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Structured Dynamics
Week 3: Module 02

Damped Force Vibration

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Welcome back to structural dynamic class so in this class we will study Damped Force Vibrations, system is vibrating because of the application of the external force which is dynamic in nature along with the damping present in the system so how the behavior of this system will be, so that we are going to study.

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Resonating Condition

rpm	w	r	R_d
0	0	0	1
100	9.10	0.05	1.00
500	45.50	0.25	1.06
1000	91.01	0.50	1.34
1500	136.52	0.76	2.39
1800	163.82	0.91	6.19
1900	172.92	0.96	15.23
1950	177.47	0.99	63.16
1960	178.38	0.997	175.09
1965	178.84	0.9996	1582.8
1970	179.29	1.0022	224.18
1980	180.20	1.0073	68.10
2000	182.02	1.0174	28.33

$$u_0 = u_{st} R_d = \frac{P_0}{k}\frac{1}{\left|1-r^2\right|}$$

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So first general equilibrium equation that is equation of motion.

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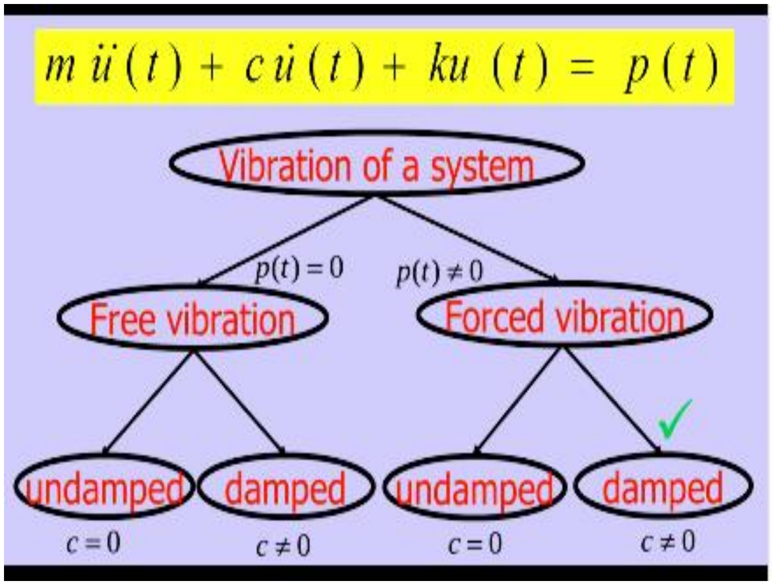

And there this so we have already studied three types of vibrations that is un-damped free vibration, damped free vibration un-damped forced vibration also we have studied, now in this class we will study damped forced vibration.

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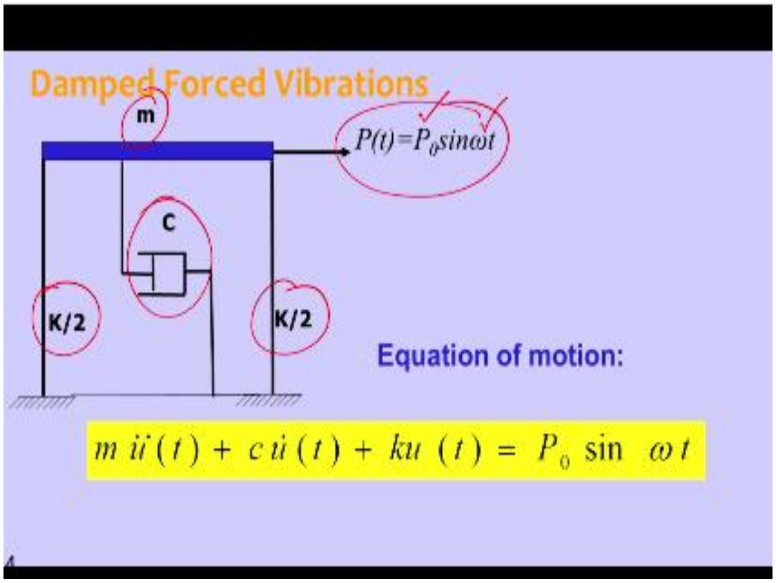

So damped force vibration looks like this so where mass of the system is given by m and then stiffness is given by k and also damping is present in the system and then external force is applied that is sinusoidal force P0 sin ω t, P0 is a amplitude of the force and ω is the frequency of the force so equation of motion looks like this one the compute equation of motion.

