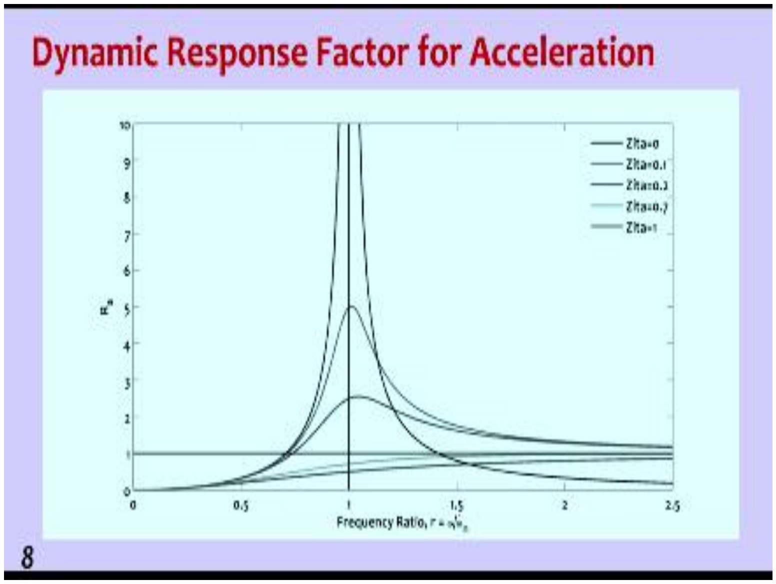

Now if we look at the plot of this dynamic acceleration we get the similar plot but there are some changes I will explain this changes little while later.

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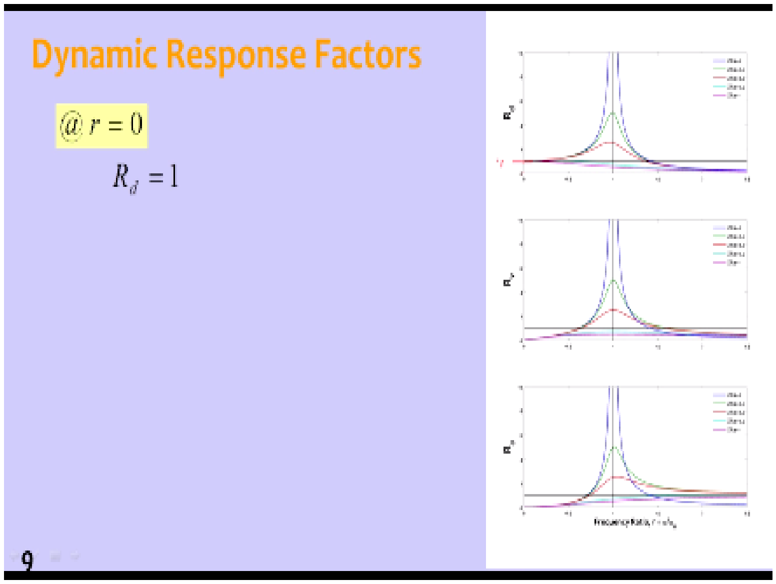

Now if you look all three at a time, okay so these graphs looks quite similar but when we observe it we will find the differences so at r=0, r=0 means it is slowly varying force are static force at this location when r=0 what is happening to R_d, R_v and R_a let us look at that. R_d is starting from 1 you can see this, this value is 1 so that means what when the body I can see this one let us take entire beam and a person or say a moving mass is, moving weight is there so if it is standing here r is less means it is static, static means directly deflection will be there, so ratio of static deflection to dynamic deflection is 1 only, so that is equal to static defection again.

