PRESTRESSED CONCRETE STRUCTURES

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Module – 2: Losses in Prestress

Lecture – 2: Friction and Anchorage Slip

Welcome back, to Prestressed Concrete Structures. This is the second lecture in Module 2 on losses in prestress.

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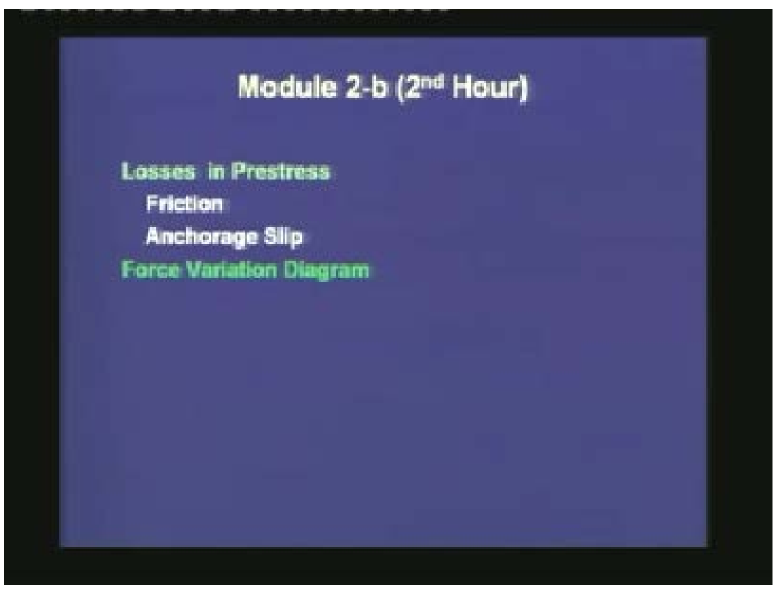

In today's lecture, we shall cover two types of losses in prestress. First is due to friction and the second is due to anchorage slip. Then we shall learn about the force variation diagram due to both these losses.

The first topic is the loss due to friction.

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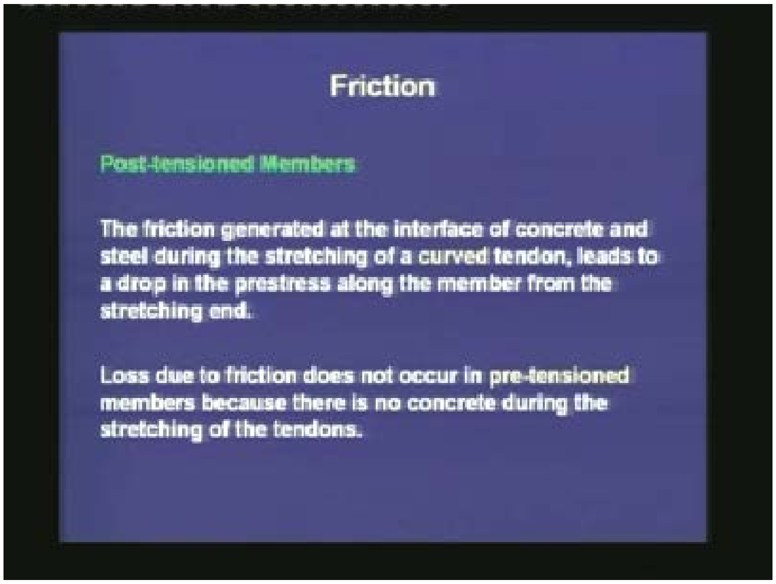

The friction generated at the interface of concrete and steel during the stretching of the curved tendon, leads to a drop in the prestress along the member from the stretching end. Unlike the elastic shortening, where the loss occurs throughout the length of the member, the loss in prestress due to friction is more towards the anchored end, and less towards the stretching end. That means, the loss progressively increases from the stretching end towards the anchored end. Loss due to friction does not occur in pre-tensioned members, because there is no concrete during the stretching of the tendons. Thus, the losses due to friction and anchorage slip both are typical phenomenon for post-tensioned members.

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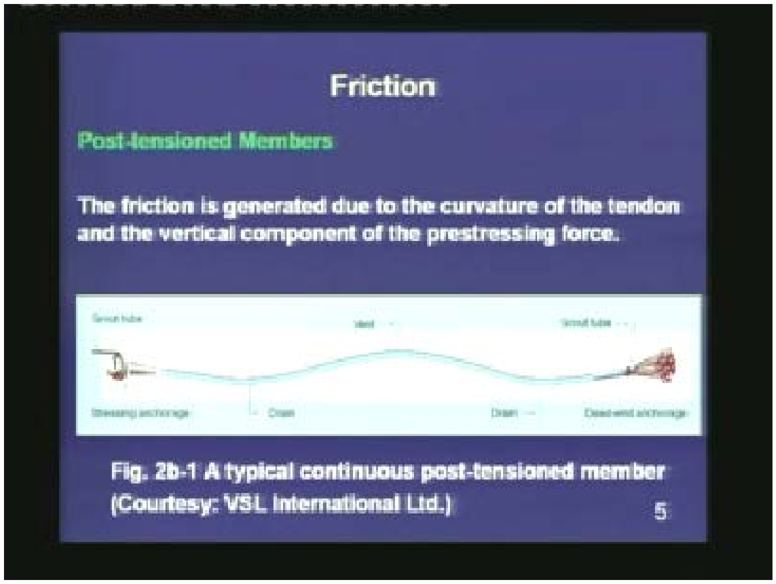

The main reason of the friction is the curvature of the tendon. This figure shows a typical profile of a curved tendon in a continuous beam. On one side, we have the jack, the stressing equipment. In the first span, the curved profile is as shown towards the left. Near the support, the curve goes upwards and then again the tendons come downwards, and finally it gets fixed at the anchored end. In a post-tensioned beam, the curvature of the tendon is the root cause of the friction in the tendon.

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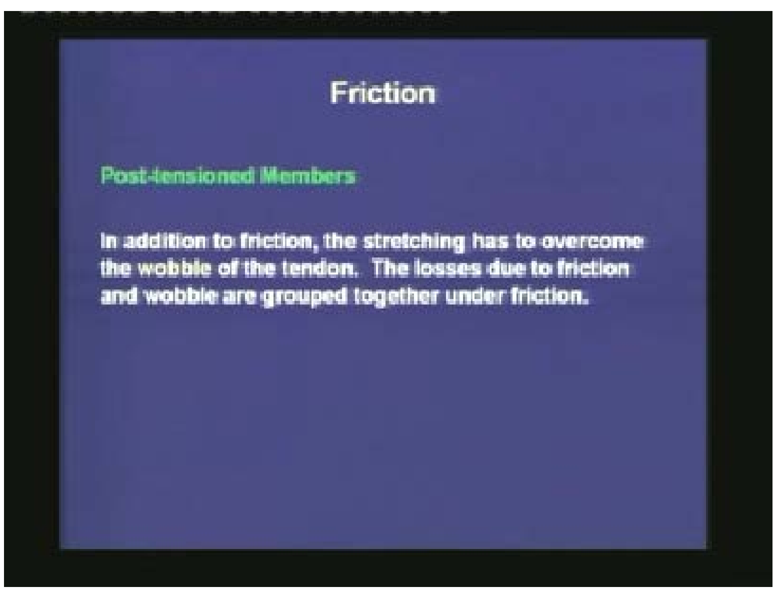

Now, in addition to friction, the stretching has to overcome the wobble of the tendon. The wobble means that, the tendon is not perfectly straight when it is stretched, but it is slightly twisted. To overcome that twisting, some additional force is needed. The losses due to friction and wobble are usually grouped together, and they are termed as the loss due to friction.

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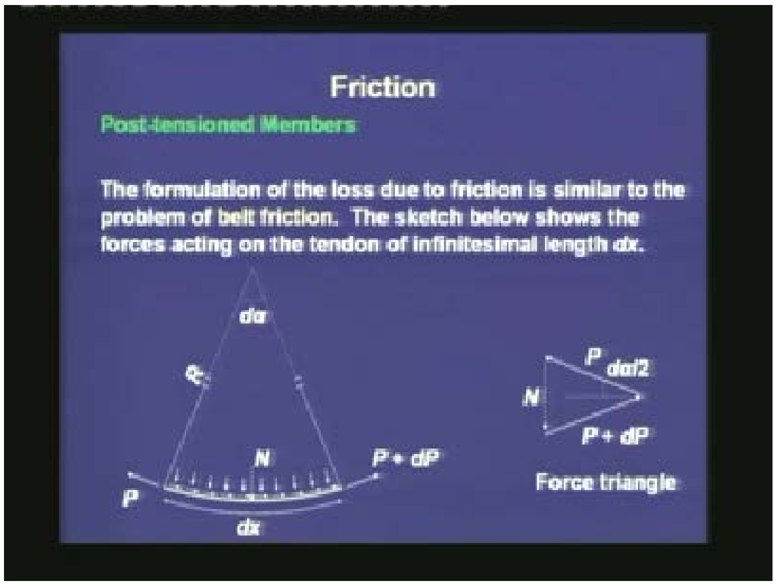

The formulation of the loss due to friction is similar to the problem of belt friction in machines. At the location of a curvature, a vertical component of the prestressing force is generated on the duct. The sketch shows the forces acting on the tendon of infinitesimal length dx, where the tendon is curved in a circular arc; R is the radius of the curvature; dα is the angle subtended by the length dx; P is the force towards the stretching end; and there is a friction in the opposite direction. The force in the other side is P + dP. Later on we shall see that dP will come out to be negative and hence, it will imply a drop in prestress on the right side. We see that the friction is generated due to the vertical component of the prestressing force, whose resultant is denoted as N. Next, P and N can be put together in the form of a force triangle, which is under equilibrium. From this, we can relate P and N.

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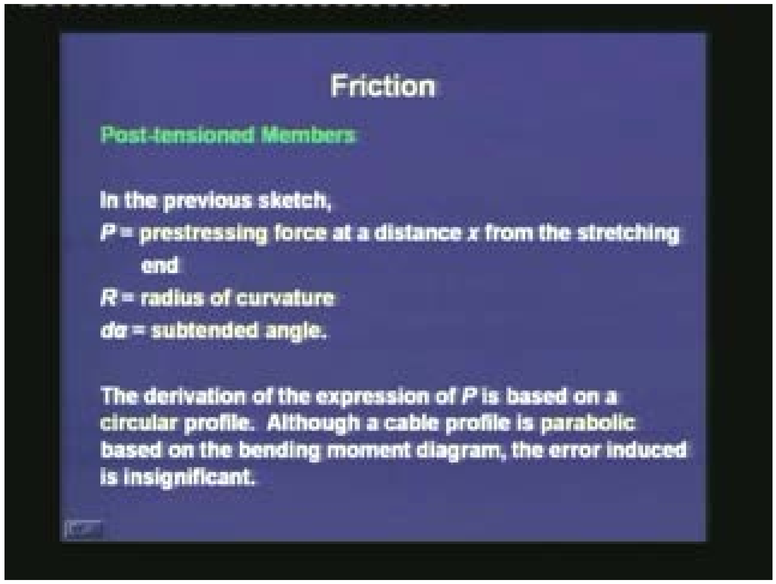

Just to summarize that in the previous sketch, P is the prestressing force at a distance x from the stretching end; R is the radius of curvature; dα is the subtended angle. The derivation of the expression of P is based on a circular profile. Although a tendon in a post-tensioned beam has a parabolic profile based on the moment diagram, the error induced is insignificant.

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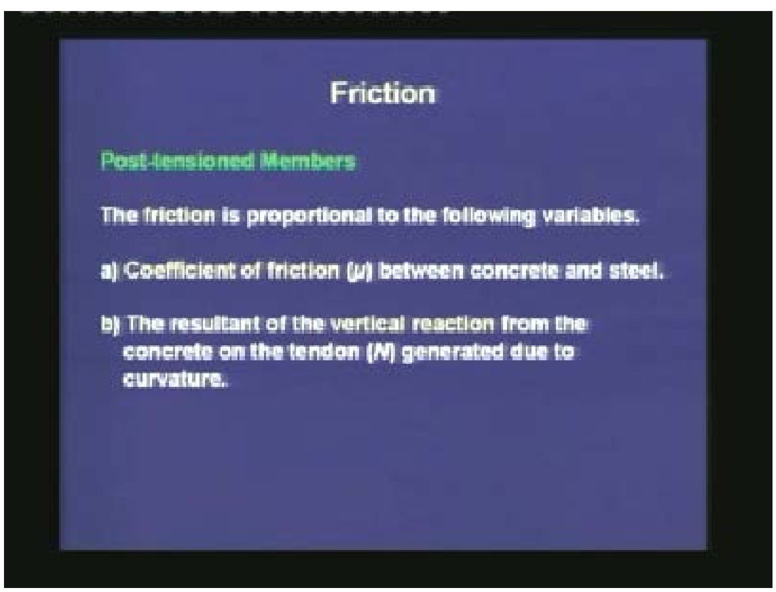

How do we quantify the friction? The friction is proportional to the coefficient of friction between the prestressing steel and the duct in the concrete (subsequently referred to as the concrete). The second factor is the resultant vertical reaction from the concrete on the tendon, which is equal and opposite to the vertical component of the prestressing force. This vertical component generates due to the curvature.

