PRESTRESSED CONCRETE STRUCTURES

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Module – 2: Losses in Prestress

Lecture – 1: Elastic Shortening

Welcome back to Prestressed Concrete Structures. Today, we are starting the second module on losses in prestress.

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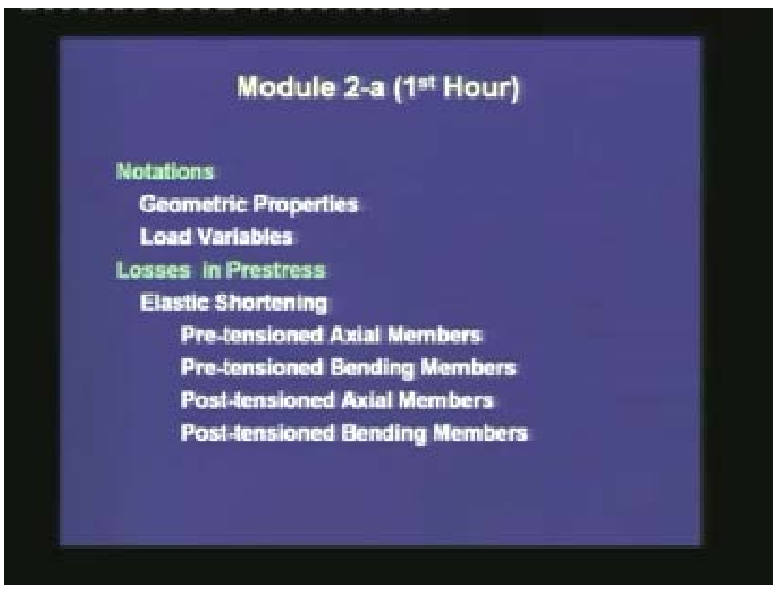

In the first lecture of this module, we shall first get familiar with the notations in the geometric properties and load variables. Next, we shall go through the first type of loss in prestressed concrete structures, that is the elastic shortening. We shall understand the phenomenon of elastic shortening for pre-tensioned and post-tensioned members. In either type of the prestressed structures, we shall look into examples of axial members and bending members.

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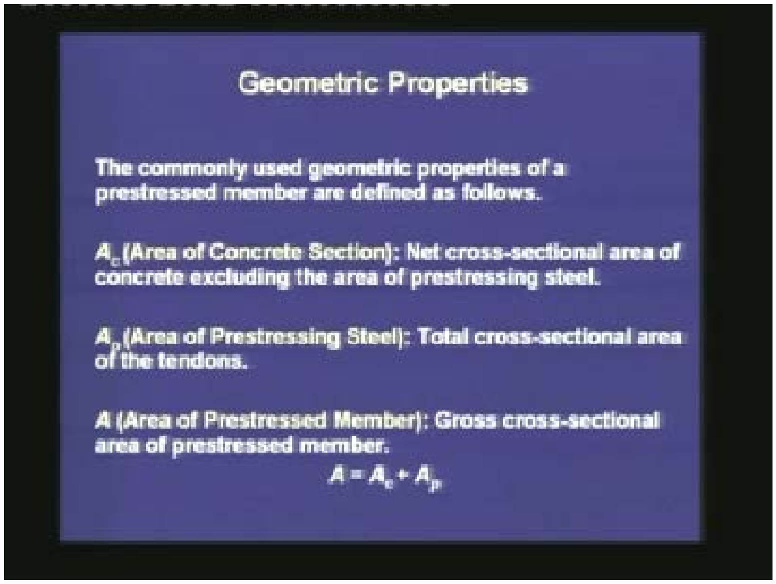

The commonly used geometric properties of the prestressed members are explained in this slide. A_c is the area of the concrete section, that is given the total sectional area of the member, if we subtract the area of the prestressing steel, then the remaining area is termed as 'A_c'. There can be of course substantial difference between A_c and the total area A, if the duct is voided and its size is large. The second notation is A_p, which is the area of prestressing steel, that is the total cross-sectional area of the tendons. The third is the area of the prestressed member, which is the summation of A_c and A_p.

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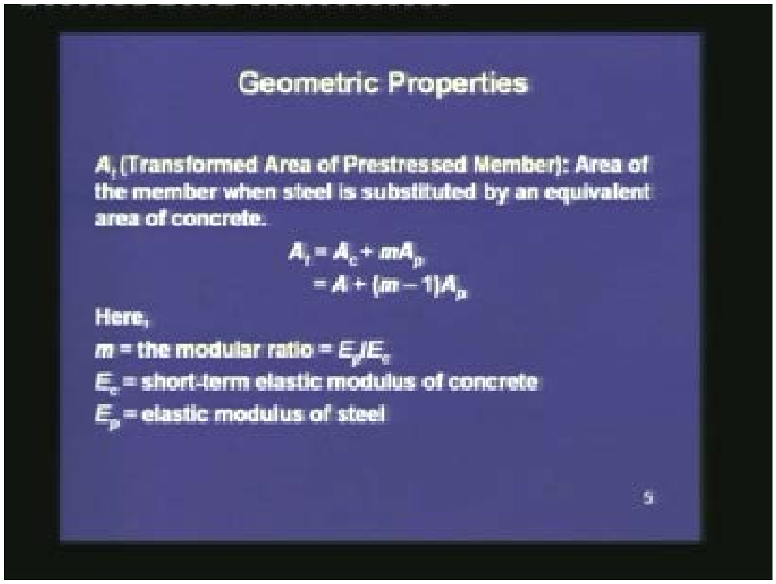

There is another definition which is used in elastic analysis, that is the transformed area of the prestressed member. This is the area of the member when the steel is substituted by an equivalent area of concrete. The transformed area is given as A_c plus the modular ratio times the area of the prestressing steel. If we substitute back the expression of the total area, then the transformed area is given as the total area plus the modular ratio minus 1 times A_p. The modular ratio is defined as the ratio of the elastic modulus of the prestressing steel divided by the elastic modulus of the concrete.

The modulus of concrete can change with time. In our elastic analysis, we may stick to the short-term elastic modulus. Then, the modular ratio is defined just based on the short-term elastic modulus. If we use the long-term elastic modulus of concrete, we are including the effect of creep in the definition of modular ratio of the member.

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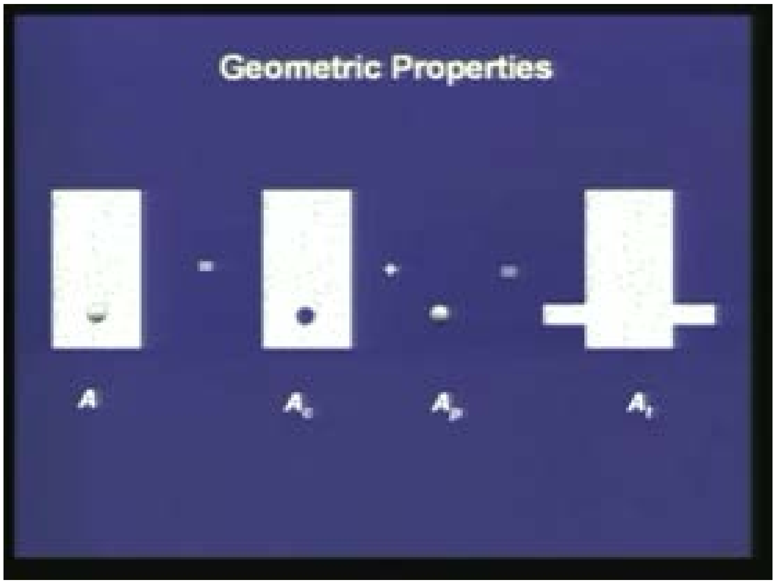

To explain it by figures, on the left is a cross-section of a rectangular prestressed member. If we look into only the net area of the concrete cross-section, it is represented by A_c. The total area of the prestressing steel, here we have denoted within one circle, is represented as A_p. This prestressed section is equivalent to a transformed section, where the full section is considered to be of concrete. That means the prestressing steel has been substituted by an equivalent area of concrete. This transformed area is considered to be made up of only one material, which is used in the elastic analysis. The analysis is same as that of an elastic analysis of a section with homogenous material.

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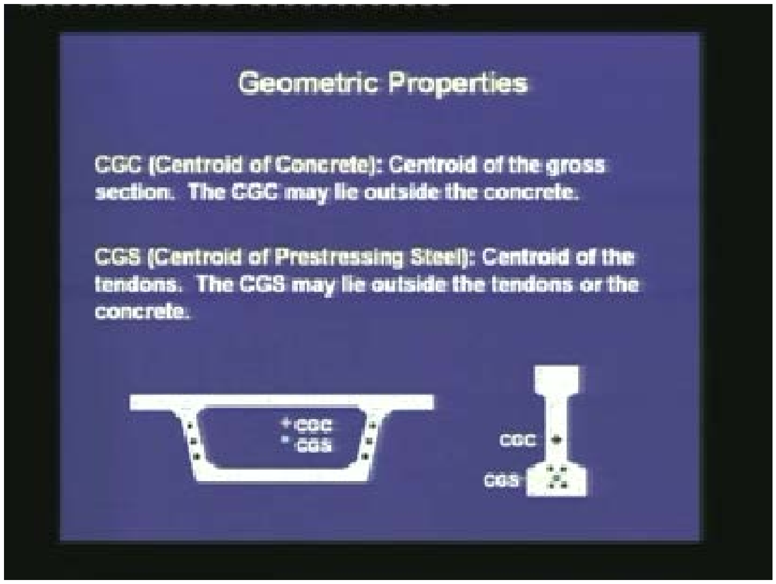

Next, we are going to learn the definitions of the centroids. The first one is the CGC, the centroid of concrete, or the centre of gravity of concrete. The centroid of the gross section can be taken as the CGC. Here, we are not deducting the area of the prestressing steel, just for simplicity in the computation. We should note that the CGC may lie outside the concrete section. An example is given for a box girder. Here, the centroid of the section is lying outside the concrete, it is lying inside the hollow space of the box girder. The second definition is the centroid of the prestressing steel, or the centre of gravity of the prestressing steel, and we shall denote that as CGS.

The CGS may also lie outside the tendons or the concrete of the section. In most of our calculations, we do not consider the individual tendons in the prestressed member. We do the calculations based on the total area A_p, and the location being considered at the CGS. There are two types of sections being shown: on the left is a box girder where both the CGC and CGS are lying inside the hollow space of the box girder. On the right hand side, we are seeing an I-girder, where the + symbol is the CGC and the blue circle is the CGS. When we do our computations, these are the two most important locations that we are concerned of.

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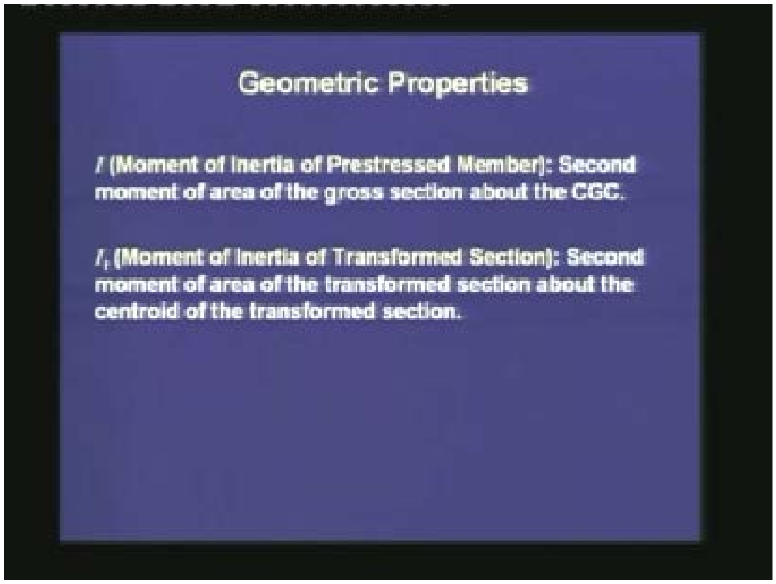

The other important geometric properties are: the first is the moment of inertia of the prestressed section, which is represented as ‘I’. This is the second moment of area of the gross section about the CGC. Remember that for an elastic section, the neutral axis lies at the CGC. The second one is a more refined calculation of the moment of inertia that is based on the transformed section, where the second moment of area is calculated about the centroid of the transformed area. The first one ‘I’ is the simplified form of the second one which is I_t.

