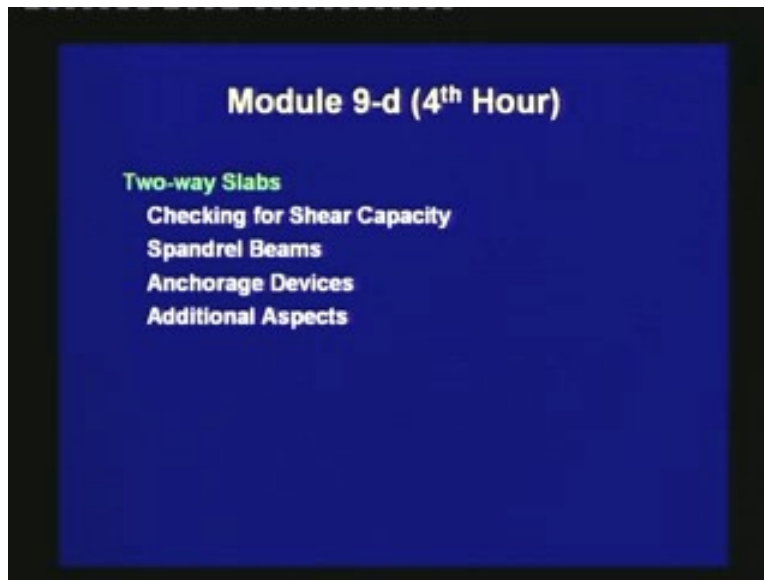


Prestressed Concrete Structures
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Lecture – 38
Two-way Slabs (Part 2)

Welcome back to prestressed concrete structures. This is the fourth lecture in the Module 9 on special topics. Today, we are continuing with the topic on the two-way slabs. In today's lecture, we shall cover the checking for shear capacity then we shall study the design of spandrel beams.

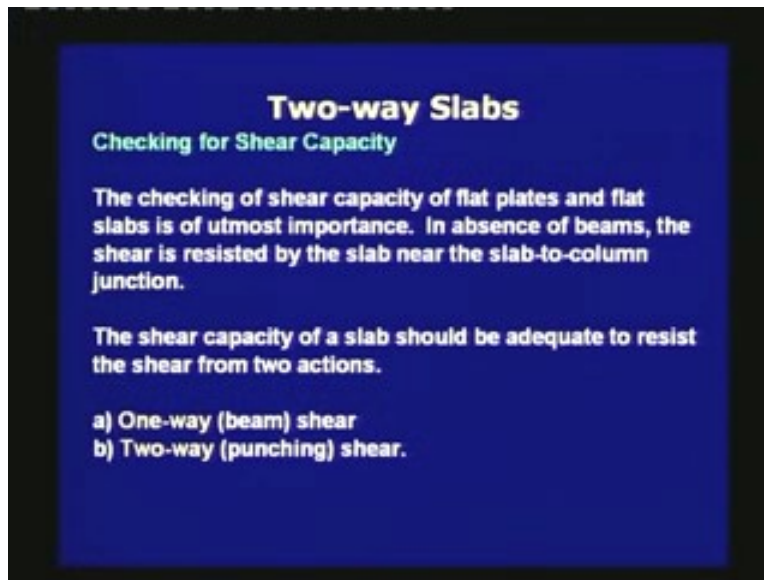
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After that we shall move on to the anchorage devices and finally we shall mention some additional aspects for the design of two-way slabs. Last time, I talked about the analysis and design for the flexural capacity and what we have learnt is that the flexural analysis is done by considering equivalent frames in each orthogonal direction. The equivalent frame is analyzed under the gravity and lateral loads and from that we calculate the moments at the critical sections. once we know the moments at the critical sections we

distribute the column strip and the middle strip then by the value of moment per unit width we design for the prestressing tendons.

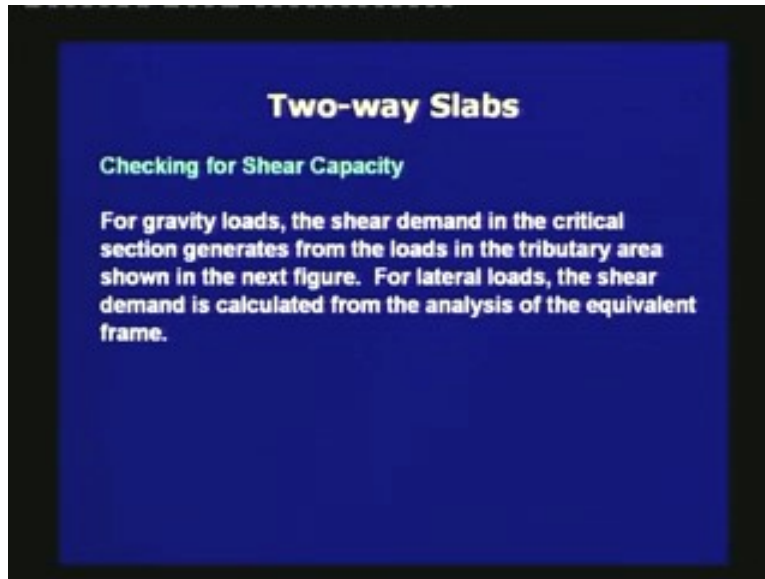
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Next, when we are looking for the shear capacity, the checking for shear capacity of flat plates and flat slabs is of utmost importance. In absence of beams the shear is resisted by the slab near the slab-to-column junction. We have to understand that when there are beams then the shear demand on the slab is much less, but in the absence of beams the shear demand on the slab near the slab-to-column junction is quite high. Hence, the checking for shear capacity in two-way slabs is extremely important, especially if the two-way slabs is of a flat plate and a flat slab.

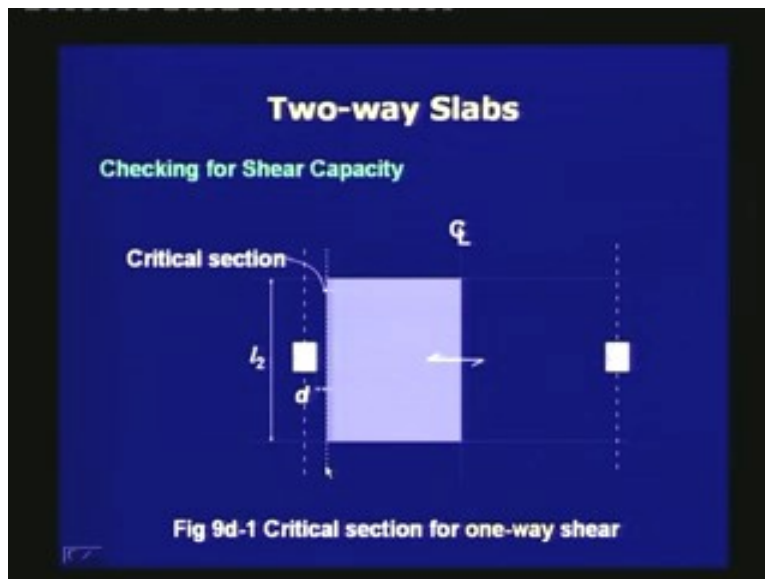
The shear capacity of the slab could be adequate to resist the shear from two actions. The first is the one-way shear, which is also called the 'beam shear' and the second is the 'two-way or the 'punching shear'. That means, for two-way slabs we analyze for two types of shear; first is the one-way shear and second is the two-way shear. The one-way shear is analogous to that generates in a beam due to flexure. This is checked in a two-way slab for each spanning direction separately. The critical section for checking the shear capacity is at a distance effective depth 'd' from the face of the column across the entire width of the frame. The critical section is transverse to the spanning direction.

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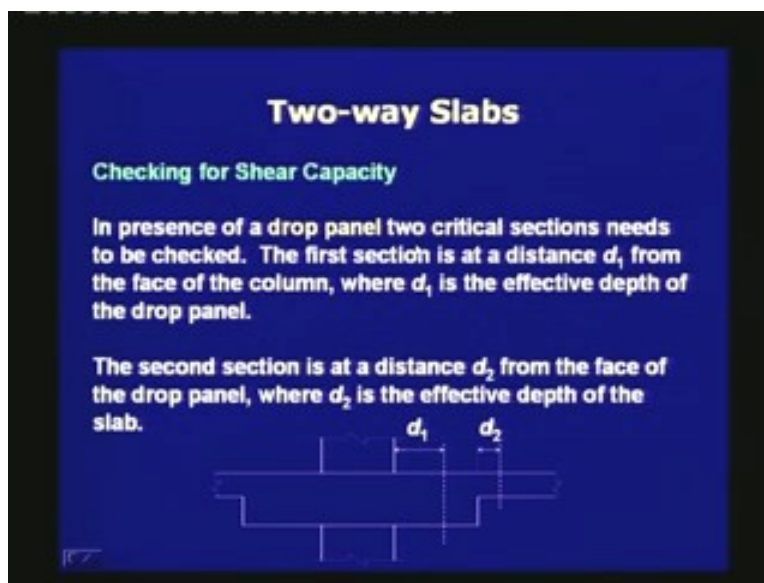
For gravity loads the shear demand in the critical section generates from the loads in the tributary area shown in the next figure. For lateral loads the shear demand is calculated from the analysis of the equivalent frame. Thus, first, once we have determined the equivalent frame and the spanning direction, the one-way shear is checked for the section which is transverse to the spanning direction and at a distance of the effective depth from the face of the support.