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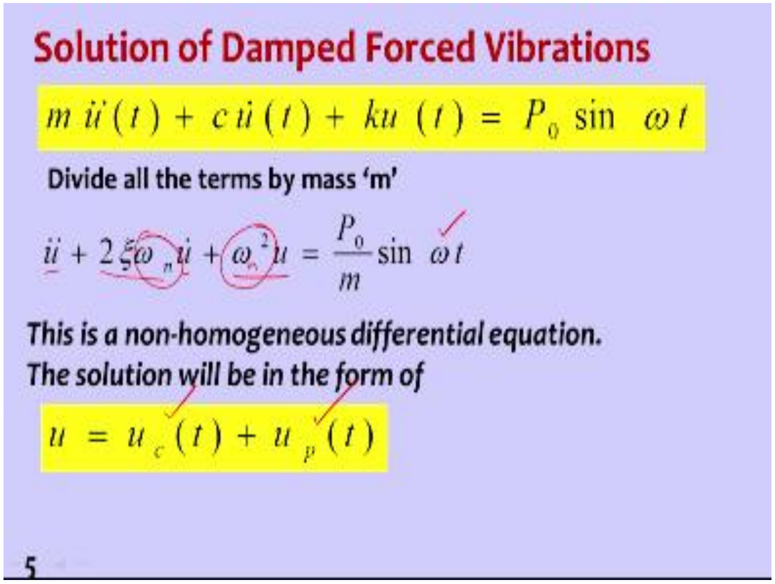

So this equation of motion consists of like two parts equation of motion has a on right on side non zero term so since it has non zero term we call it as non homogenous differential equation this linear differential equation with constant co-efficient non homogenous type so if you divide this equation by mass so we get U.. + 2ζ ωn U . + ωn^2 u and Po/m sin ωt so this ω is forcing frequency this ωn this ωn is a natural frequency of the system which depends on system properties.

That is stiffness as well as mass, so solution since it is non homogenous differential equation so solution has two parts one is complementary function and the second one is vertical integral.

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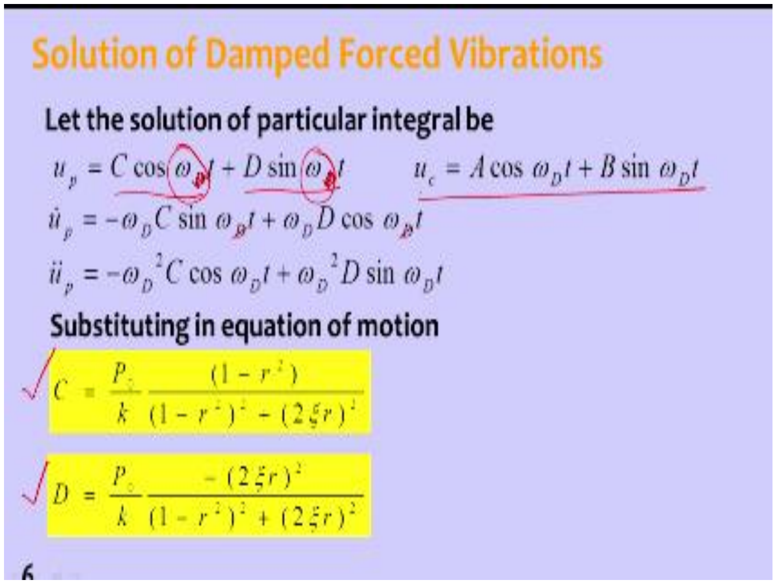

So first of all let us find out how the particular integral looks like, so since sinusoidal wave is applied as a forcing frequency so we can find the solution or we can assume the solution to be of two component that is C cos ωdt and D sin ωDt so this ωD is a damped natural frequency, okay. So U complementary is Cos ωdt Sin ω dt sorry this one will not have damping present in this one if we differentiate it with respect to time so UP that is ω D C sin ωt ωD . C cos ωt substituting in the equation of motion we get C constant and D constant.

