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Structural Dynamics
Week 1: Module 03

Degree of Freedom

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In this module we will study degrees of freedom or kinematic indeterminacy. The outline of this module is.

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We will discuss about static indeterminacy, kinematic indeterminacy and then we will define degrees of freedom and we will formulate the stiffness matrix for a given structure and then like wherever mass is not associated to any degree of freedom, so how to remove those degrees of freedom. So these things we will study in this module, the first what is the static indeterminacy.

So as well all know that we have three equations of equilibrium so that is sum of all forces in horizontal direction must be equal to zero, and sum of all vertical forces should be equal to zero.

And sum of all moments should be equal to zero, so these are three equations of equilibrium in static condition.

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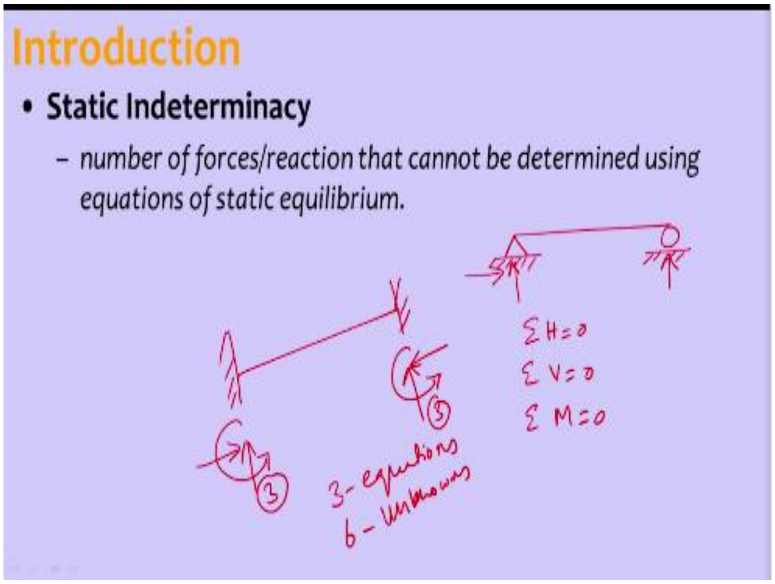

So static indeterminacy means number of forces or reactions that cannot be determined using the equations of static equilibrium. So for example, o if I take a simply supported beam here hinge is there and a roller is there. So roller is vertical reaction comes here and here vertical reaction as well as horizontal reaction comes. So three unknown reactions are there, three unknown reactions and we have three equations of equilibrium $\sum H=0$, $\sum V=0$, $\sum M=0$.

So that means it is statically determined structure, whereas if we take the fixed beam, so we have vertical reaction, horizontal reaction and a moment, I mean vertical reaction here, horizontal reaction here and moment. So here three unknowns are there here and three unknowns here, total six unknowns are there, but we have three equations of equilibrium, three equations, six unknowns.

So this is statically independent structure, so this is called static indeterminacy degree is three. Now that comes to kinematic indeterminacy, what is kinematic indeterminacy.

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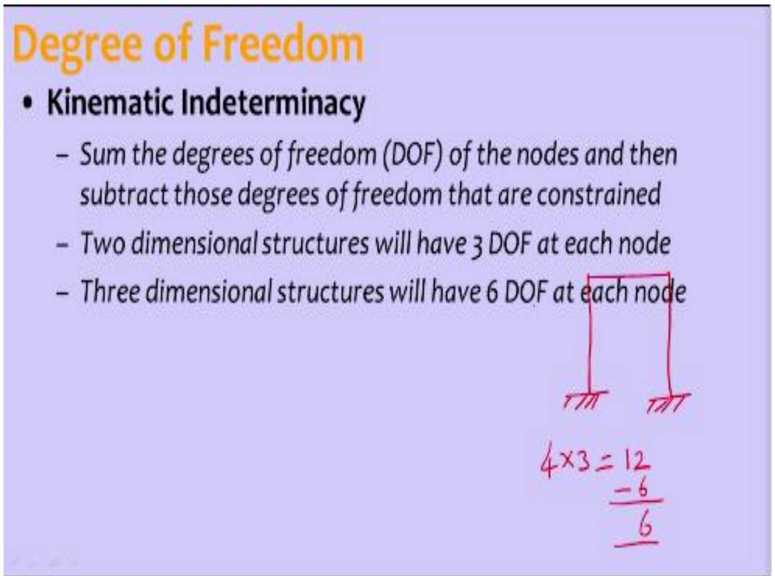

So we take sum of degrees of freedom of the nodes and then sum all the degrees of freedom of the nodes and then subtract those degrees of freedom that are constrained. So it is something like say, I have a frame in this so, 1, 2, 3, 4, 4 nodes are there, 4 nodes. So 4 nodes in 2D plane frame structure each node will have three degrees of freedom. So 4 nodes into three degrees of freedom, so that will make 12.