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In this sketch you can see the critical section marked by the dash line is at a distance 'd' from the face of the column. Note that, the spanning direction is running east-west, whereas the critical section is running north-south. The shear demand that comes in this critical section can be determined based on this tributary area which is shown shaded for gravity loads. But if there are lateral loads then we get the shear in the critical section from the analysis of the equivalent frame. This shear is analogous to the shear that generates in a beam under reflections.

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In the presence of a drop panel two critical sections need to be checked. The first section is at a distance d_1 from the face of the column where d_1 is the effective depth of the drop panel. The second section is at a distance d_2 from the face of the drop panel, where d_2 is the effective depth of the slab. Thus, if there is a variation of the thickness of the slab due to the drop panel, the first critical section is at the distance d_1 from the face of the column or the column capital, where d_1 is the effective depth of the drop panel. The second critical section is at a distance d_2 from the face of the drop panel where d_2 is the effective depth of the slab. Thus, whenever there is a variation in the depth slab we need to have multiple critical sections checked.

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Two-way Slabs

Checking for Shear Capacity

The calculations can be for unit width of the slab. The shear demand due to gravity loads per unit width is given as follows.

$$V_u = w_u (0.5l_n - d) \quad (9d-1)$$

Here, l_n is the clear span along the spanning direction.

The calculations can be for unit width of the slab. The shear demand due to gravity loads per unit width is given as follows: $V_u = w_u (0.5l_n - d)$. Here, ' l_n ' is the clear span along the spanning direction. Thus, if we know the tributary area, the length of the tributary area is 1/2 the clear span minus the effective depth d and then, the shear demand per unit width of the slab is given as w_u which is the factored gravity load per unit area times the length of the tributary area. Now, this is the shear demand that we are calculating for the gravity loads.

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Two-way Slabs

Checking for Shear Capacity

The shear capacity per unit width is given as follows.

$$v_{ur} = v_c \quad (9d-2)$$

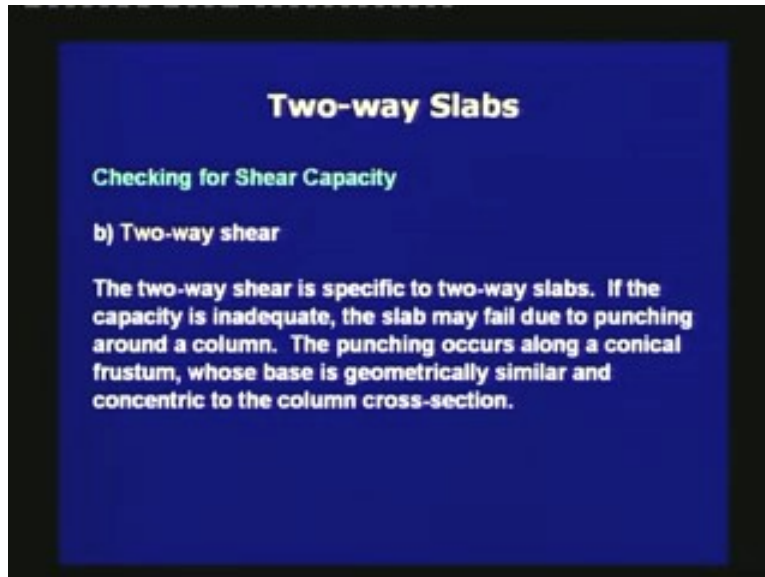
v_c is the shear capacity of uncracked concrete of unit width of slab. The expression of v_c is given in the Module of "Analysis and Design for Shear and Torsion".

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The shear capacity per unit width is given as follows: $v_{ur} = v_c$ where v_c is the shear capacity of uncracked concrete of unit width of slab. The expression of v_c is given in the "Module of Analysis and Design for Shear and Torsion". In slab design, conventionally we do not place shear reinforcement. The shear capacity is given only by the shear capacity of concrete. In the shear capacity of concrete there are two expressions given in the code; one for uncracked sections and another for cracked sections. Usually, near the support the uncracked section governs and once we know v_c , we equate that to v_{ur} , the resistance for shear. For adequate shear capacity we need to have v_{ur} greater than or equal to v_u . That means, the capacity should be greater than or equal to the demand. If this is not satisfied it is preferred to increase the depth of the slab to avoid shear reinforcement along the width of the slab.

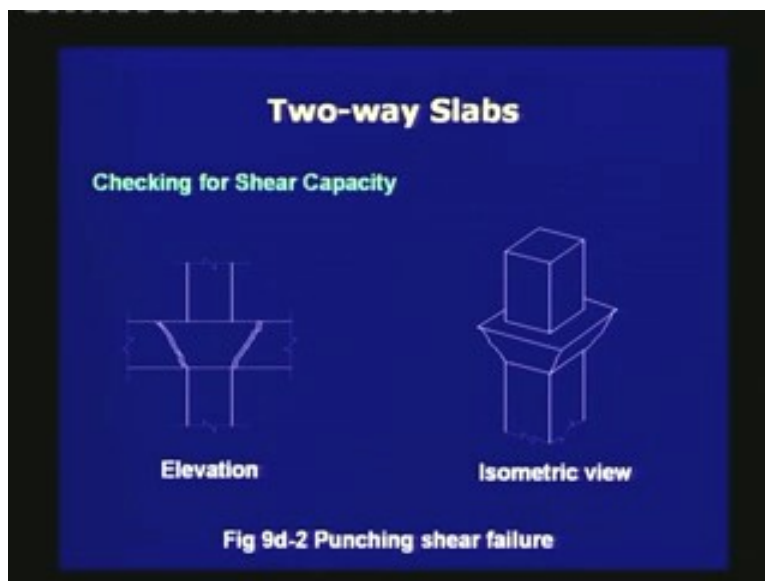
Thus, if in absence of drop panels and if we have a shear demand which is exceeding the shear capacity then we can provide a drop panel that means we can thicken the part of the slab near the column and we can have v_{ur} greater than or equal to v_c . That means, without providing shear reinforcement if we can increase the thickness of the slab then we may have the shear capacity greater than the shear demand. This is preferred to avoid shear reinforcement in the slab.

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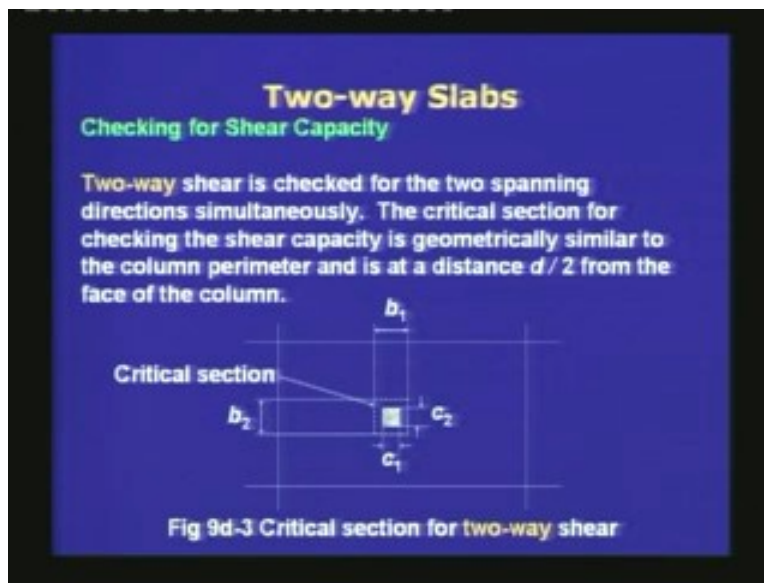
Next, we move on to two-way shear. The two-way shear is specific to two-way slabs. If the capacity is inadequate, the slab may fail due to punching around a column. The punching occurs along a conical frustum whose base is geometrically similar and concentric to the column cross-section.

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The punching shear failure is typical for two-way slabs and in absence of beams. In this case, under the gravity loads or if there are moments to be transferred from the slab to the column, there can be cracking in the slab which appears like a conical frustum whose base is similar to the column cross-section. Thus, in this figure, you can see the elevation where you see that there is conical crack in the slab and the slab tries to drop down from the slab and the column junction. The failure surface looks like a conical frustum. Note that the failure section is geometrically similar to the column cross-section.

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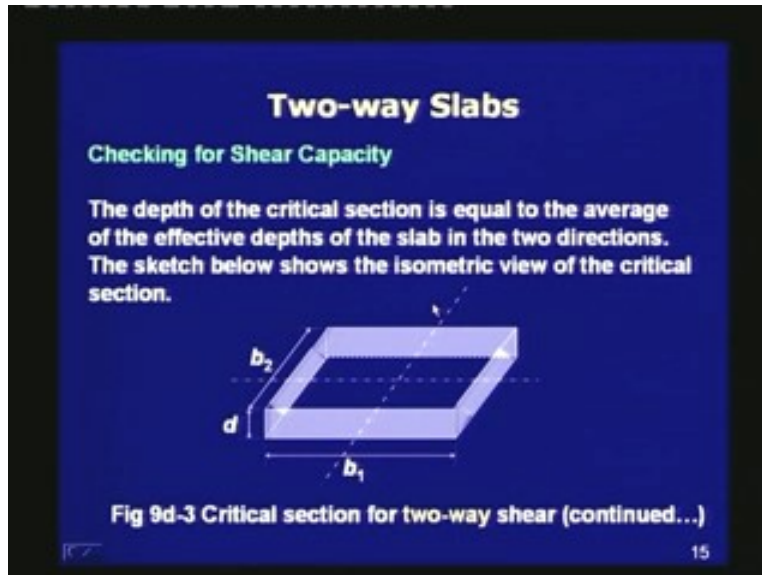


Two-way shear is checked for the two spanning directions simultaneously. So this is unlike one-way shear where we check the shear capacity individually in each of the spanning direction. But in two-way shear we check the capacity for both the directions simultaneously. The critical section for checking the shear capacity is geometrical similar to the column cross-section and is at a distance of d by two from the face of the column. Thus, based on the observed behavior under punching we select a critical section which is all around the column. It is geometrically similar to the column cross section and it is at a distance of $d/2$ from the face of the column or column capital.