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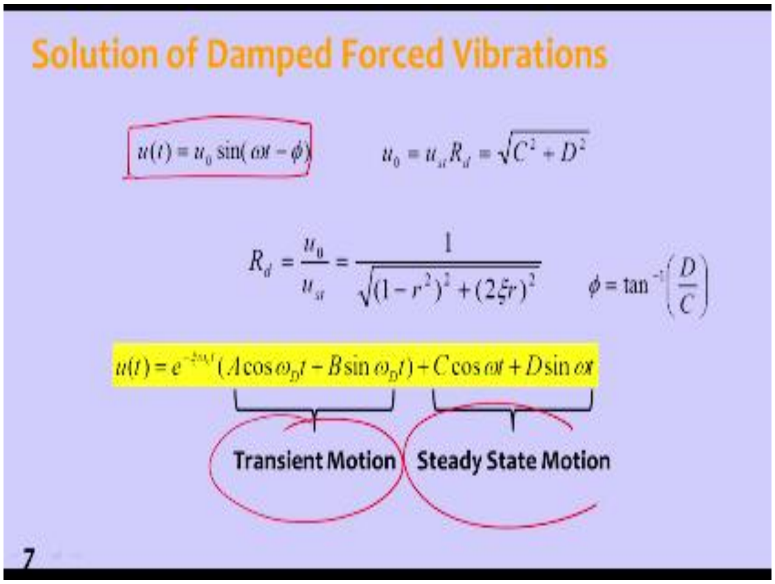

Than if we write it back the solution of the differential equation that is linear differential equation with constant co-efficient non homogenous in nature the response will be u(t) = u= sin (ω t – Ø) so here it is showing phase so phase is like when we are applying force you can look at it so this is a cantilever beam and then a person is standing sat the end and slowly like sitting up and down and then the rod is deflecting.

So your rod is also oscillating because of the persons moving up and down so what happens is for sometime the motion of the person and the motion of the rod are the cantilever beam a blank will be in phase suddenly at some point of time it moved out of phase as a person is trying to come down the plank is applying force in the upward direction so that is called out of phase kind of motion.

So this Ø denotes that so in phase out off phase so in phase is less than 90 out of phase is more than 90. So the complete solution looks like this it has two components one is transient motion and the second one is steady state motion.

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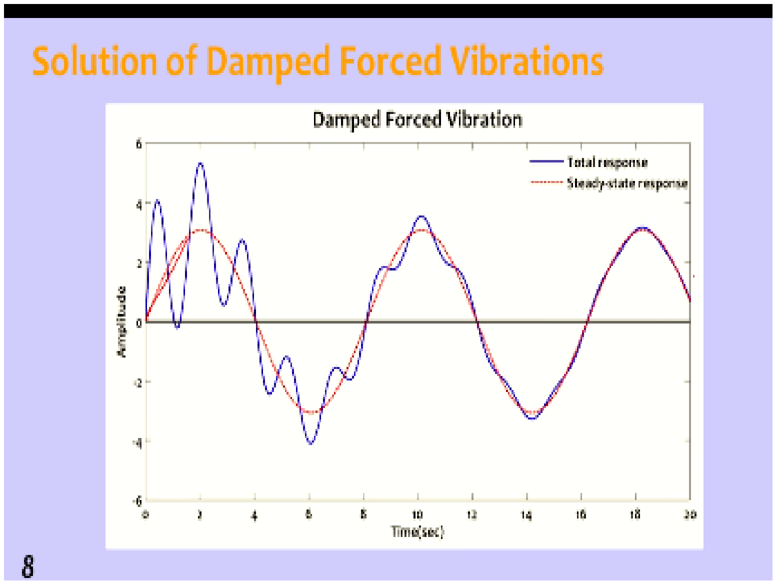

