

**NPTEL  
NPTEL ONLINE COURSE**

**Structural Dynamics  
Week 1: Module 02**

**Types of Analysis**

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This module we are going to discuss about types of analysis.

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So the outline of this module is Classification according to the force analysis, Classification of analysis according to how the force is acting and Classification of analysis according to the behavior of material and then Classification of analysis according to the level of deformation so these three classifications will discuss in this module.

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## Introduction

- **Vibration is an oscillatory motion**

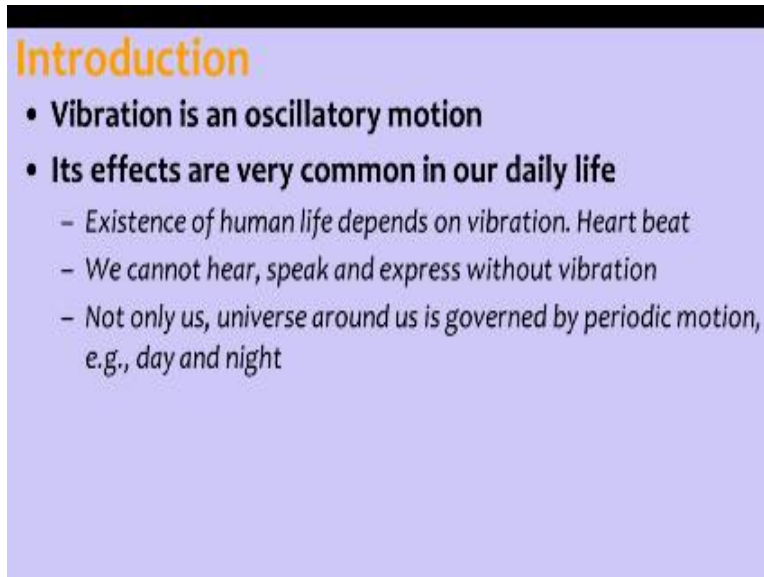
So if we look at the vibration so vibration is an oscillatory motion so something like simple harmonic motion it is if I pull the pendulum.

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And leave it so it is set to vibrate so this is called simple harmonic motion so this kind of motions we see in our day-to-day life so while we walk we do to and fro motion of our hands so our heart beats so many things our legs are in rhythm so what we are trying to do is we are balancing ourselves.

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**Introduction**

- **Vibration is an oscillatory motion**
- **Its effects are very common in our daily life**
  - Existence of human life depends on vibration. Heart beat
  - We cannot hear, speak and express without vibration
  - Not only us, universe around us is governed by periodic motion, e.g., day and night

So this vibrations its effects are very common in our daily life so if you look at the existence of human life it completely depends on say vibration so for example heartbeat so heartbeat is a the heartbeat is a vibration so it is continuously happening and we cannot hear we cannot speak and we cannot express without vibration so vibration is most essential part of our life not only we human beings all around us are also dependent on vibration okay so for examples universe around us is governed by a definite principles so there is periodic motion so day and night all these things are anyway happening.

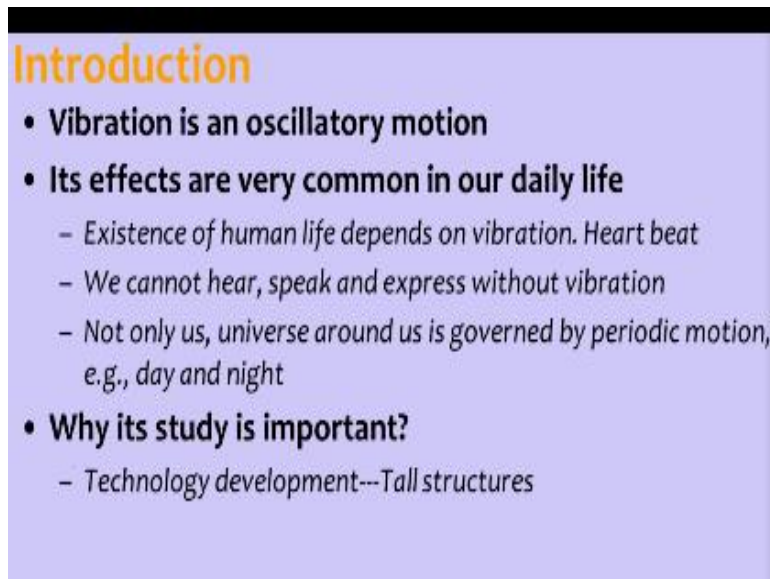
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## Introduction

- **Vibration is an oscillatory motion**
- **Its effects are very common in our daily life**
  - *Existence of human life depends on vibration. Heart beat*
  - *We cannot hear, speak and express without vibration*
  - *Not only us, universe around us is governed by periodic motion e.g., day and night*
- **Why its study is important?**

If you look at it we understand now why its study is important especially for us civil engineers.