In our following expressions we shall use these notations, c_1 is the dimension of the column in the one reaction; c_2 is the dimension of the column in the other orthogonal

direction; b_1 is the width of the critical section, which is parallel to c_1 ; and b_2 is the dimension of the critical section, which is parallel to c_2 . Thus, the perimeter of the critical section is twice $b_1 + b_2$.

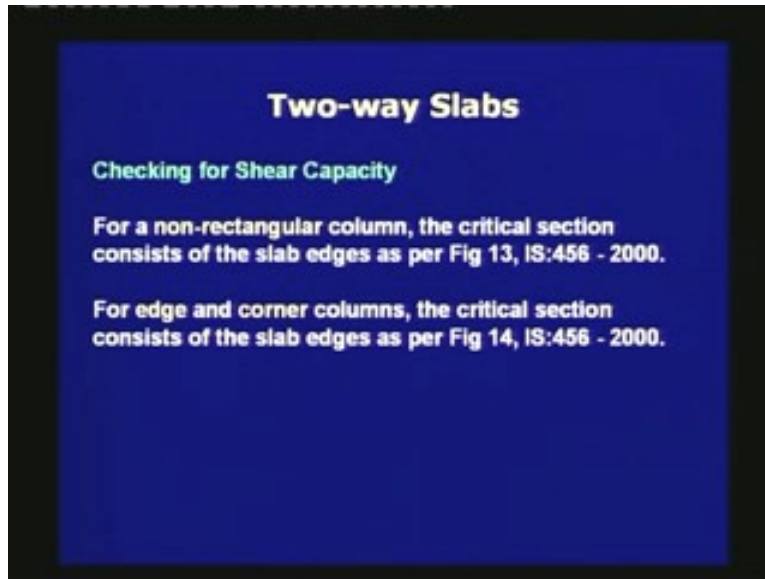
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The depth of the critical section is equal to the average of the effective depths of the slab in the two directions. We may have different effective depths in the two spanning directions because the steel in the two directions will lie above each other. But for computational simplicity we consider an average effective depth for both the two directions and that is the value of d , we use to compute b_1 and b_2 . The sketch below shows the isometric view of the critical section. Thus, for a rectangular column we have rectangular critical section with dimensions b_1, b_2 and the depth equal to the average of the effective depth, which is d .

The lengths of the sides of the critical section along and transverse to the spanning direction are denoted as b_1 and b_2 respectively. Thus, $b_1 = c_1 + d$, because each side the section is at a distance of $d/2$. Hence, $b_1 = c_1 + d$ by 2 on the left plus d by 2 on the right. Hence, $b_1 = c_1 + d$. Similarly, $b_2 = c_2 + d$ and here, c_1 is the dimension of the column or column capital in the spanning direction and c_2 is the dimension of the column or column capital in the transverse direction.

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For a non-rectangular column, the critical section consists of the slab edges as per figure 13, IS: 456-2000. There can be columns other than rectangular columns and in such situations, the code IS: 456-2000, gives us guide lines, how to select the critical section for non-rectangular columns. For edge and corner columns, the critical section consists of the slab edges as per figure 14, IS: 456-2000. That means, if a column is close to an edge and in that case, the critical section has the slab edge as one of its sides and how to select a critical section in such a situation is also given as IS: 456-2000.

We are not going to the details of special cases but we are focusing on the calculating shear demand and the shear capacity for a critical section of an interior rectangular column.

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Two-way Slabs

Checking for Shear Capacity

The demand in terms of shear stress is given as follows.

$$\tau_v = \frac{V_u}{b_0 d} + \frac{M_{uv|_{2-2}} \left[\frac{b_1}{2} \right]}{J_{2-2}} + \frac{M_{uv|_{1-1}} \left[\frac{b_2}{2} \right]}{J_{1-1}} \quad (9d-5)$$

Here,
 V_u = shear due to gravity loads from the tributary area
 M_{uv} = fraction of moment transferred about an axis
 b_0 = perimeter of the critical section = $2(b_1 + b_2)$.
 J = polar moment of inertia of the critical section about an axis

The demand in terms of shear stress is given as follows: $\tau_{u,v}$ is equal to v_u divided by $b_0 d$ plus M_{uv} about axis₂₋₂ access times b_1 by 2, divided by j about axis₂₋₂ access plus a M_{uv} about axis₁₋₁ access times b_2 , divided by two divided by j about axis₁₋₁ access. Let us try to understand this generic expression by the individual terms. Note that here, we are calculating the shear demand in terms of the shear stress $\tau_{u,v}$. Thus, v_u is the shear due to gravity loads from the tributary area.

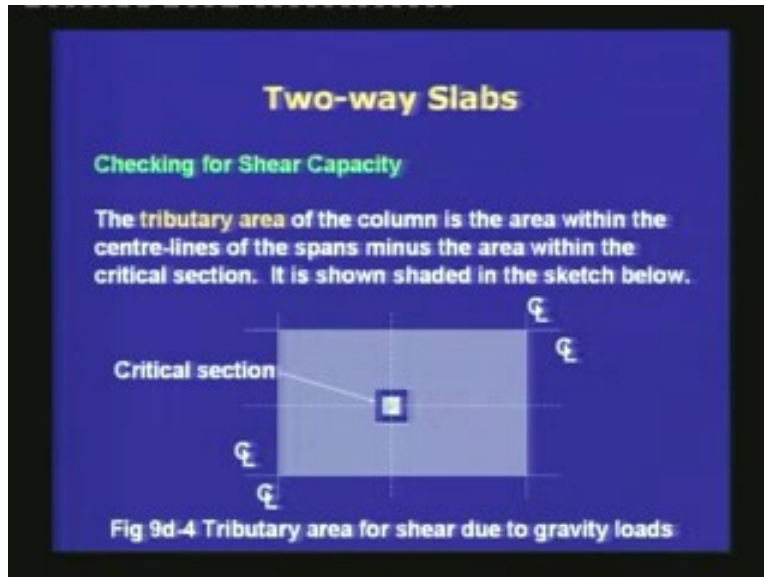
What is the tributary area?

We shall discuss later. The first term v_u , divided by $b_0 d$ comes from the gravity loads in the tributary area. M_{uv} is the fraction of the moment transferred about an axis.

For flat plates and flat slabs, the moments from the slab are transferred to the column by the slab-to-column junction and part of the moment is resisted by the shear. That, we are determining by a special expression, **we shall see later**. We are denoting the part of the moment that is transverse by shear as M_{uv} . When the moment is acting about the axis₁₋₁ the corresponding notation of the moment which generates shear stress is M_{uv} about axis₁₋₁. Similarly, when the moment acts about the axis₂₋₂, the corresponding moment generating the shear stress is denoted as $M_{uv|_{2-2}}$. Hence, b_0 is the perimeter of the critical section which is equal to twice $b_1 + b_2$. This we saw earlier, for a rectangular cross-

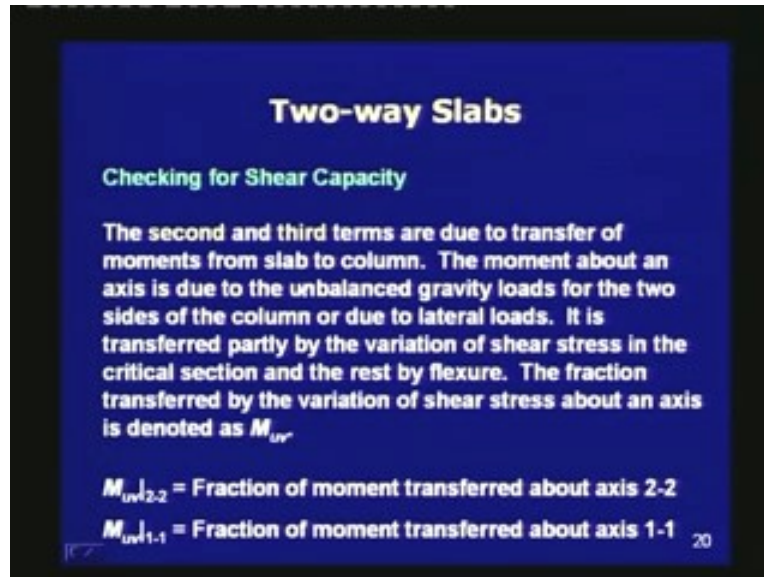
sectional column we have a rectangular critical section. In that case, $b_0 = 2(b_1 + b_2)$ and j is the polar moment of inertia of the critical section about an axis. We shall see what is the expression of j , again we have to calculate j about 2 axis; about₁₋₁; about₂₋₂ axis.