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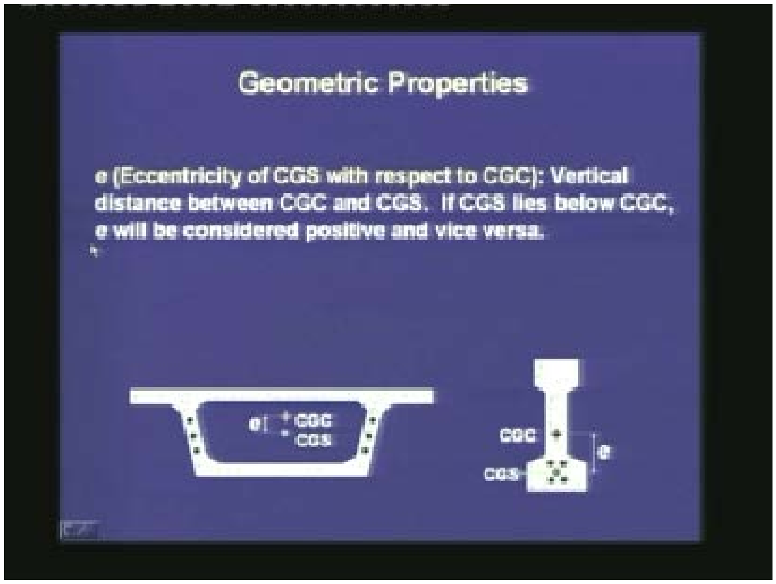

Another most important variable that we shall use in our calculations is the eccentricity, which is the vertical distance between the CGS and the CGC. If the CGS lies below the CGC, then we shall consider the eccentricity to be positive. If the CGS lies above the CGC, then the eccentricity will be considered to be negative. For the box girder, we see that 'e' is the distance between the CGC and the CGS and here, it is positive. Similarly, for the I-girder also, e is positive because the CGS is lying below the CGC.

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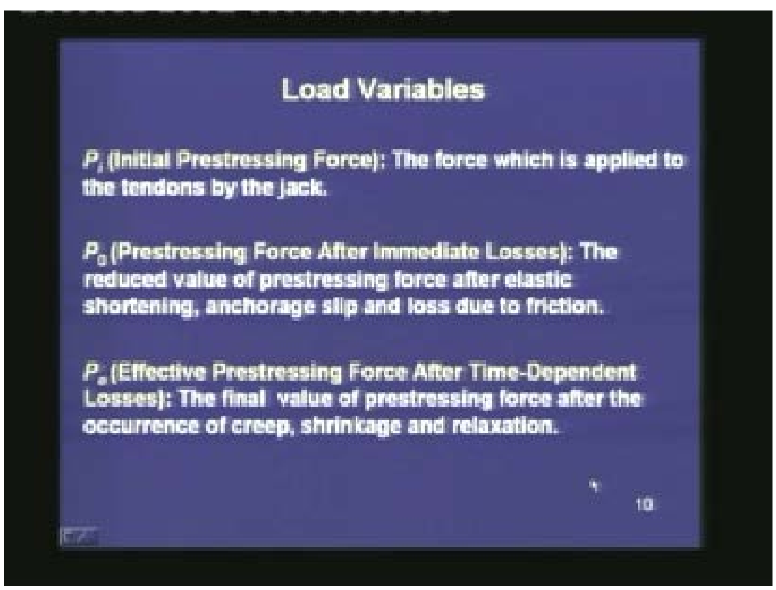

Among the load variables, P_i represents the initial prestressing force which is applied by the jack. This force is recorded by the gauge in the jack. The second one is P_0, which is the prestressing force after immediate losses. That means, the actual prestressing force that is transferred to the concrete section is lower than the value which is recorded by the gauge in the jack. It is lower because of the immediate losses due to elastic shortening, friction and seating of the anchorage, which we shall study subsequently. The third one is P_e, which is the effective prestressing force after the time dependent losses. As we have learnt earlier, the prestressing force drops with time and after several years, it gets stabilised to a final value. That value will be referred to as the effective prestressing force, and it will be denoted as P_e.

Next, we are moving on to the losses in prestress.

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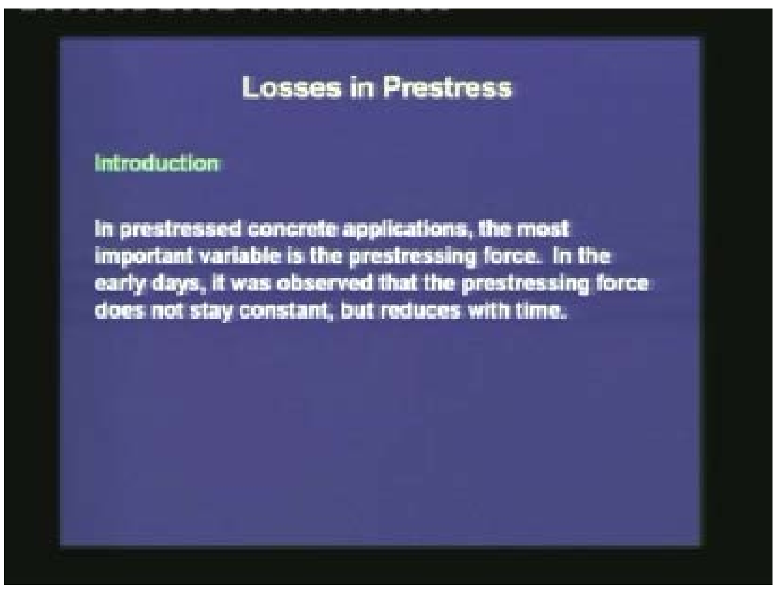

Unlike reinforced concrete member, the strength of a prestressed concrete member is not constant throughout its lifetime. Even if we neglect the deterioration, the prestressing force drops with time due to the time dependent losses. Hence, the calculation of the losses is very important in prestressed concrete applications. In the prestressed concrete applications, the most important variable is the prestressing force. In the earlier days, it was observed that the prestressing force does not stay constant, but it reduces with time.

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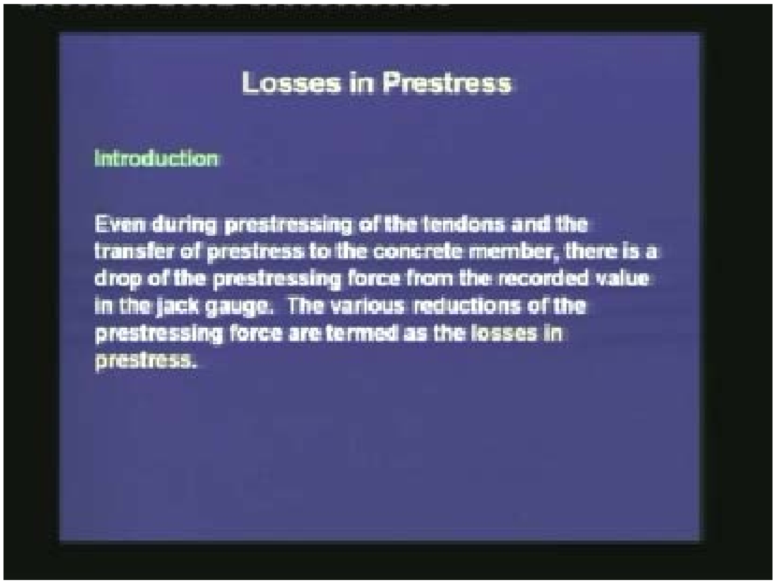

Even during prestressing of the tendons and the transfer of prestress to the concrete member, there is a drop of the prestressing force from the recorded value in the jack gauge. The various reductions of the prestressing force are termed as losses in prestress.

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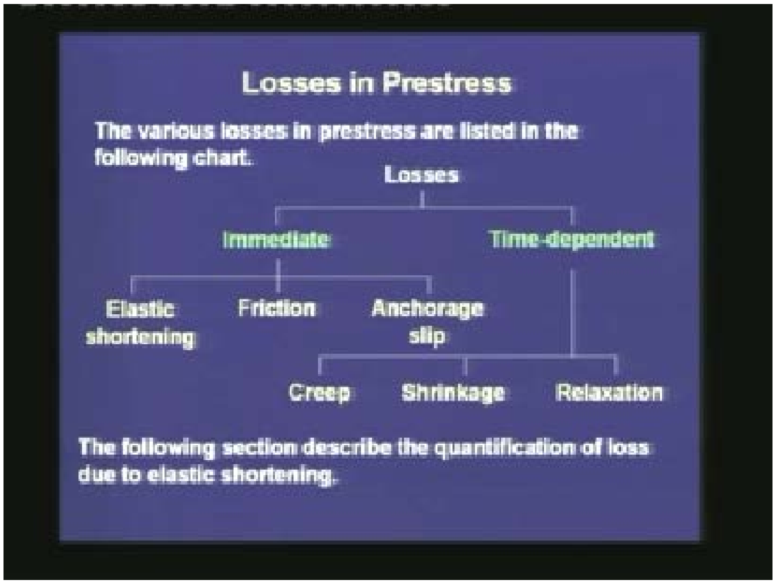

In this chart, we are trying to understand the various losses under different sections. The losses in prestress can broadly be classified under two groups: one is the immediate losses, which are shown on the left side. The other is the time dependent losses, which takes several years till the prestressing force gets stabilized. Out of immediate losses, the important one is the elastic shortening, which is the shortening of the concrete member when the prestressing force is transferred to it. It is an immediate shortening. Then, the second one is the friction. The friction is the drop in the prestressing force along the length of the prestressing tendon, because of the curvature in the prestressing tendon. The third one is the anchorage slip. After the jacks have pulled the tendons and the wedges are placed, the tendons are released. At that instant, the wedges and the anchorage block seat in the prestressed member. During the seating, there is some loss till the wedges get locked in the anchorage block. The loss due to this seating of the anchorage is called the 'anchorage slip'.

Among the time dependent losses, we have already studied the phenomena under the material properties. Creep and shrinkage are typical behaviour of concrete. Creep is the deformation with time under a constant load. Shrinkage is the deformation with time due to loss of moisture. Relaxation is a property of the prestressing steel, which is the drop in the stress under a constant strain, with time. In today's lecture, the following section will quantify the loss due to elastic shortening.

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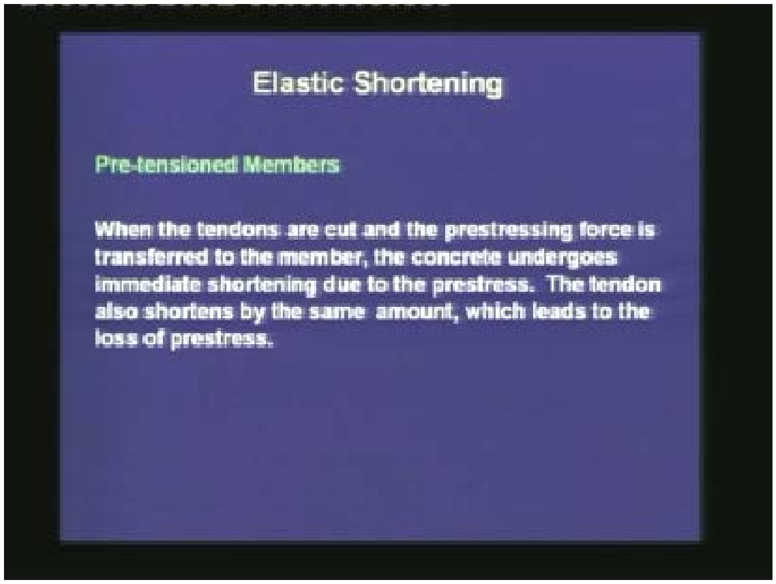

In pre-tensioned members, when the tendons are cut and the prestressing force is transferred to the member, the concrete undergoes immediate shortening due to the prestressing force. The tendons also shorten by the same amount, which leads to the loss of prestress. The elastic shortening is more of a concern in a pre-tensioned member.

