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Structural Dynamics
Week 3: Module 01

UnDamped Forced Vibration

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Welcome back to structural dynamics course. So this week, in this module we will study undamped forced vibration. So what is undamped forced vibration or what is forced vibration. So earlier we had studied that a structure is there which is set to oscillate by giving initial disturbance. So structure is oscillating, so where force is not taken into consideration only initial displacement or initial velocity is taken into consideration.

So that is called free vibration. So now this week we will study about the forced vibration where force is continuously acting on the structure. So this is called forced vibration. So we need to take force also into our equation of motion when solving it. So the outline of this module where we are discussing undamped free vibration is like how to find the solution for undamped free vibration and we will solve an example problem on that.

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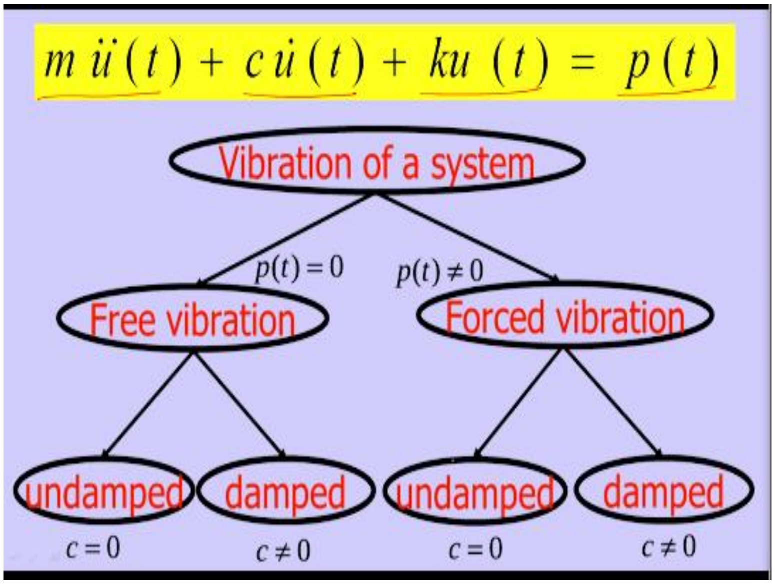

So this is general equation of motion or dynamic equilibrium equation. So in this dynamic equilibrium equation we have three terms that is inertia term, damping term, stiffness term as well as externally applied force term. So these four are there inertia, damping and stiffness and externally applied force. And the types of systems are like if force is not acting then we call it as free vibration.

If force is acting we call it as forced vibration. And when damping is not present it is undamped vibrations, and if damping is present it is damped vibration. So undamped free vibration and damped free vibration, so these two things we have already discussed and now in this module let us discuss undamped forced vibration.

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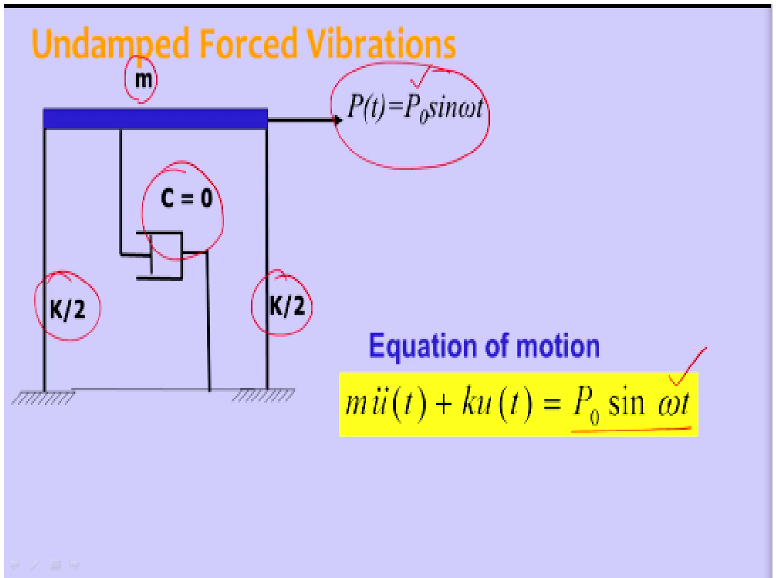