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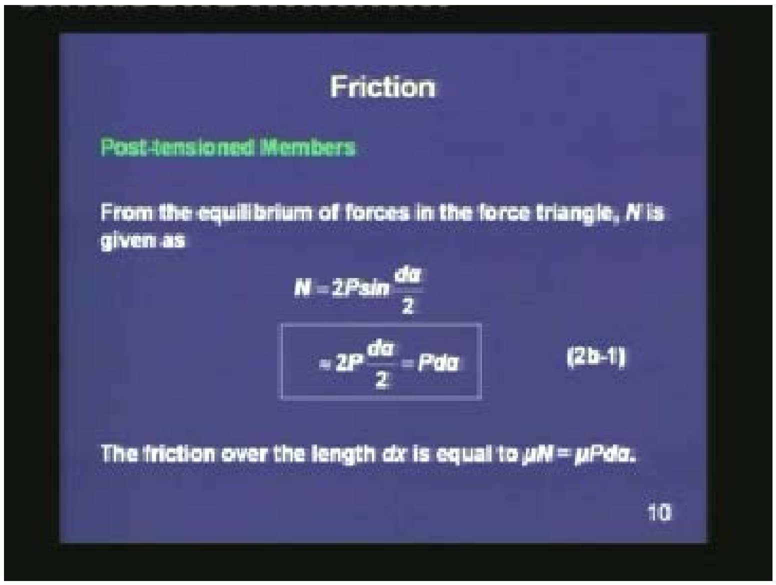

From the equilibrium of the forces in the triangle, we can write that the vertical component N = 2P times sine of the angle $d\alpha/2$. From a Taylor series expansion, an approximate relationship is 2P $d\alpha/2$. Hence, the vertical reaction is equal to P times $d\alpha$. That means, the vertical reaction is proportional to the prestressing force, and also it is proportional to the angle subtended, which is a measure of the curvature. The more curved a profile, the higher will be $d\alpha$. The friction over the length dx is equal to μ times N, equal to μ times $Pd\alpha$.

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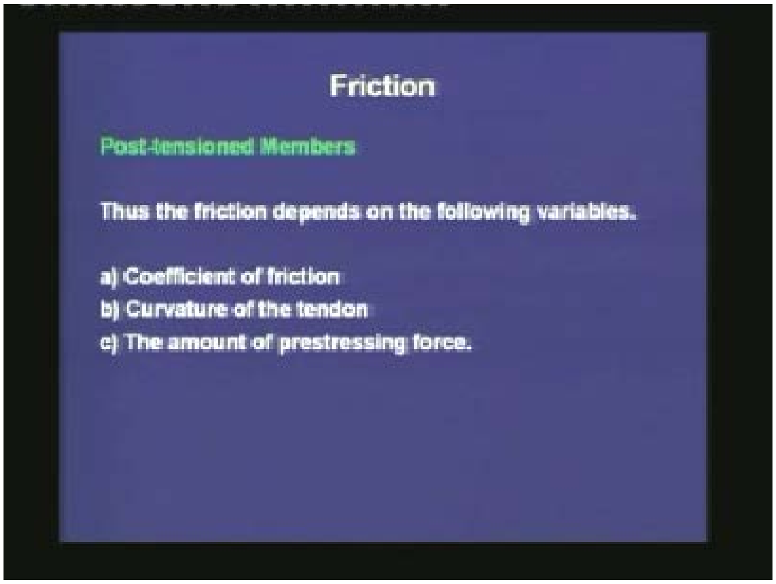

Thus, the friction depends on the following variables: first, the coefficient of friction, second, the curvature of the tendon, and third, the amount of prestressing force.

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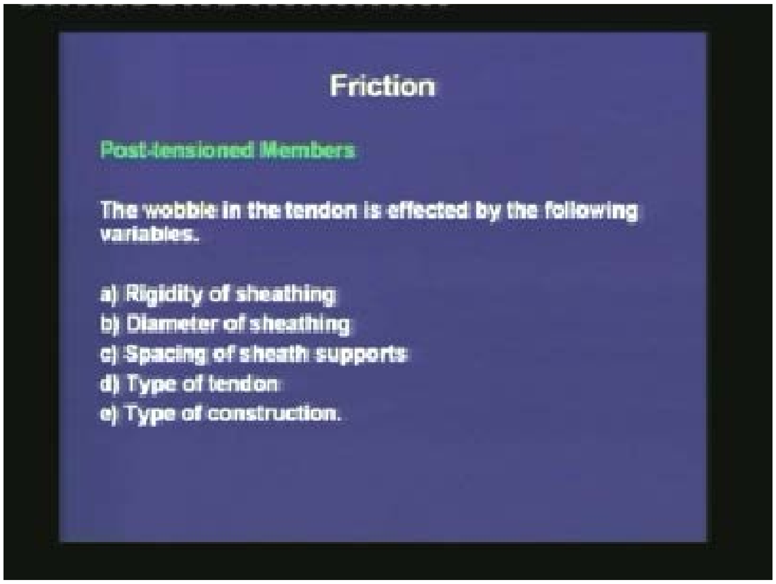

The other phenomenon, wobble is affected by the following variables: first, rigidity of the sheathing; second, diameter of the sheathing; third, spacing of the sheath supports for a continuous beam. Usually, the sheaths are supported by some means to have the curved profile. The spacing of the supports is also influential in determining the wobble. The type of tendon and the type of construction, whether segmental construction or not, influence the wobble.

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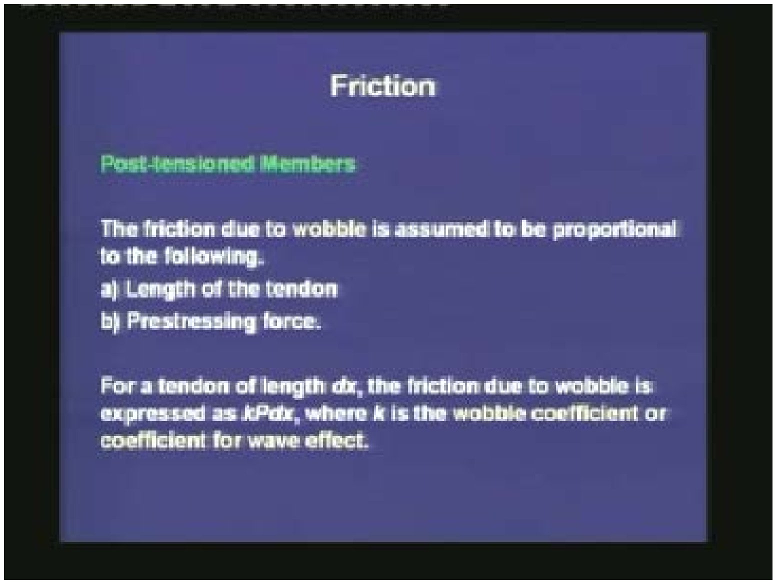

The friction effect due to wobble is assumed to be proportional to the following: first the length of the tendon; that means, the longer the tendon, the longer will be the effect of the wobble. Second, the amount of prestressing force; that means, the more prestressing force we want to apply, the more resistance will be due to the wobble effect. For a tendon of length dx the friction due to wobble is expressed as k times Pdx, where k is the proportionality constant termed as the wobble coefficient or coefficient of wave effect. Based on the equilibrium of forces in the tendon for the horizontal direction of the sketch that we have shown earlier, the following equation can be written.

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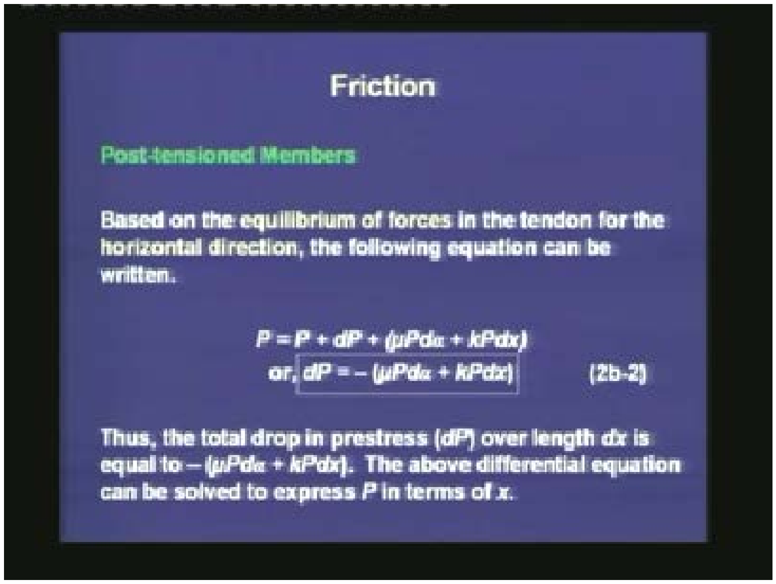

On the left side, we have P acting towards the left; on the right side, we have P + dP acting towards the right, and then we have the total frictional force which is also acting towards right. The total frictional force is a summation of the friction due to the curvature and the wobble effect. From there, we get this differential equation $dP = -(\mu Pd\alpha + kPdx)$ The equation can be solved to express P in terms of x.

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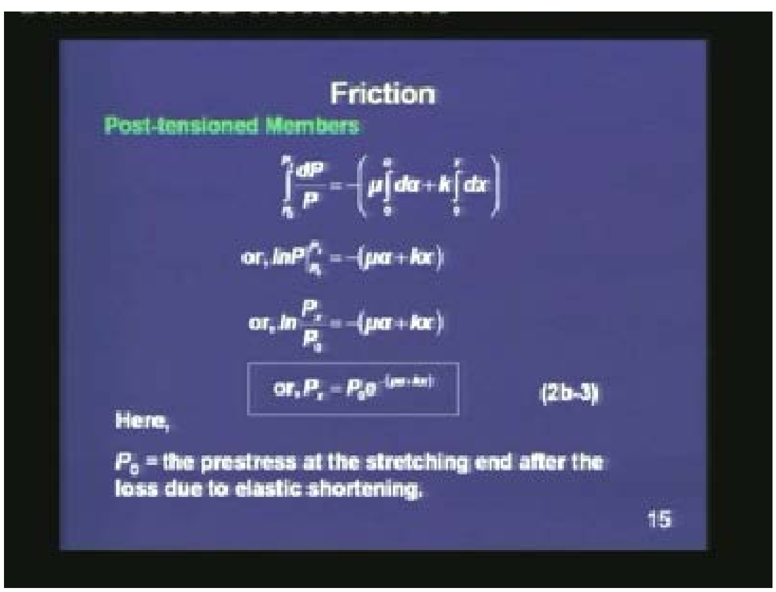

The solution procedure is conventional. We are dividing both sides by P, and we are integrating the expressions on the left and right, from a distance zero to a distance x. At x = 0, the prestressing force is denoted as $P_{0.}$ At x, the prestressing force is denoted as $P_{x.}$ The angle subtended between 0 and x is equal to α. Once we integrate, then we have the logarithm of P, varying from P_0 to P_x, = −(μα + kx). Finally, we have $P_x = P_0\ e^{-(\mu\alpha + kx)}$. Here, P_0 is the prestress at the stretching end, after the loss due to elastic shortening, if any.

As I said that there can be elastic shortening in post-tensioned members, due to sequential stretching of the tendons. Now here, we have got an expression which gives the variation of the prestressing force along the length. Due to the friction and wobble, the prestressing force is not constant over the length of the post-tensioned member. It drops in an exponential form.