And from this we need to subtract those degrees of freedom that are constrained. So in this three here at the base are constrained, three on the other right side support are constrained. So that means 6 are constrained so we can remove. So this structure can be six degrees of freedom structure, so this frame can be six degrees of freedom frame. Six degrees of freedom are required to represent the systems behavior or structures behavior.

So in two dimensional structure we will have three degrees of freedom at each node and whereas three dimensional structure we will have six degrees of freedom at each node.

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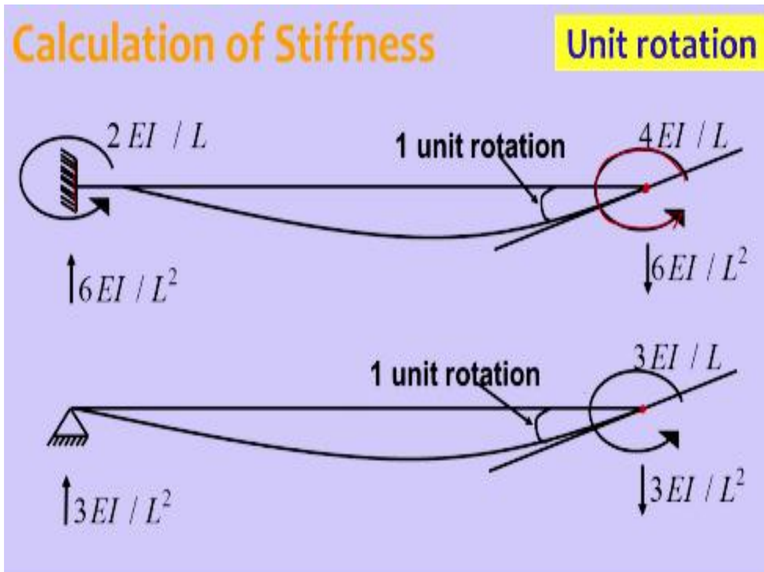

Now how do we calculate a stiffness of the member which is required for finding out the stiffness of the structure or frame. So here we have like one unit rotation, so a figure is shown on the slide, if I say take this a member and fix it on one side and apply rotation on another side. If I apply rotation on one side the moment required to produce one unit rotation at this end whereas the far end is fixed, the stiffness coefficient required is 4EI/L and half of that moment is transferred to the other end to EI/L.

And if the far end is fixed and if we are applying one unit rotation at this end so the far end is not fixed it is hinged, so then we need 3EI/L. So here if the far end is fixed we need 4EI/L and the moment transfer to this one or the carry over is 2EI/L. So 4EI/L and 2EI/L both are in the same direction that is anticlockwise moment, anti-clockwise rotation, anti-clockwise rotation both put together will give us 6EI/L total anti-clockwise rotation.

But for mentoring the equilibrium of the member we need to give a couple or the couple is developed at the two ends. So 6EI/L anti-clockwise rotation must be balanced by $6EI/L^2$ force with the lever arm of length, that means here you can see $6EI/L^2$ on the right side downward force and $6EI/L^2$ on left side upward force so that will give a balanced force to this one unit rotation.

And when it comes to hinged okay, where far hinged end is hinged so we are giving 3EI/L as far end is hinged at hinge location there will not be any moment. So that is why the total unbalanced moment is 3EI/L and the reactions or the couple what will form with that is the EI/L^2. So $3EI/L^2$ downward on the right end and $3EI/L^2$ upward on the left end, so this is calculation of the stiffness coefficients when one unit rotation is given to one end of the member.