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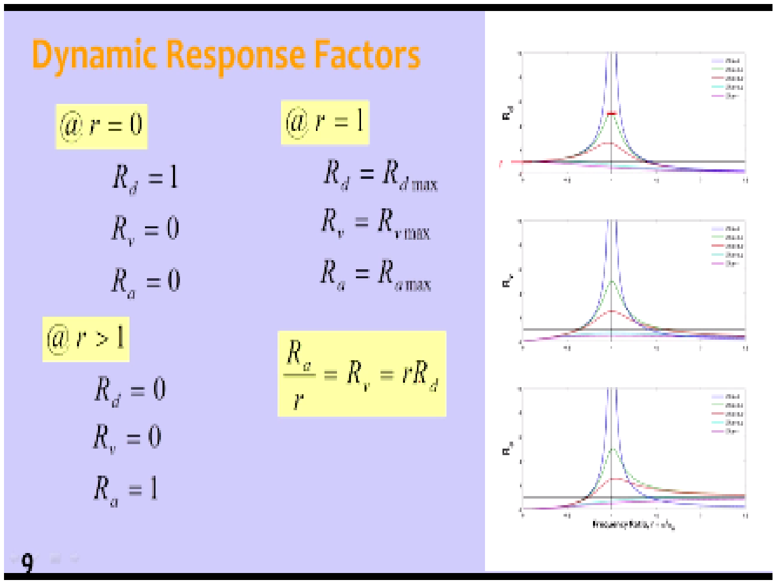

And if you look at R_v velocity since there is mass is not moving that force is not moving R_v will be 0 and similarly R_a will also be 0, so at r=0 at lower frequencies or at rest condition ω/ω_n that ω is 0 or nearly equal to 0 we get r=0, so R_d=1, R_v=0, R_a=0. When we go to exchange values okay, or when we come to resonating frequencies what is going to happen. Resonating frequencies along with damping R_d=R_{dmax} value so something like this at this location.

And then R_v is R_{vmax} and Ra is R_{amax} so a point need to be observed in this one is all this R_{dmax}, R_{vmax} and R_{amax} values they are not occurring exactly at the same value but they are nearly maximum at that value, nearly maximum. Because of that results of damping that slightly shifts slightly away from this one mark, and when r value is much, much greater than 1 so that means r is a 10, 20 something like that.

So what happens is R_d goes to 0 you can see here, so at larger values R_d goes to 0. And R_v also goes to 0 and then R_a is equal to 1, so that means what we can clearly see that at lower values R_a R_d is sensitive at higher values of frequency ratio R_a is sensitive and let resonating frequencies R_v is sensitive, so if we look at the relationship bit among these three things R_a divided by frequency ratio is equal to R_v that is again equal to r.R_d.

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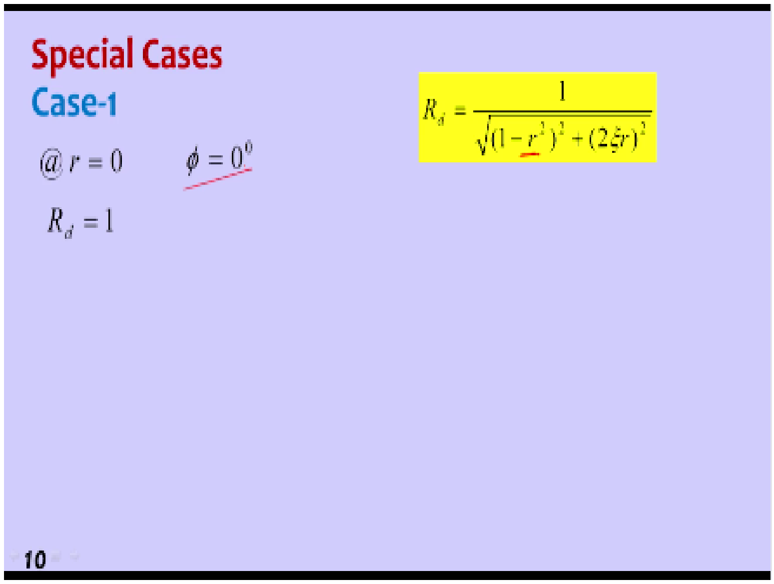

Now let us look at some special cases in this, what are this special cases, special case is if we discuss in detail that when r=0 that is at rest condition, so at rest condition if you look at this R_d equation $R_d=1/(1-r^2)^2+(2\xi r)^2$ now when r=0 if you substitute in this one so what remains is so if all these values become 1 so R_d becomes 1, and this phase is 0 so that means what at rest condition or near rest condition so when the forcing frequency is applied like this.