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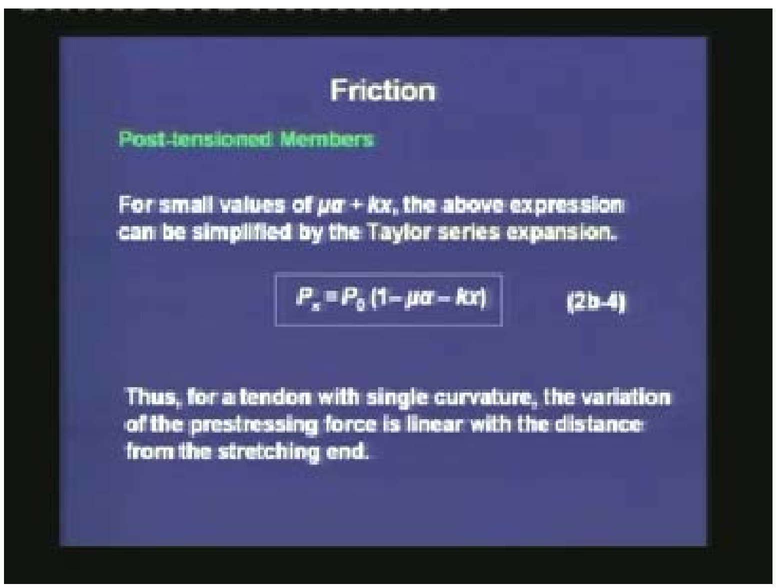

We have a small value of μα + kx, because α is expressed in radians and k, the wobble coefficient, is itself a small value. Then the exponential term can be simplified by the Taylor series expansion. We have a simplified expression, which is $P_x = P_0\ (1 - \mu\alpha - kx)$. Here, we can see that the variation of P with x is linear, which drops from P_0 as x increases. Thus, for a tendon with single curvature, the variation of prestressing force is linear with the distance from the stretching end.

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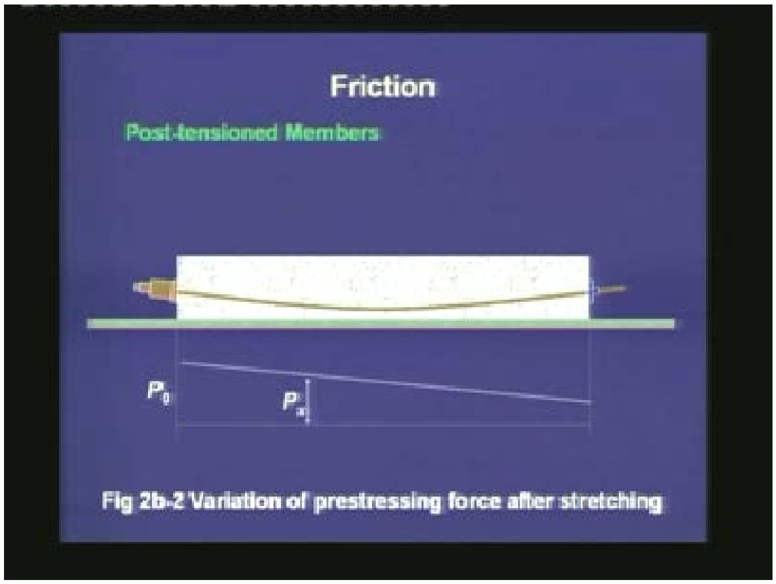

We are plotting the variation of the prestressing force along the length of a post-tensioned beam with a curved tendon, where the jack is at the left end. The force is P_0 at the left end and then, we have a dropping value which is denoted as P_x at distance x from the stretching end.

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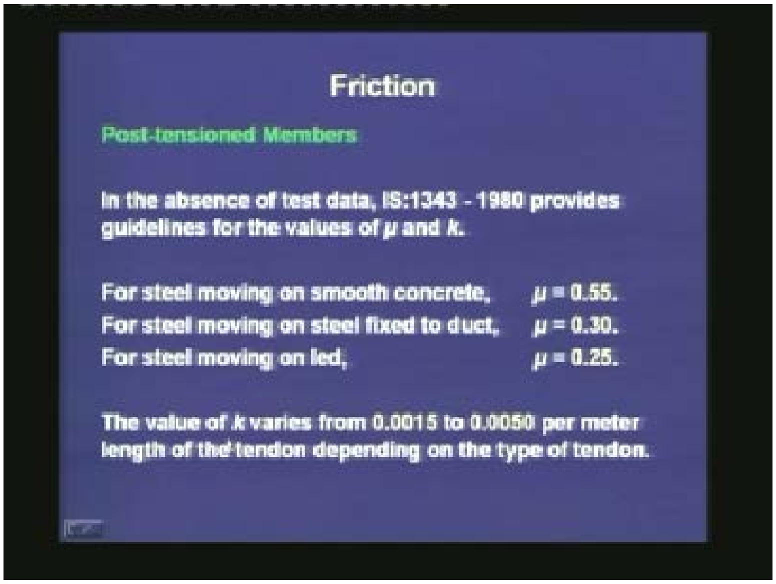

In order, to get the value of P_x, we need to the know the friction coefficient μ and the wobble coefficient k. In the absence of test data, IS: 1343-1980 provides some guidelines for the values of μ and k. For steel moving on smooth, concrete μ = 0.55. For steel moving on steel fixed to the duct, μ = 0.30. For steel moving on lead, μ = 0.25. Thus, the coefficient of friction drops if the interface of the steel with the duct becomes smoother. The value of k varies from 0.0015 to 0.0050 per meter length of the tendon, depending on the type of tendon. Usually, these values will be provided by the suppliers of the tendon and the duct.

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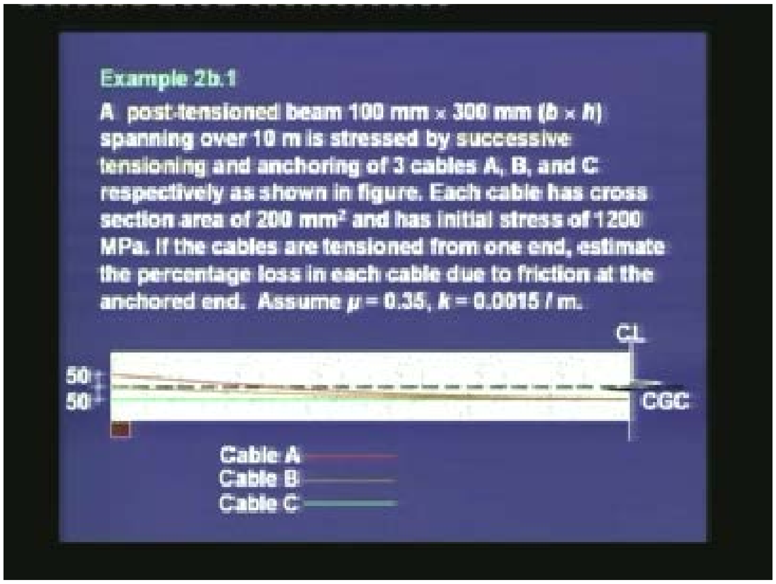

Let us work out a problem, to find out the loss due to the friction for a post-tensioned beam. Here, we see a beam whose cross-section is 100 mm × 300 mm, spanning over 10 m. It is stressed by successive tensioning and anchoring of three cables A, B and C, as shown in the figure. Each cable has a cross-section area of 200 mm^2 and has an initial stress of 1200 MPa. If the cables are tensioned from one end, estimate the percentage loss in each cable due to friction, at the anchored end.

For the beam, $\mu = 0.35$ and k = 0.0015/m. The sketch shows that Cable A has a negative eccentricity at the ends; that means, the CGS is located above CGC. At the centre, it is having a positive eccentricity of 50 mm. Cable B does not have any eccentricity at the ends, but has a positive eccentricity at the centre. Cable C does not have any curvature throughout. The eccentricity is constant, which is 50 mm throughout the length of the beam. Thus, Cable A is more curved as compared to Cable B, which is again more curved as compared to Cable C.

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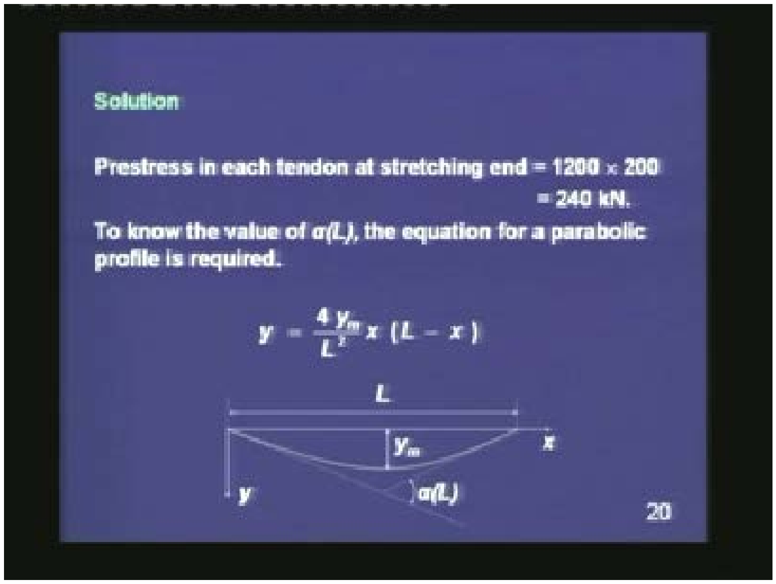

The prestress in each tendon at the stretching end is equal to 1200 × 200 N = 240 kN. This value does not include any loss due to elastic shortening. To compute the prestress loss due to friction, we need to know the value of α, which is also equal to the change in the slope of the curved profile of each tendon. To know the value of α, the equation for a parabolic profile is required. The equation is given in terms of y, which is the displacement of the CGS from the level at the ends; $y = (4y_m / L^2)\, x\, (L - x)$. This is a second order equation, which gives the equation of the parabola. The subtended angle is same as the change in the slope between the two ends of the parabola, which is represented here as α(L).

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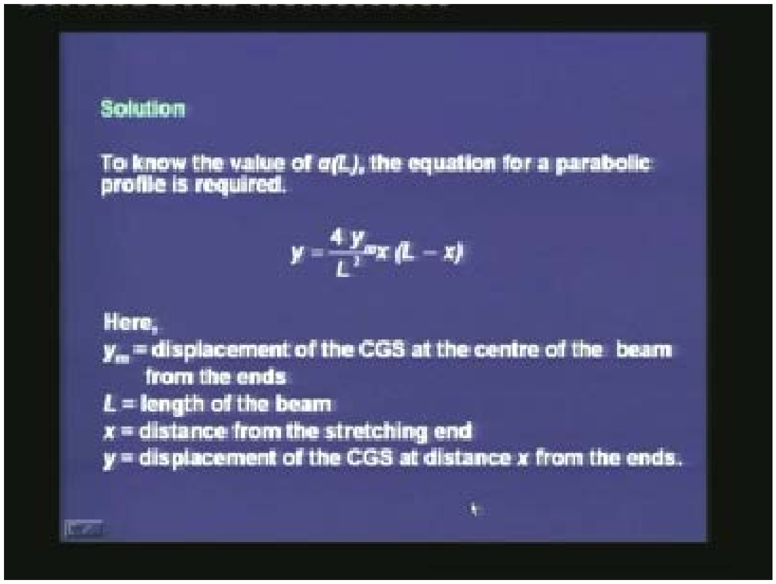

Here, y_m is the displacement of the CGS at the centre of the beam from the level at the ends, L is the length of the beam, x is the distance from the stretching end, and y is the displacement of the CGS at distance x from the ends.

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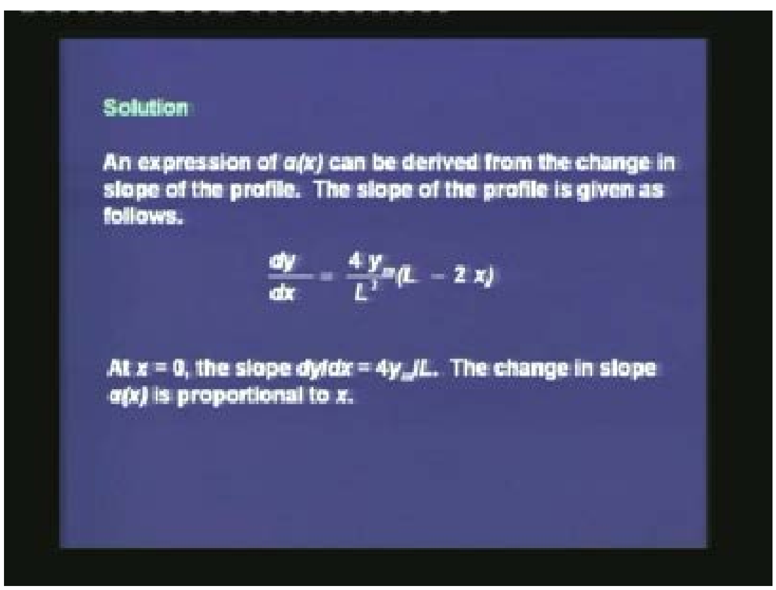

An expression of α(x) can be derived from the change in slope of the profile. The slope of the profile is given by the first differential, which is $dy/dx = (4y_m/L^2)(L - 2x)$. At x = 0, the slope $dy/dx = 4y_m/L$. The slope at any other point can be computed, and the difference between the slopes at two points gives the value of the subtended angle α. Since the expression of slope is linear, that means it is a first order function of x, the change in slope will also be linear with x.