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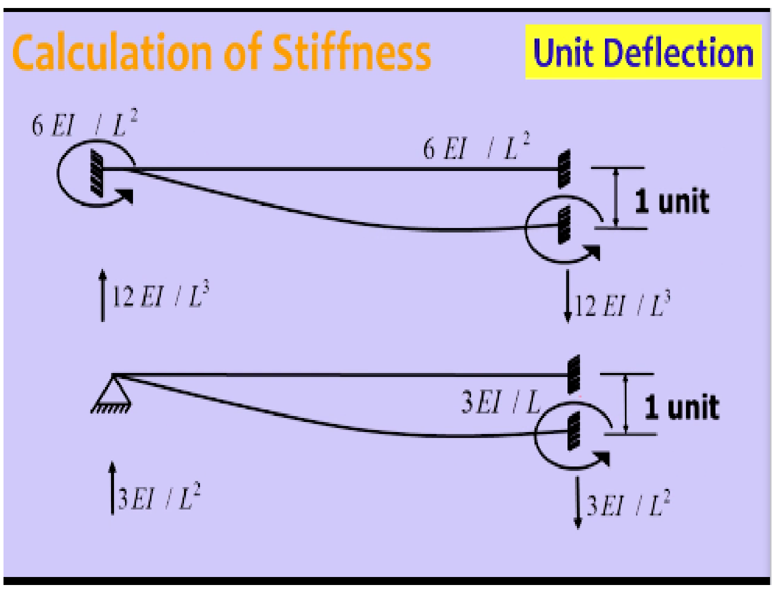

Now when it comes to one unit deflection given at one end of the member, as you can see here like a member where far end is fixed and the near end is rigid, so to that we are giving one unit displacement because of this one unit displacement to maintain zero angle here a rotation have to be applied so that rotation value is $6EI/L^2$ and the same rotation need to be applied on the other end also $6EI/L^2$.

To maintain this position and $6EI/L^2$ and $6EI/L^2$ together is $2EI/L^2$ unbalanced movement to balance this we need to apply clockwise couple okay so that will be $2EI/L^3$ on the right end downward and $2EI/L^3$ on the left side support upward so that will make the whole member in equilibrium and the next one is like one unit displacement with a far end hinged so there we need to apply our rotation of a moment of $3EI/L^2$ on the right side.

And on the left side that is the hinged support so moment is not needed there so total unbalanced moment will be $3EI/L^2$ so unbalanced moment is $3EI/L$ then couple with forces $2EI/L^2$ on the right side downward and $3EI/L^2$ on the left side upwards.

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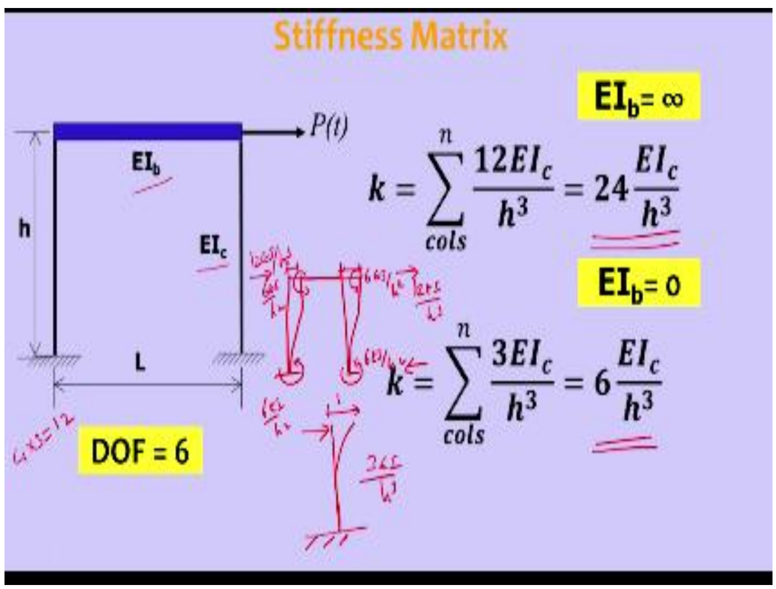

Now let us develop stiffness matrix for a given structure given flame frame structure, in this you can see like being is having length L and height of the column is H EI_C is given this is fractural visibility of column and fidelity edge of column one and column 2 both are same and flexural visibility of being EI_b is given and a force is applied at this level at the roof level so in this one the degrees of freedom are 6 so how does degrees of freedom are 6?