Now however a system and the equation of motion looks like, it will be like this. So mass is there at the roof level and stiffness that is a resisting elements and damping is not present in the system and the excitation force which is in the form of sinusoidal excitation with constant force P0. So that is how it is written P0sinωt. So ω here is a force in frequency, it is not natural frequency do not get confused with natural frequency.

Natural frequency is a system property which depends on mass of the structure and stiffness of the structure, forcing frequency is a, the frequency at which force is getting applied see like this frequency at which force is getting applied is forcing frequency. So what is going to happen if a force is dynamic and it is acting. So if you can see this is the cantilever beam or you can assume this as a diving road of a swimming pool.

So if a person is standing, so if a person is standing here then because of the self mate of the person there will be some deflection. But all of a sudden if person starts moving up and down, up and down, up and down so what will happen to deflection surely it will increase and sometimes it will decrease. So what happens is a person is coming down going up, coming down going up so with his knees bent up, down knees while knees is bending so a person can go up down, up down kind of thing while his foot is resting on the edge of the diving board.

So then motion will take place in this one which is more than static equilibrium position and sometimes it is less than static equilibrium position. So how this dynamics is taking place that we are going to study in this module.

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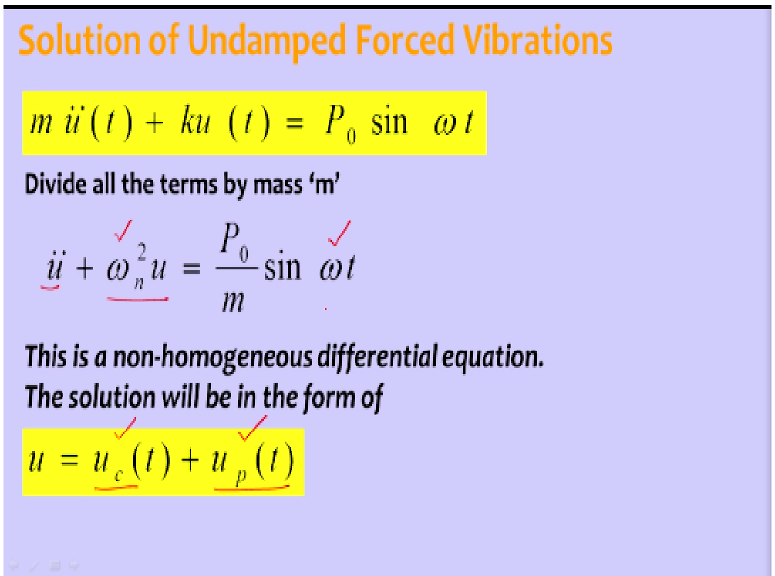

The equation of motion we have set, so we need to find the solution for this equation of motion. So what we are going to do is like in this equation we first divide all the terms by mass, if we divide all the terms by mass, so equation looks like this U.. and k/m is nothing but ωn^2P0/m and sinωt so this ω is a forcing frequency and ωn is a natural frequency of the system.

Now this is like non-homogeneous differential equation, homogeneous differential equation means what, where right hand side is 0, so this right hand side is nonzero so that means we call it as non-homogeneous differential equation and this is linear differential equation. So the solution of this one we will have two components.

So one is complimentary component and second one is particular integral component. So this complimentary component depends on the natural properties of the structure system and particular integral depends on the force being applied. Now let us evaluate this particular integral and complimentary function.