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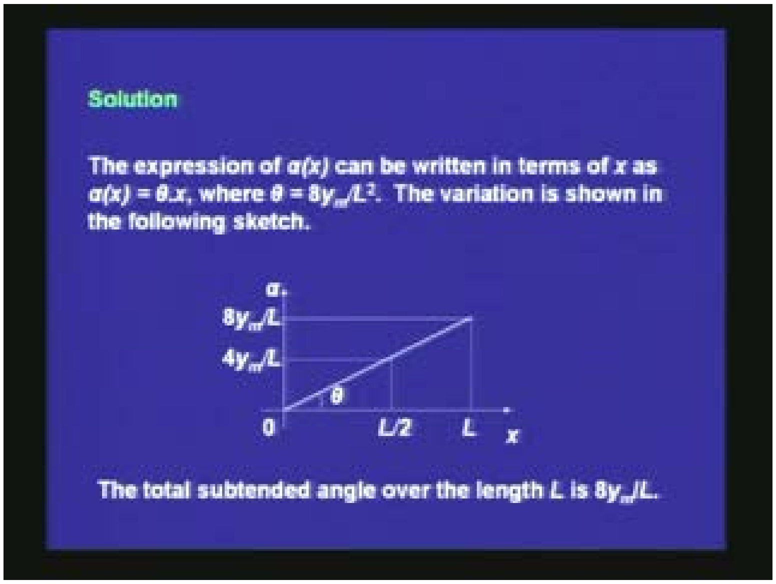

The expression of $\alpha(x)$, the subtended angle can be written in terms of x as $\alpha(x) = \theta\, x$. This means $\alpha(x)$ is proportional to x, where the proportionality constant $\theta = 8y_m/L^2$. The variation is shown in the sketch. Thus for a parabolic profile, as we are moving from the stretching end towards the anchoring end, the subtended angle or the change in slope is linear with respect to the distance from the stretching end. The total subtended angle for the profile over the length L is given as $8\, y_m/L$.

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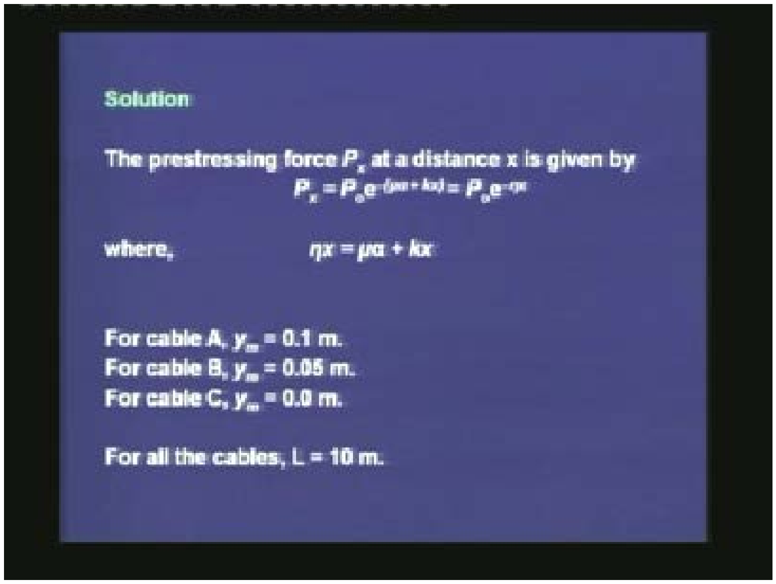

The prestressing force P_x at a distance x is given by this expression: $P_x = P_0\ e^{-(\mu\alpha + kx)}$, which is simplified as equal to $P_0\ e^{-\eta x}$, where $\mu\alpha + kx$ has been substituted by a function ηx. α in the first term and kx, both are proportional to x, the distance from the stretching end. Hence, we can substitute $\mu\alpha + kx$ as a term ηx which is also proportional to x.

To calculate ηx, we need to know the value of y_m, the displacement of the CGS at the middle from the level at the two ends. For Cable A, the total displacement is 50 mm at the left plus 50 mm at the middle, which gives a total of 100 mm or 0.1 m. For Cable B, y_m is equal to 0.05 m. For Cable C there is no displacement; it is a straight cable and hence, $y_m = 0$. For all the cables the length (L) is equal to 10 m.

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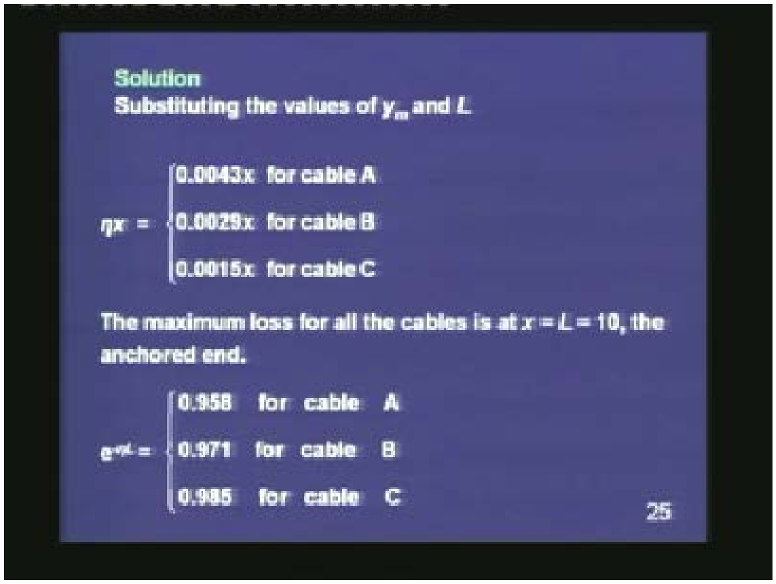

If we substitute the values of y_m and L, then we get the expression of ηx for each cable. ηx = 0.0043x for Cable A, 0.0029x for Cable B, and 0.0015x for Cable C. ηx is a measure of the drop in the prestressing force. What we can observe is that, it is highest for Cable A, which is the most curved profile out of the three. ηx is lower in Cable B as compared to Cable A, and it is the least in Cable C because, the only drop in the prestressing force is due to the effect of wobble. The maximum loss for all the cables is at x = L = 10, the anchored end. For that location, $e^{-\eta L}$ is equal to 0.958 for Cable A, 0.971 for Cable B and 0.985 for Cable C.

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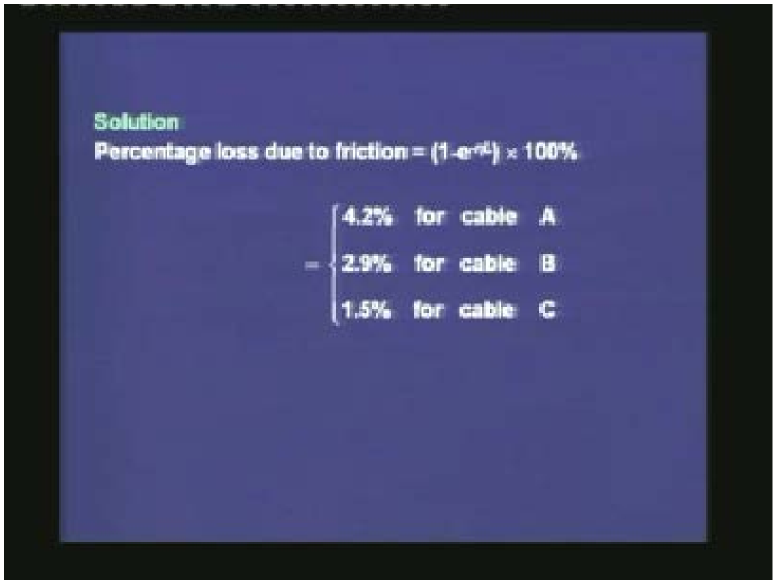

The percentage loss due to friction at the anchored end is given as $(1 - e^{-\eta L}) \times 100\%$. That means, first, we are calculating the prestressing force at the anchored end. Next, we are subtracting that from the value at the stretching end, and then we are dividing the difference by the force at the stretching end. We are getting the percentage loss by multiplying the result by 100. Once we do the computations, the results are 4.2% for Cable A, 2.9% for Cable B and 1.5% for Cable C. That means, the percentage loss in Cable A is maximum. The percentage loss in Cable B is in-between Cables A and C, and the percentage loss in Cable C is the minimum.

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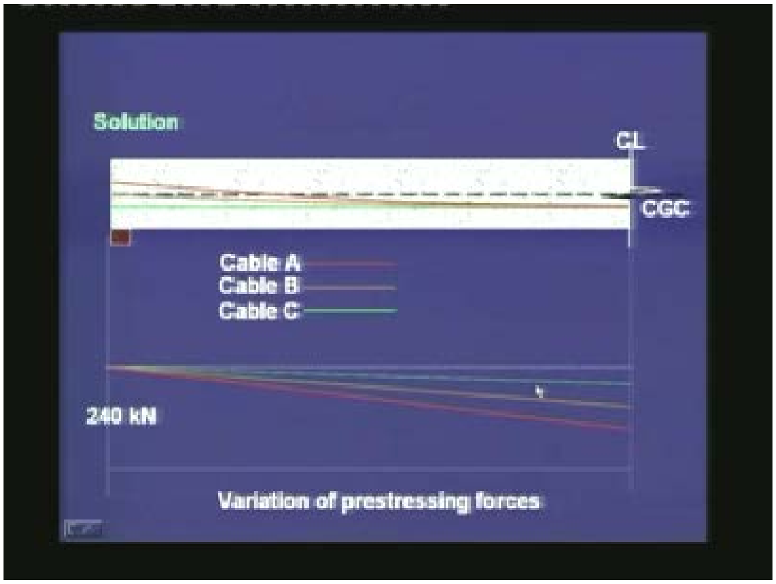

If we plot the variation of the prestressing force, then we find that, from the stretching end the prestressing force drops the least for Cable C (the green line). The drop in Cable B (the brown line) is in between those for Cable A and Cable C, and for Cable A the drop is maximum (the red line). This confirms the concept that the prestressing force will drop more for cables with higher curvature.

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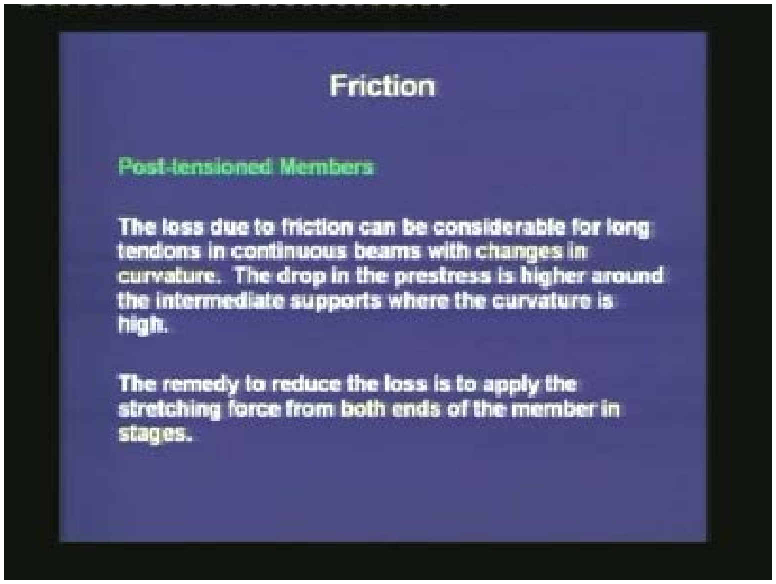

The loss due to friction can be considerable for long tendons in continuous beams with changes in curvature. The drop in the prestress is higher around the intermediate supports, where the curvature is high. Let me explain this by a sketch.