They are like 4 nodes are there 1, 2, 3 and 4 nodes are there so each node will have three degrees of freedom so 4 x 3 = 12 minus the sum of the reactions if you remove so that will become 6 degrees of freedom system so let us look at some special cases in this one so EL_b that federal visibility of this B if it is infinity so what is going to happen so federal visibility of this mean is infinity than the reaction coming from one column or the forced required to produce one unit deflection at the tip of the column or the end of the column is 12 EI/L^3 that is H^3.

So two columns are there so two columns means column 1 column 2 so that will give us $24EL/H^3$ so in this one what given condition is federal visibility of the column is infinity so in that case what happens it behaves like this. So this is one unit displacement so this will be 6 EI/H^2 here 6 EI/H^2 here $6EI/H^2$ so this anti clockwise 6 EI/H^2 so we need to apply clockwise couple that is 126 EI/H^3 and here also same 12 EI/H^3.

So this $12EI/H^3$ together will get 24 EI/H^3 this is a combined stiffness of all the I means both the columns so that is well federal visibility is infinity so rigid the slab is rigid deem is rigid then we can go for this condition okay and in on the other hand if say there is no federal visibility so it is 0 so in that case so what is going to happen so in that case stiffness is $3EL/H^3$ is a stiffness offered by one column so it is something like this.

So force applied so this will be for one unit deflection okay that stiffness co-efficient is $3EL/H^3$ so two columns are there so that force will be $6EI/H^3$ so EI_b the federal visibility of been is 0 is one extreme federal visibility of beam is infinity there is another extreme but usual cases it is neither 0 nor infinity but usual cases it is somewhere in-between so if it is somewhere in-between so how do we develop stiffness matrix.

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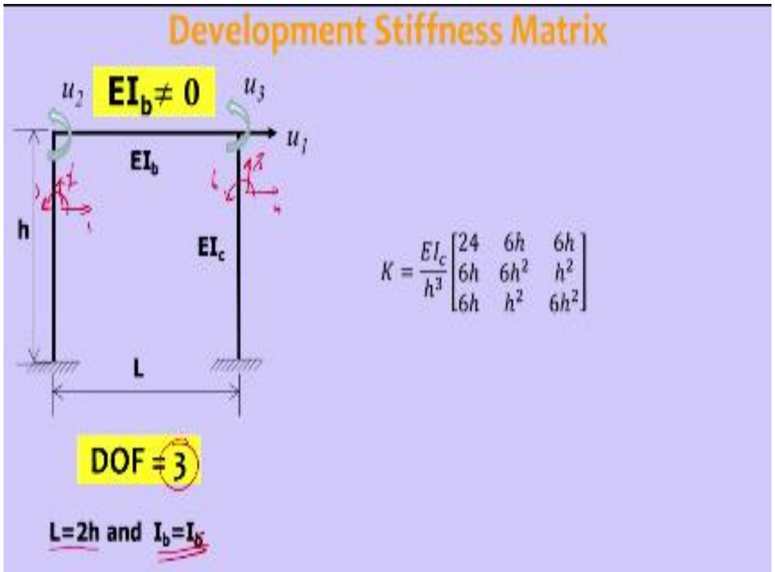

So that is what we are going to see in this slide, as you can see here okay let me explain you the 6 degrees of freedom I told you in this previous slide now if I come to this slide so degrees of freedom are 3 only, so 3 degrees of freedom at this node and 3 degrees of freedom at the right mode so if I take this 3 degrees and 3 degrees it is something like this 1, 2 and 3 degrees of freedom.

And 1, 2, and 3 degrees of you so 1, 2, 3, 4, 5, 6 so how this has become 3 degrees of freedom, it is like actual extension of columns are neglected so that will give you this degrees of freedom two degrees of freedom 5 so that will give you one now total 6 degrees of freedom will become 4 degrees of freedom it rotation 3 will be there and rotation 6 will be there and this translation that is one translation degree of freedom 1 and translation degree of freedom 4.

Both are one in the same so there is no axial compression or extension in the beam itself okay so that will make this 2 as 1 only so the total number of degrees of freedom will be 3 degrees of freedom so 3 degrees of freedom structure will have like if you have to develop a stiffness matrix for this 3 degrees of freedom is we will get 3 x 3 matrix so let us discuss how this matrix can be developed.