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The tributary area of the column is the area within the center-lines of the spans minus the area within the critical section. It is shown shaded in the sketch below. When we are calculating v_u , we are considering this shaded area which is the area bounded by center-line of the adjacent spans. From that we are deducting the area which is within the critical section. Thus, this shaded area generates the shear force v_u . The second and third terms are due to transfer of moments from slab to the column.

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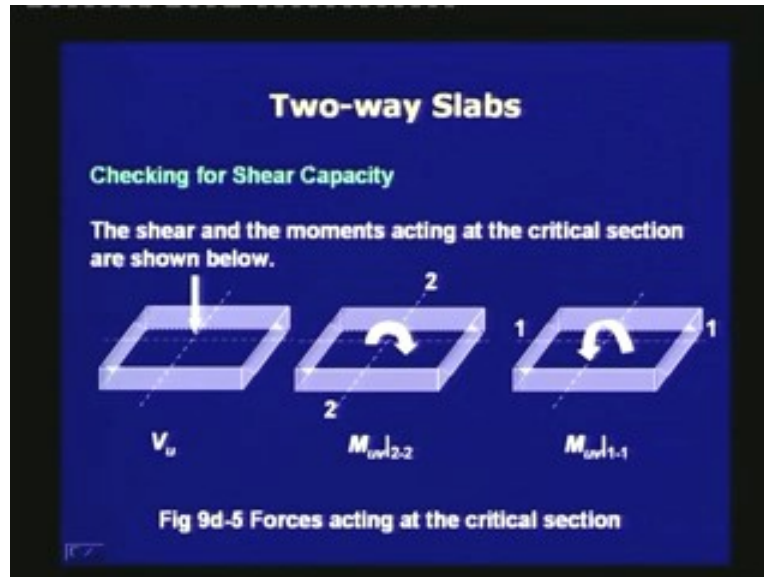


The moment about an axis is due to the unbalanced gravity loads for the two sides of the column or due to lateral loads. It is transferred partly by the variation of shear stress in the critical section and the rest, by flexure. The fraction transferred by the variation of shear stress about an axis is denoted as M_{uv} .

Thus, to summarize the second and third terms are related with the moment that gets transferred from the slab to the column. This moment can generate due to unbalanced loads on the two sides of the spans. If the spans are of different length then we can have a moment generated even if the load is uniformly placed and if the spans are of same length, then also we can have a moment if the live-load is placed only on one side. Thus, first, we need to calculate the moment that gets transferred from the slab to the column.

If there is lateral load acting then this moment is available from the analysis of the equivalent frame. From that moment, part of it is resisted by the variation of the shear stress in the critical sections. We are denoting that fraction as M_{uv} . Thus, M_{uv} about₂₋₂ is the fraction of moment transferred about axis₂₋₂. M_{uv1-1} is the fraction of moment transferred about axis₁₋₁.

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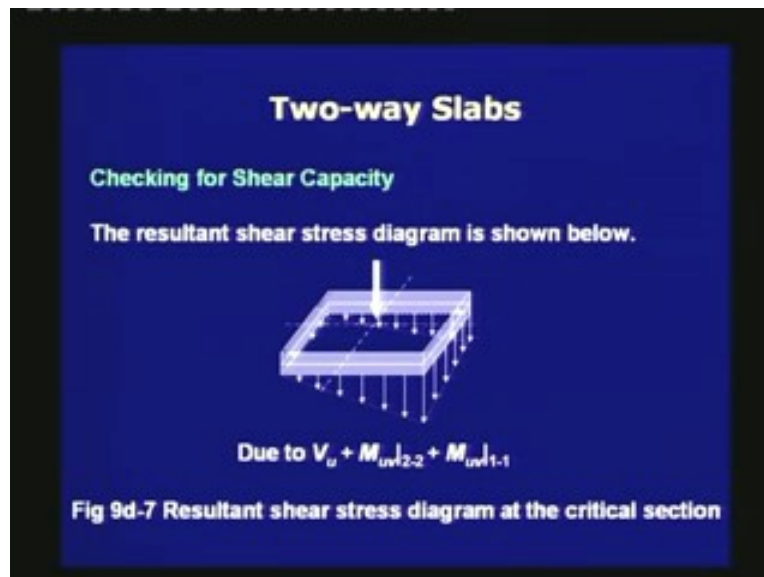
The shear and moments acting at the critical section are shown below. If you see the first diagram, the shear force, V_u acts at the center of the column. In the second diagram, M_{uv} is the part of the moment that acts about the axis₂₋₂. In the third diagram, you can see M_{uv1-1} is the part of the moment that acts about axis₁₋₁. Thus, these are the three forces that are acting in the critical section, which generates shear. The shear stresses due to these individual forces are represented below.

V_u generates a uniform shear stress all around the critical section. M_{uv} about axis₂₋₂ generates a varying shear stress, which varies from the left to right as we cross the axis₂₋₂. It is of course, uniform in the faces, which are parallel to the axis₂₋₂. Similarly, the shear stress that is, generated by M_{uv1-1} varies in the faces, that is perpendicular to₁₋₁ and they are uniform about the faces that are parallel to₁₋₁.

Thus, once we understand the mechanism of the transform of shear from the slab to the column, we can develop the expression of the shear stress that was given before. That means, the first term of the shear stress is V_u divided by the area in the perimeter which is b_0d . The second term is due to the moment acting about axis₂₋₂ and the third term, due to the moment acting about axis₁₋₁. Finally, we add them up for the maximum shear stress that occurs anywhere in the critical section.

If we note in the sketch the right-hand corner, the near corner in that case, say, if I pick up this corner, then the shear stress is downwards for due to V_u . Similarly, due to M_{uv2-2} , that is also downwards and the maximum value. For the shear due to M_{uv1-1} , that is also downwards and the maximum value. Thus, in this corner, all the shear stresses are additive. Hence, we are adding all the terms to get the maximum shear stress that generates in the critical section.

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The resultant shear stress diagram is shown below. Here, you can see that the stresses have added up in this closer right-hand corner and it is less in the other corners. But it is the closer right-hand corners which determine the shear demand. Hence, in all the terms of a shear demand we are adding the values due to the three terms. The fraction of moment transferred by the variation of shear stress about an axis which is denoted as M_{uv} is given in terms of the total moment transferred M_u as follows:

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Two-way Slabs

Checking for Shear Capacity

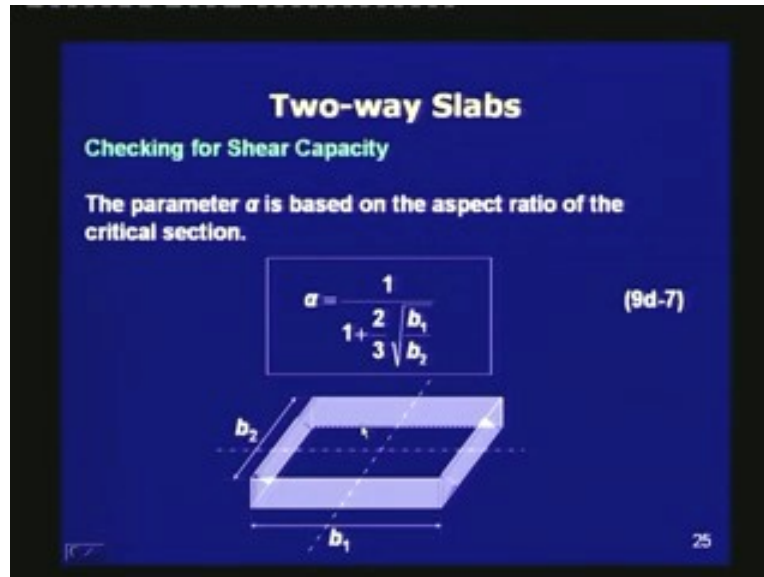
The fraction of moment transferred by the variation of shear stress about an axis (M_{uv}), is given in terms of the total moment transferred (M_u) as follows.

$$M_{uv} = (1 - \alpha)M_u \quad (9d-6)$$

The value of M_u due to unbalanced gravity load is calculated by placing live load on one side of the column only. The value of M_u due to lateral loads is available from the analysis of the equivalent frame.

As I said earlier, first, we need to calculate M_u . For gravity loads, it is calculated from the unbalanced loads on the two spans on the two sides. For lateral loads, M_u is calculated from the analysis of the equivalent frame. Now, once we know M_u then we can calculate M_{uv} by this expression, $M_{uv} = 1 - \alpha (M_u)$. The value of M_u due to unbalanced gravity load is calculated by placing live load on one side of the column only. The value of M_u due to lateral loads is available from the analysis of the equivalent frame. Then, we can combine the effects of the gravity loads and live loads based on the load combinations that we are familiar with. The parameter alpha is based on the aspect ratio of the critical section.

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It has been found that, when the critical section is square then the resistance to shear is better for this punching shear and hence this parameter has been developed to quantify the shear capacity for non-square sections. That means, for non-square sections, it will be less than the value corresponding to a square critical section. The parameter alpha depends on the aspect ratio of the critical section.