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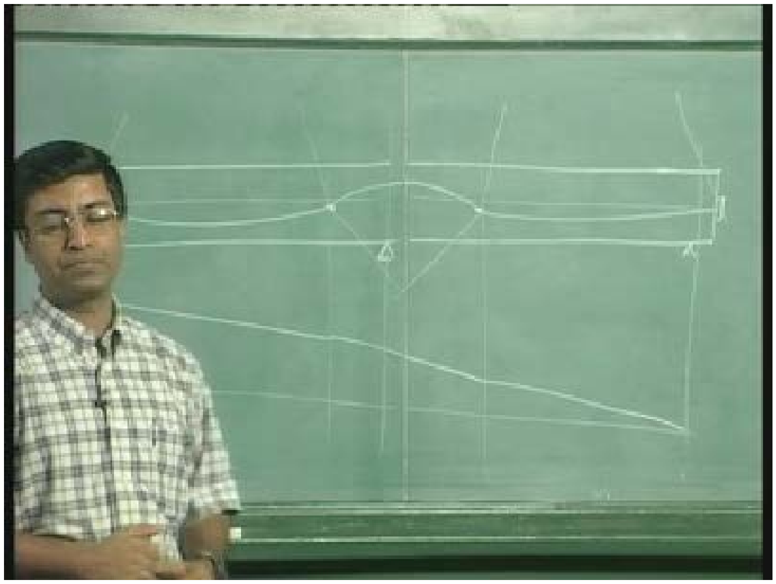

What we see in the simplified sketch is that in a continuous beam, the curvature changes. For the first span, the profile is going down, and then over the support the curvature is in the reverse direction. There is a point of contraflexure between these two curves, and again when we go to the second span the curvature is in the same direction as in the first span. Thus, for a continuous beam the cable changes curvature, and this leads to a higher drop in the prestressing force. There is one angle subtended by the first curve, there is another angle which is subtended by the second curve and there is a third angle, which is subtended by the third curve. What we observe is that the intermediate angle will be the highest out of all the three.

As we said earlier, that the drop in the prestressing force is proportional to the length x and is also proportional to the curvature of the cable. Since the curvature in the span is lower than that of the support, the rate of drop is relatively low. But over the support, the rate of drop is high. Then, again when we go into the span, the rate of drop is low.

To summarize, if we are stretching the cable from one end, the drop is uniform for the first curvature. It will be steeper over the supports and again uniform for the third curvature. The longer the beam becomes, the more spans and curvatures the beam has, and there will be larger drop in the prestressing force.

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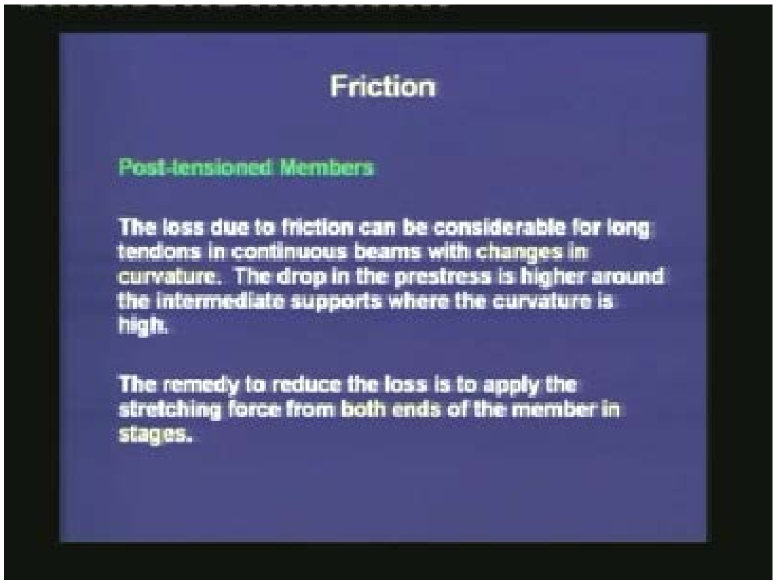

The remedy to reduce the loss is to apply the stretching force from both ends of the member in stages. That means, to reduce the drop in the prestress due to friction, one way is to first stretch from one side. Next, bring the jacks to the other side and, stretch from the other side. Repeat this process and get a more or less uniform prestressing force. If, more than one jack is available then the stretching can be done from both the sides.

Next, we shall learn the concept of anchorage slip.

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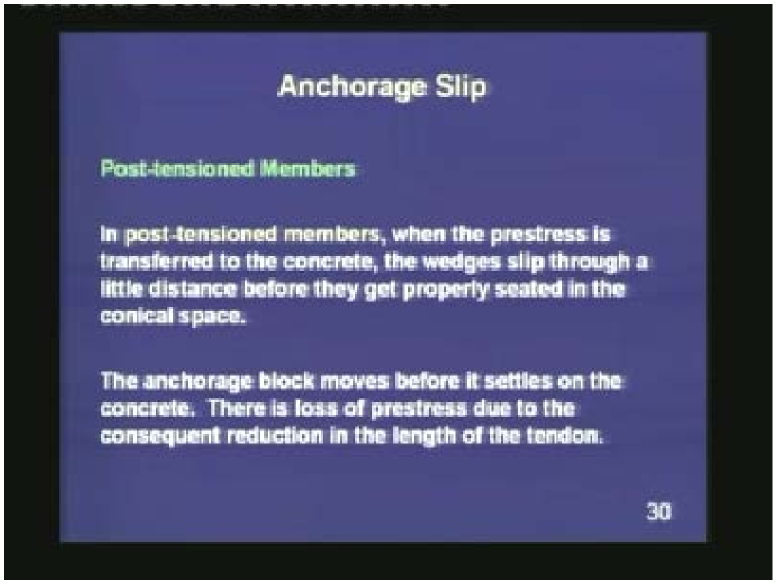

In post-tensioned members, when the prestress is transferred to the concrete, the wedges slip through a little distance before they get properly seated in the conical space. The anchorage block moves before it settles on the concrete. There is a loss of prestress due to the consequent reduction in the length of the tendon.

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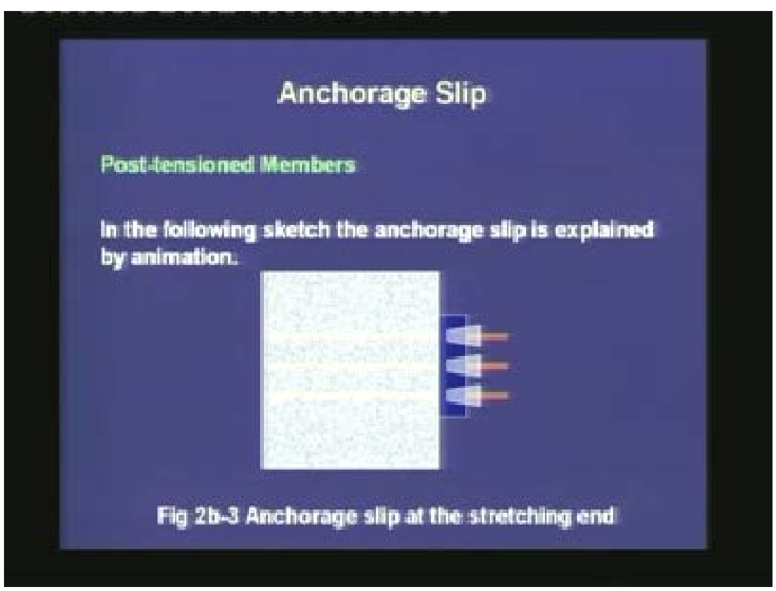

The above sketch animates the process of the loss due to anchorage slip. What we can notice is that, as the prestressing tendon is released from the jack, the wedges seat in the conical space. The anchorage block gets slightly depressed within the concrete. The sum total effect is called the anchorage slip, which causes a reduction in the length of the prestressing tendon and leads to a drop in the prestressing force. Thus, the anchorage slip is another phenomenon which leads to a loss in the prestressing force.

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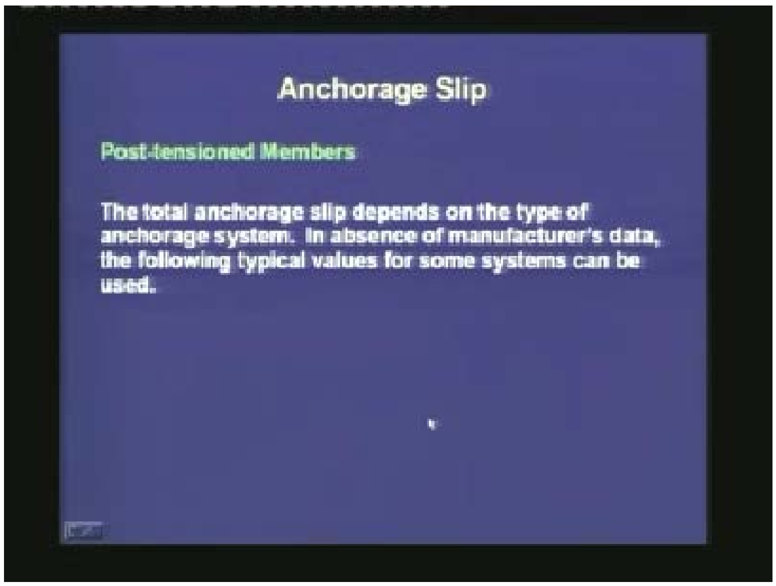

The total anchorage slip depends on the type of the anchorage system. Usually, the suppliers of the anchorage block will provide some data to calculate the anchorage slip for their particular system. In absence of manufacturer’s data, typical values of some systems can be used.

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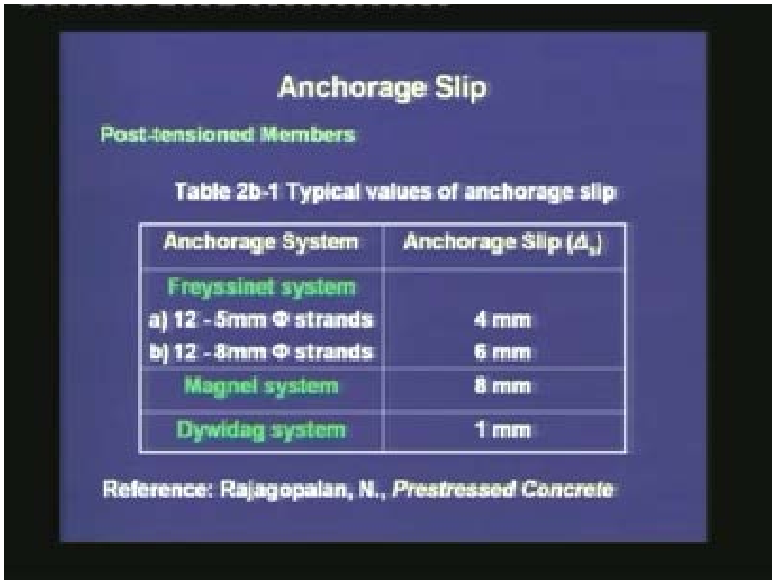

The anchorage slip is given by the total slip, the distance by which the prestressing tendons shortens because of the setting of the anchorage block. It depends on the system, like for a Freyssinet system with 12 numbers of 5 mm strands, the slippage is by 4 mm. For 12 numbers of 8 mm diameter strands, the slippage is by 6 mm. For Magnel system it can be 8 mm, for Dywidag system it can be as low as 1 mm.

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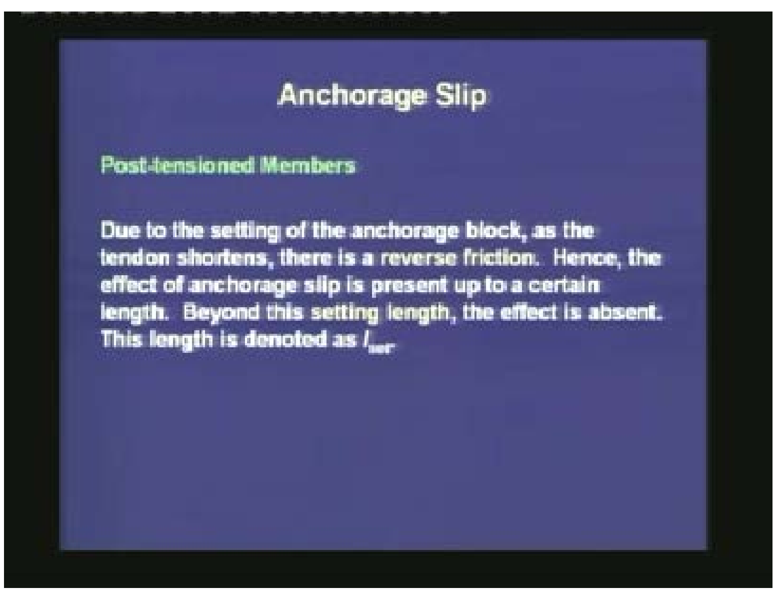

Due to the setting of the anchorage block as the tendon shortens, there is a reverse friction. This is an interesting phenomenon, that when we are first stretching the tendon there is a friction in the opposite direction of the stretching force; that means, if we stretch towards the left the friction acts towards the right. But when we are releasing the jack, and the anchorage block is setting on the post-tensioned member, the tendon is trying to move from left towards the right. For a certain distance near the stretching end, the friction reverses, and this phenomenon is called reverse friction. That means, it does not allow the movement to occur throughout the full length, but the shortening happens only in a limited region of the tendon. Hence, the effect of anchorage slip is present only up to a certain length of the member. Beyond this setting length, the effect is absent. This length is denoted as ℓ_{set}.