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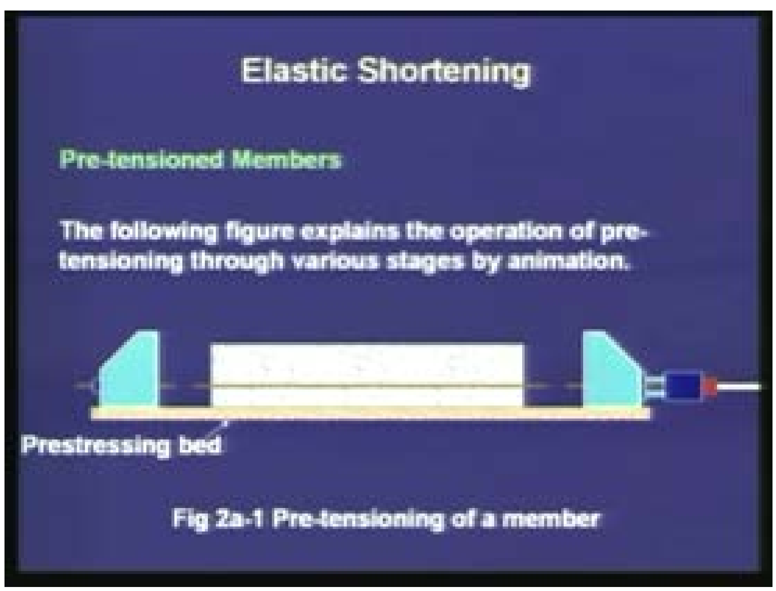

Let us try to understand this by the animation which we had seen before. This animation will clarify the phenomenon of elastic shortening during the transfer of prestress. At first in the prestressing bed, the tendon is anchored at the bulk heads. Next, we position the jack at one end and then we apply the tension in the tendon. After the shuttering is placed, the concrete is cast. It is cured and hardened to the desired strength, and then the tendon is cut. When the tendon is cut, note that the concrete will shrink from its original length to a reduced length. That shortening is called the 'elastic shortening'. Note this process carefully.

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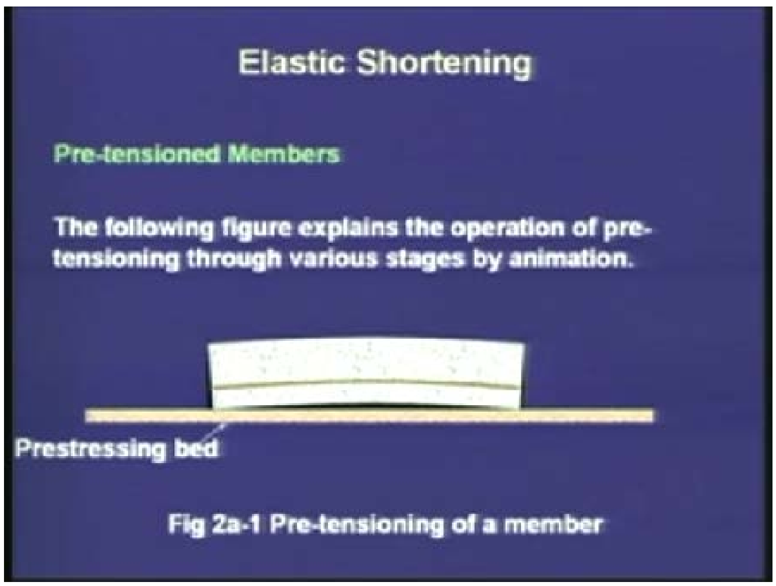

As the prestressing is being transferred, this concrete member is reducing in length. That reduction is called the 'elastic shortening'. If the tendon is placed eccentrically, then it will also deflect upwards which is called 'cambering', along with the elastic shortening. In a pre-tensioned member, the elastic shortening occurs when we are cutting the tendons at the end, and the prestressing force is getting transferred to the concrete. The system comes into an equilibrium after the elastic shortening.

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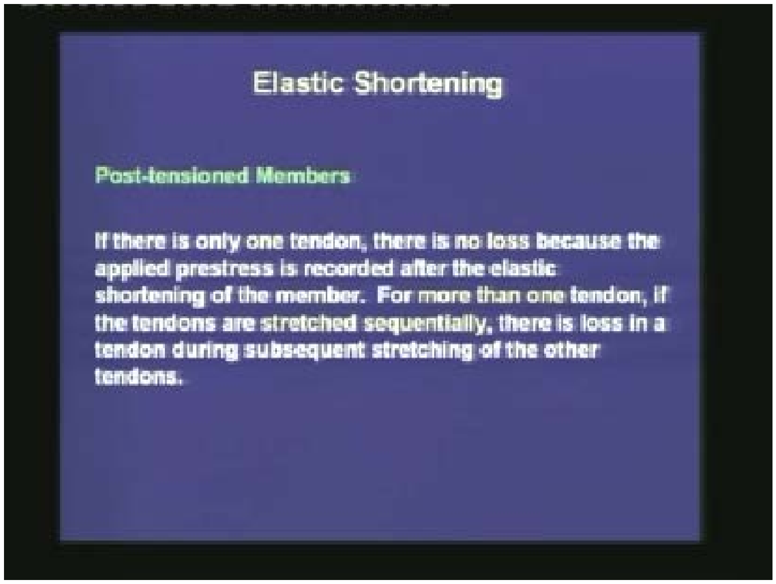

In post-tensioned members, the phenomenon of elastic shortening is different from that in pre-tensioned member. If there is only one tendon, then there is no loss because the applied prestress is recorded after the elastic shortening of the member. In post-tensioned members, as the jack gets the reaction from the concrete member itself, when the jack is applying tension in the tendon, the member is shortening. After the shortening stabilises, we are recording the final force. Hence, we do not consider the elastic shortening as a loss, because we are recording the prestressing force after the concrete member has shortened. This is true, if we have just one tendon for post-tensioning the member. But, if we are having more than one tendon, then when we are applying tension to a subsequent tendon, the tendons which we have tensioned earlier, they will be undergoing some elastic shortening. For more than one tendon, if the tendons are stretched sequentially, there is loss in a tendon during subsequent stretching of the other tendons.

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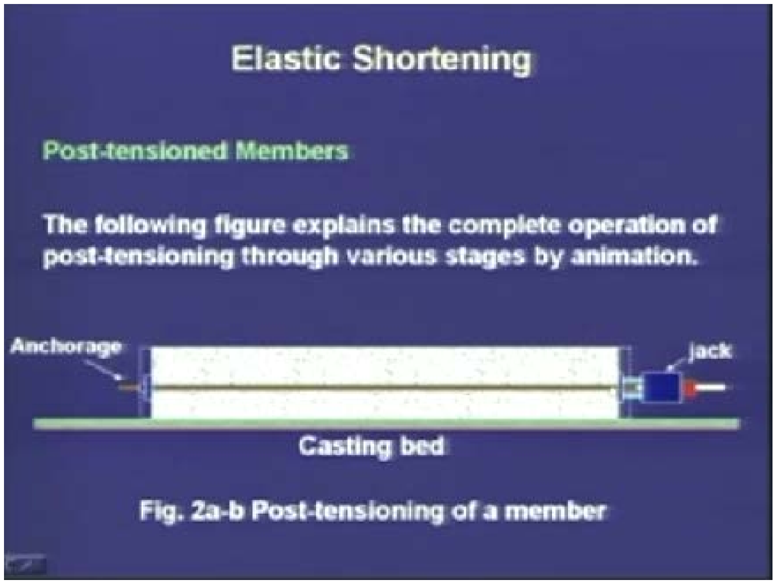

In this figure, we shall try to understand the process of elastic shortening in a post - tensioned member. In a post-tensioned member, first the concrete is cast on a casting bed.

There is a duct, which creates a hole in the concrete member. Through the duct, we pass the tendon and then we apply the anchorage at one end. We position the jack at the other end. When we are applying tension in the jack, notice that this member is undergoing elastic shortening.

Once this member has come to a stable length, we record the jacking force. Hence, whatever shortening has occurred will not get reflected in the value of the force that is recorded by the jack. Hence, if we are just having only one tendon, then there is no elastic shortening from the value of the force that is recorded by the jack. But, if we have more than one tendon, then the tendons which are tensioned earlier will have a loss during the tensioning of the subsequent tendons.

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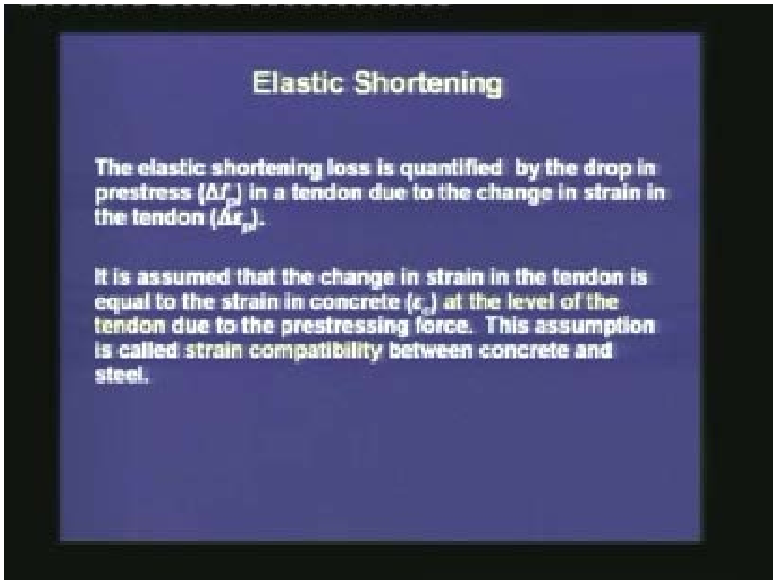

How do we calculate the loss in prestress due to elastic shortening? The elastic shortening loss is quantified by the drop in the prestressing force in a tendon, due to the change in strain in the tendon. It is obvious from Hooke's law, that if we have a change in strain, there will be a change in the stress. What we are trying to quantify is the change in the strain. It is assumed that the change in strain in the tendon is equal to the strain in concrete at the level of the tendon due to the prestressing force. This assumption is called strain compatibility between concrete and steel.

For a pre-tensioned member, there is bond between the prestressing tendon and the concrete. For a post-tensioned member, there is bond when we are grouting the post-tensioned members and hence, we can consider strain compatibility between the concrete and the steel. Whatever the strain the concrete undergoes at the level of the steel, the same strain is reflected in the prestressing tendon. The prestressing tendon also undergoes the same change in strain as equal to the strain in the concrete.

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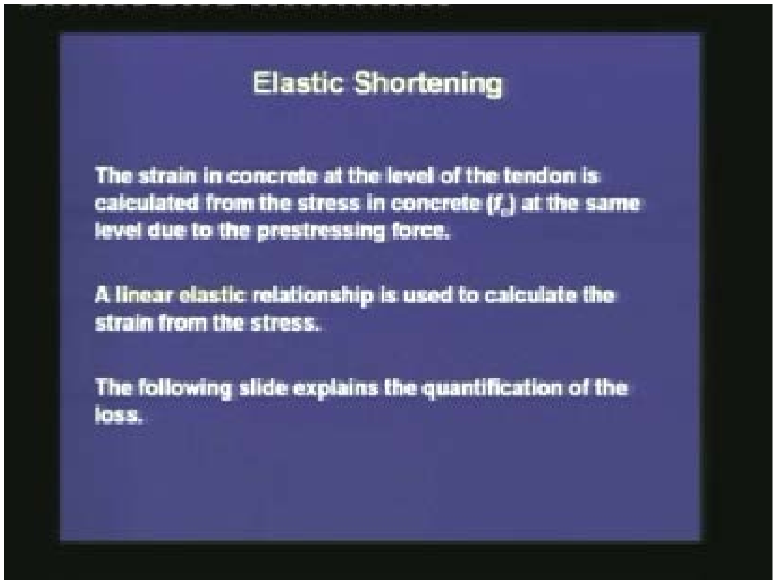

Our next step is how do we calculate the strain in the concrete? The calculations are based on the prestressing force, which we are recording. The strain is calculated from the stress that is generated in the concrete at the level of the prestressing steel, due to the prestressing force. A linear elastic relationship is used to calculate the strain from the stress. The following slide explains the quantification of the loss.