So in this the condition given is length of the beam = 2 times height and moment of inertia beam = moment of inertia of column both are same. So in that how do we develop this stiffness matrix.

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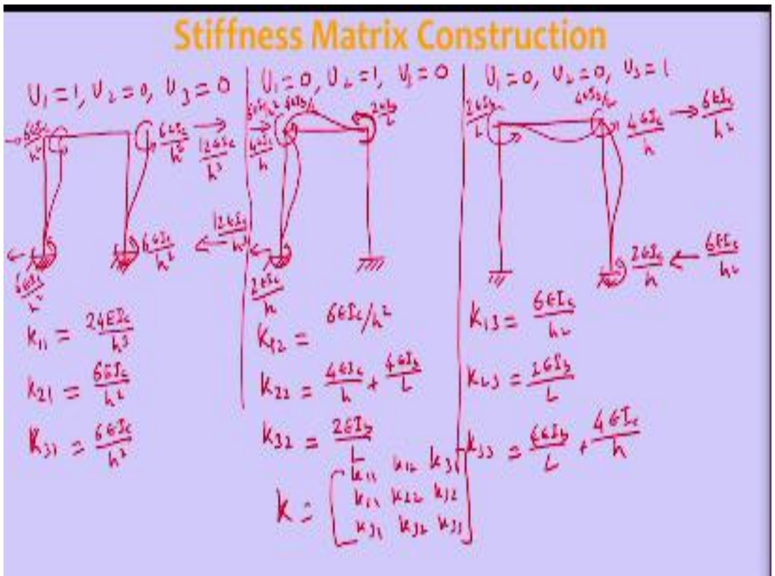

The first case is like given unit displacement to one degree of freedom and rest two degrees of freedom should be restrain and force required to maintain this system is a stiffness element, if you write say $U_1=1$, $U_2=0$ and $U_3=0$ in that case so if write this so U_1 is 1 so that means degree of freedom 1 is given unit displacement as you can see here so degree of freedom 1 U_1 is translation U_2 is rotation and U_3 is also rotation, so U_1 translation is given one unit so one unit so that means structure is moving up to this so that means what it is essentially converts structure like this one unit here, one unit here, okay so this.

So that means this the moment required here is $6eIc/h^2$ and this $6EIc/h^2$ here $6EI/h^2c$ here $6EIc/h^2$ so this $6EIc/h^2$ $6EIc/h^2$ together will create unbalanced moment of $12EI/h^2$ $12EI/h^3$ so that is in this direction $12EI/h^3$ column definition and this reaction is also $12EI/h^3$ similarly on the other side. So now we need to write the forces at this degrees of freedom levels so we have U_1, U_2, U_3 degrees of freedom so let us write stiffness coefficients what is K_{11}.

So K_{11} will be $24EIc/h^3$ this is first coefficient K_{21}, K_{21} so 2 this one so that means $6EIc/h^2$ and K_{31} is $6EIc/h^2$ so that completes first column of stiffness matrix. Now let us go to the second column so how do we generate second column of stiffness matrix second column can be like U_1 is 0, U_2 is 1 and U_3 is 0, so in this case it is something like this. So U_2 is one unit rotation so given one unit rotation to this joint so it is something like this.

And here one unit anti-clockwise rotation so and I give one unit anti-clockwise rotation here something like this. So where far end is fixed how much is needed, how much moment is needed if you look at only beam so it is 4EI beam by length, and here 4EIc/h and here 2EIb/l and here 2EIc/h so these are the forces which will act. Now if we develop stiffness coefficients from this one K, K_{12}=, K_{22}, K_{32}, K_{12}, K_{22}, K_{32}.

So K12 is so this is anti-clockwise moment here so this is anti-clockwise here, anti-clockwise here so this is a beam reaction, beam moment. So this is $4EI/h^2$ 4EI/h and 2EI/h together it will 6EI/h unbalanced so we need to produce force which is $6EIc/h^2$ so it will be K_{12} is $6EIc/h^2$ and K_{22} is the addition of this two moments 4EIc/h+4EIb/l and K_{32} is 2EIb/l so this is the second column of stiffness matrix and the third column we can consider to U_1=0, U_2=0 and U_3=1.