The aspect ratio means the ratio of one side divided by the other side of the critical section. Alpha is equal to 1 divided by 1 plus two-thirds of square root of b_1 divided by b_2 . Here, b_1 and b_2 are the dimensions of the critical section parallel to axis₁₋₁ and axis₂₋₂. Then, from the aspect ratio, we can calculate the value of the parameter alpha. Next, once we know M_{uv} , the next quantity we need to know is the polar moment of inertia.

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Two-way Slabs

Checking for Shear Capacity

The polar moments of inertia of the critical section, about the axes are given as follows.

$$J_{1-1} = 2 \left[\frac{1}{12} b_2 d^3 + \frac{1}{12} d b_2^3 + b_2 d \left(\frac{b_1}{2} \right)^2 \right] \quad (9d-8)$$
$$J_{2-2} = 2 \left[\frac{1}{12} b_1 d^3 + \frac{1}{12} d b_1^3 + b_1 d \left(\frac{b_2}{2} \right)^2 \right] \quad (9d-9)$$

The polar moments of inertia of the critical section about the axes are given as follows:

J_{1-1} is equal to 2 times $\frac{1}{12} b_2 d^3 + \frac{1}{12} d b_2^3 + b_2 d \left(\frac{b_1}{2} \right)^2$ divided by 2 whole square. Similarly J_{2-2} is equal to 2 times $\frac{1}{12} b_1 d^3 + \frac{1}{12} d b_1^3 + b_1 d \left(\frac{b_2}{2} \right)^2$ divided by 2 whole square. The expressions of the polar moments of inertia have been determined based on the parallel axis and perpendicular axis theorems. These theorems are covered in the undergraduate structure analysis courses. From those theorems, we can develop these expressions of the polar moments of inertia.

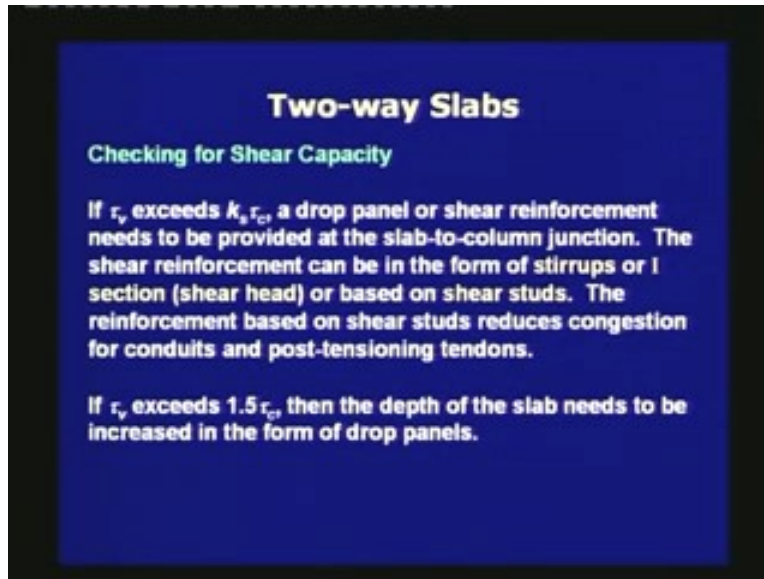
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The slide is titled "Two-way Slabs" in yellow text on a blue background. Below the title, it says "Checking for Shear Capacity" in green. Then, "For adequate shear capacity" is written in white. A white-bordered box contains the formula $\tau_v \leq k_s \tau_c$ with "(9d-10)" to its right. Below this, it states "The shear stress capacity of concrete for a square column is given as follows." in white. Another white-bordered box contains the formula $\tau_c = 0.25 \sqrt{f_{ck}}$ with "(9d-11)" to its right. At the bottom, it explains: "Here, f_{ck} is the characteristic strength of the concrete in the slab. The effect of prestress is neglected." in white.

For adequate shear capacity, the τ_v , which is the resultant shear stress demand due to the three forces should be less than or equal to $k_s \tau_c$. The shear stress capacity of concrete for a square column is given as follows: τ_v is less than or equal to $0.25 \sqrt{f_{ck}}$, where f_{ck} is the characteristic strength of the concrete in the slab. The effect of prestress is neglected. Thus, the shear capacity τ_c is given as $0.25 \sqrt{f_{ck}}$, neglecting the effect of prestress. The factor k_s accounts for the reduced shear capacity of non-square columns.

Again as I said that, for a cross-section, which is non-square, it has been found that the shear capacity is lower than that of a square cross-section. That is taken into account by these factor k_s , where k_s is equal to $0.5 + \beta_c$. The value of k_s should be less than one and β_c is the parameter based on the aspect ratio of the column cross-section. It is the ratio of the short side to long side of the column or column capital. Again, β_c is the ratio of the short side to long side of the column. To that, we are adding 0.5 to get the value of k_s . We need to make sure that k_s is less than or equal to 1. Then, we are multiplying k_s to τ_c , which is equal to $0.25 \sqrt{f_{ck}}$ and by that, we are getting shear capacity of the critical section at a point. For adequate shear capacity, the shear demand τ_v has to be less than or equal to τ_c .

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If τ_v exceeds $k_s \tau_c$, a drop panel or shear reinforcement needs to be provided at the slab-to-column junction. Thus, if we find that the shear demand τ_v is greater than the capacity $k_s \tau_c$. Then, we have two options in hand.

The first option is, we can thicken the slab around the column and that is called a drop panel. The second option is, which is applied for flat plates that to provide shear reinforcement inside the slab. The shear reinforcement can be in the form of stirrups or I section, which is sometimes called shear head or based on shear studs. The reinforcement based on shear studs reduces congestion for conduits and post-tensioning tendons. There have been various types of shear reinforcement. Some of it, we shall cover in this lecture and depending on the suitability, that is, how much reinforcement we have in the column, depending on that, we can select what type of shear enforcement we need to provide at the slab-to-column junction.

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Two-way Slabs

Checking for Shear Capacity

The stirrups are designed based on the following equation.

$$A_{s_v} = \frac{(\tau_v - 0.5\tau_c)}{0.87f_y} \quad (9d-13)$$

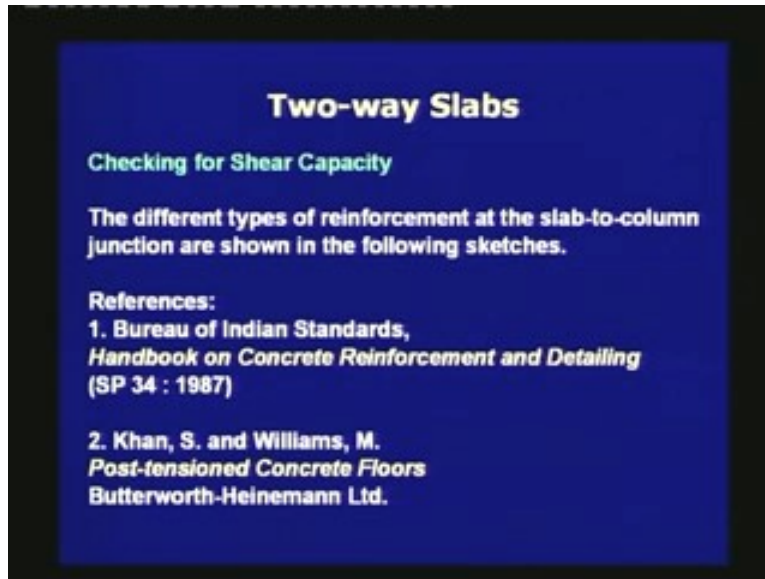
The stirrups are provided along the perimeter of the critical section. The first row of stirrups should be within a distance of $0.5d$ from the face of the column. They can be continued in outer rows (concentric and geometrically similar to the critical section) at an interval of $0.75d$, till the section with shear stress $\tau_v = 0.5\tau_c$.

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If τ_{ov} exceeds $1.5 \tau_{oc}$, it is an upper limit; in that case, the depth of the slab needs to be increased in the form of drop panels. That means, the code says that if the shear demand is very high, which is like 1.5 times τ_{oc} , then we need to increase the depth of the slab around the columns which is known as drop panels. The stirrups are designed based on the following equation: A_{s_v} is equal to τ_{ov} minus $0.5 \tau_{oc}$ divided by $0.87 f_y$. Thus, we have calculated the shear demand, τ_{ov} . From that, we are subtracting only half of the shear capacity of the concrete, which is $0.5 \tau_{oc}$. We are dividing that, by the permissible stress in the stirrups, which is $0.87 f_y$ to get the value of A_{s_v} , which is the shear reinforcement around the critical section.