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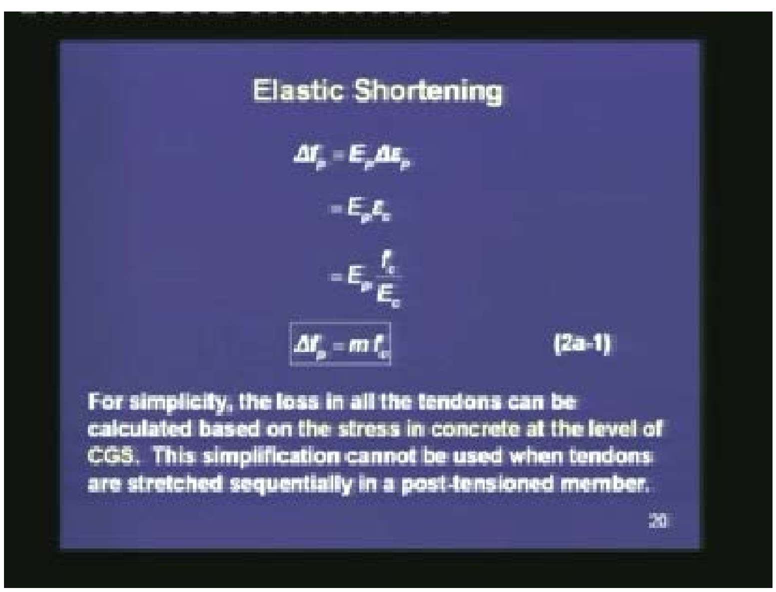

The drop in prestress is equal to the elastic modulus of the prestressing steel times the change in strain. The change in strain in the prestressing tendon is equal to the strain in the concrete. The strain in the concrete is equal to the stress in the concrete, divided by the elastic modulus of the concrete. The ratio of the two elastic moduli, that of prestressing steel divided by that of the concrete is denoted as the modular ratio 'm'. Hence, the basic equation to calculate the loss in prestress due to elastic shortening is equal to the modular ratio times the stress in the concrete at the level of the tendon.

As we know that there need not be a single tendon, there can be several tendons in a concrete member. For simplicity, the loss in all the tendons can be calculated based on the stress in concrete at the level of CGS. Here comes the utility of the CGS, that we are considering as if all the tendons are concentrated at that location. We are calculating the stress in the concrete at the level of the CGS. From there, we are calculating the loss in the prestress. This simplification cannot be used when the tendons are stretched sequentially in a post-tensioned member, because in a post-tensioned member, the calculation is more involved. It is a sequential process. Hence, we cannot club together all the tendons to be located at the CGS.

Let us first look into the pre-tensioned axial members.

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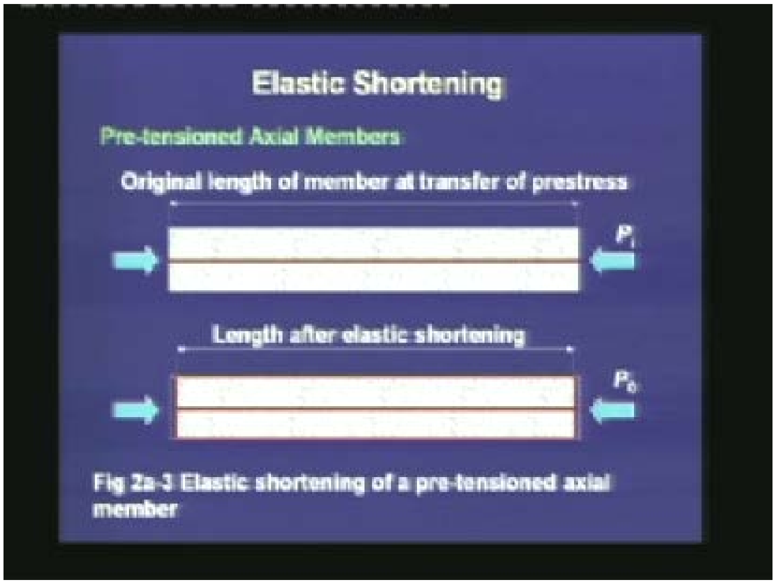

The axial member means that we are neglecting the eccentricity of the CGS, as if the member will just shorten elastically and there will be no effect of camber. In order to calculate the stress in the concrete at the level of CGS, we are trying to understand the static equilibrium. When the prestressing force is transferred, the concrete member has an original length. During the process of transfer of prestress, this member comes to equilibrium with the reduced length after elastic shortening, and the prestressing force also drops from the initial value to a value P_0.

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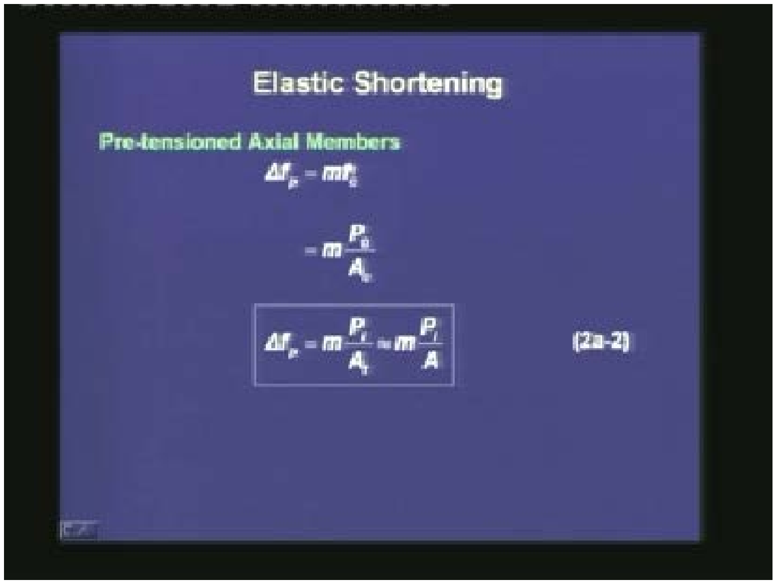

The stress in the concrete is given as (remember, this expression is at the static equilibrium) P_0 divided by the area of the concrete, which we are denoting as A_c. That means, Δf_p is equal to the modular ratio times P_0 divided by A_c. Here, there is a difficulty in using this expression. We do not know P_0 correctly, because the value that we record by the jack is P_i. It is easier, if we can transform this equation in terms of P_i. We shall see later that this expression can be again written as the modular ratio times $P_{i,}$ which is the force we are recording at the jack, divided by the transformed area. This can be simplified to a product of the modular ratio times P_i divided by the gross area. The difference between the gross area and the transformed area may not be significant, if the amount of steel is small.

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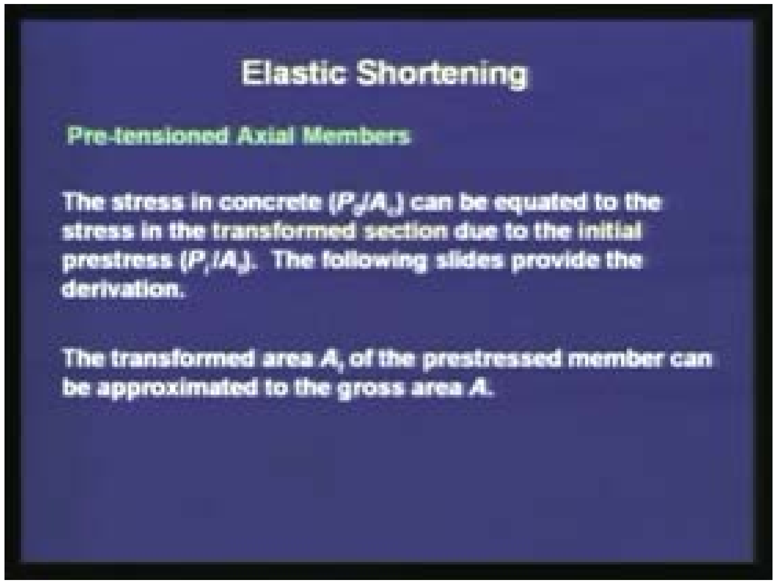

Just to summarize, the stress in the concrete which is P_0 divided by A_c can be equated to the stress in the transformed section due to the initial prestress. The following slides will provide this derivation. We have also seen that the transformed area A_t can be approximated to the gross area A, for computational simplicity.

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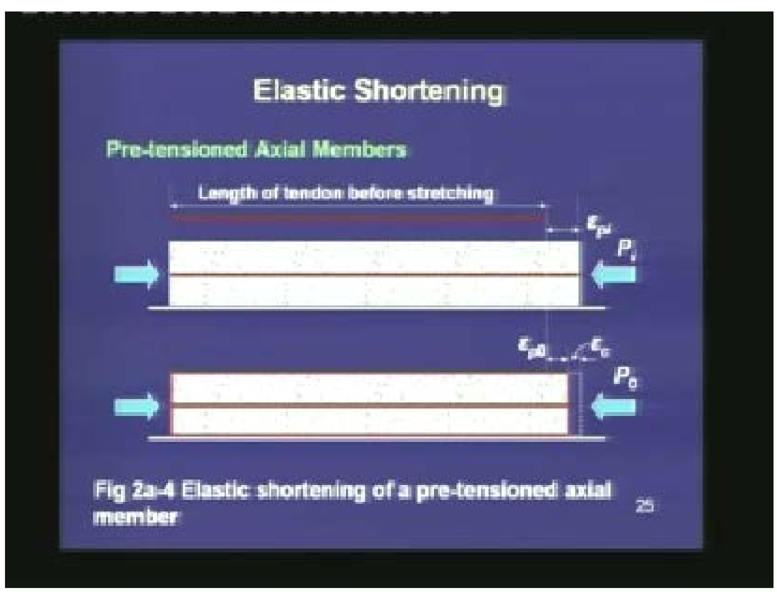

We are trying to understand the change in strain due to the elastic shortening. When we are stretching the steel, there is an initial strain in the prestressing strand. This strain is calculated based on the original length of the tendon before stretching, and is denoted as ε_{pi}. Next, after elastic shortening, the concrete undergoes a strain of ε_c. The final strain in the prestressing steel is denoted as ε_{p0}, which is calculated from the final length of the concrete member and the original length of the prestressing tendon.

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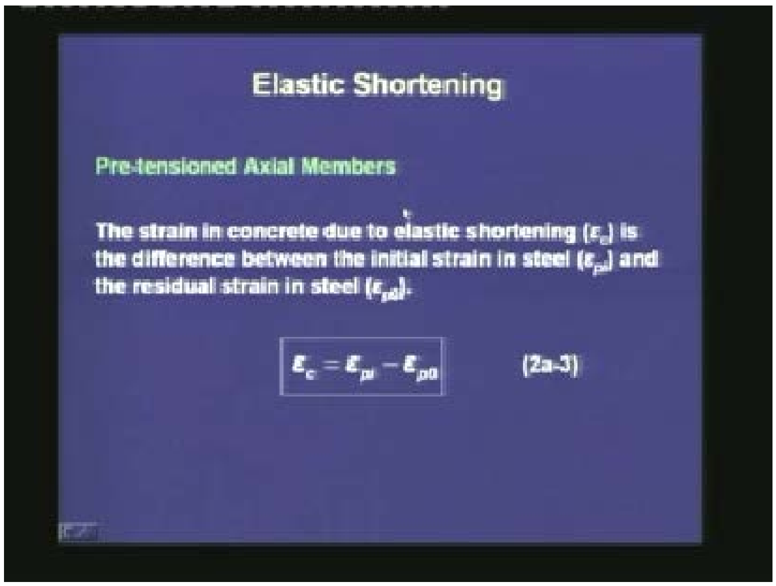

The strain in concrete due to elastic shortening ε_c is the difference between the initial strain in the steel, which is ε_{pi} and the residual strain in the steel, which is ε_{p0}. Hence, ε_c is equal to ε_{pi} minus ε_0.This is a strain compatibility relationship.