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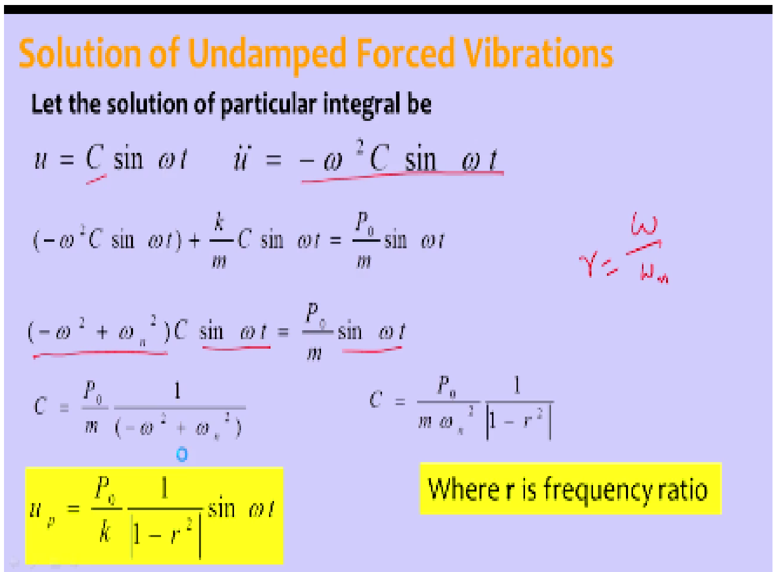

So how the particular integral will be, so usually if the forcing frequency or forcing function is sinωt so particular integral will also of the form sinωt with some constant C which we need to evaluate that constant. So if U is a assumed solution then let us derive U.., so U.. is two time differentiation of this term. So if you differentiate this two times, so what happens is like ωcostωt and again if you differentiate it will become –ωsinωtxω so we get –ωCsinωt.

So let us substitute U and U.. in the original equation, so if you substitute U and U.. in the original equation. So this is a u.. so this system and a k/m u so right is equal to p/m sin ωt so how to solve this one, so the easy method is we need to equate this like we need to take common this sin terms and then equate the rest of the thing with the quotient as you can see this one if you re-arrange this above equation.

We get $-\omega^2 + \omega n^2$ C sin ωt and then Sin ωt so this sin ωt is common so we can take out from this C from the above equation we get constant c as P0/m x 1/- $\omega^2 + \omega n^2$ now if you re-arrange this terms by taking ωn from this bracket common so then what happens is $\omega n^2 / \omega n = 1$ and this $\omega^2 / \omega n^2 = r$ so here we call r is equal to frequency ration that is ω/ ωn frequency ration, so then the equation can be re-arrange like this. So u particular integral will be P0/K so this is C is derived as P0/k $1/1\text{-}r^2$ x Sin ωt so this is particular integral.

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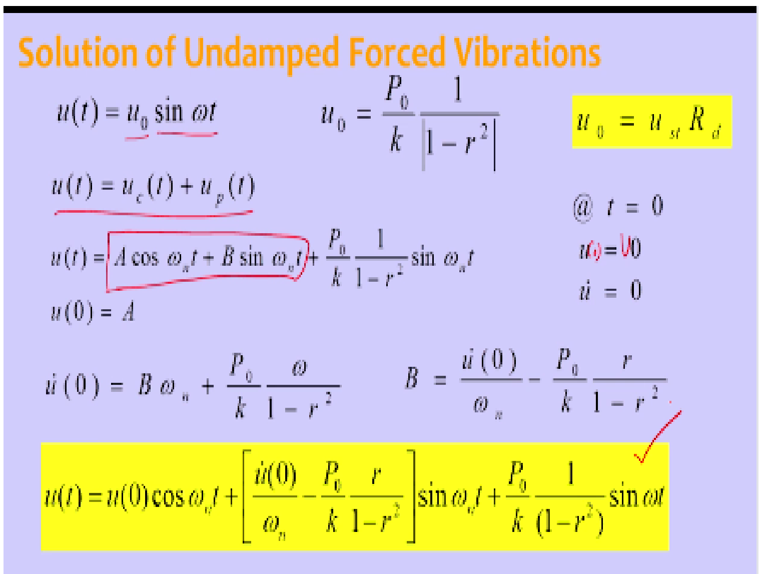

So now we need the full solution so u(t) = u(0) sin ωt so this u(0) is giving amplitude to us and then sinusoidal part is Sin ωt so usually there will be some phase in this one but for un-damped vibration phase will be only when ω is less than ωn it will vibrate in phase and ω is greater than ωn it will be out of phase so we will discuss about this phase a little while later.