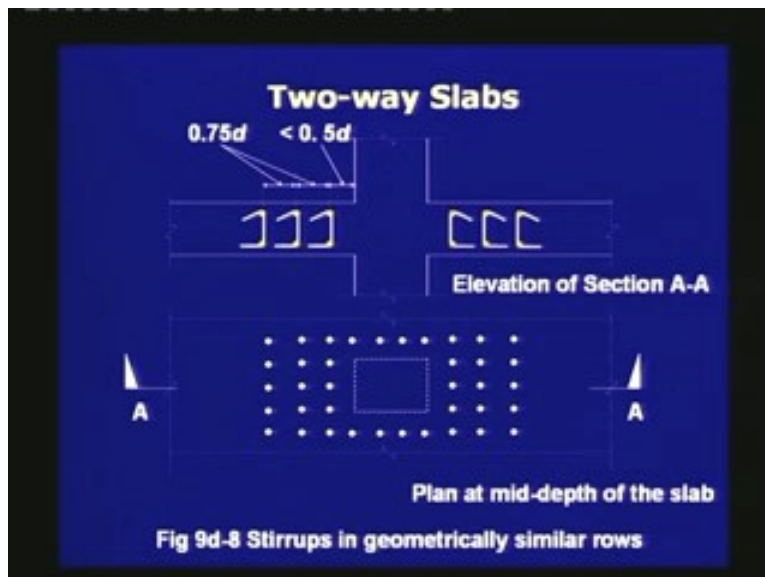
The stirrups are provided along the perimeter of the critical section. The first row of stirrups should be within distance of $0.5 d$ from the face of the column. They can be continued in outer rows, which are concentric and geometrically similar to the critical section at an interval of $0.75 d$ till the section with the shear stress τ_{ov} is equal to $0.5 \tau_{oc}$. Let us understand this by the help of a sketch.

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The different types of reinforcement at the slab-to-column junction are shown in the following sketches: These sketches have been taken from Bureau of Indian Standards Publication, Handbook on Concrete Reinforcement and Detailing, which is SP 34: 1987. The second reference is Khan and Williams and the title of the book is Post-tensioned Concrete Floors and it has been published by Butterworth and Heinemann limited.

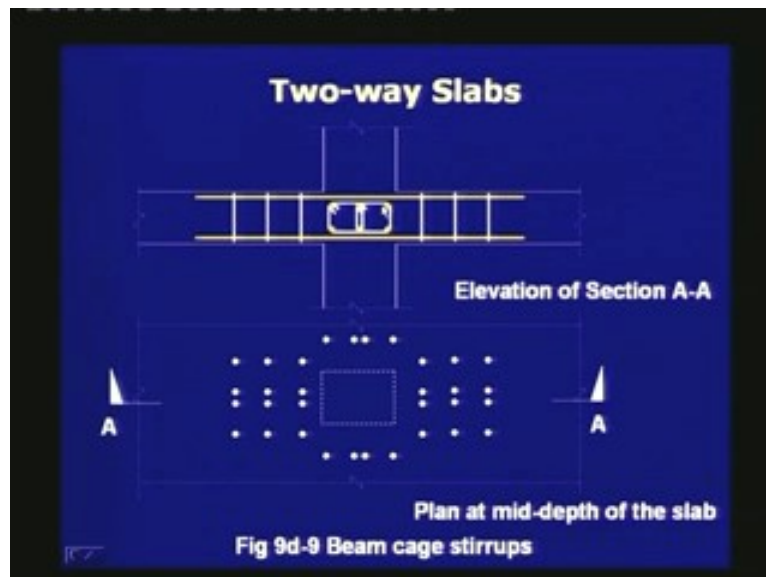
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In this figure, we see that around the column section, we have identified the critical section and we are providing the stirrups in a perimeter, which is geometrically similar and concentric to the column cross-section. The first row of stirrups should start within a distance point, $5d$, so that at least there is one row of stirrups that intercepts the punching shear. Then, we may provide subsequent rows of stirrups till the shear demand comes below $0.45 \tau_{ou_c}$. In this figure, we have seen that we are providing these stirrups in two more rows and you can see that these stirrups are also placed in a perimeter, which is geometrically similar and concentric to the column cross-section.

We have to provide some holder bars for the stirrups incase for pre-stress slabs and these stirrups have to be placed in and perimeter, which is geometrically similar to the column cross-section. The second type of shear reinforcement is by the beam cage reinforcement. In this figure, we are showing some stirrups which are analogous to beam stirrups,

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These are closed stirrups placed about the holder bars and if I take a cross section at the mid-depth of the slab, then the vertical legs of the stirrups appeared this way. Again, we see that the stirrups are around the column, which is geometrically similar and concentric to the column cross-section. This is another alternative of placing the stirrups around the columns. The third alternative is by providing bent-up bars. This is suitable in absence of

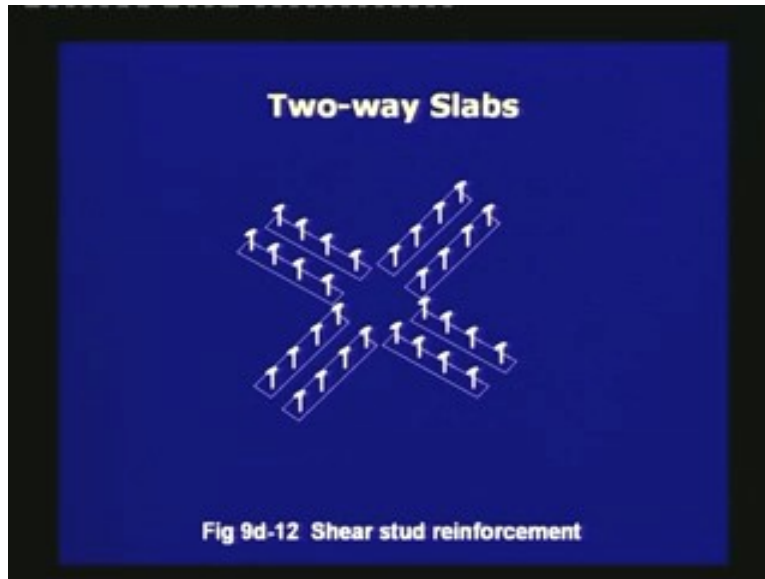
lateral loads and if the amount of shear reinforcement is less. We can provide some bars which have inclined legs and these inclined legs can carry the vertical component of the shear force. Hence, again if I take a section at the mid-depth of the slab, then we see that the legs of the stirrups appear to be about a perimeter, which is geometrically similar and concentric to the column cross-section.

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The fourth type of the shear reinforcement in a flat plate, especially can be given by shear head or which are I sections welded together. In this sketch, we find that there are two I sections, which are welded at the center and this shear head reinforcement can be placed at the slab-to-column junction to sufficiently increase the shear capacity of the junction.

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Finally, we are coming to the reinforcement based on shear studs. Here, shear studs are welded to plates and this assembly is placed at the slab-to-column junction. The advantage of shear stud reinforcement is that, it does not intercept the main reinforcement through the column and the placement of the prestressing tendons conduits is easier, if we have this shear stud reinforcement. Thus, depending on the column reinforcement, the layout of the prestressing tendons, any other obstruction, whether it is an electrical conduit; in that, depending on this, we are selecting the type of shear reinforcement to enhance the shear capacity of the slab-to-column junction.

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Two-way Slabs

Checking for Shear Capacity

The residual moment transferred by flexure (M_{uf}), is given in terms of the total moment transferred (M_u) as follows.

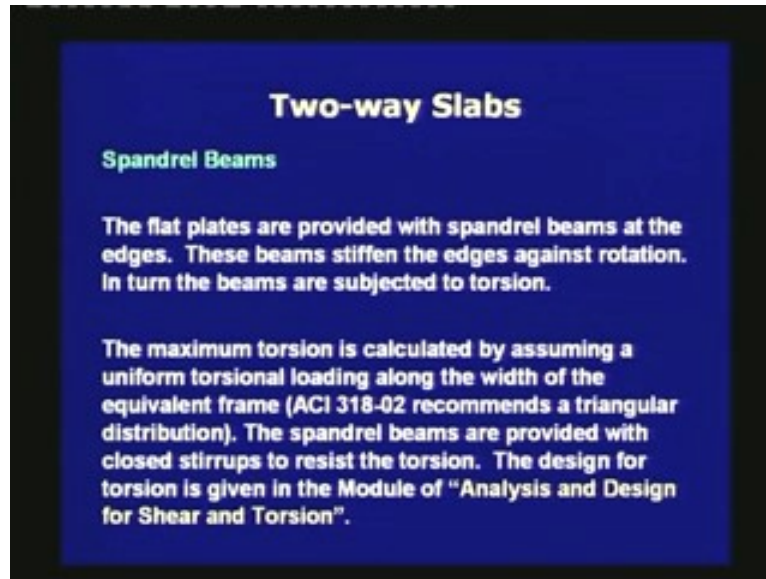
$$M_{uf} = \alpha M_u \quad (9d-14)$$

Additional non-prestressed reinforcement is provided at the top of the slab over a width $c_2 + 3h$ (centred with respect to the column) to transfer M_{uf} .

The residual moment transferred by flexure, which is denoted as M_{uf} is given in terms of the total moment transferred M_u as follows: M_{uf} is equal to alpha times M_u . Thus, what we have seen is that, the moment to be transferred is M_u . Part is transferred by shear, which is one minus alpha times M_u and we have denoted that as M_{uv} . The other part which is transferred due to flexure is denoted as M_{uf} and M_{uf} is equal to alpha times M_u . This M_{uf} will be resisted by additional flexural reinforcement.