For friction, what we have seen is that, the drop in the prestressing force is occurring throughout the length of the member. But for anchorage slip, the reverse friction restricts the drop to a certain length of the member, which is adjacent to the stretching end. This particular length is called the setting length, which is denoted as ℓ_{set}.

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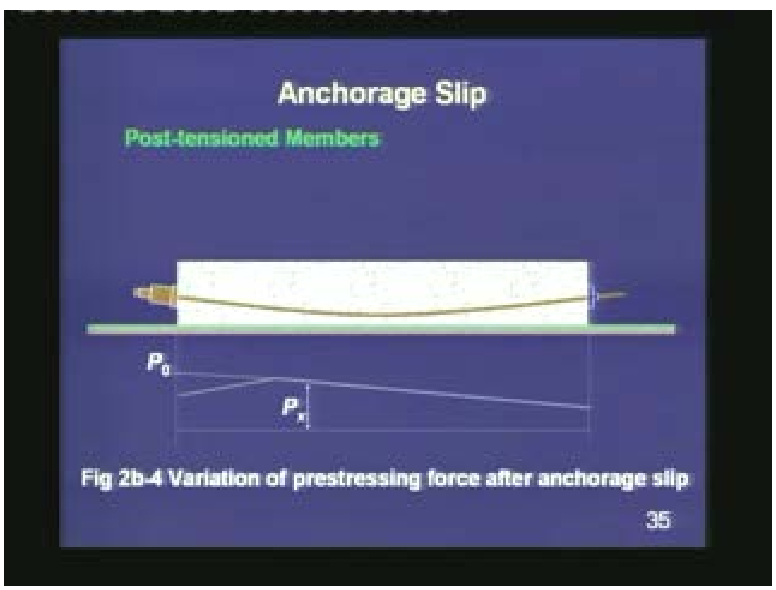

If we plot the force variation after anchorage slip, what we observe is that before the tendons were released, the prestressing force was dropping linearly. But after the tendons have been released, and the anchorage block has set, there is a drop in the prestressing force at the end. But this drop is not reflected throughout the full length because of the reverse friction. This drop reduces, as we go away from the stretching end, and after a certain distance this drop is not present.

Now, both the friction and the anchorage slip cause a variation of the prestressing force along the length. We can plot the variation of the prestressing force in a diagram, which is called the force variation diagram.

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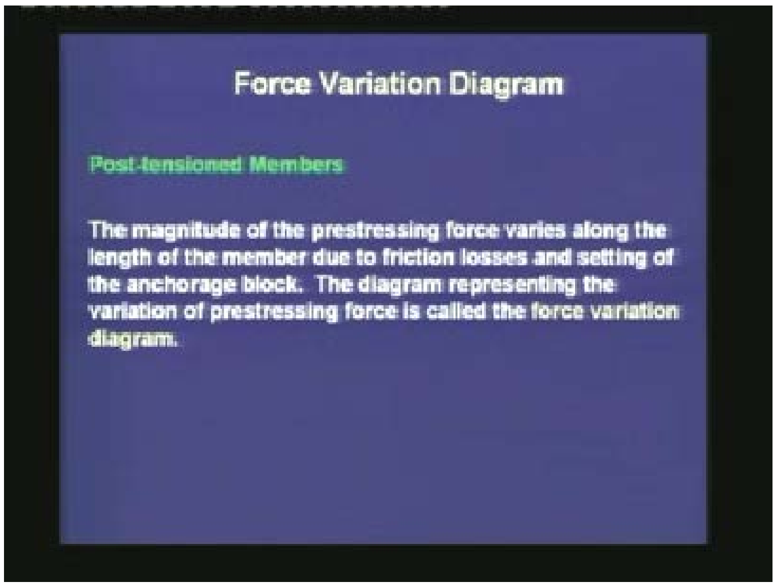

For a post-tensioned member, the magnitude of the prestressing force varies along the length of the member due to friction losses and setting of the anchorage block. The diagram representing the variation of prestressing force is called the force variation diagram. This diagram is helpful if we are interested in the loss of the prestressing force throughout the length. It gives an idea at what location we have higher drops in the prestressing force, and how can we improve the variation with stretching the cable from the two ends either sequentially, or simultaneously. The force variation diagram helps us to determine the stretching process for a continuous beam, because in a continuous beam the force variation is quite significant. Unless we take a rational measure to do the stretching, we will not be having an idea how much to stretch at each of the ends.

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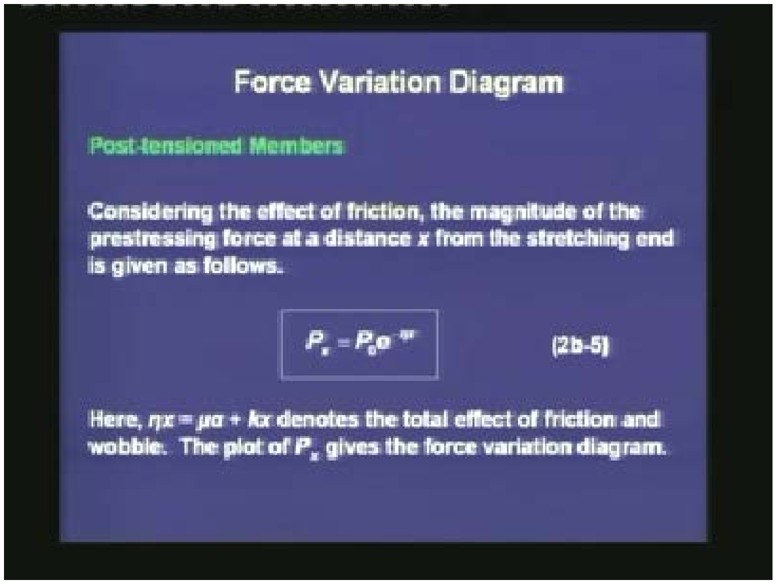

To draw the force variation diagram, we are again reverting back to the expression of P_x. Considering the effect of friction, the magnitude of the prestressing force at a distance x from the stretching end is given by the expression $P_x = P_0 e^{-\eta x}$, where ηx has both the effect of friction, as well as the wobble. ηx is equal to $\mu\alpha + kx$, which denotes the total effect of friction and wobble. The plot of P_x itself gives the force variation diagram.

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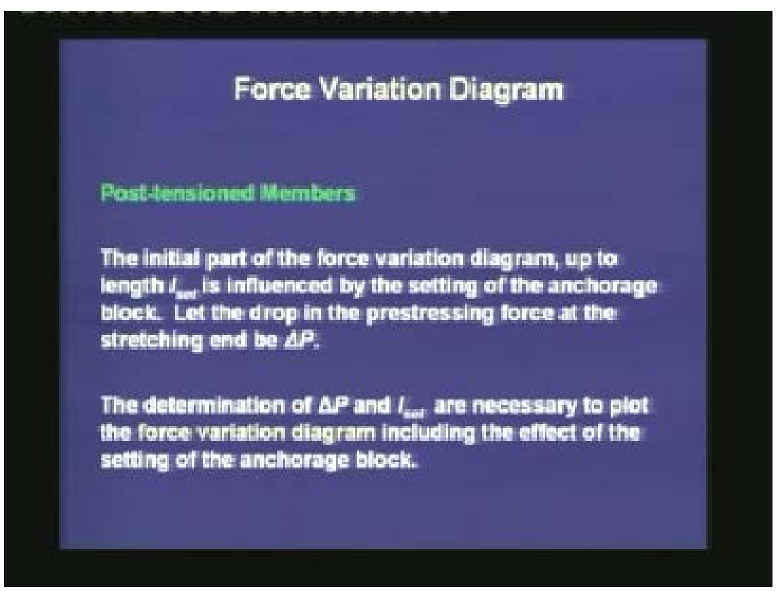

The initial part of the force variation diagram, up to length ℓ_{set} is influenced by the setting of the anchorage block. Let the drop in the prestressing force at the stretching end be denoted as ΔP. The determination of ΔP and ℓ_{set} are necessary, to plot the force variation diagram including the effect of setting of the anchorage block. We have an expression of P_x, which is satisfactory to plot the force variation diagram for most of the length of the beam, except near the stretching end where over a certain distance there is a drop in the prestressing force from P_x due to the anchorage slip.

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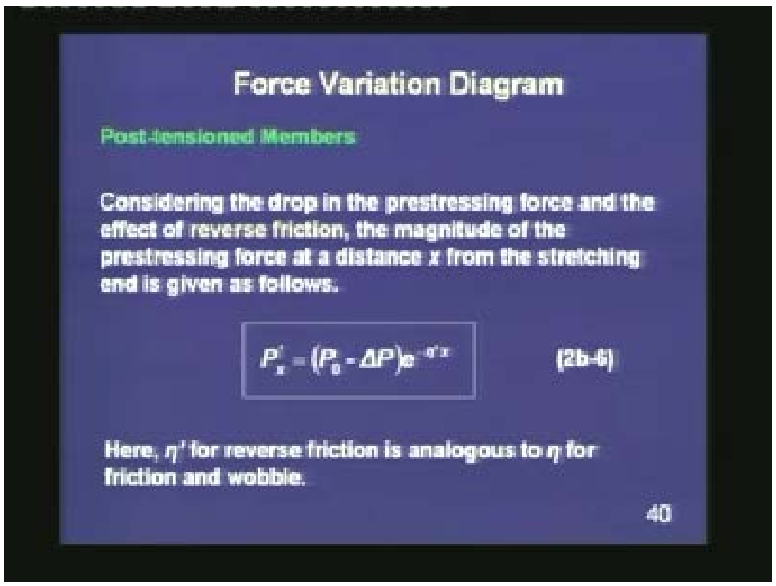

Considering the drop in the prestressing force and the effect of reverse friction, the magnitude of the prestressing force at a distance x from the stretching end is given as follows. This equation is analogous to the equation that we have seen for friction. The difference is that the new value of the prestressing force, which is denoted as $P_x^{/}$, is in terms of the reduced value of the prestressing force at the stretching end, and is given as $(P_0 - \Delta P)\ e^{-\eta' x}$. Here, $\eta^{/}$ is for reverse friction; that means, the friction is occurring in the opposite direction, and is analogous to η for friction and wobble. Now remember that, for reverse friction $P_x^{/}$ increases with the distance from the stretching end.

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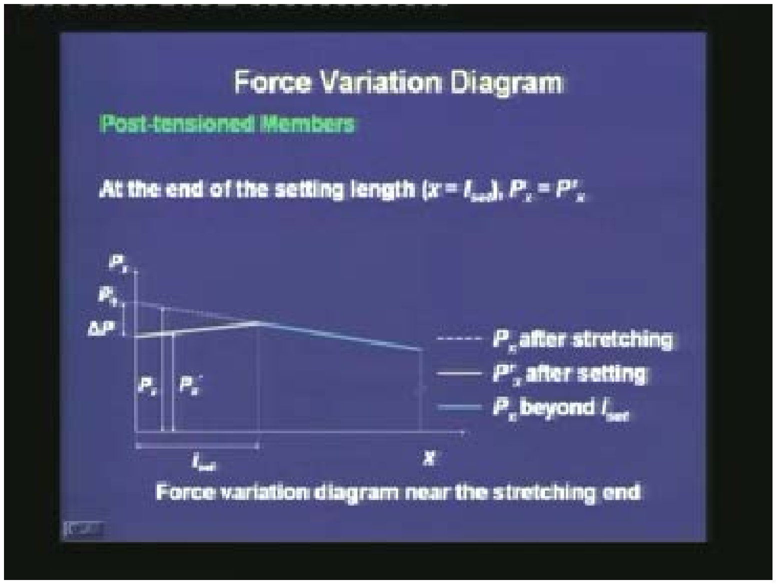

If we plot the two equations, the first equation gives a dropping curve from the value of P_0 with x the distance from the stretching end; the second equation gives an increasing curve, which corresponds to the reverse friction. At a certain distance both these equations give the same value of the prestressing force, and the two curves intersect. The distance from the stretching end to this point is the setting length, which is denoted as $\ell_{set.}$ In the plot, the dashed line is the original force variation diagram after the stretching. The solid white line is the variation of the prestressing force after the anchorage slip. The blue line is the variation of the prestressing force beyond the setting length, which is due to the friction effect.