So let us concentrate on the amplitude of vibration so amplitude of vibration that is u(0) = P0/k so P0/k is nothing but static deflection when mass is at rest static defection multiplied by this dynamic amplification factor component so then the total solution will be u is equal to complementary function plus particular integral so now you need to find out the complementary function also so this complimentary function originally we know from the un-damped free vibrations is.

A cos ωnt + B sin ωnt together with a particular integral part, so now we need to get rid of A and B constants so will get rid of this A and B constant by using initial conditions which we already know so at time t = 0 displacement is u0 at time t = 0 velocity is u.0 so a time t = 0 if displacement u = u0 if you substitute that in this one then u0 will be A and then at velocity component is so u.0 will get B ωn + P0/k ω/1- ω^2.

So using this we can get B constant B as this term and then if you substitute back in the original equation so this is a solution of un-damped forced vibration as you can see that if you remove forcing frequency from this one it will go back to un-damped free vibration situation only.

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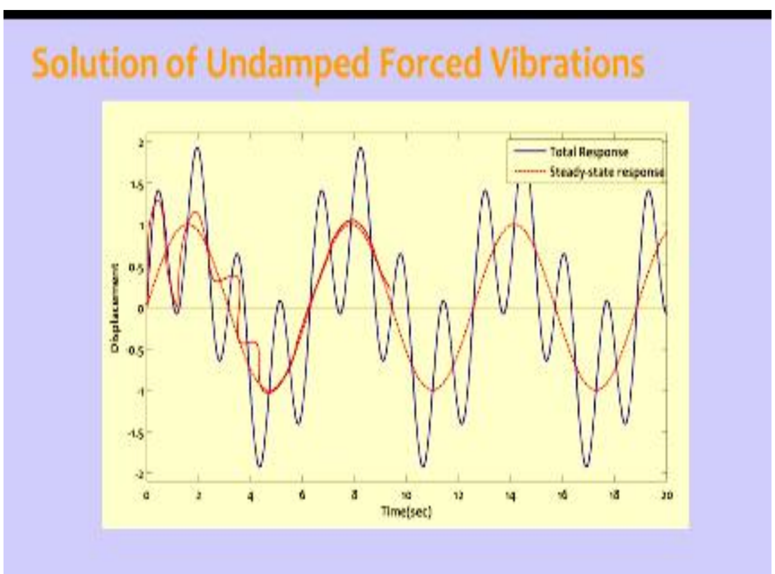

Now how it looks like so it as two components in it one is like transient component and second one is steady state component so if you look at this dark blue line so dark blue line is showing us the total response whereas this dotted red line is showing the steady state response. So steady state response continues forever as long as the force is there and the total response is in addition steady state response plus transient response.

Usually when damping is not present this transient response will continue forever along with the steady state response but in cases where damping is present it usually it dies out something like this it merges with a steady state response.