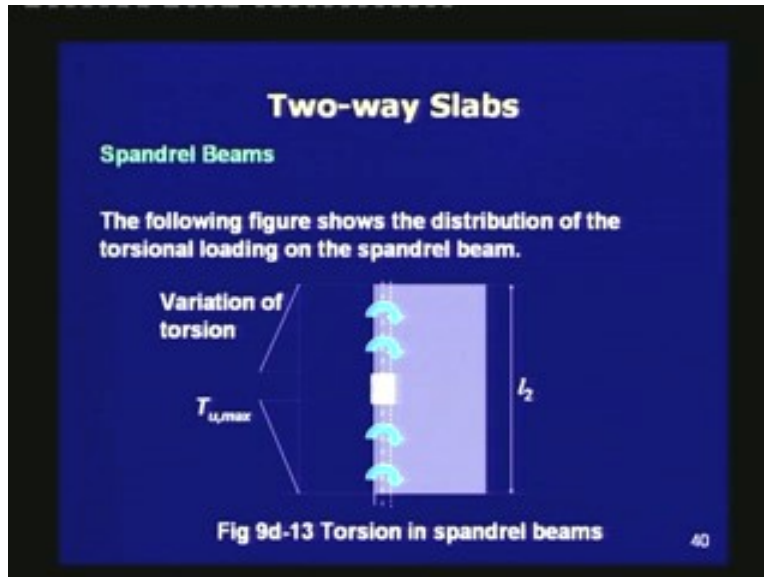
Additional non-prestressed reinforcement is provided at the top of the slab over a width c_2 plus 3 h centered with respect to the column to transfer M_{uf} . Thus, once we know M_{uf} , we can calculate the amount of non-prestressed reinforcement that is required to transfer M_{uf} in each orthogonal direction. Also, reinforcement is provided and it is distributed over a width c_2 plus three times the depth of the slab. In that width, this additional reinforcement is banded to transfer the part of the moment by flexure. Next, we are coming to the design of spandrel beams.

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The flat plates are provided with spandrel beams at the edges. These beams stiffen the edges against rotation. In turn, the beams are subjected to torsion. When we earlier studied about flat plates, we said that the flat plate option is selected to have a flat bottom beneath the slab, so that it does not create any obstruction for the conduits. Even for plates, usually a beam is provided around the edge of the building which does not intercept the conduits and this beam stiffens the edges for the slab against rotation. These special beams are termed as 'spandrel beams'. In turn, the spandrel beams are subjected to torsion, for which they have to be analyzed and designed, if required. The maximum torsion is calculated by assuming a uniform torsional loading along the width of the equivalent frame. Of course, IS: ACI 318-02 recommends a triangular distribution. The spandrel beams are provided with closed stirrups to resist the torsion. The design for torsion is given in the module of 'Analysis and design for shear and torsion'.

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In this figure, we are showing the torsion variation along the spandrel beam. We have selected the width of the equivalent frame and we are assuming the torsional load is uniformly distributed along this length of the spandrel beam, which spans in the transverse direction along l_2 . If the torsional loading is uniform, then the maximum torsion will be at the face of the column. We are denoting this maximum torsion as $T_{u,max}$. The maximum torsion $T_{u,max}$ is given as follows:

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Two-way Slabs
Spandrel Beams

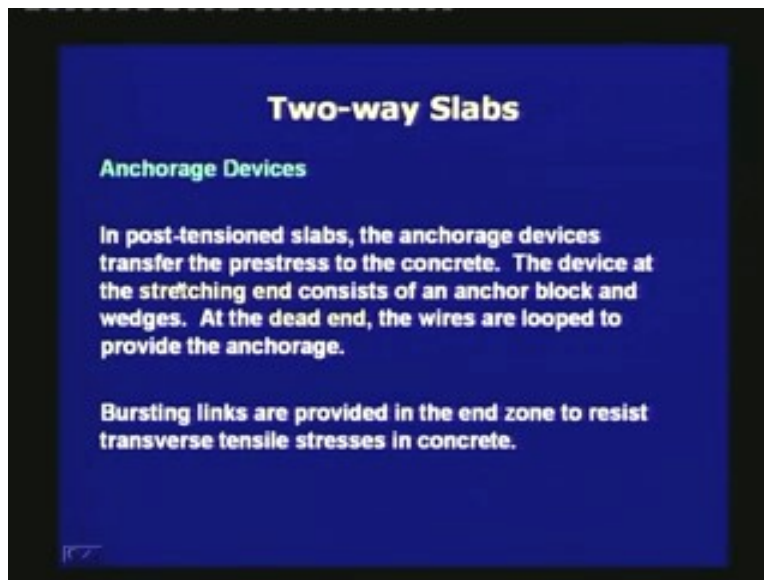
The maximum torsion ($T_{u,max}$) is given as follows. Here, $M_{u,e}$ is the moment at the exterior support of the equivalent frame.

$$T_{u,max} = \frac{l_2 - c_2}{2} \frac{M_{u,e}}{l_2} \quad (9d-15)$$

$T_{u\max}$ is equal to l_2 , which is the width of the equivalent frame minus c_2 , which is the width of the column parallel to l_2 divided by 2, that means, we are dividing that area into two parts times M_e minus divided by l_2 . This is the moment negative moment at the exterior support, which is uniformly distributed about l_2 . Thus, the maximum torque depends on the moment per unit length which is denoted as M_e minus l_2 times the half of the tributary length, which is l_2 minus c_2 divided by 2. Once we calculate $T_{u\max}$, then we need to make sure that shear capacity $T_u R$ is greater than or equal to $T_{u\max}$. The torsion design was covered in our previous module on analysis and design for shear and torsion'.

We need to make sure that the stirrups are closed in a spandrel beam, because torsion generates a circulatory shear around the periphery of the beam. Next, we are moving on to the anchorage devices of the flat slabs and flat plates.

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In post tensioned slabs, the anchorage devices transfer the prestress to the concrete. The device at the stretching end consists of an anchor block and wedges. At the dead end, the wires are looped to provide the anchorage. Bursting links are provided in the end zone to resist transverse tensile stresses in concrete. Earlier, when we studied anchorage devices for beams, we had seen that in post tensioned members, the anchorage device is of extreme importance, because the prestress is transferred to the concrete at the ends. The

similar is true for post-tensioned slabs. In the stretching end, we have an anchorage block with wedges against which the tendon rests on the concrete and in the dead end, the tendons can be opened up to form a loop. This loop itself is sufficient to transfer the prestress from the tendon to the concrete.

We also provide bursting links near this anchorage zone, so as to check cracking due to the transverse tensile stresses that is generated due to the stress concentration.

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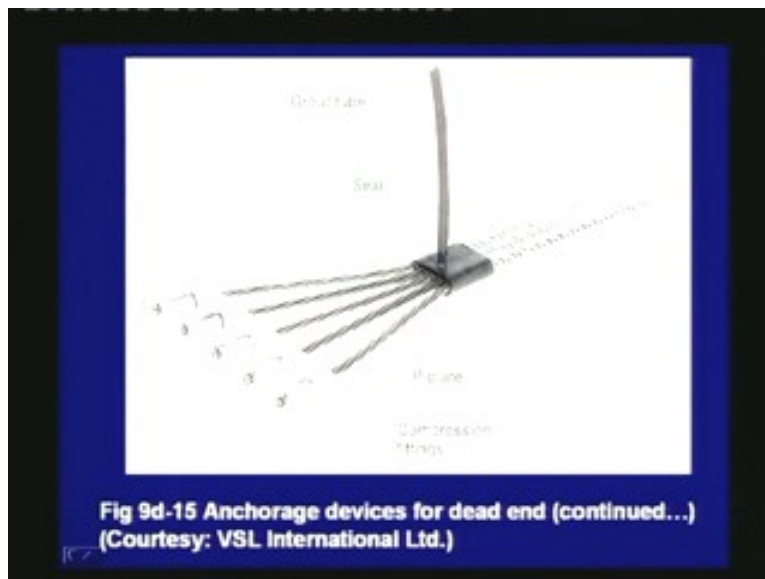
In this figure, we can see the anchorage device at the stretching end, that the tendons after they are passed to the duct. Then they pass through casting, which is a funnel shaped and then we have the anchorage block within which there are wedges and against these wedges, the strands are hold. There is also the provision for **tube**, by which they can pass on the grout after the post-tension operation has been done. The **research** former is used so that after the post-tensioning operation is done, we can put some concrete around this anchorage block and cover it up, so that it cannot be seen from outside. Thus, the **research** former is just for an esthetic purpose and not for a functional use.

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The anchorage device for dead end appears that after the duct ends, the strands are opened up to form a loop. In this loop, a bar is passed and this loop rests against the concrete, which helps to transfer the prestress, which is lower in the dead end. This special form can avoid any wedge action that is necessary, if we have to provide any block at the dead end. There can be also a grout tube to pass grout throughout the dead end.

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This is another sketch of the anchorage device for the dead end, that we can have a plate against which the strands are hold. This plate rests against the concrete surface.