So in the direction of force displacement will be there, so that is what is the meaning of in phase so 5= 0 means in phase and 5= up to 90 is in phase only the suddenly if it shift and goes to 180 that is out of phase.

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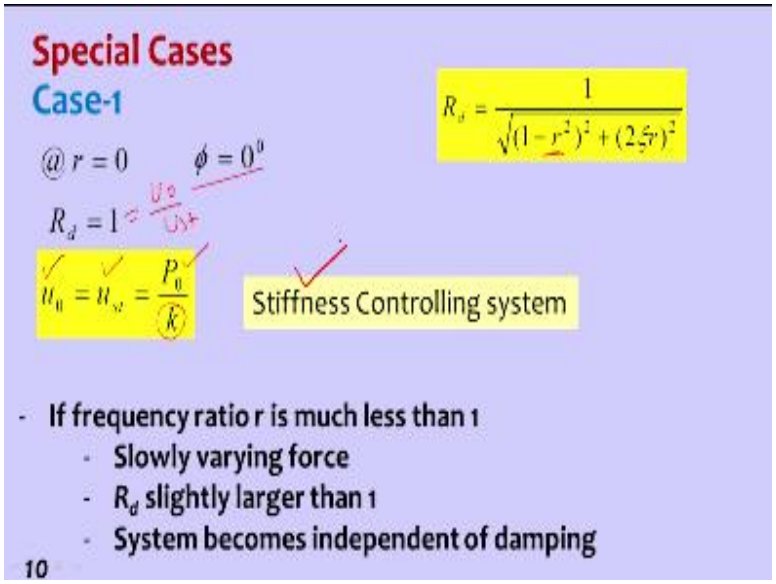

Now if we substitute this value what happens is rd =1 rd means u0 / ust if it is = 1 there it gives us u0 = ust so that means what ust u0 us = p0/k so what we are understanding from this is p0 is the externally applied force amplitude, amplitude of externally pallid force then k is stiffness control in so that means what k is stiffness so when r – 0 or nearly 0 vibration is control by stiffness so that means stiffness control is system it is.

So if frequency ratio r is much less than one it is slowly varying force rd is slightly larger than one so that means what it can be 1, 1.0, 11.00 1 or 1.12 something like that the system becomes independent of damping her damping here damping as no role to play whatever damping maybe present ion the system but deflection is control by stiffness so more stiff less displacement less stiff more displacement. So this is first special case where force is slowly varying.

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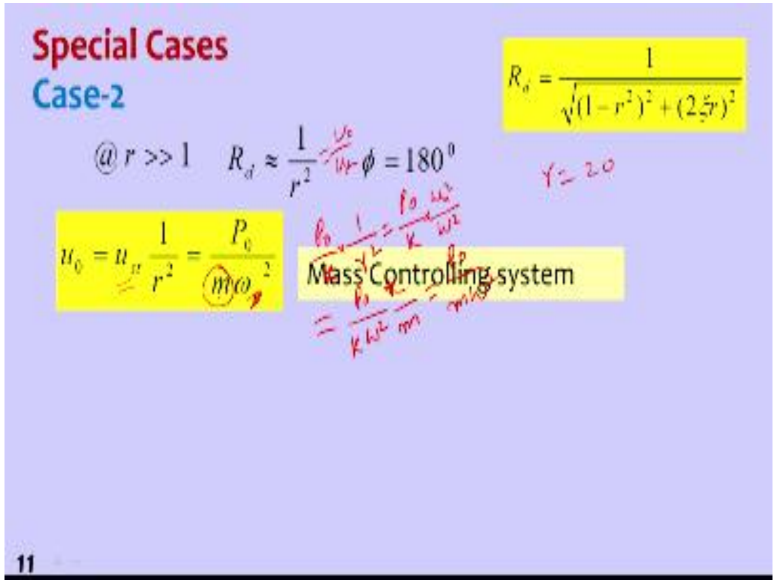