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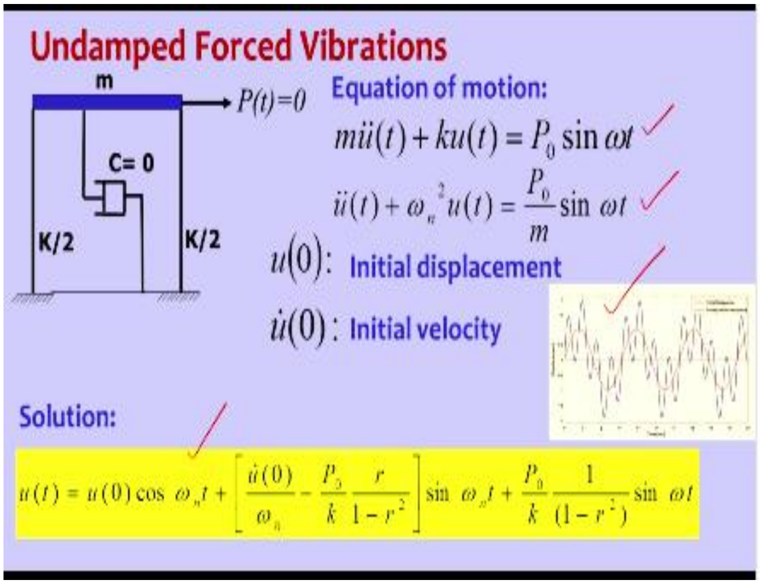

Now let us look at the overall un-damped free vibrations in actual this is a equation of motion of un-damped forced vibrations and then this is a simplification of that equation and by solving it we get this as a solution and the plot is this.

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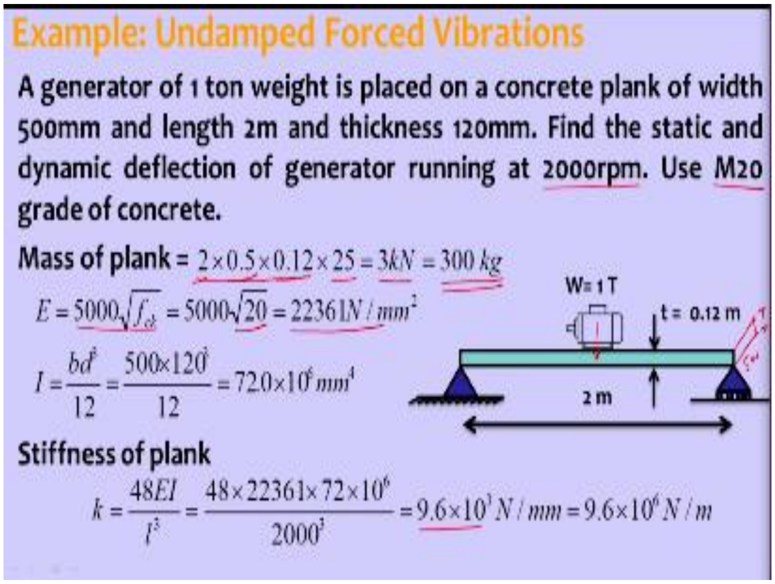

Than let us solve the example problem one problem application of it say here I have a problem I generator of one ton where it is placed on a concert plank of width 500mm and length 2m and thickness of 120mm at the dimensions of the plank are given and find the static and dynamic defection of generator running a 2000rpm, and use M20 grade concrete for the that plank so it looks like this so it is simply supported.

Length is 2m and weight of the motor the weight of the generator is 1 ton and the thickness of slab is 120mm so we are placing this at the center we are placing with this centre so if this we placed at the center then it will automatically cause the deflection so using that we can find the stiffness of the system so let us first find the system properties.

First let us find out mass of the plank. So mass of the plank is 2 meters length multiply by 0.5 meter width multiplied by 120mm thickness so all this three things is volume and then this is weight density of concrete so we get 3kn so if we convert in to kg that we get 300kg as a the mass of the system.

And then modeless of the velocity of the system where according to is 456 we get 5000√fck since patristic strength is given as 20n per mm^2 so we have 5000√ 20 so we get 22361 n/mm^2 is modeless of velocity. Then we find out moment of inertias so that is bd^3 / 12 so b width of the plan this on the other side and you can see this one width of the plank is 500 and then this thickness is this.