So total response if we plot it will be this red dotted line is a steady state response and a transient motion along with steady state motion is given in the form of blue line as you can see in this one blue line is slowly merging into transient, so because this transient motion which is responsible which is coming from the initial conditions will remain only till some point so when damping is present it will die out soon.

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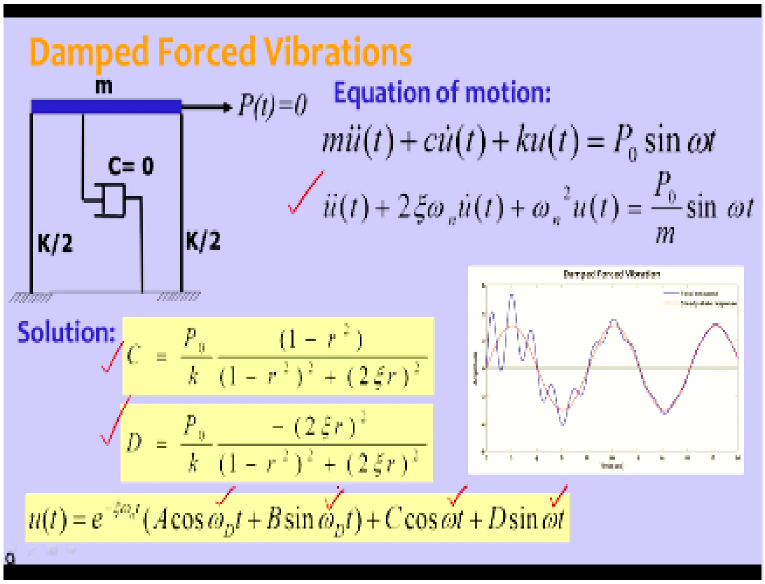

So in overall summary if we look at it equation of motion simplified equation of motion and the solution constants and then finally A cos ωD, B sin ωDt and C cos ω + B sin ω so this ω, ω is a forcing frequency and this ωD on ωD damped in natural frequency and this is a response of that.

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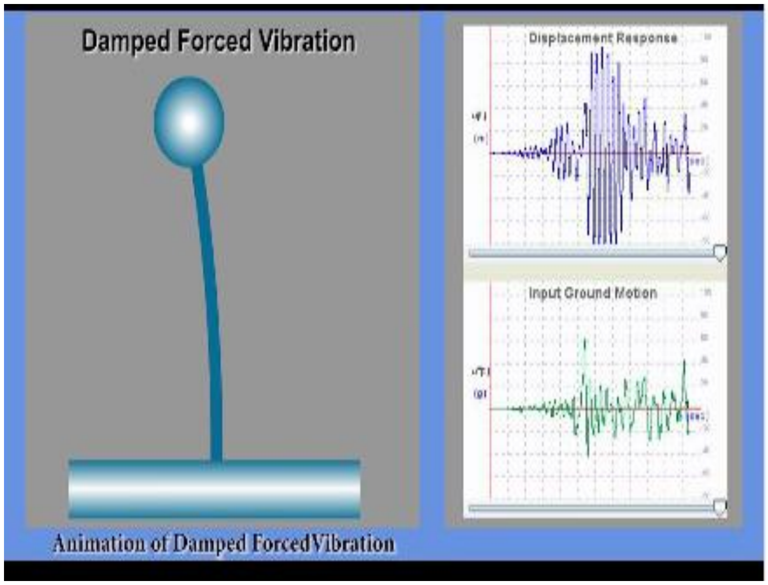