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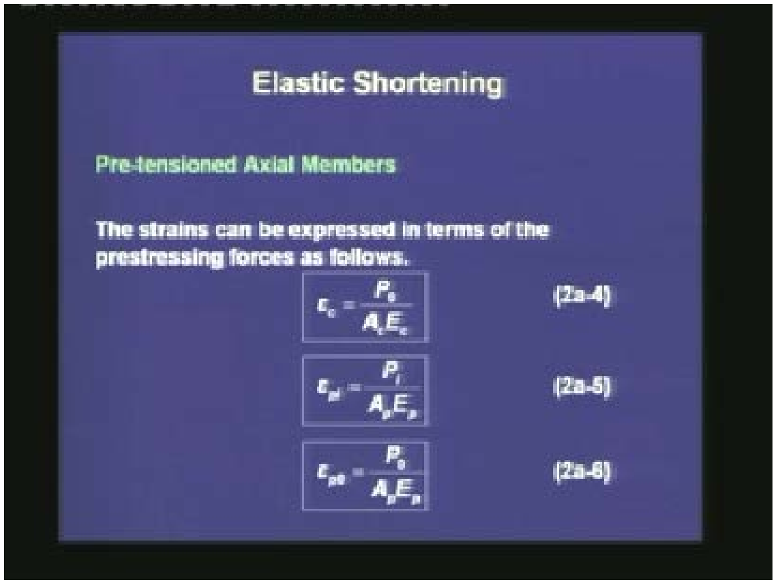

Each of these strains can be equated to the corresponding forces, by the elastic equations: ε_c is given by the force which is occurring after the member has come to static equilibrium, divided by the area of the concrete and the modulus of the concrete. ε_{pi} is given by the initial prestressing force which is applied to the jack, divided by the area of the prestressing steel and the modulus of the prestressing steel. ε_{p0}, which is the residual strain in the prestressing steel, is equal to the residual prestressing force P_0 divided by the area of the prestressing steel and the modulus of the prestressing steel.

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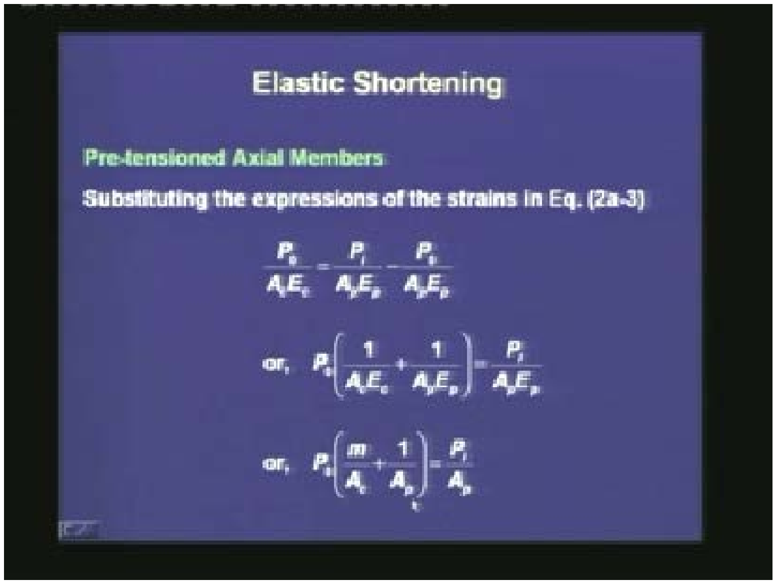

Substituting these expressions in the expression of the strains, $\varepsilon_c = \varepsilon_{pi} - \varepsilon_{p0}$, we get the top equation. We transpose the terms, that is, we are bringing the terms with P_0 on one side and the terms with P_i on the other side, and we are also substituting the ratio of E_p and E_c by the modular ratio.

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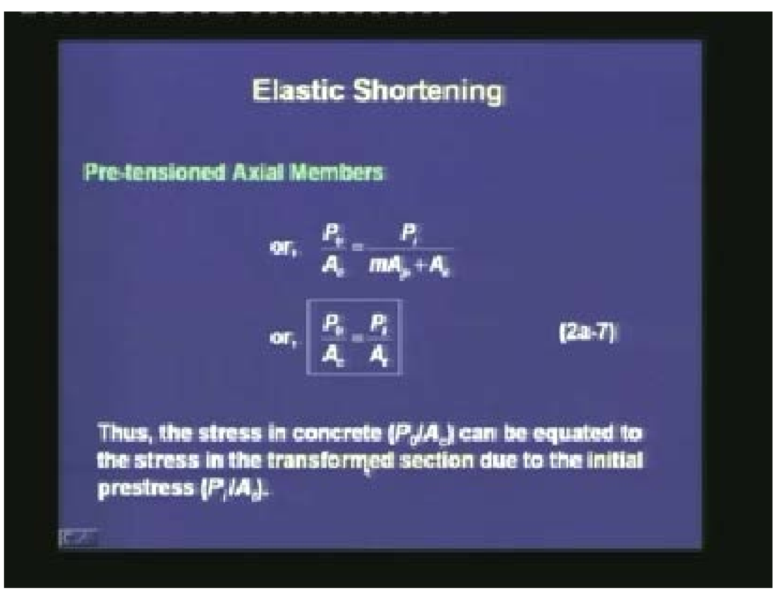

Once we do this transposition and substitution, we come to the expression which gives the stress in the concrete (originally calculated based on the stress at transfer P_0 divided by A_c) equal to the initial prestress that we record by the jack gauges, divided by the transformed area. This equation is helpful because, we do not need P_0 any more, and we are able to calculate using P_i. Hence, we can substitute P_i / A_t in place of P_0 / A_c.

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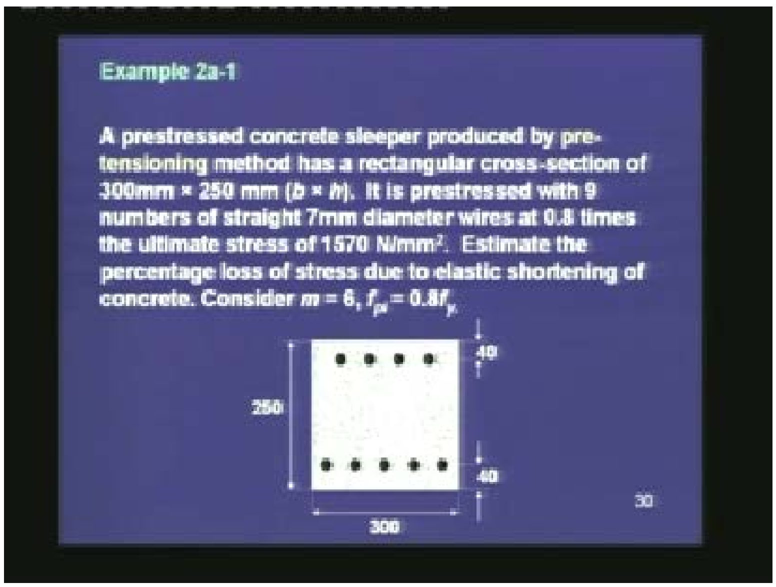

Let us use this expression to solve a problem. A prestressed concrete sleeper produced by the pre-tensioning method has a rectangular cross-section of 300 mm × 250 mm. It is prestressed with 9 numbers of straight 7 mm diameter wires, at 80% of the ultimate strength (f_{pk}) of 1570 N/mm^2. We have to estimate the percentage loss of stress due to elastic shortening of concrete. Consider the modular ratio (m) to be 6. We have already seen that f_{pi} is equal to 0.8 times f_{pk}.

Here, the prestressing tendons are divided into two groups: 4 tendons are lying at the top and 5 tendons are lying at the bottom. Since, the tendons are divided into two groups, one above the CGC and the other group lying below the CGC, we are calculating the stress in the concrete at the two levels separately.

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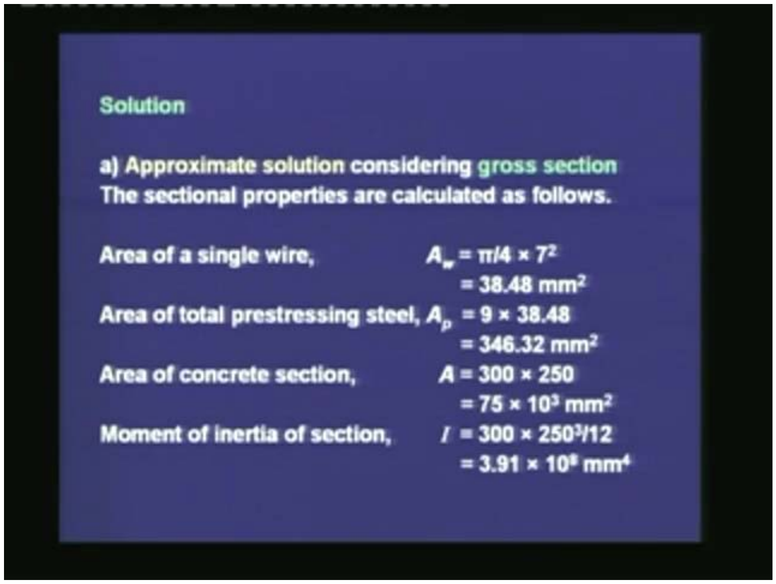

The first procedure will be an approximate solution based on the gross section. We are calculating the area of a single wire, which is π/4 times the diameter square. Then, the total area of the prestressing steel is given as 9 times the area of a single wire. The area of the gross section (A) is $300 \times 250 = 75 \times 10^3\ mm^2$. The moment of inertia of the gross section (I) is equal to $300 \times 250^3/12 = 3.91 \times 10^8\ mm^4$.

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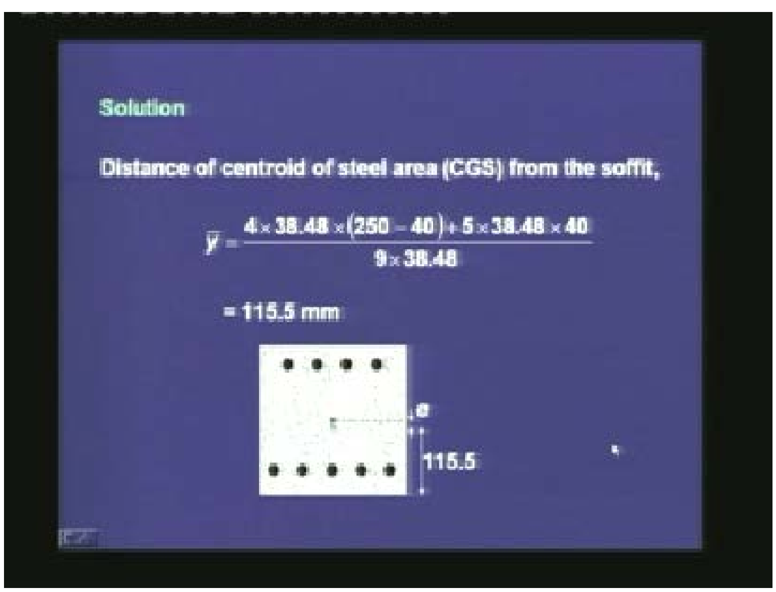

Once we know these geometric properties, we can calculate the location of the centroid of the steel. This expression comes from the concept of the first moment of area. That is, we are calculating the location of the CGS, by calculating the location of each of the tendons. Once we get the location of the CGS, we can calculate the eccentricity, which is the distance between the CGS and the CGC. Here, for the gross rectangular cross-section, the CGC is at the centre. What we can see is that, the CGS is located 115 mm from the bottom. Since there are 4 tendons at the top and 5 tendons at the bottom, the CGS is shifted a little bit down from the CGC.

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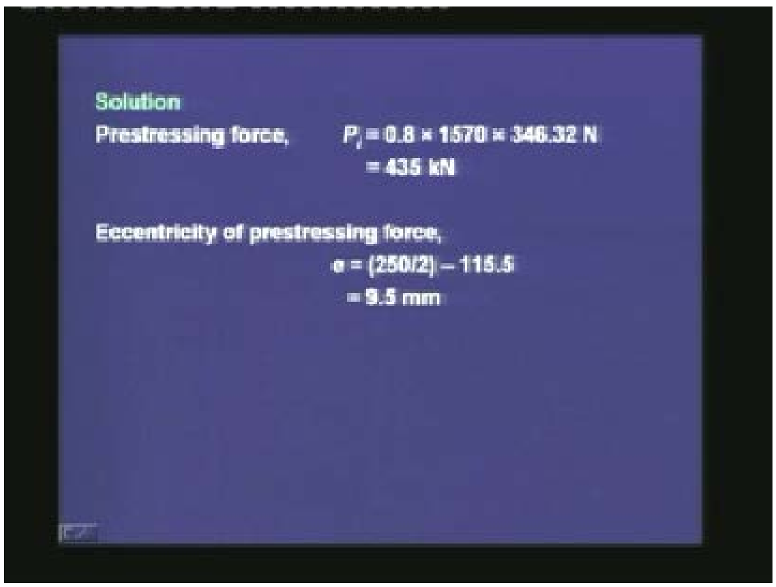

The prestressing force is 80% of the ultimate strength times the area of the prestressing tendons. Thus, the total prestressing force applied is 435 kN. The eccentricity of the prestressing force is calculated from the location of the CGC, which is half of the total height, minus the location of the CGS. The value of eccentricity (e) is 9.5 mm.