So b is that 120 mm thick bd^3 /12 so that is 72 x 10^6 mm^4 and then stiffness of the plank so first we find the stiffness of the plank k because it is simply supported of $48EI/l^3$ so substitute all the values 48 e value as in here and i value as in here and then length is a2000mm so we get stiffness as 9.6 x 10^3 n/mm and that can be rewritten as if we convert the units so 9.6 x 10^6 n/m.

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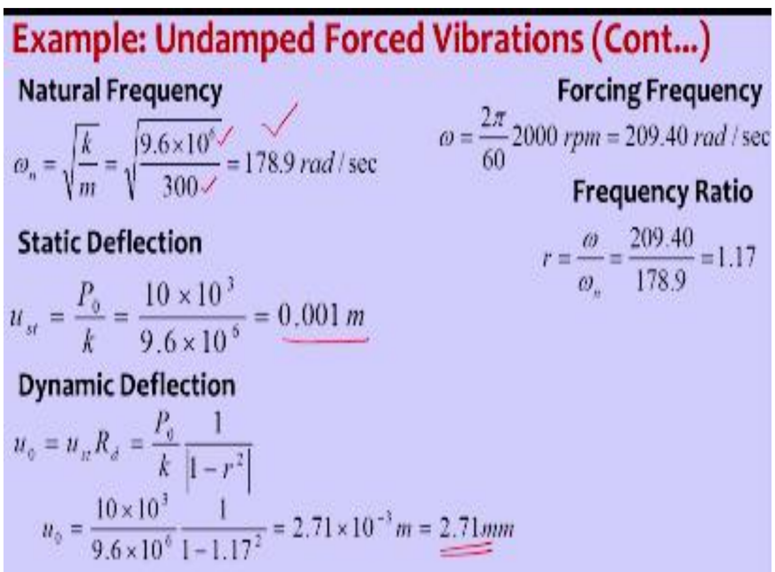

Now let us find out the natural frequency of the plank so natural frequency of the plank depends on the system properties that is √k/m so k is known value that is 9.6 will be 10^6 and mass is 300kg so that gives us 178.9 rad/per first find out the static deflection so static deflection is use static is p0/k so it is something if we not by it gets starting deflection so p is the load of the generator or motor that is the ton is converted in to new ton.

So that is 10 x 10^3 /k k is also known to us 9.6 x 10^6 so that gives us 0.001 m so that is equal to 1mmm and then we need to find out the dynamic deflection so for that if the equation of the dynamic is U_0 that is amplitude that U_{st} x R_d is amplitude of dynamic deflection so use U_{st} x R_d so U_{st} is nothing but P_0/k R_d is 1/1-r^2 so if we substitute all these values in this one we are getting 2.71mm as a dynamic deflection as suppose to 1mm as a static deflection.

So that means because of the running of the motor so deflection is becoming higher because of the running of the motor but here we need to understand one case well simply supported beam is there when motor is kept on the center of the beam motor is running at the operating speed of 200rpm but what happens is when you switch on the motor starts it takes time to this operating frequency or operating rpm so it starts from a 0 and then 10rpm 20 rpm 30 rpm like that.
So it is going and then reaching 2000 rpm case so what is going to happen in between this 0 to 2000 so let us discusses that.

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Resonating Condition

rpm	w	r	R_d
0	0	0	1
100	9.10	0.05	1.00
500	45.50	0.25	1.06
1000	91.01	0.50	1.34
1500	136.52	0.76	2.39
1800	163.82	0.91	6.19
1900	172.92	0.96	15.23
1950	177.47	0.99	63.16
1960	178.38	0.997	175.09
1965	178.84	0.9996	1582.8
1970	179.29	1.0022	224.18
1980	180.20	1.0073	68.10
2000	182.02	1.0174	28.33

$$u_0 = u_{st} R_d = \frac{P_0}{k}\left[\frac{1}{|1-r^2|}\right]$$

$$R_d = \frac{U_0}{U}$$

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Now if you look at this one so in this we will discuss the resonating condition so resonating condition arises when r becomes 1 so here rpm is 0 so that means it I in static, then ω so ω is a forcing frequency the relationship between that rpm and forcing frequency is this so 2000 rpm so

2π/ 60 p/ m is there so 2π. 60 x rpm will give us the forcing frequency in terms of radiance for second. So exactly that we are calculating so this is rpm of the machine this is forcing frequency of the mnachi9ne and this r is = ω / ω n and then rd is you calculated using this equation.

