Prestressed Concrete Structures

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Lecture - 37

Two-way Slabs (Part 1)

Welcome back to the prestressed concrete structures. This is the third lecture of module nine on special topics. In this lecture, we shall study two-way slabs.

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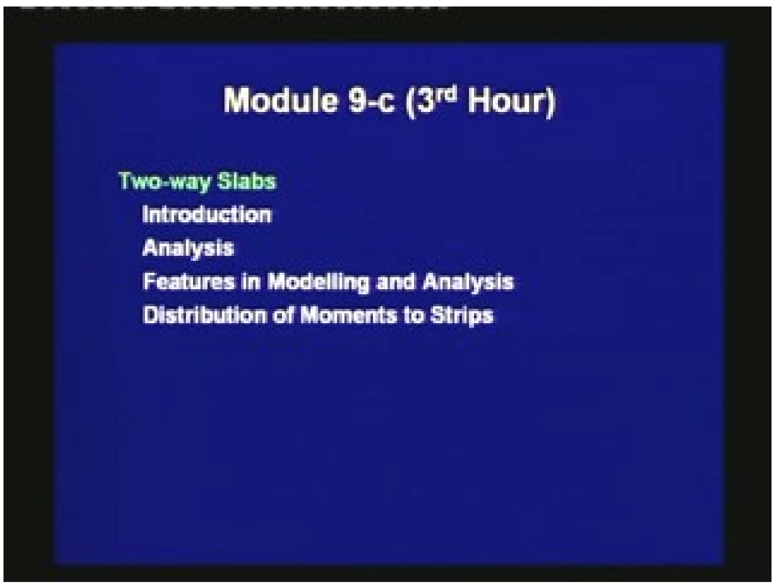

First, we shall introduce the different types of two-way slabs; next, we shall learn about the analysis of two-way slabs. Then, we shall move on to the features in modelling and analysis; what are the different aspects in the two-way slabs which are different from that of one-way slabs and then, we shall move on to the distribution of moments to the strips in the two-way slabs.

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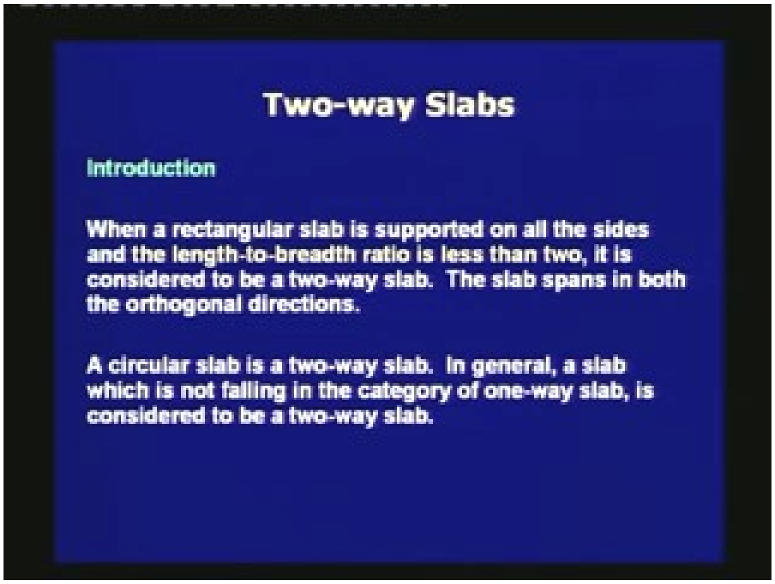

When a rectangular slab is supported on all the sides and the length-to-breadth ratio is less than 2, it is considered to be a two-way slab. The slab spans in both the orthogonal directions. What we had seen last time for a one-way slab is that we design the slab for only one direction and in the transverse direction, we provide nominal reinforcement. But for two-way slabs it has to be designed for both the directions. A circular slab or other types of rectangular slabs with different types of support condition which do not fall in to the category of one-way slab, are considered as a two-way slab. That is a rectangular slab which is supported on three sides or the two adjacent sides are also categorized under two-way slabs.

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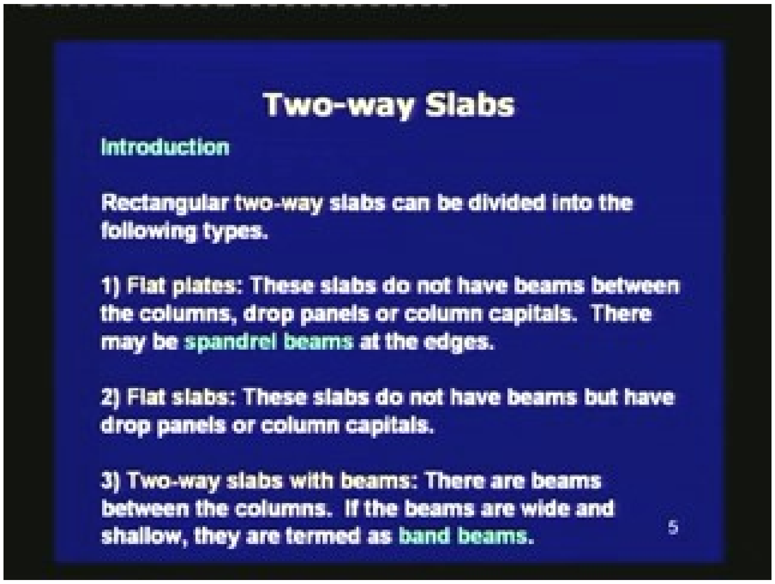