So in this case we will do it similar operations so U_3 is this one so anti-clock one unit anti-clockwise rotation here, so it behaves like this and it rotates like this for rotating this one unit rotation for making one unit rotation in the column we need 4EIc by height of the column so that is h^2 and sorry, 4EIc/h and here 4EIb/l and half of that will be transferred here that is 2EIb/l and column 1 will transfer half that is 2EIc/h so the unbalanced moment at this column is 4EIc/h and 2EIc/h together will be $6EIc/h^2$ here $6EIc/h^2$.

So now if you write the coefficients K_{13}, K_{23}, K_{33} so k13 is that is 6 $_{EI}$ columns by h^2 and K23 is 2aib/ L and K33 is 43EI /L + 4 EIc/ h so this constitutes stiffness matrix so k11 k21 k31 k12 k22 k32 k31 k32 here k33 okay so this is how we can form the stiffness matrix and now we can see.

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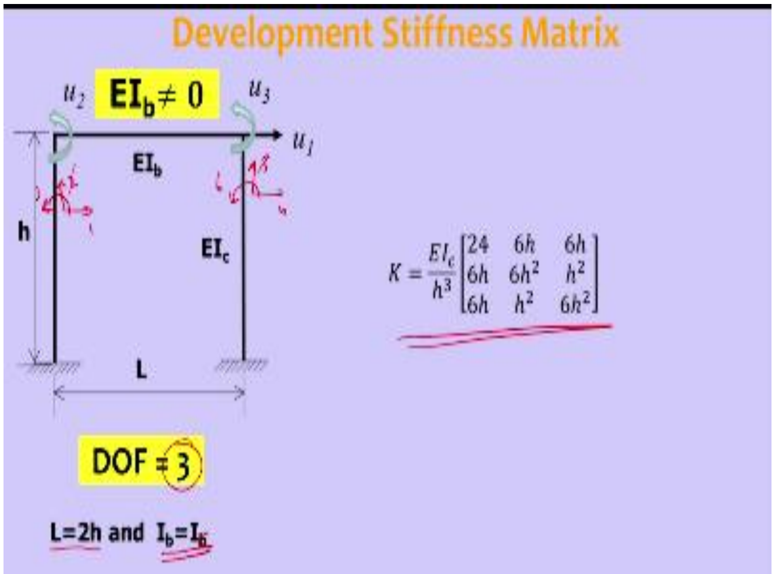

By using l= 2h and moment of in h of column is = moment of in h of I_b so up open simplification the total 3/3 matrix is this, now after finding out after generating the stiffness matrix if there are no forces associated with the degrees of freedom available it is something like this you can see in this one.

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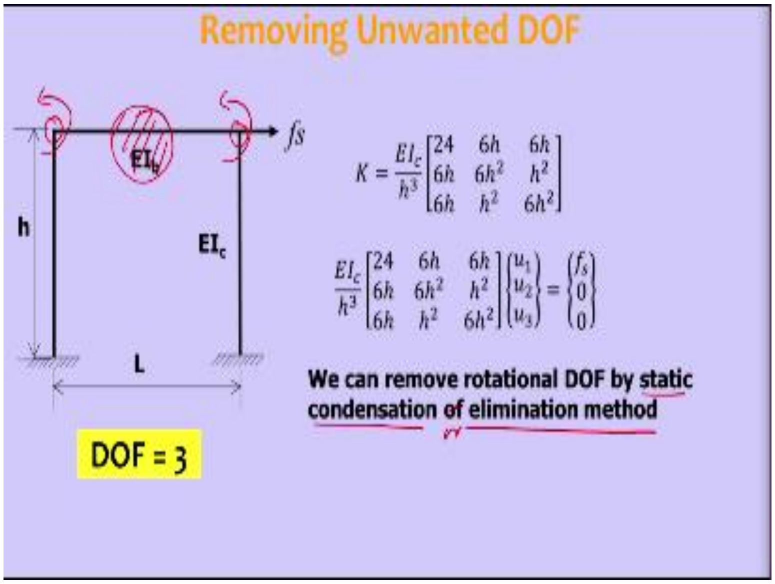

The degrees of freedom is here degree of freedom is here rotation is here rotation is here and translation so translation means entire mass is concentrated this one so that means translational degree of freedom that is degree of freedom one has mass associated with it but degree of freedom 2 has 0 mass and degree of freedom 3 has 0 mass so in this situation how do we remove unwanted degrees of freedom from the equation.