So if it is a trust then rd value is 1 so that means dynamic deflection and static deflection are same now when rpm is slowly achieved 100 rpm then at that time so on forcing frequency is 9.1 and then at that time r is so this frequency ratio is 0.05 and the rd is very near to 1 so if you look at this if when it is increase it to 500 then ω is 45.5 then r as increased frequency ratio is increase little bit further but still it is at one only rd is one because that means what there is not much difference between static deflection and dynamic deflection.

So when rpm become 1000 then frequency has become that forcing frequency is 91 and the ra as become 0.5 so here you can see that there is 30% increase between static deflection dynamic deflection in a way you can take rd as U_0 / U_{st} so that means like what by how much amount the percentage increase compute to static deflection.

So it is 34% now when, when RPM has become 1500 it is 2.39 and when RPM is become 1800 then it is 0.9 is R then you can see this became six times and RPM became 1900 it became 15 times when RPM became 1950 it became 63 times and RPM became 90 you can see this it's very increasing very fast and suddenly it has dropped here.

Suddenly it has dropped so that means something is happening between R value 0.999 and R value so just before 1 and just after 1 something is happening so if we discuss that using the graph is something like on X axis let’s take R and on Y axis let’s take RD which is dynamic amplification factor or ratio of dynamic deflection to static deflection so when R is 1. So before R is 1 it is starting from with 1 then R is 0 then it is starting from 1 then slowly it is increasing gradually it's increasing.

And suddenly it is increasing when it is reaching 1 and then after that it is falling down, after that it is falling down so this is dynamic amplification curve so if it's exactly =1 this value goes to infinity. This value goes to infinity and then comes back so what happens is practically it is not possible why because if you can look at Rd dynamic displacement is starting displacement multiplied by Rd.

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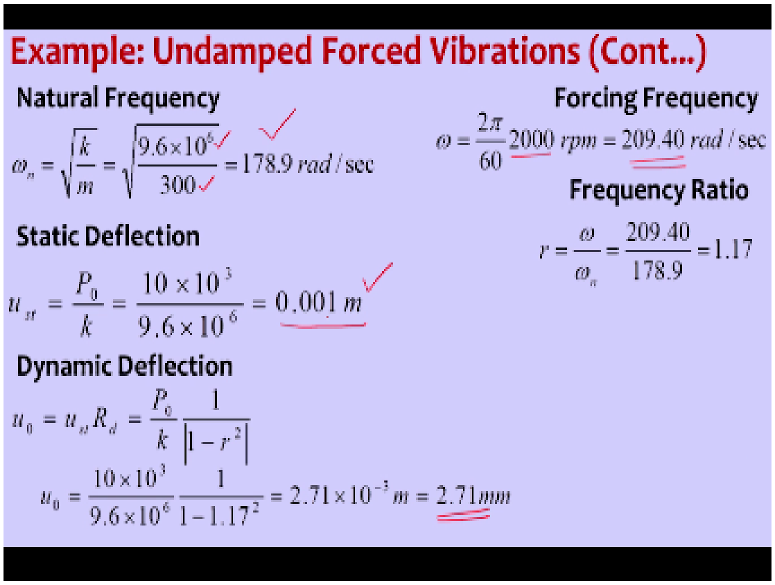