And the second special case is when force is raptly varying so when force is raptly varying means what high frequency forces are acting on the system when we look at the rd response curve so 1 / √(1- $r^2)^2$ + 2 zeta or whole square so that means what r value is much much larger so if r value is much much larger this second will be significance so √ and square that becomes only $1/r^2$ so this is approximately equal to 1/ r^2 if you put some numerical numbers and you check so you will get the similar value.

Maybe you can put say r = say 20 and check approximately that will be same you will get in this one and zeta any value zeta will become in sensitive there like 1/ r^2 rd = $1/r^2$ and this shows that it is out of phase so when r value is more than one vibration amplitude vibration becomes out of phase when r value is less than one vibration amplitude is in phase it is something like when force is applied when this force is applied displacement in the same direction.

But when the r value is very large when force is applied displacement comes like this so it is goes like this okay using this rd = $1/r^2$ relationship as you can see so this is equal to u0/ ust so that means what u0 = $1/r^2$ x ust, ust x $1/r^2$ so that means $1/r^2$ means p_0 / ust x $1/r^2$ so ust is t_0 / k and if you expand this one if you open this one what is going to happen is something like this p_0/k x 1/r^2 so that is equal to p_0/k x ω n^2 / ω^2.

So you note by k ω^2 x ω nm^2 means √ k /m, will be left with p_0 / $m\omega^2$ sorry this n is not there so p_0/ m ω^2 and id we look at this one this is controlled by mass so that means what, what system parameter is there is in this one mass is there so more mass less amplitude less mass more amplitude in the earlier case more stiffness less displacement less stiffness more displacement that is a.

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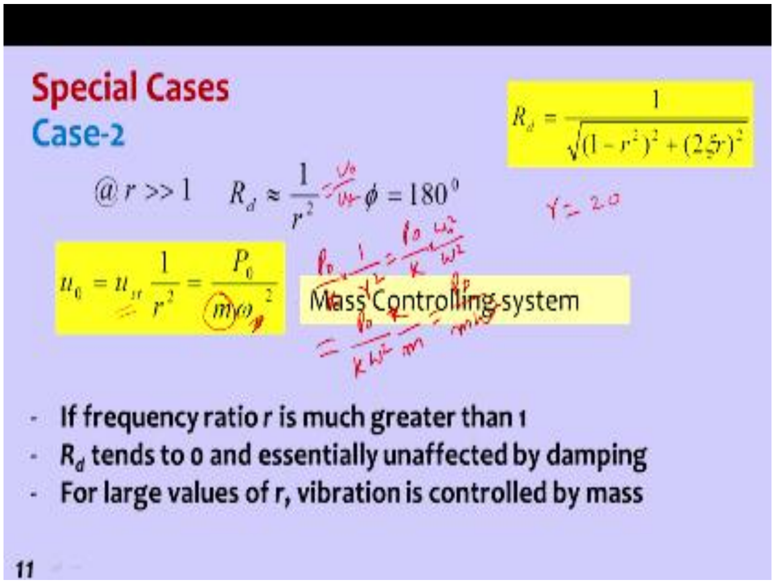

Special case one this is a special case two so in this one what we can summaries is the frequency ratio is much greater than 1 that is rapidly varying force then already tents to 0 and it is it essentially unaffected by damping so higher value means it is very, very less value $1/R^2$ for large values of R vibration is controlled by mass so mass controlled system so slowly varying force Stiffness control system rapidly varying force mass control system.