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**Introduction**

- **Vibration is an oscillatory motion**
- **Its effects are very common in our daily life**
  - Existence of human life depends on vibration. Heart beat
  - We cannot hear, speak and express without vibration
  - Not only us, universe around us is governed by periodic motion, e.g., day and night
- **Why its study is important?**
  - Technology development---Tall structures

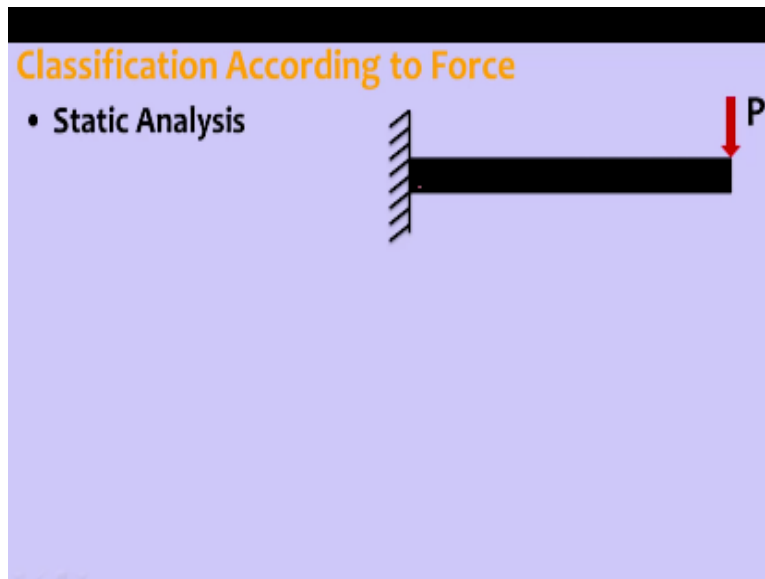
So it is important for us because like these days tall structures are coming, daring structures are coming, and there is a possibility that these structures are subjected to like severe vibrations so in normal structures also sometimes when machines are installed so they are set to vibrate so we need to study the dynamics of the floor on which machine is installed so there is a floor vibration so because of that whether floor is safe or not so we need to understand what is the relationship between that floor properties and the Machine properties.

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So his RPM is proper or not and then a tall building so which is subjected to wind vibrations so that is also oscillatory motion that also we need to understand and sometimes this building is subjected to earthquake ground motion so that is also vibration so that also we need to understand so it is very important for us to understand the dynamics of structure when it is subjected to force.

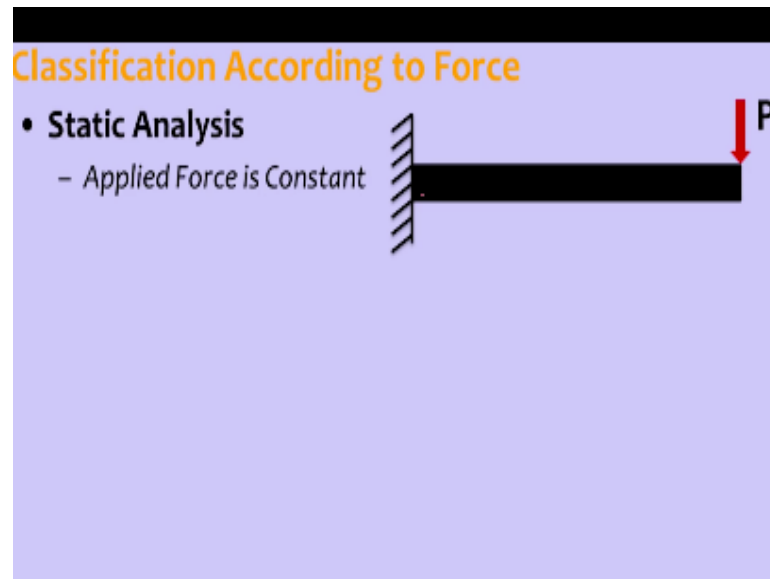
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So now classification of analysis according to force so we have two kinds of analysis one is called static analysis and another one is dynamic analysis so what is static analysis static analysis is in which you can see this example so there is a cantilever and force is acting at the tip of the cantilever so because of the application of this force the member that cantilever beam deflects and if you increase the load more will be deflection if you further increase the load more will be deflection but whatever amount of load you kept deflection will be constant.