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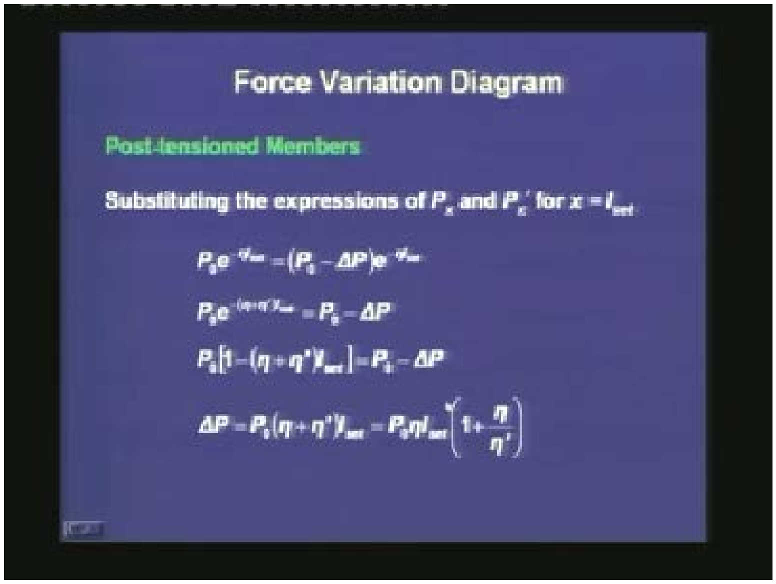

Our next task in hand is to derive an expression of the setting length. In order to do that, we are substituting x is equal to ℓ_{set} in both the expressions of the forces. On the left, we have the expression of P_x. On the right, we have the expression of $P_x^{/}$, and the equality is satisfied at x equal to ℓ_{set}. We are transposing the terms related with the exponential, and here we are again using the Taylor series expansion. We get the third form of the equation. We are writing an expression of ΔP, which is the drop in the prestressing force at the stretching end, as equal to P_0 $(\eta + \eta^{/})$ ℓ_{set} . We can express the reverse friction in terms of the friction in the tendon.

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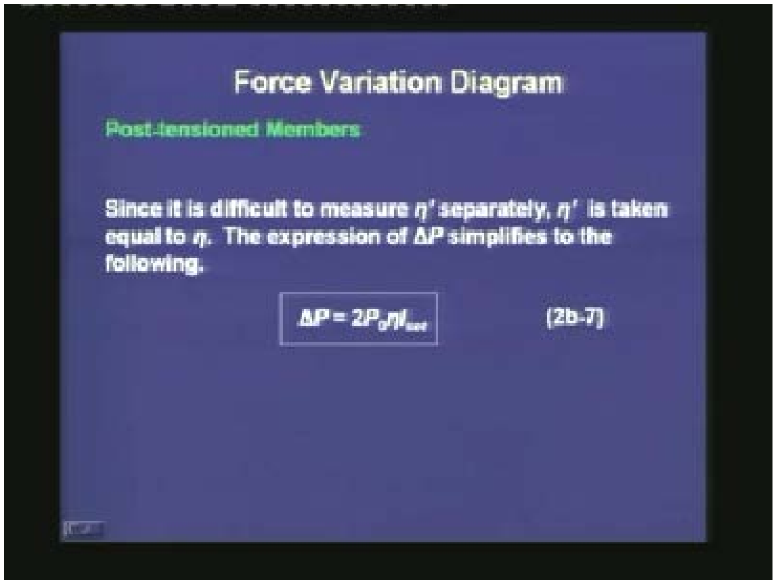

Since, it is difficult to measure the reverse friction separately from the friction that we have seen earlier, the reverse friction is usually taken equal to the frictional value that we use in our conventional calculation of the prestressing loss. If we take $\eta^{/} = \eta$, then the expression of delta P simplifies to the following. That means, the drop in the prestressing force at the stretching end is equal to $2P_0\eta\ell_{set}$.

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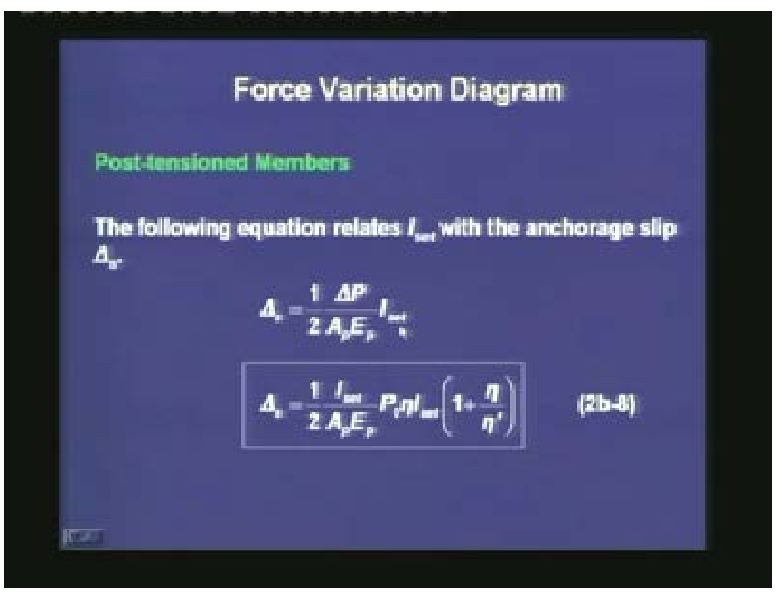

The following equation relates ℓ_{set} to the anchorage slip. We can calculate ΔP from the slip that is provided by the manufacturer of the anchorage block. The slip (Δ_s) is given as $½(\Delta P/A_pE_p)\ \ell_{set.}$ This is the expression of the slip, which relates it to the setting length. Once, we substitute the expression of ℓ_{set} from the previous expression, we can find an expression of the slip in terms of the stretching prestressing force P_0, the friction term η and the reverse friction term $\eta^{/}$.

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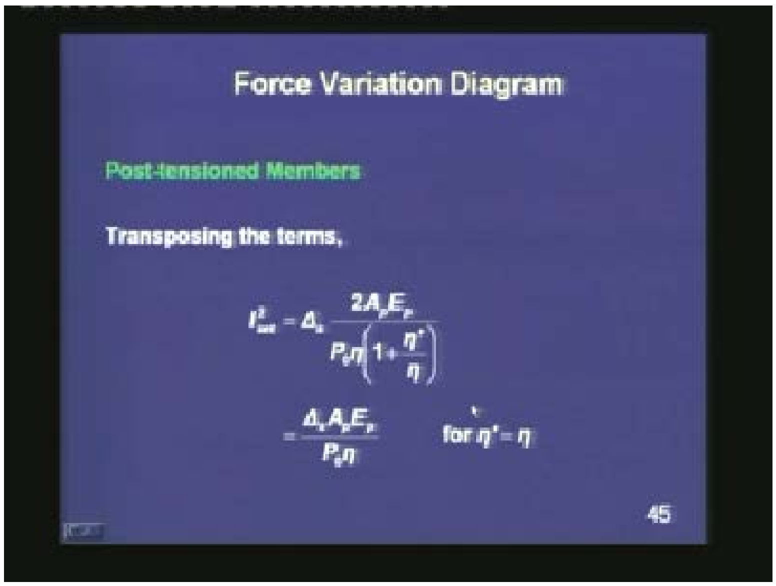

Transposing the terms, we have brought ℓ_{set} on the left and Δ_s (which is available from the manufacturer) on the right. We are also simplifying to $\eta^{/} = \eta$.

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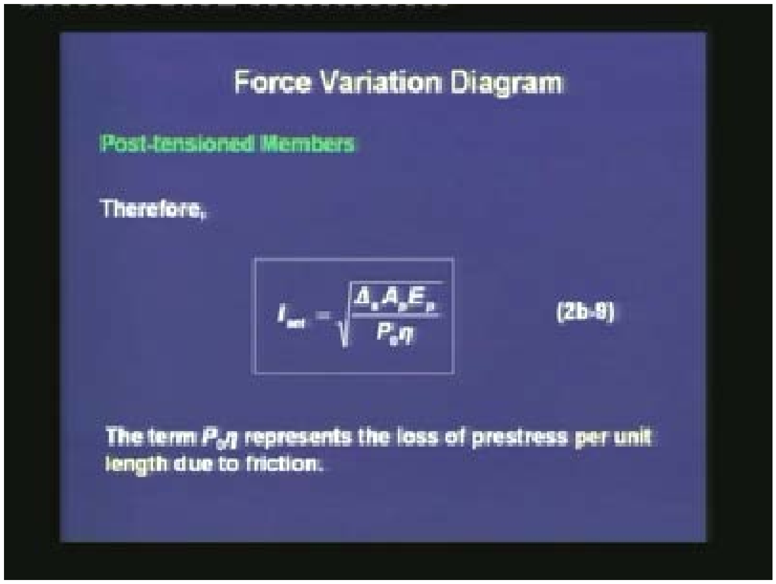

From this, we get an expression of $\ell_{set.}$ The ℓ_{set} is equal to the square root of $(\Delta_s A_p E_p/P_0\eta)$. We can observe that the setting length is proportional to the square root of the slippage. That means, the more slip we have for a particular type of anchorage block we shall have a longer $\ell_{set.}$ But the variation is a square root function. The term $P_0\eta$ represents the loss of prestress per unit length due to friction. This, we can derive from the expression of P_x.

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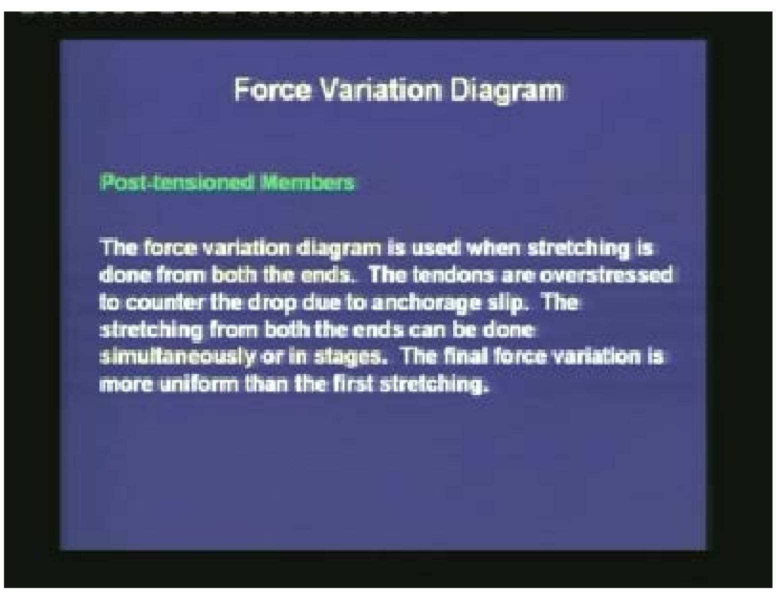

Once we have determined ℓ_{set} from the value of the anchorage slip, and we know the drop in the prestressing force ΔP, we can draw the force variation diagram including the effect of anchorage slip. The force variation diagram is used when stretching is done from both the ends. It is a rational tool to determine how much force we need to apply at each end, and what is the sequence we should follow. Finally, what is the prestressing force we shall get along the member, is available only if we draw the force variation diagram. The tendons are over stretched to counter the drop due to anchorage slip. The stretching from both the ends can be done simultaneously, if we have more than one jacks, or it can be done in stages if we just have one jack. The final force variation is more uniform than the first stretching.

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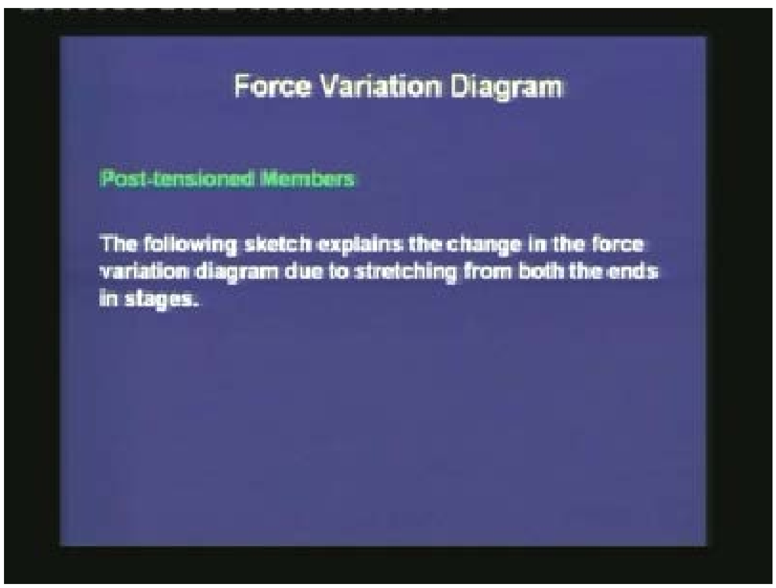

The following sketch, explains the change in the force variation diagram due to stretching from both the ends in stages.