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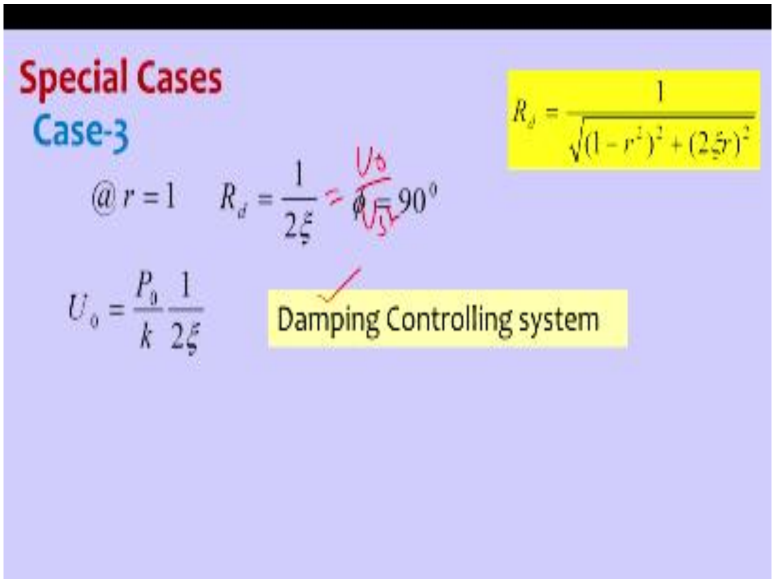

Now let's look at the third special case third special case is when r=1 so that means what then forcing frequency is very near to the resonating frequency okay. so if you look at the this one and substitute R=1 here so will be left with 2ξ so Rd = 2 ξ so that's nothing but U0/USR Rd= ratio of dynamic to static displacement. So and then 5 is 90 so faces is exactly 90 that is the switching of location so p0 /k 1/2ξso here ξ is damping ratio that means we call it as vibration is controlled by damping.

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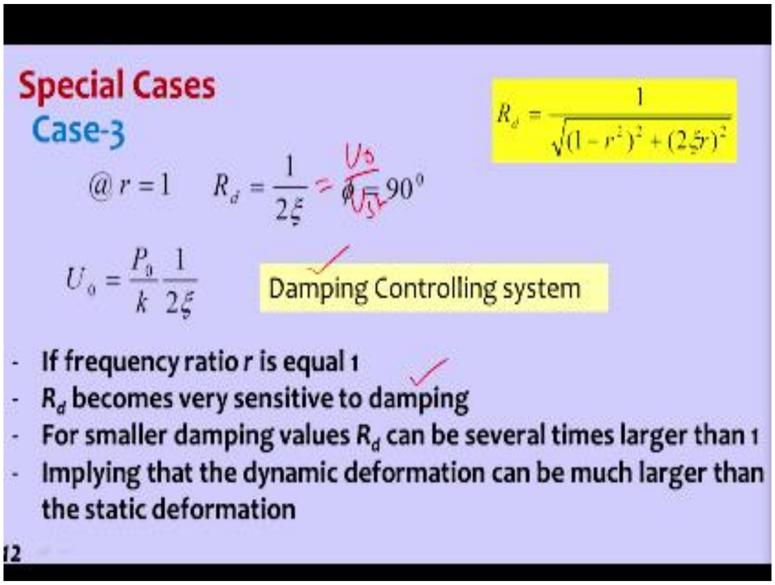

So if frequency ratio is =1 RD becomes very sensitive to damping and for small values of damping Rd can be several times larger than 1 implying that dynamic differ measure can be larger than static deformation at this range so as in summary we have studied like RD dynamic response factor for displacement dynamic response factor for velocity dynamic response factor for acceleration. And how these three things are related that way of studied in this class and then in dynamic amplification curve Rd.

So different places the sensitivity we have studied so when force is slowly varying okay that means R value is near one then deformation is controlled by stiffness and when force is rapidly varying then R value is very, very large so that is rapidly varying force. Deformation are controlled by mass and then when R is around resonating frequency around 1 that means forcing frequency and natural frequency are same nearly equal and then deformation is controlled by damping.

So K control system N control system and C control system so these three things three controls when they come into picture so that we have discussed in this class.

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