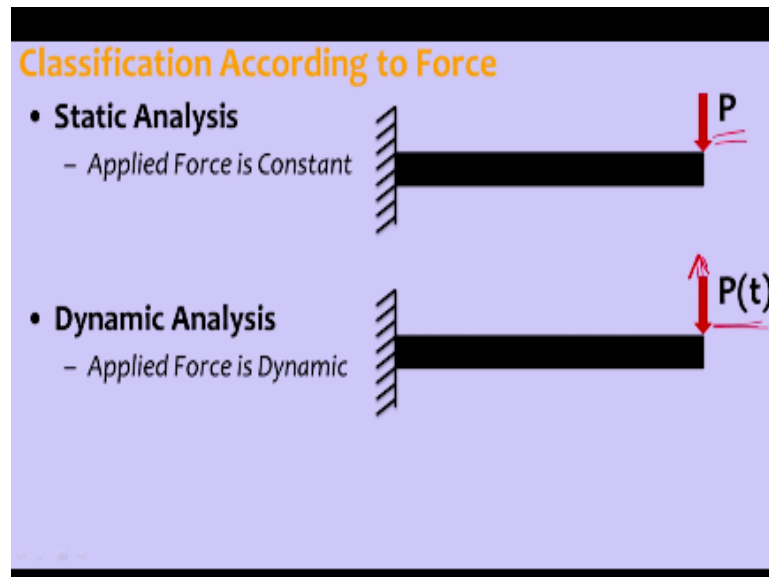


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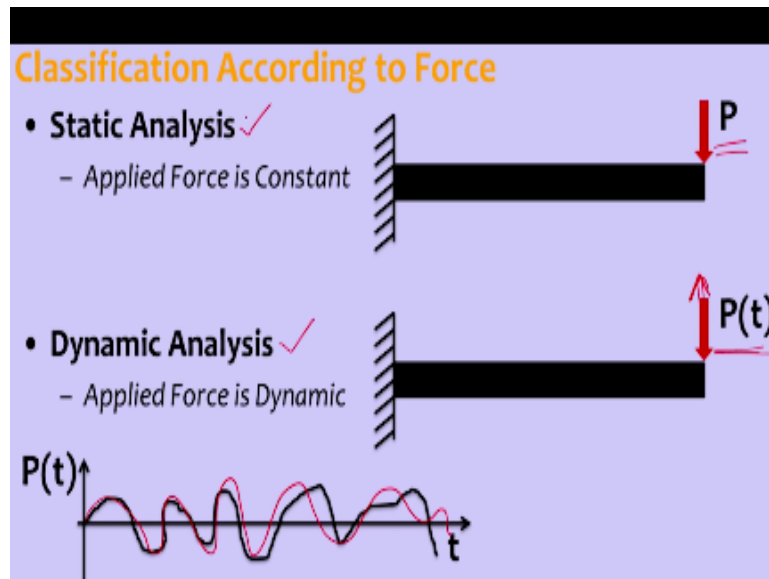
The deflection is not varying so as long as applied force is constant the response of the structure that is displacement is constant because stiffness is a fixed quantity and in this the next one dynamic analysis.