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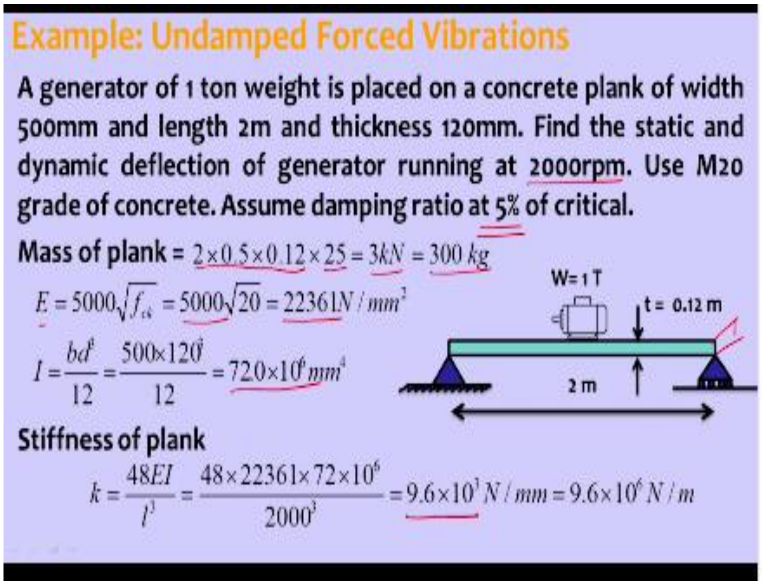

And then if we take the example problems same example problem what we have discussed earlier so a generator of one ton is placed on a concrete plank of width 500 mm and length 2m and thickness 120 mm, find static and dynamic deflection of generator running at 2000rpm , use M20 grade concrete assume damping ratio at 5% of critical so now damping is present in the system.

Earlier case damping is 0, so if we derive or find out mass of the plank this is volume of the concrete plank and then density of the concrete that gives us weight 3KN so that is equal to 300kg and then modulus of elasticity of that is 5000 under root FCK which is 22361N/mm^2 and moment of inertia I = BDQ/12 so B is 500 width is 500 and depth is 120m /Q/12 that is 72 x 10^6 and the stiffness of the plank is 48 EL/l^3 so when we substitute all the above value all the values we get 9.6 x 10^3 N/mm or 9.6 x 10 ^{6} N/m.

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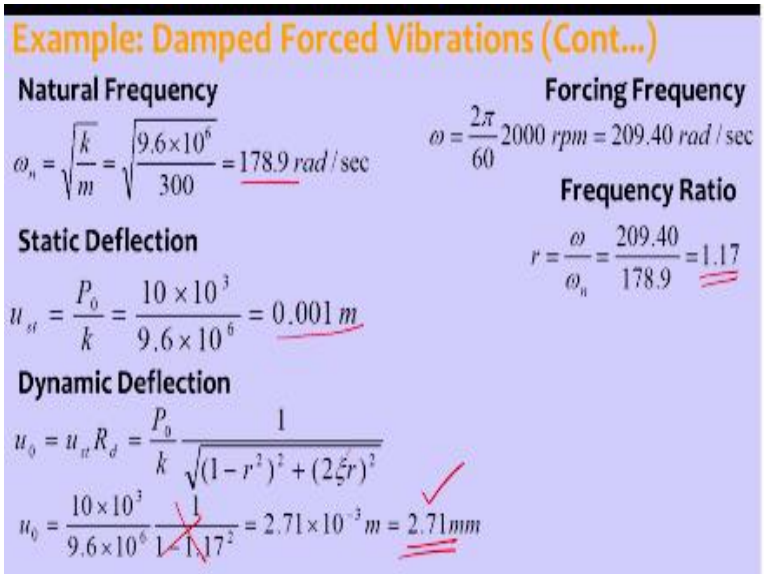

So if we calculate the natural frequency the ωN which is under root K/M so this is 9.6 x 10^{6} / 300 so we get 178.9 in radians/sec this is a natural frequency of the system, as the machine is operated at 2000rpm so we can convert that rpm into radians per second that is 2Π/60 into given rpm so which we get 209.4 rad/sec, so the frequency ration you will use it so frequency ration R is ω/ωn.