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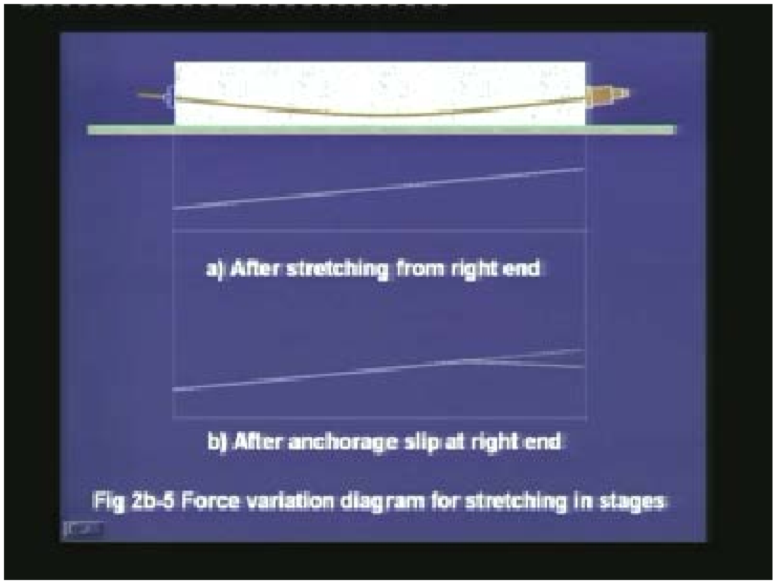

For this post-tensioned beam, the stretching is done from the right. We can see that there is a variation of the prestressing force as we move from the right towards the left, due to friction. Once the tendons are released on the right side, there is an anchorage slip which leads to a drop in the prestressing force near the stretching end. But the drop is reduced till the drop becomes zero at the distance ℓ_{set} from the stretching end. The second sketch is the variation of the prestressing force after the anchorage slip at right end.

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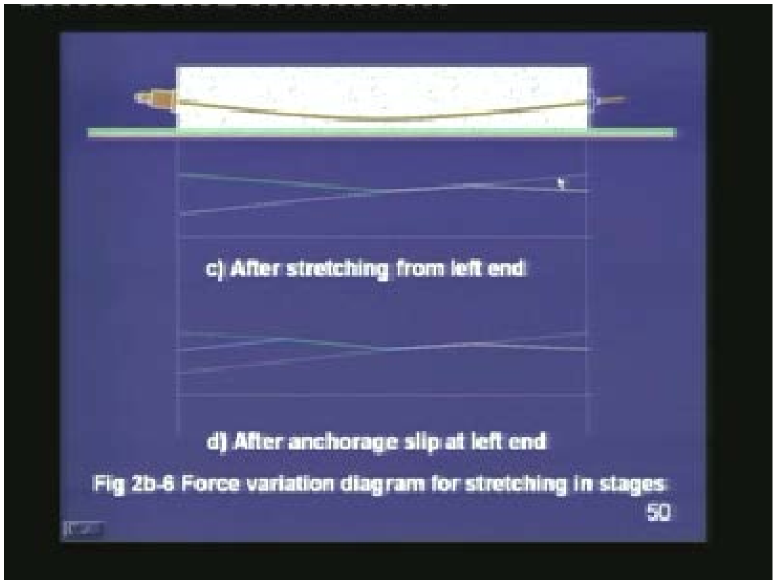

Next, we are moving the jack to the left, and we are re-stretching the tendon up to a value which is equal to the original value on the right side. When we are applying the tension on the left side, there is a drop in the prestressing force along the length due to friction, just like as it happened when we stretched from the right end. The green curve meets the previous curve near the centre of the beam. Once we release the jack at the left end, there is a drop in the prestressing due to the anchorage slip, and that drop again reduces. It becomes zero at a distance ℓ_{set} from the left end, which is now the stretching end.

Thus, what we observe is that for this single span beam, if we are stretching it from both the ends sequentially, the force variation diagram is more uniform compared to what we had got after the first stretching from the right end.

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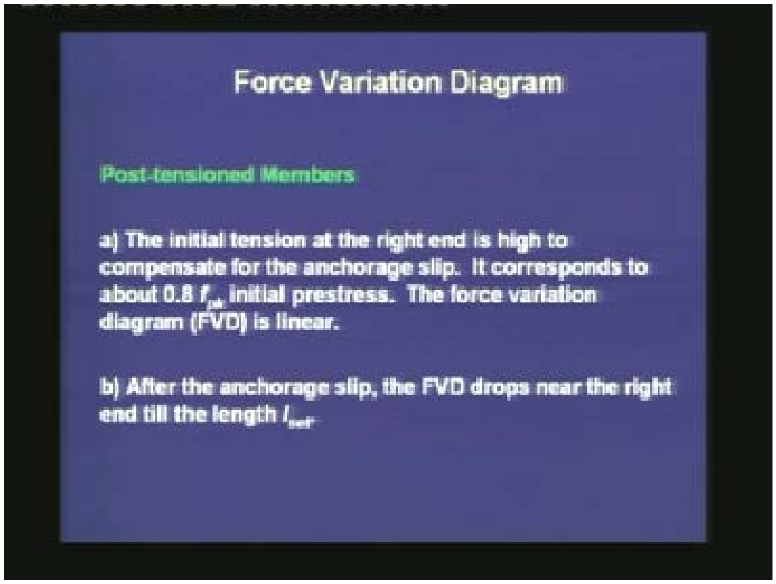

To summarize, the initial tension at the right end is high to compensate for the anchorage slip. It corresponds to about 80% of the characteristic strength and the force variation diagram (FVD) is linear. After the anchorage slip, the FVD drops near the right end till the length ℓ_{set}.

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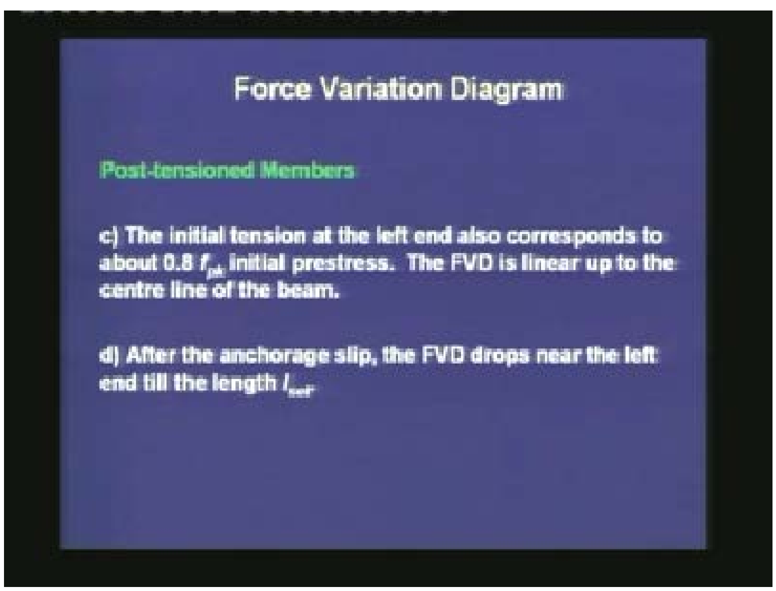

Then, the initial tension at the left also corresponds to about 80% of the characteristic strength. The force variation diagram is linear up to the central line of the beam. After the anchorage slip, the FVD drops near the left end till the distance ℓ_{set}.

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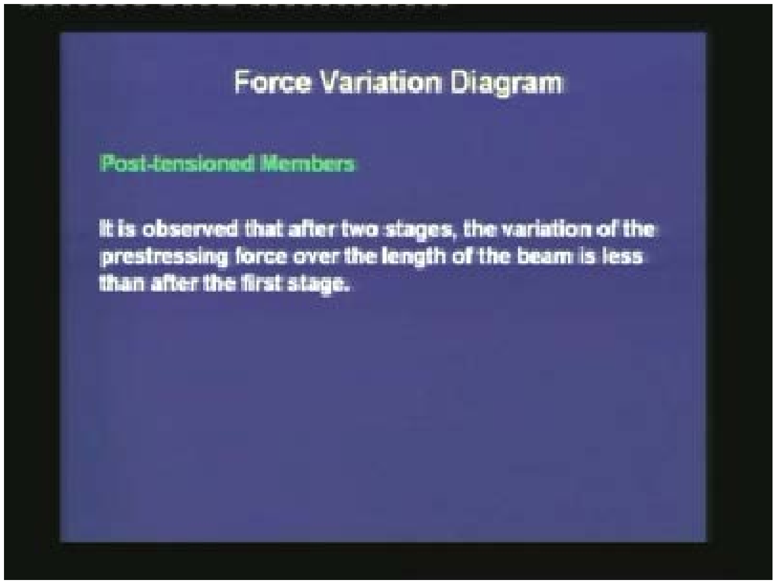

It is observed, that after two stages, the variation of the prestressing force over the length of the beam is less than after the first stage. If we do a few more stages, we shall observe that the variation will be still lower, and the prestressing force will be more uniform than the variation which we had obtained after the first stretching. This is more relevant in a long continuous beam, where the drop in the prestressing force is even higher. There sequential prestressing or a simultaneous prestressing from both the ends is the method to reduce the effect of the drop in the prestressing force, due to friction over the length of the member. If we are stretching from both the ends then, the force variation is more or less symmetric about the centre line of the beam, and that is desirable when the beam is loaded symmetrically.

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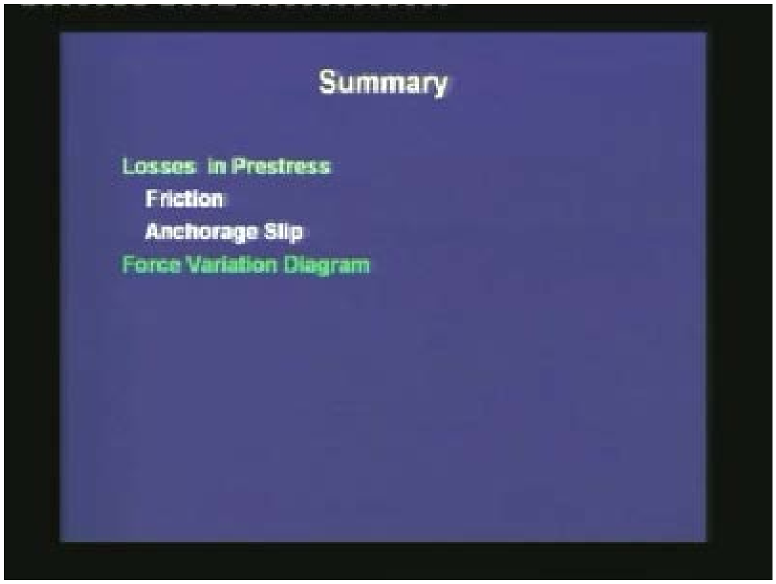

To conclude, today we talked about two more immediate losses of prestressing force. The first cause was the friction. Due to friction, the force drops from the stretching end as we move away from the stretching end. This can be severe for longer beams, and for continuous beams. We have found that the drop in prestressing force depends on first, the amount of the prestressing force; second, the length of the prestressing beam; and third, the curvature of the prestressing tendon. For a long continuous beam, there is reversal of curvature of the prestressing tendon. The drop in the prestress is significant near the intermediate supports because, there the curvature is high. Hence, there can be a significant difference between the prestressing force at the stretching end, and the prestressing force at the anchored end.

The other type of immediate loss that we have observed is the anchorage slip. This occurs due to the seating of the wedges in the conical space, and also the setting of the anchorage block in the concrete. The anchorage slip is usually given by the manufacturer of the anchorage system. The effect of the anchorage slip is limited within a certain length, called the setting length, from the stretching end.

The force variation diagram can be drawn to understand the variation of the prestressing force along the length. We have solved a differential equation to express the prestressing force in terms of x, the distance from the stretching end. We have seen that the drop is almost linear with distance. If we incorporate the drop due to the anchorage slip, then we have an increasing curve near the stretching end up to a distance ℓ_{set} from the stretching end, and after that the force drops down. The force variation diagram can be rationally used to have a more uniform prestressing force throughout the length. If the external loading is symmetric, then it is always preferred to have a symmetric prestressing force variation in the member. If we are loading it sequentially, what we can see is that, we can achieve a force variation diagram which has less variation than compared to the diagram after single stretching.

With this we finish the immediate losses of the prestress. In our last lecture, we had covered elastic shortening. Today, we covered friction and anchorage slip. All these three losses occur during the prestressing process, and it is felt immediately during the stretching. Hence, all three are grouped under immediate losses. In the next lecture, we shall start the other group of loss of prestressing force, which is the long term loss, which depends on time. These are affected by the creep and shrinkage of concrete (which are the variations of the length of the concrete member with time), and by the relaxation of the steel (which is the drop in the prestressing force under a constant strain). These long term losses become functions of time, which we need to monitor to get the final effective prestress.

Thank you.