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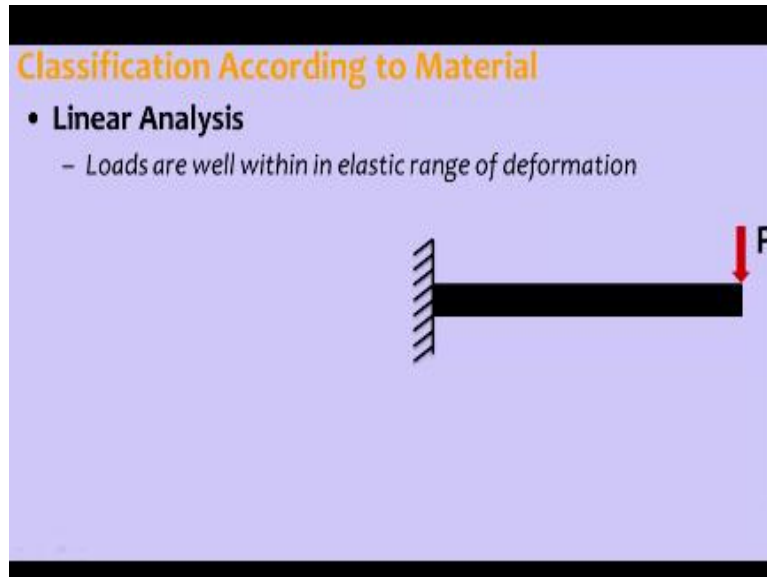
So in dynamic analysis what is happening is force is varying so you can see here the difference between the force here and difference between force here in fact it should be written arrow up and down up and down so this is going means increasing decreasing increasing decreasing so how it is governed you can see applied forces dynamics so that means it is the force is not constant force is varying in time so as you can see this one so.

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On x-axis it is time scale and on y-axis it is the amplitude of the force so this force is continuously varying so it can be as random as possible but it is varying with time so, so the analysis in which force is considered as varying force that goes into dynamic analysis the analysis in which force is a static or constant that is called static analysis so this is a classification of analysis according to the force type or type of force.

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Now the next one is classification of analysis according to the behavior of the material so in this we can classify it into two parts that is linear analysis so in linear analysis once again we discuss with this example so cantilever beam force is applied so loads are well within the elastic deformation range so loads are will with inelastic deformation range means what so material obeys Hooke's law material obeys Hooke's law.

So Hooke's law means stress is proportional to strain and then superposition principle is applicable so the superposition principle is applicable means like there is a cantilever beam and I apply say 1 kilo Newton load here.

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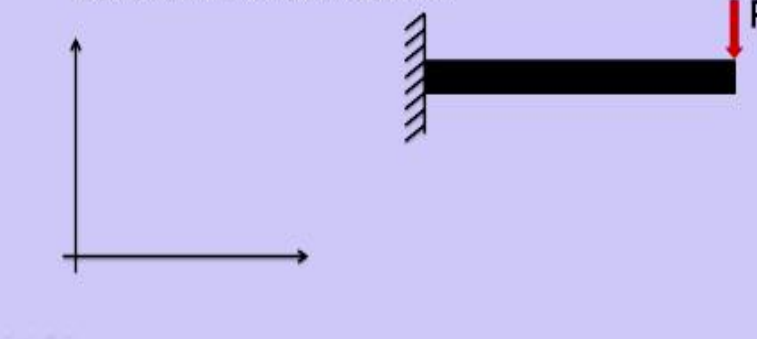
And it deflects and then I apply say another 1 kilo Newton load it deflects so if I add 1 kilo Newton load deflection and, and second 1 kilo Newton load deflection so the addition of that will be equal to if I put 2 kilo Newton load then the addition will be same as 1 kilo Newton load deflection plus 1 kilo Newton load deflection or you put 1 kilo Newton load at the beginning and then after that remove that 1 kilo Newton load add 2 kilo Newton put 2 kilo Newton load.

So deflection due to 1 kilo Newton load plus deflection due to 2 kilo Newton load that will be equal to if you put 3 kilo Newton that will be equal to deflection due to 1 kilo Newton load plus deflection due to 2 kilo Newton load. So that means when both are same we say that superposition principle is applicable.

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### Classification According to Material

- **Linear Analysis**
  - Loads are well within in elastic range of deformation
  - Superposition Principle is applicable




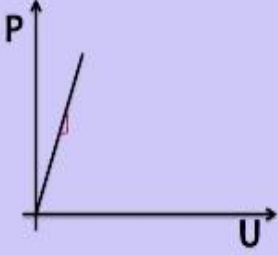
The diagram shows a cantilever beam fixed at the left end and free at the right end. A downward force  $P$  is applied at the free end. To the left of the beam is a coordinate system with a vertical  $y$ -axis and a horizontal  $x$ -axis.