So that is what is required to be discussed so it is something like this so stiffness k u = f so k has all elements in it but this placement need to be formed out but f is only applied only at one place so whereas these two degrees of freedoms so rotation there is no force applied sop that means force is applied only ion the translational direction. So that means we need to remove unwanted degrees of freedom for simplification of solution.

So how do we remove this unwanted degrees of freedom, this unwanted degrees of freedom can be remove ion a different methods so that is by statics condensation or elimination method so there are this two methods starting condition method also can be used or elimination method also can be used so what is this method let us discussed.

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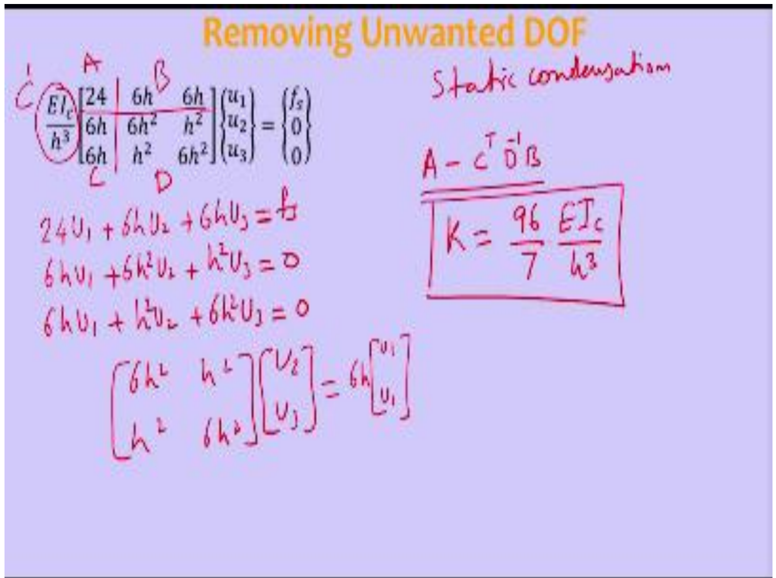

Now you can see this one so if I take this equation so like let us make this as some constant C okay so 24 U_1 + 6k U_2 + 6h U_3 = f_s ,6hU1 I am getting simultaneous equation $6h^2$ U_2 + h^2 U_3 = 0 so 6h U_1+h^2U_2 + $6h^2$ U_3 = 0, so we have four associated force is applied in one and only equation whereas in digital equation it is 0. So it is better to substitute U2 and U3 in the form of U1 so like what we can do is we can formulated matrix like this.

So $6h^2$, h^2 h^2 and $6h^2$ x U_2 and U_3 = $6hU_1$ U_1 okay so assuming that this is a known one and we can find out U_2 in terms of U_1 so in this we can in this manner we can eliminate and reconstitute ion this one so this is one method of eliminating it there is another method which is called statistic condensation, so in this static condensation the entire matrix can be divide in to four parts we can say this as a this as b this as c and this as d.

So this matrix can be formulated like c is any there here we can write this as c' so A – C^T D inverse B okay, so if we work out this matrix operation okay and then the as well get only finally this equation so that final equations will be that k equivalent is 96/7 x EI c / $h^{3,}$ so this will be the equivalent stiffness after this matrix operation in summary what we have discussed in this module is degrees of freedom first we discussed static indeterminacy and second we discussed kinematic indeterminacy.

So kinematic indeterminacy means the number of independent degree of freedom which are required for defining the system for example something like pendulum is there only Δ is needed for writing the displacement of this pendulum, so like l is fixed if you have Δ we can say x and y coordinate of the systems at any point of time, so like that this is single degree of freedom system so if you take multi story building so multi story building can be likely represented in N number of degrees of freedom.,

So each note will have in 3d freedom each note will have 6 degrees of freedom and in 2d analysis we take each note have been 3 degrees of freedom so we have also discussed how to formulate a stiffness matrix for pone story frame so that is 3/3 matrix and if franc is associated to the degree of freedom we have studied how to remove this unwanted degrees of freedom where mass is not associated by static condensation or elimination method.