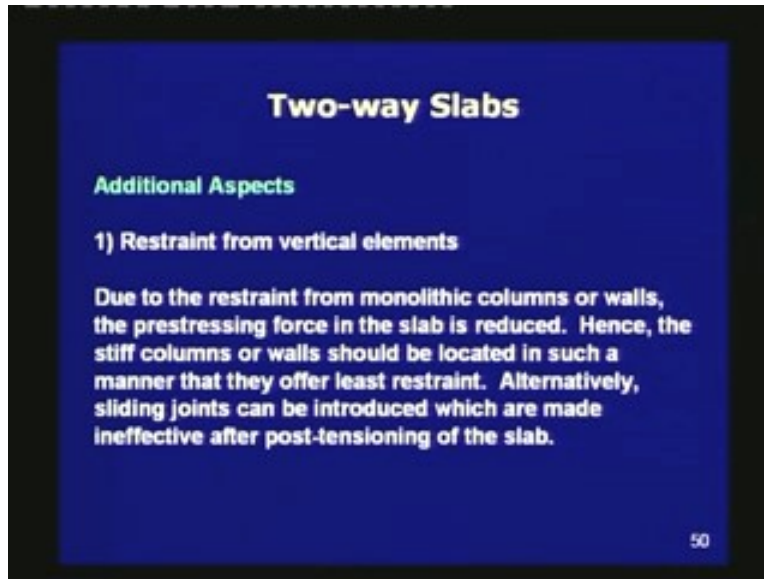
This is the figure of the end of the post-tensional slab and at the stretching zone, you can see that this is the funnel, the casting piece and then, we have the anchorage block inside. We can also observe the bursting links, which checks the transverse tensile stresses and note that in the spandrel beam, there are closed stirrups, which help to carry torsion. This spandrel beam is required for the flat plates in order to stiffen the edge of the slab against rotation.

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This is the figure for the dead end and here, you observe that after the duct, the strands have been exposed; they have been spread out; and they have been opened up to found a bulk or loop. This loop resists against the concrete and which provides the anchorage at the dead end. We have also provided bursting links near the duct and you note that in the spandrel beams, the stirrups are closed stirrups. Next, we are moving on to some additional aspects of analysis and design of two-way slabs. First, we are trying to understand that what is the effect of prestress on the other components of the buildings?

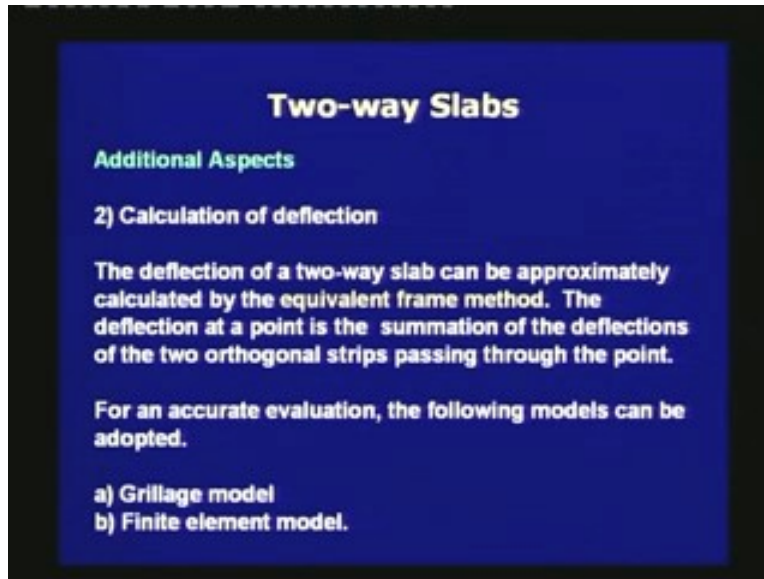
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The slab rests on the columns. Now, due to the restraint from monolithic columns or walls, the prestressing force in the slab is reduced. That means, when, if the column and the slab are cast together and then after that we are post-tensioning the slab, the prestressing operation is facing a restraint from the columns or the walls which have been integrally cast with the slabs. Hence, the stiff columns or walls should be located in such a manner that they offer least restraint. Alternatively, sliding joints can be introduced which are made ineffective after post tensioning of the slab.

Since the stiff columns and walls provide some restraint in the prestressing operation, we need to locate them in such a way so that it creates the least resistance. There is another alternative option that we can have sliding joint between the slab and the column, which will be removed after the post-tensioning operation has been done. In that way, we can reduce the restraint from the vertical elements on the prestressing force.

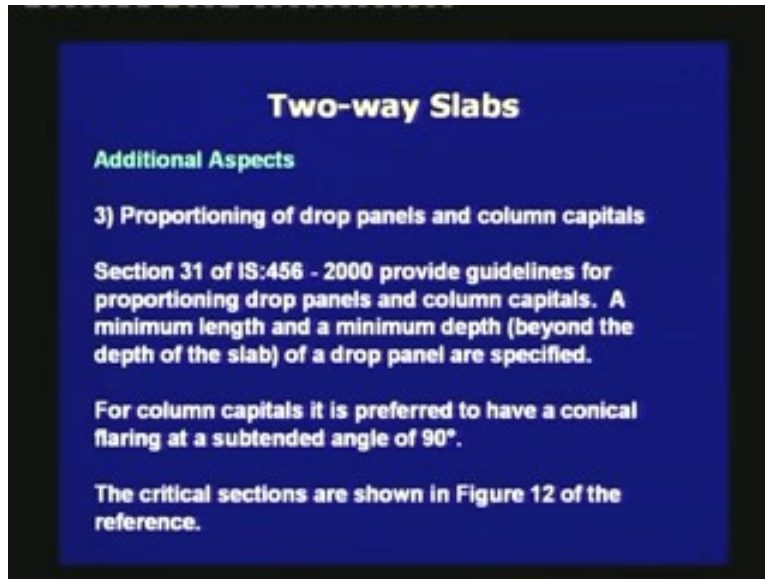
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The second is the calculation of deflection. Deflection of a two-way slab can be approximately calculated by the 'equivalent frame method'. We have studied that the equivalent frame method is one option to analyze and design a two-way slab. This method itself can be used to calculate the deflections. The deflection at a point is summation of the deflections of the two orthogonal strips passing through the point.

Thus, if we take the middle of the slab, there are two strips passing through a point and the deflections of these two strips are approximately added up to get the total deflection at the point. Now, this is an easy way to find out the deflection from the analysis of the equivalent frame method. There are of course refined analysis to find the deflections. For an accurate evaluation, the following models can be adopted: a grillage model and a finite element model. These models are based on the finite element concept and where the slab is divided into small plate elements or beam elements in a privilege model and this model can be used to find out the deflection more accurately.

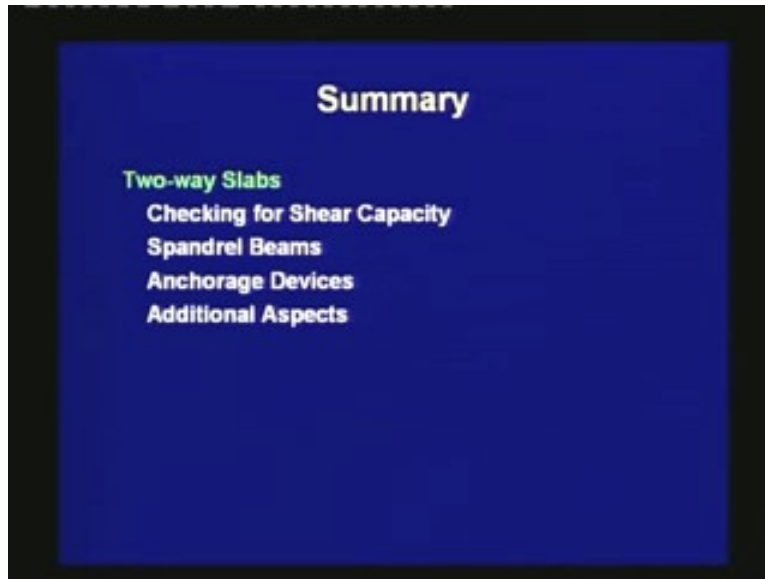
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Third is proportioning of drop panels and column capitals. Section 31 of IS: 456-2000 provide guidelines for the proportioning of drop panels and column capitals. A minimum length and a minimum depth beyond the depth of the slab of a drop panel are specified. That means, even if we provide drop panels or column capitals, we need to have them adequate enough to function properly. The code IS: 456 gives some guidelines on proportioning the drop panels and column capitals. For drop panels, they have to be a minimum length and a minimum depth beyond the slab. For column capitals, they have to be a proper shape, so that the concrete in the column capital is effective. For column capitals, it is preferred to have conical flaring at a subtended angle of 90 degrees, that means, whenever we are providing a column capital, it is preferred that the subtended angle in the column capital should be 90 degree. If there is material outside this 90 degree, then that has to be neglected in the analysis of the slabs.

Thus, once we proportion the drop panels and column capital properly, then only we can take their full advantage. The critical sections are shown in figure 12 of the reference. IS: 456 gives us guidelines that what are the critical sections if we have a drop panel or if we have a column capital. Those details are not being mentioned here, but you can refer to IS: 456 to go through these details.

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Thus, in this lecture, we have covered the two-way slabs. After refreshing the flexural design, we moved on to the analysis and design for shear. We have found that for two-way slabs, there are two types of slabs which need to be checked. The first is the one-way shear, which is similar to the beam under flexure and that is checked for each orthogonal direction separately. The second type of shear is the punching shears which is checked for both the directions simultaneously. The punching shear is checked for a critical section, which is geometrically similar and concentric to the column cross-section. If the shear capacity is not adequate, then we provide shear reinforcement and the type is selected based on how much column reinforcement we have and how much shear reinforcement we need. We also studied the anchorage devices for the post-tensioned slabs. Finally, we moved on to the additional aspects like the restraints due to the vertical elements and the deflection. With this, we are ending the module on two-way slabs. Thank you.