So this is 1.17 so this is slightly higher than the resonating condition, now if we find the static deflection which is P0/K so that is equal to 10 x 10^3 / 9.6 x 10^6 that gives us 0.001 m which is 1mm then dynamic deflection is U is equal to u is a starting deflection component multiplied by dynamic amplification factor so the dynamic amplification factor in this case is 1/(1 – R^2 $)^2$ + $(2\zeta r)^2$ under root.

So if you substitute all these terms here or this is without dynamic part this is a substitute all these terms here we get if you substitute if we take 0 damping in this case so we will get 2.71 mm but let us look at the case with the 5 % damping.

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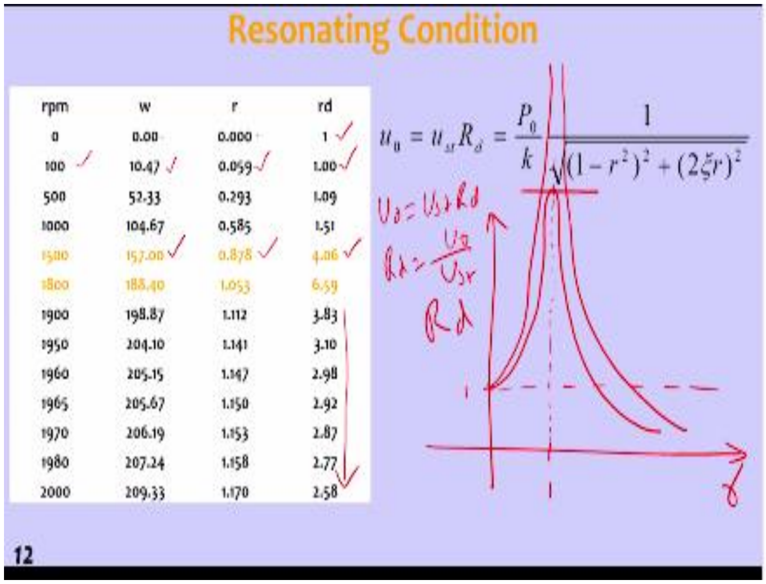

So when we put 5% damping and then when we increase the rpm from 0 to 2000 so how it looks like let us see 0 rpm then what is in frequency also 0 then r is 0 rd will become 1 and if rpm increases slightly which is 100 so frequency is 10.47, 0.059 and again is very neat to 1 so as when it increase up to 1500 ω is 157 and 0.878 is r and 4 so as you can see this is like 4 times more than static defection so you can see here u0 = u_s t x Rd.

So that means Rd = U0/U_st so that means it is 4 times more than static deflection and when it comes to 1800 it is 188 it is above 1.053 is that means what in-between this one something is happening so what is that resonating condition is happening but still immediately after that displacement have reduced before that displacement is higher.

So that is amplification is high but on the reducing direction that is in path you can see this one so if we plot that frequency ration r and Rd so this resonating frequency is 1 and this is static deflection okay Rd is U_0y U_s t so here if it is 0 at static it is 1 and then it goes like this up and then suddenly it comes down like this, okay. So what damping does is, damping controls the amplitude of vibration if it is un-damped vibration then it would have been something like this.

It will go to infinity and make the system what a break during the vibration summary, what we have discussed in this class we have discussed how to find out the solution to damped forced vibration where forcing frequency is ω and natural frequency is ωn and damped natural frequency is ωd so we get a equation consisting of two part one is transient part and a steady state path.

So transient part will die out because of the presence of damping and steady state part will continue and we have studied one example problem also of how we apply this damping in damped forced vibration.

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