Now you can see this graph where displacement is on  $y$  axis and load is on  $x$  axis this is linearly proportional so force and displacement are proportional with some constant term that constant term the slope of this line is called as stiffness of the system  $P = KU$  we can also derive from this one that is  $U = PL^3 / 3EI$  so if you want to write the same displacement relationship in the form of deflection relationship in this form so that will be equal to  $P = (3EI/L^3)U$ .

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### Classification According to Material

- **Linear Analysis**
  - Loads are well within in elastic range of deformation
  - Superposition Principle is applicable


$$P = kU$$
$$U = \frac{P^3}{3EI}$$
$$P = \left(\frac{3EI}{L^3}\right) \cdot U$$

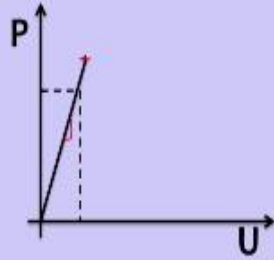
This is a relationship in this one so here we can see that up to this point and that load is proportional load and displacement are proportional or stiffness is constant so that we call it as linear analysis, analysis well within the limit of this proportionality goes into linear analysis.

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## Classification According to Material

- **Linear Analysis**

- Loads are well within in elastic range of deformation
- Superposition Principle is applicable



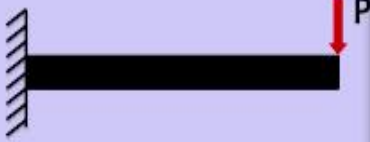
$$P = kU$$
$$U = \frac{PL^3}{3EI}$$
$$P = \left(\frac{3EI}{L^3}\right) \cdot U$$



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**Classification According to Material**

- **Non-Linear Analysis**
  - Loads are *beyond elastic range of deformation*




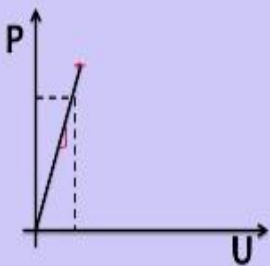
The diagram shows a horizontal black line representing a cantilever beam. The left end is fixed to a vertical wall, indicated by a hatched pattern. The right end is free. A red arrow labeled 'P' points downwards from the free end, representing an applied load.

Now the next analysis is nonlinear analysis this is dependent on the non-linearity of the material so the same thing cantilever beam will be a point or applied at the end so in this one loads are beyond elastic range or deformation.

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### Classification According to Material

- Linear Analysis
  - Loads are well within in elastic range of deformation
  - Superposition Principle is applicable


$$P = kU$$
$$U = \frac{P^3}{3EI} = 0.1 \text{ mm}$$
$$P = \left(\frac{3EI}{k}\right) \cdot U$$

1
10
100
<u>1000 mm</u>

So for example in this case say if deflection of the beam when some force is applied is say 0.1 millimeter 0.1 millimeter so length of the beam let us say an example say 2 meters so if we apply say 10 P 10times P then the deflection will become 1 millimeter and if we increase the load and further and apply then 10 times more so 100 P then deflection will become 10 millimeter and if we further apply increase the load and go further then it becomes 100 millimeters or maybe 1000 millimeters so 1,000 millimeters means it is equal to nearly half the length of the beam so what happens is before it is reaching this 1000 millimeter usually it fails.

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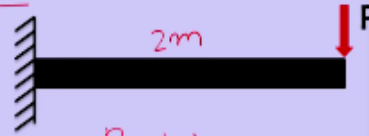
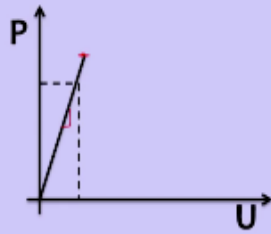
If I go on increasing the load here so that will not be any, any more linearly proportional it will break at that's at some point so where it will end where the first crack comes so from that point onwards this linear analysis is not applicable and the superposition principle is not applicable.