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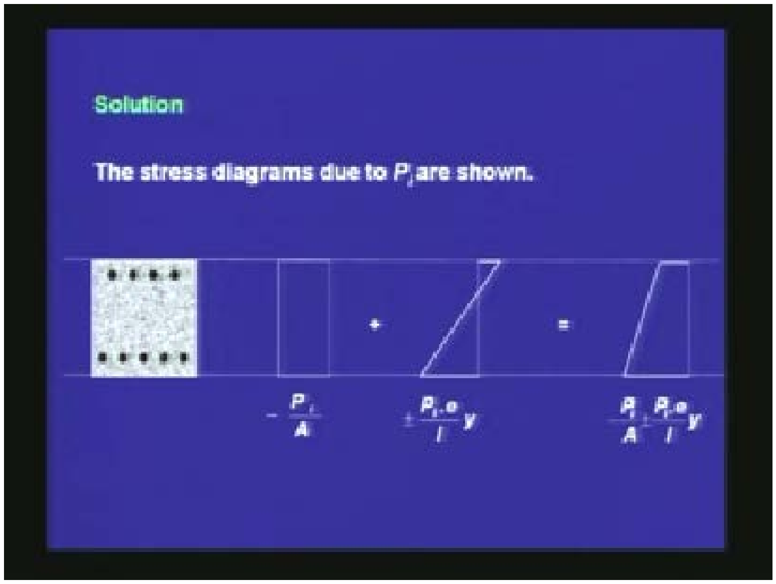

Under the prestressing force, the stress diagrams are as follows: there is a uniform compression if we consider the prestressing force acting at the CGC, plus there is a varying force because of the eccentricity of the CGS from the CGC. The constant value is given as P_i /A. The varying stress is given as the moment due to the eccentricity of the CGS (which is $P_i \times e$) times the distance from the CGC, divided by the moment of inertia. Once we add these two terms, we get the final stress profile due to P_i over the depth of the cross-section. We can see that the compression at the bottom is higher than the compression at the top, because the CGS is lower than the CGC.

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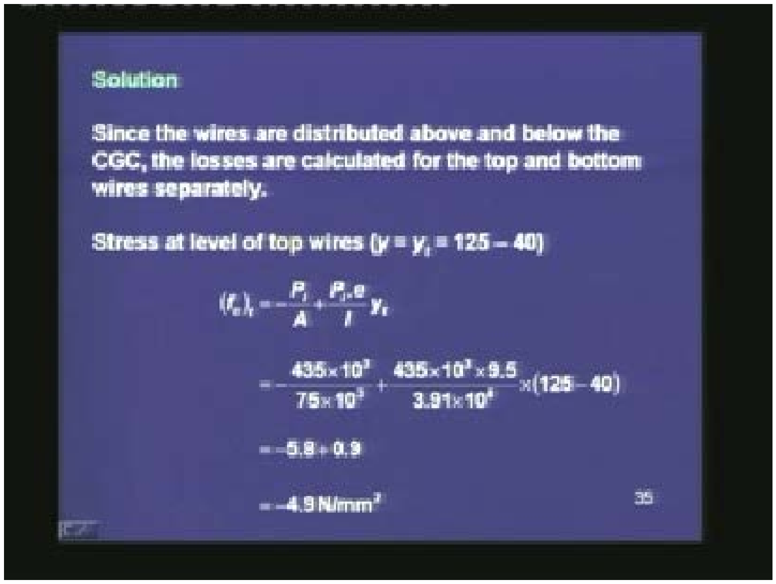

Our next task in hand is to calculate the stresses in the concrete at the two levels of steel. We are calculating the stresses based on the stress profile that we have seen before. For the top wires, the distance is given by half the depth of the section minus the effective cover. Once, we substitute that in the expression of the stress, we get the stress at the level of the top steel. For calculating the stress, the first term is the uniform stress, and the second term is the varying stress which comes due to the eccentricity of the prestressing force. The stress at the level of the top wires is -4.9 N/mm^2. We are using a negative value for a compressive stress.

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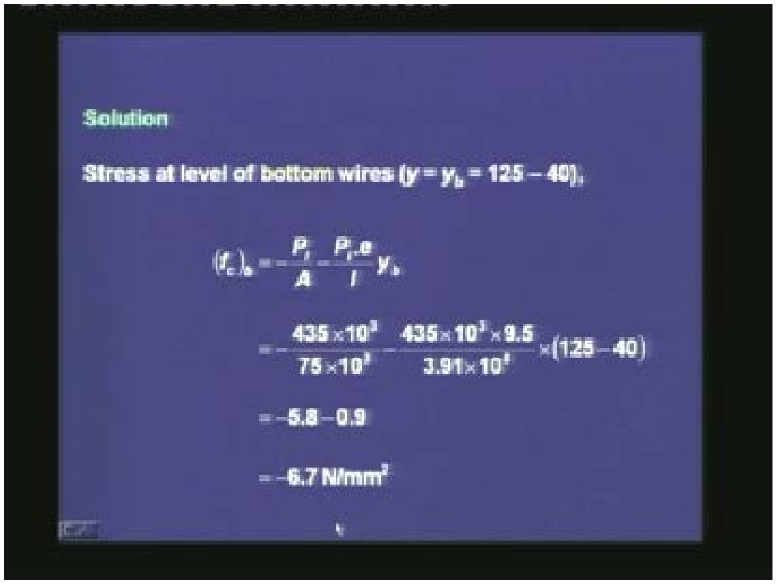

Similarly, the stress at a level of the bottom wires is given by substituting the distance of the bottom wires from the CGC. The distance is same as that of the top wires, which is half the depth minus the effective cover. When we substitute the value of y in the expression of stress, we get the stress at the level of the bottom steel which is −6.7 N/mm^2. We see that the stress at the level of the bottom steel is higher than the stress at the level of the top steel.

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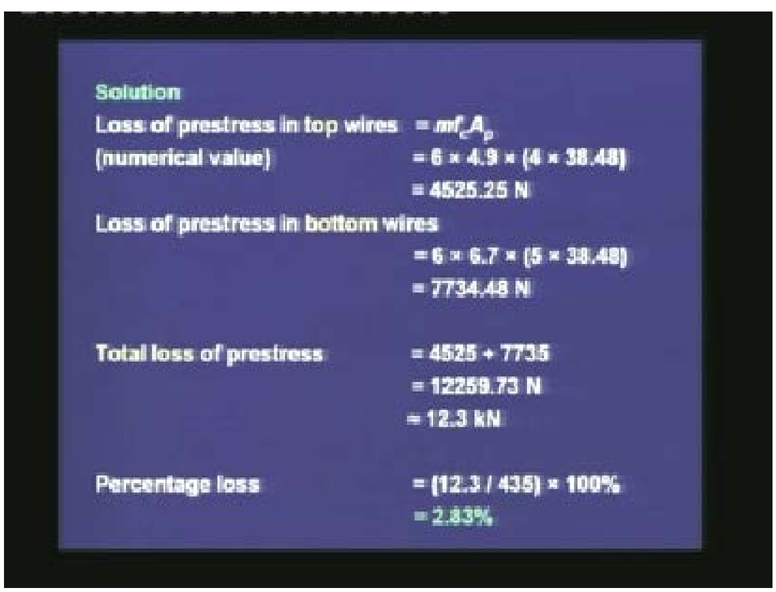

Once we have calculated the stresses in concrete at the level of the steel, we are calculating the losses in the tendons at each level. The loss of prestress in the top wires is equal to the modular ratio times the stress in the concrete, times the area of the prestressing steel at the top. We are multiplying the drop in the prestress times the area of the prestressing steel, to get the total loss in the prestressing force. Similarly, we can calculate the loss of prestress in the bottom wires, which is the modular ratio times the stress in concrete, times the area of the prestressing steel at the bottom. The total loss of prestress is summation of the two terms, that is 12.3 kN. Our original prestressing force was 435 kN. From that, we have lost 12.3 kN due to the elastic shortening. The percentage loss due to elastic shortening is given as 12.3 divided by 435 times 100, which is 2.83%. We have lost 2.83% of the prestressing force due to elastic shortening.

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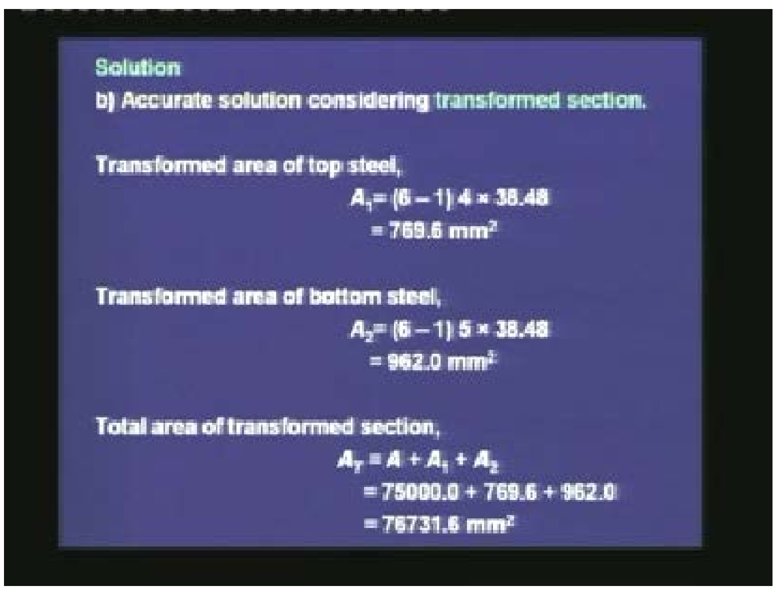

The same problem can be done by the more accurate method, using the transformed section. The transformed area for the top steel is given as the modular ratio minus one, times the area of the prestressing steel. Similarly, we can calculate the transformed area for the bottom steel. Then, the total area of the transformed section is equal to the area of the gross section plus the equivalent areas corresponding to the two levels of prestressing steel. Note that, the transformed area is larger than the original area of 75,000 mm^2.

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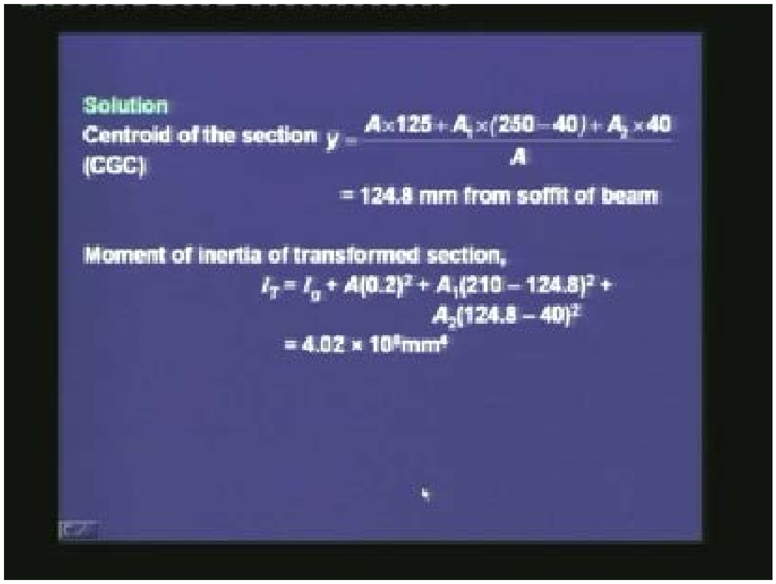

We have to calculate the centroid of the section. This time, it is not at the mid-depth of the section, since we are considering a transformed section, and the steel is not symmetric about the horizontal axis. The centroid of the section, the CGC itself is shifted from the previous location. We can see it is 124.8 mm from the bottom of the beam, which is slightly different from 125 mm which we had seen earlier. The moment of inertia of the transformed section can be found out by using the principles of the parallel axes theorem. First, we are calculating the moment of inertia of the gross section. Then we are adding to that the terms, which are the products of the areas times the shifts in their centroids from the CGC of the transformed section. Once we use the parallel axes theorem, we get the moment of inertia of the transformed section. Note that, this is also slightly different from the earlier moment of inertia that we had calculated for the gross section.