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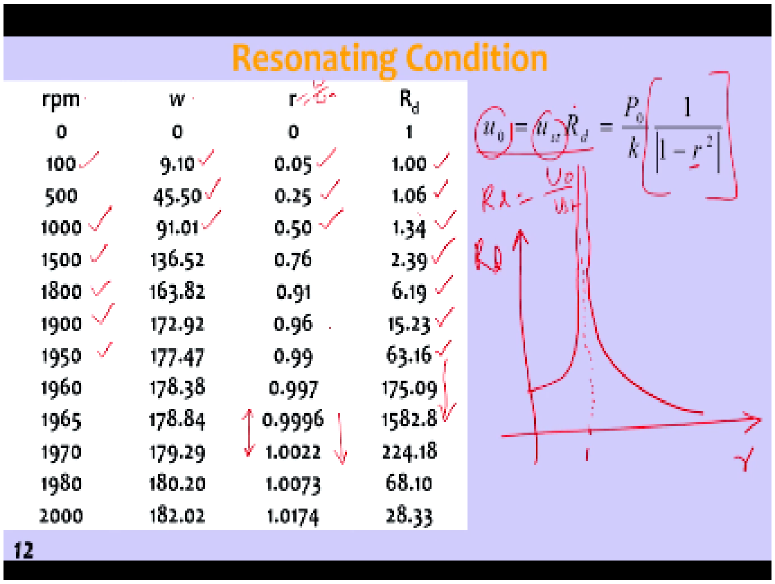

rpm	w	r	R_d
0	0	0	1
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1980	180.20	1.0073	68.10
2000	182.02	1.0174	28.33

So what is static displacement one millimeter only so then what happens is this one millimeter multiplied by this Rd so that means what 1582 millimeters so that means length of the plank is.

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2 meters that deflection is 1.5 meters so this is not possible so what is going to happen is either motor is going to fall outside of the system and there is condition or the plank is going to break so this tells us the presence of damping or the tells us the important of damping to be present in the vibrating system especially dually.

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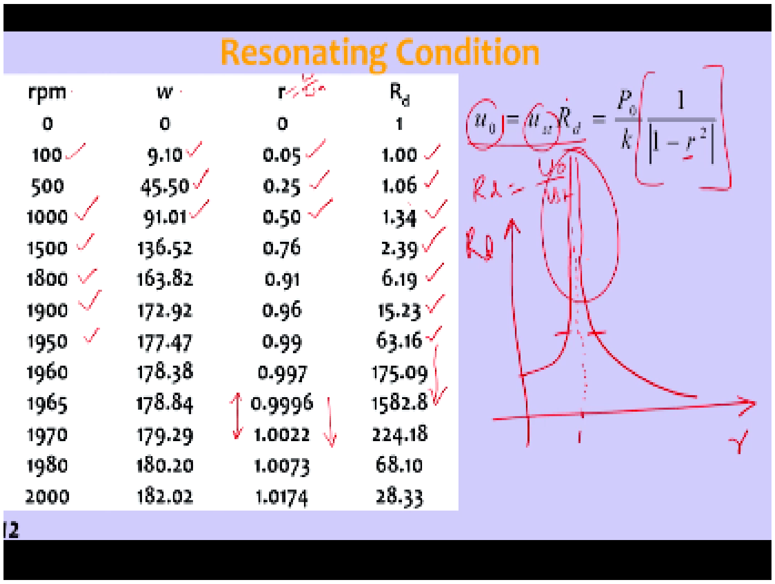

Near the resonating conditions so here it is in the limit perfectly in the limit limits of deflection but when it is near nearing the resonating condition it is deflections are very high and system A breaks so damping is very important to be included in the system were vibrations are natural were the frequency of vibrations is very close to natural frequency of the system in summary so what we have studied in this module so we have developed a equation of motion and try to find the solution of the equation of motion.

So solution of equation of motion contains two parts two components one is a complementary component which depends on the system parameters and particular integral which exclusively depends on the applied force so if you found out the complementary function as well as particular integral and then applied to one example problem and understood the resulting condition also.

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