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## Classification According to Material

- **Linear Analysis**

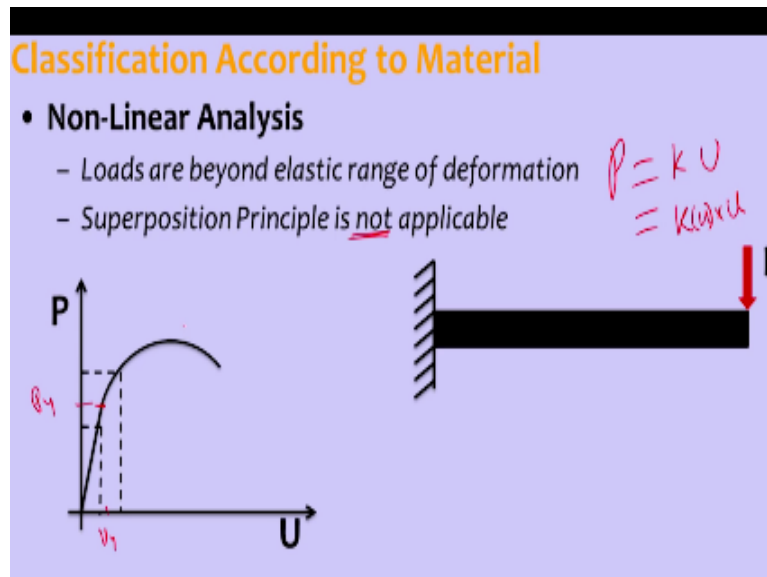
- Loads are well within in elastic range of deformation
- Superposition Principle is applicable



$$P = kU$$
$$U = \frac{P l^3}{3EI} = 0.1 \text{ mm}$$
$$P = \left( \frac{3EI}{l^3} \right) \cdot U$$

1  
10  
100  
1000 mm

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So that is where this nonlinear analysis comes that is loads are beyond elastic range of deformation so then this equation  $P = KU$  will not hold good in its form so some changes are needed so what changes are needed the changes are stiffness is dependent on deformation multiplied by deformation so superposition principle is not applicable so you can see this graph so in horizontal axis is displacement vertical axis is load it is proportional only up to this level so this we can call, call as yield force or yield displacement  $U_y$  so up to that this linear analysis is applicable.

And beyond that it is nonlinear analysis as you can see after yield tangent of that the slope of the tangent at every point is reducing so that is considered in non linear analysis.

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And the next classification of analysis based on the range of the deformation so even in nonlinear deformation nonlinear analysis so it can say member is cracking member is deflecting member is cracking but member is not collapsing number is not collapsing so that means what the deformation of the member is within the limits of allowable deformation so that we call it as small deformation analysis.

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### Classification According to Deformation

- **Small Deformation Analysis**
  - Analysis confined to primary effects
    - Material nonlinearity only
- **Large Deformation Analysis**
  - Analysis considers secondary effects also




The diagram shows a horizontal black bar representing a beam. The left end is fixed to a vertical wall, indicated by a hatched pattern. The right end is free. A red arrow labeled 'P' points downwards from the right end of the beam, representing an applied load.

So analysis is confined to primary effects in that and we consider only material non-linearity in it and then in large deformation so analysis considers secondary effects also so that means what the secondary effects include like  $p-\Delta$  effects and buckling effects those things are considered so here geometric non-linearity is also included in this analysis so it will be slightly complicated like time involving process in this large deformation analysis the small deformation analysis consists of linear nonlinear static dynamic small deformation analysis theory.

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### Classification According to Deformation

- **Small Deformation Analysis**
  - Analysis confined to primary effects
    - Material nonlinearity only
- **Large Deformation Analysis**
  - Analysis considers secondary effects also
    - Geometric nonlinearity is included



The diagram shows a horizontal black bar representing a beam. The left end is fixed to a vertical wall, indicated by a hatched pattern. The right end is free. A red arrow labeled 'P' points downwards from the right end of the beam, representing an applied load.

And in large deformations usually nonlinear static and dynamic analysis will be there in large deformations total six types of analysis, analysis are there not so in summary.



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In this module we have discussed the classification of analysis based on force type based on the material and based on the deformation range so we have discussed the static analysis and dynamic analysis dynamic analysis where force is a function of time the static analysis wave force is constant and the second classification is in linear analysis and non linear analysis.

So linear analysis is like up the deformation is up to a stage up to a point where Hooke's law is applicable and the nonlinear analysis is beyond that point and then the third classification is classification based on the range of the deformation that is small deformation analysis and large deformation analysis so usually small deformation analysis primary effects are taken into consideration and large deformation analysis secondary effects are also taken into consideration so like say collapse analysis of building and like progressive collapse breaking of some elements in building so all those things come under large deformation analysis

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