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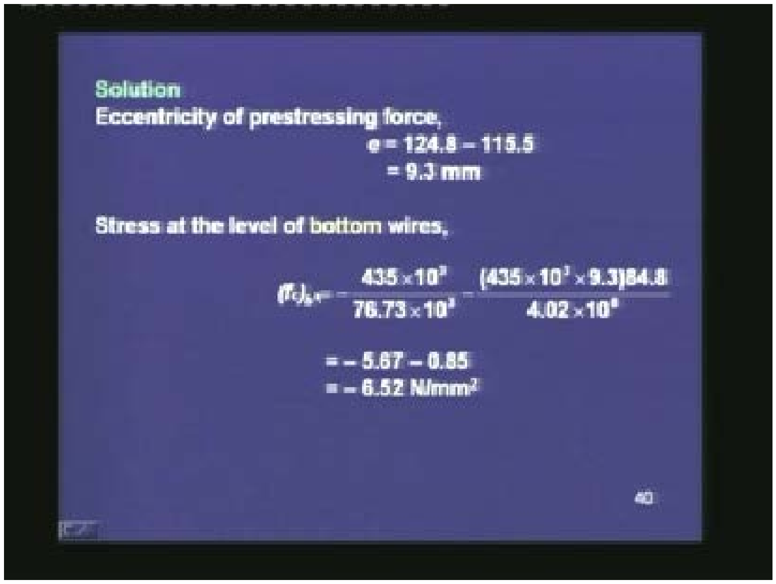

The eccentricity of the prestressing force is the difference between the locations of the new CGC and the CGS. The calculation of CGS that we had done earlier remains same for this method of solution also. The difference between the CGC and the CGS gives the eccentricity (e) which is 9.3 mm. Once we have got these accurate values, we are calculating the stress at the bottom. It is the same expression, but with more accurate values of the area, the moment of inertia and the eccentricity. With the accurate values, we find that the stress at the bottom is −6.52 N/mm^2.

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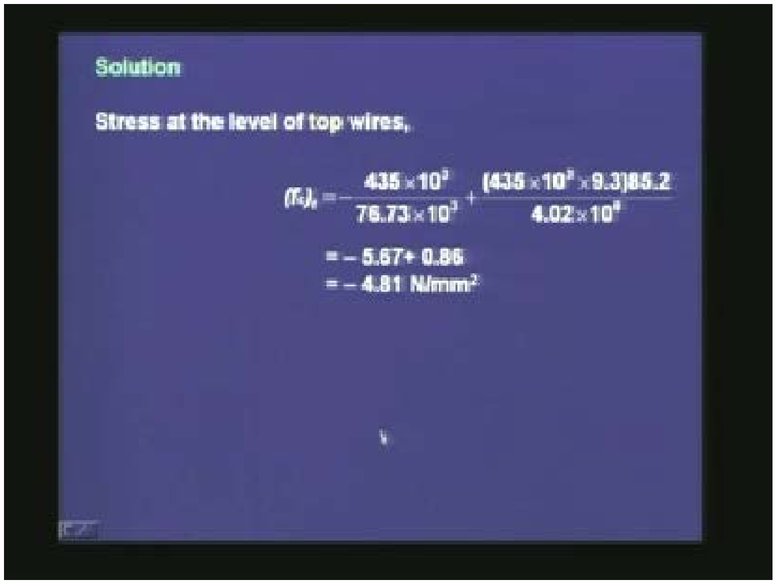

The stress at the top is −4.81 N/mm^2. Both the values of stress are slightly different from the values that we had seen earlier. From the stresses in the concrete, we can calculate the loss of prestress in the top wire, which is the modular ratio times the stress in the concrete, times the area of the prestressing steel at the top. Similarly, we are calculating the loss of prestress at bottom wires. When we add these two, we get the total loss of 12 kN.

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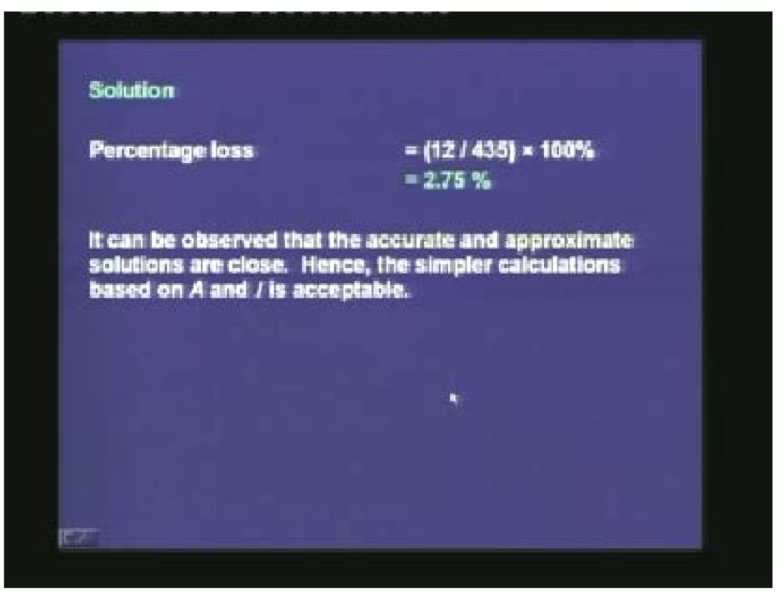

The percentage loss is 12 divided by 435 times 100, which is 2.75%. It can be observed that the accurate and approximate solutions are very close. Hence, the simpler calculations which are based on the area and the moment of inertia of the gross section, are acceptable. The additional accuracy that we get by using the area and the moment of inertia of the transformed section may not be warranted, because it leads to more computation. The solution that we get based on the gross section is sufficiently good.

We are quickly reviewing the calculation of the loss for the other types of members.

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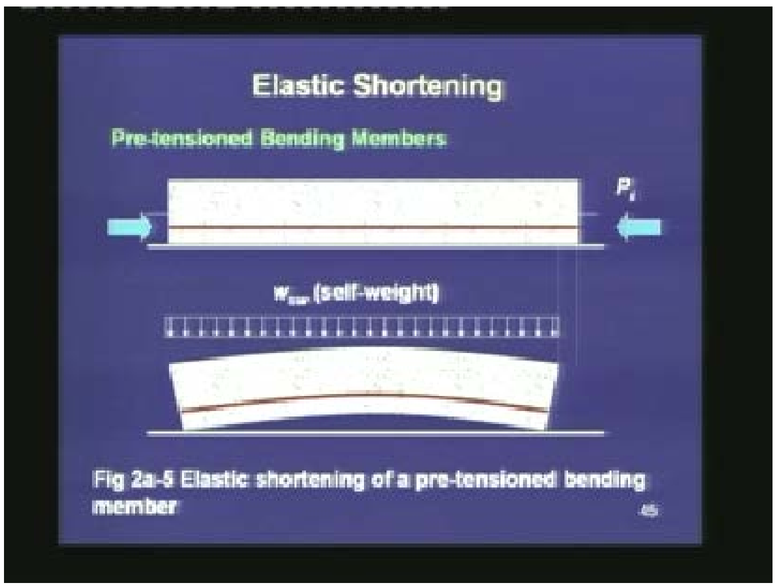

A pre-tensioned bending member is subjected to camber. We also have to consider it's self-weight in the calculations. At transfer, when it comes under a static equilibrium, it is not resting on the prestressing bed throughout its length, but it is resting only at the two ends. Hence, it is subjected to bending along with axial compression. Thus, for a bending member, we need to have additional terms for the stress, which consider the effect of bending.

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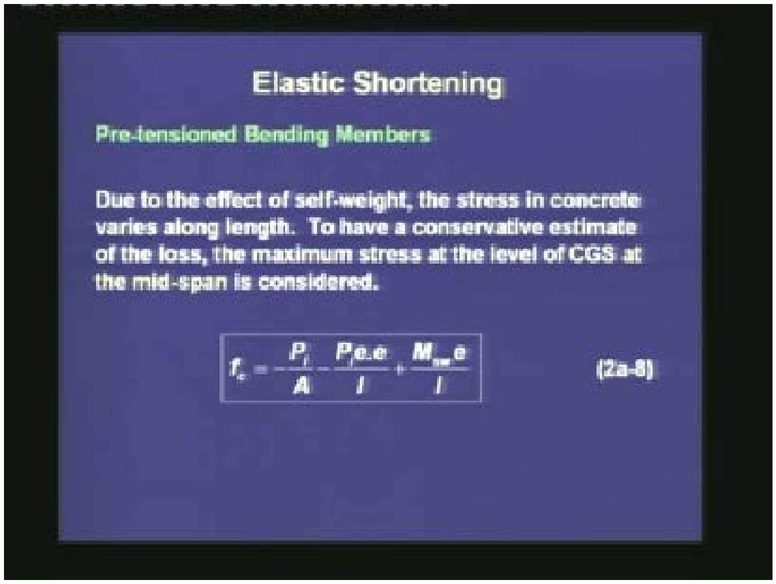

Due to the effect of self-weight, the stress in concrete varies along the length. To have a conservative estimate of the loss, the maximum stress at the level of CGS at the mid-span is considered. For a bending member, since the stress in the concrete at the level of CGS varies along the length of the member, the elastic shortening also varies. To simplify the calculation, we consider the stress at the mid-span as the representative stress for the concrete at the level of the CGS.

The expression of the stress in the concrete at the mid-span, at the level of the CGS consists of the following. The first term is the constant stress. The second term is the varying stress, due to the eccentricity of the prestressing force. Since we are calculating the stress at the level of the CGS, the distance of investigation is same as the eccentricity. Then, there is a third term, which is due to the self-weight of the member. Since the member is hogging up, the self-weight is also creating stress.

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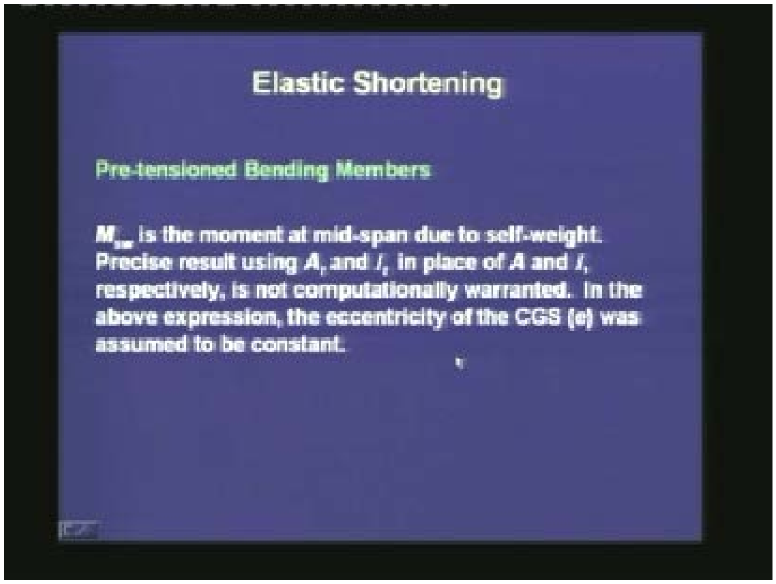

In the earlier expression, M_{sw} is the moment at mid-span due to the self-weight. The precise result using A_t and I_t of the transformed section, in place of A and I of the gross section, is not computationally warranted. In the above example, the eccentricity of the CGS was assumed to be constant throughout the length of the member.

Next, we move on to the post-tensioned axial members.

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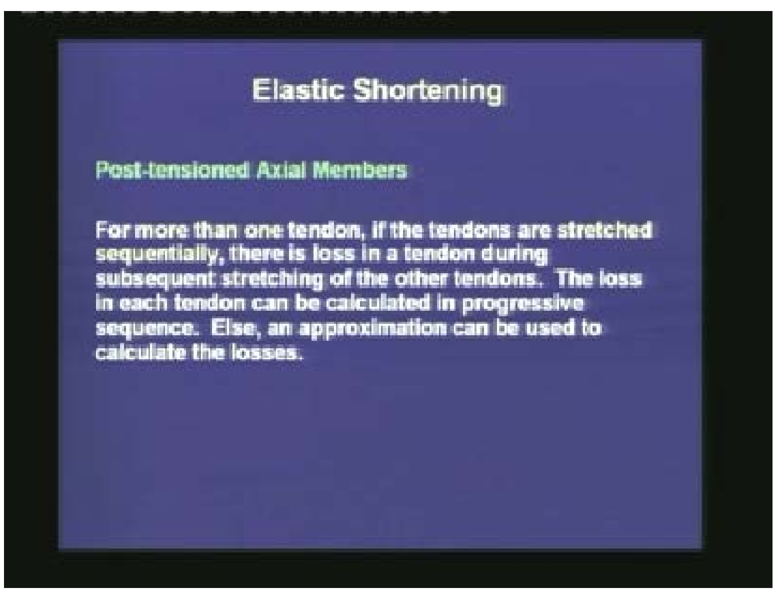

To recollect, for more than one tendon, if the tendons are stretched sequentially, there is a loss in a tendon during subsequent stretching of the other tendons. The loss in each tendon can be calculated in progressive sequence or else, an approximation can be used to calculate the losses. The elastic shortening is of concern in a post-tensioned member if there is more than one tendon, and the tendons are stretched sequentially. Since the losses in each tendon due to the stretching of the subsequent tendons vary, the calculation is more involved than a pre-tensioned member. It needs a sequential calculation for each tendon separately. We can approximate this calculation by considering an average elastic shortening for all the tendons.

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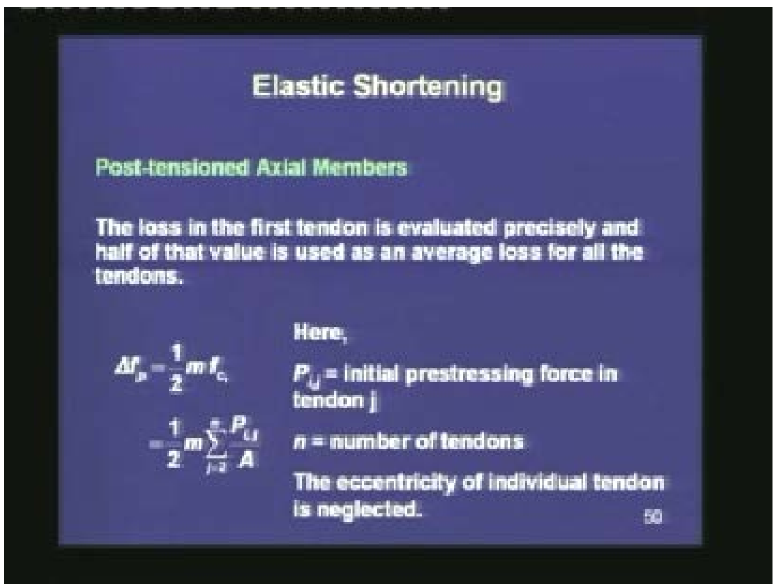

The loss in the first tendon is evaluated precisely and then, half of that value is used as an average loss for all the tendons. That means, we are calculating the stress in the concrete at the level of the first tendon, and multiplying that by the modular ratio. This is the loss in the first tendon. In our approximate calculation, what we are assuming is that, the average loss in the prestress of all the tendons is half of the loss of the tendon which has been stretched first. The expression can be written in terms of the initial prestressing force that is applied in the jack. Then, we are summing this up for all the tendons that are being stretched. To summarize, Δf_p is equal to half times the modular ratio, times the summation of the prestressing force in each tendon, divided by the area of the cross-section. The summation is done from 2 to the total number of tendons (n). The summation starts from 2, to consider the tendons stretched subsequent to the first tendon.

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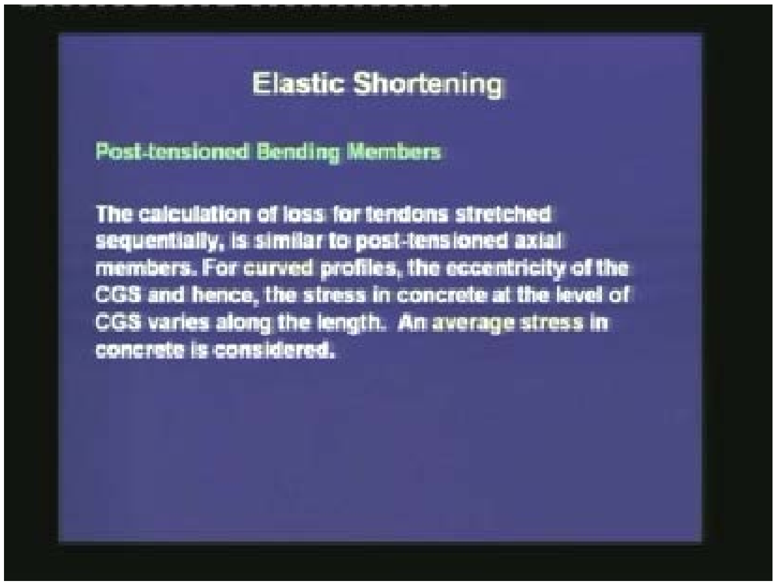

For a post-tensioned bending member, the procedure is similar. The calculation of loss for tendons stretched sequentially is similar to that of the post-tensioned axial members. For curved profiles, the eccentricity of the CGS and hence, the stress in concrete at the level of CGS varies along the length. This is another complication for a post-tensioned member, since usually the eccentricity varies along the length of the member. Most of the time, the profiles of the tendons are parabolic, that means the eccentricity at the mid-span is different from the rest of the length. To consider an average elastic shortening over the full length of the member, an average stress in the concrete can be considered.

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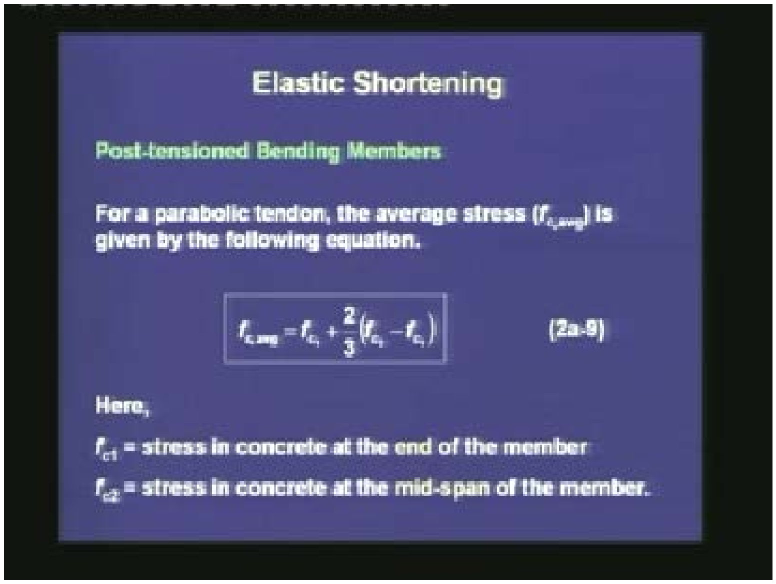

The above expression is specifically for a parabolic tendon. The average stress is given as the stress at the end of the member, which has a lower value, plus two third of the difference of the stresses at the mid-span and at the end. Thus, if the stress varies in a parabolic fashion from f_{c1} at the end to f_{c2} at the mid-span, the average stress in the concrete can be considered to be f_{c1} plus two third times the difference of f_{c2} and f_{c1}. This expression can be derived from the equation of the parabolic variation of the stress in the concrete at the level of the CGS.

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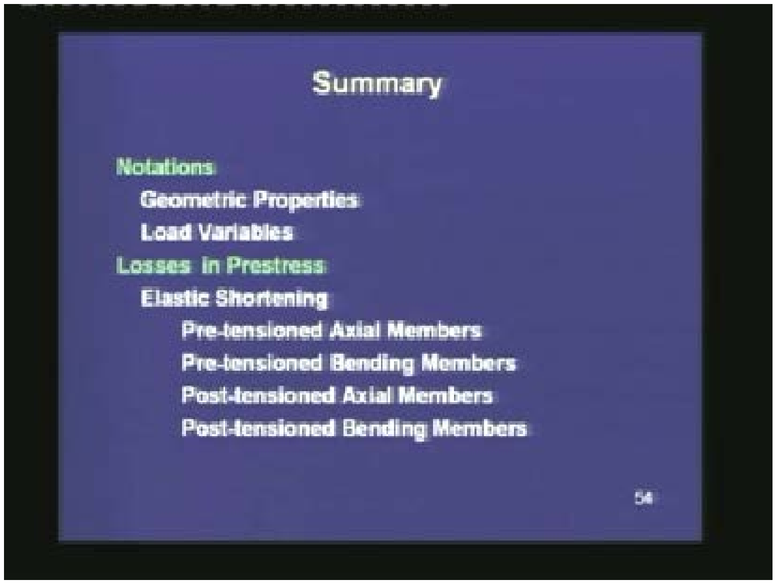

First, we studied the notations that we are using in our equations. There were two sets of notations. One set was the geometric properties: there we have learnt the notations of A, the area of the gross section, A_c the area of the concrete, A_p the area of the prestressing steel and A_t the area of the transformed section. We learnt, I the moment of inertia of the gross section, I_t the moment of inertia of the transformed section, and e the eccentricity, which is the distance between the CGS and the CGC. The value of e is considered to be positive, when the CGS is below the CGC. The CGC is the centroid of the gross section and CGS is the centroid of the tendons. We have to be aware that either of CGC or CGS may lie outside the concrete section itself. We have shown that for a hollow box girder section.

Next, we have learnt the notations for the force variables: P_i, which is the initial force as recorded by the gauges in the jack, P_0 is the force which drops from P_i due to elastic shortening for a pre-tensioned member, and P_e is the effective prestress, which occurs after several years after the long-term losses of the prestressing force. Next, we moved into the losses of prestress, we have seen that the losses can be grouped into two divisions: the immediate losses, under which we have elastic shortening, friction and anchorage slip, the second group is the time dependent losses, which generates due to creep, shrinkage and relaxation.

In today's lecture, we covered the first immediate loss, which is due to the elastic shortening. We saw that this is of a concern for pre-tensioned members. We saw the example of a pre-tensioned axial member in details. The calculations of a problem showed that, if we use the properties of a gross section, the values are close enough compared to the value which is derived by considering a transformed section. Hence, in our subsequent calculations we shall use the gross section instead of the transformed section for a member. Then we saw the expression of loss for a pre-tensioned bending member where, the eccentricity of the prestressing force and the stress created due to the self-weight of the member should also be included.

For a post-tensioned member, we have seen that if there is a single tendon then we need not have to consider any loss due to elastic shortening, because the force in the jack is recorded after the elastic shortening has occurred. But, if there is more than one tendon, and if the tendons are stretched sequentially, then there will be losses in the tendons which are stretched earlier and the calculation of the loss is done sequentially for each tendon.

The process can be simplified, if we consider an average loss for all the tendons, which are stretched sequentially. The average loss is given in terms of the loss in the tendon which is tensioned first, and the average loss is considered to be half that is seen in the first tendon. In calculating the stress of the concrete at the level of the CGS, we do the summation for all the tendons, except the first one. Hence, the summation is from 2 to the total number of tendons. Like that, we get an approximate expression of the loss due to elastic shortening in post-tensioned members, where the tendons are stretched sequentially. In a post-tensioned member, if the tendons have a parabolic profile, then the computation becomes a bit more involved. The eccentricity varies along the length of the member and hence, the stress also varies along the length of the member. For a parabolic profile, we have seen a simplified expression for an average value of the stress in the concrete. It is an average between the stress at the end and the stress at the mid-span. From the average value of the stress in the concrete at the level of CGS, we can calculate an average loss in prestress in a post-tensioned bending member.

To summarize, elastic shortening needs to be calculated for a pre-tensioned member because the pre-tension force drops from the value applied by the jacks. It needs to be calculated for a post-tensioned member, when there is more than one tendon and the tendons are stretched sequentially. In our next lecture, we shall move on to the other two types of immediate losses, which are due to the friction and the anchorage slip.

Thank you.