Rectangular two-way slabs can be divided in to the following types. First, flat plates - these slabs do not have beams between the columns, drop panels or column capitals. There may be spandrel beams at the edges. Let us first know these terms a bit more properly. If a column has a flared portion at its top, that is called a column capital and if the slab is thickened around the column, it is called drop panel. In a flat plate, there are no beams, no drop panels or column capitals. There may be some beams at the edges at the exterior of the building and those beams are called spandrel beams. These are called by a special name because it needs to be designed for torsion.

The second type of two-way slabs is the flat slabs. These slabs do not have beams but they have drop panels or column capitals. Thus, the difference between a flat plate and a flat slab is that a flat plate does not have even column capitals or drop panels, but a flat slab has those in the building.

The third type is the two-way slabs with beams. There are beams between the columns. If the beams are wide and shallow, they are called band beams. That means, a two-way slab with beams has beams running in between the columns and if those beams are shallow and wide then these are called as band beams.

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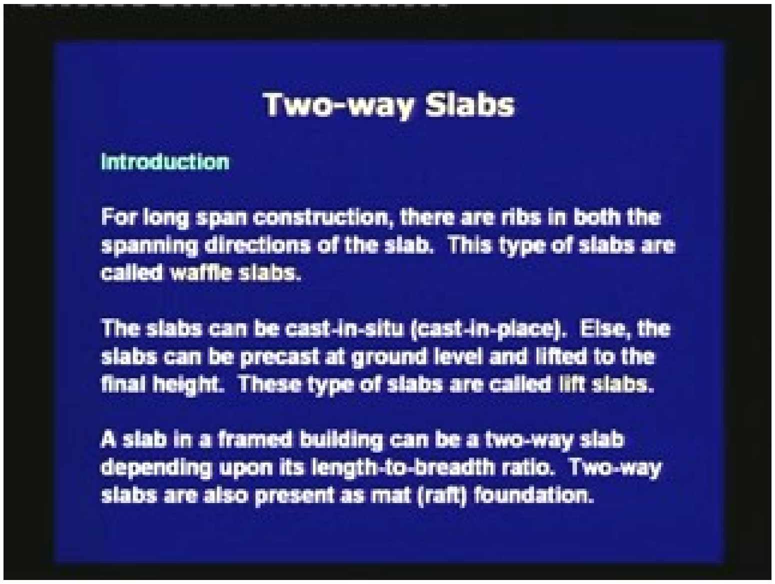

For long span construction, there are ribs in both the directions and these types of slabs are called waffle slabs. The slabs can be cast-in-situ which is also called cast-in-place. Else, the slabs can be precast at ground level and lifted to the final height. These types of slabs are called lift slabs. This is a modern construction method where to reduce the amount of the form work, the slabs are cast at the ground level and then they are jacked up to their final height and these types of slabs are called lift slabs.

A slab in a framed building can be a two-way slab depending upon its length-to-breadth ratio. Two-way slabs are also present as mat or raft foundation. It is given a building plan. First as a designer, we have to identify which slabs are one-way slabs and which slabs are two-way slabs. If the slabs are supported on all the four sides, then the decision is based on the length-to-breadth ratio.

Two-way slabs are not only present as floors or roofs; they are also present as mat or which is also known as raft foundation. In this lecture, I focus only on the floor and roof slabs.

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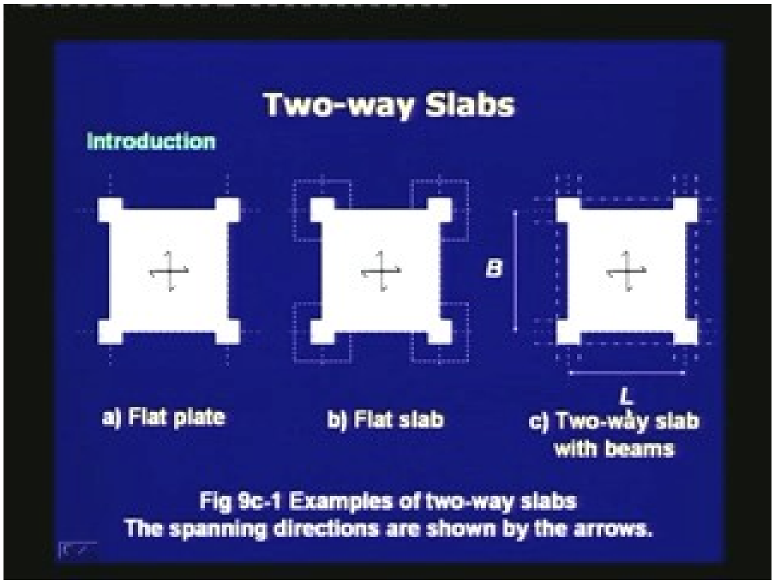

In this figure you can see the different types of two-way slabs, rectangular two-way slabs. In the first figure, it is a flat plate where there are no column capitals or drop panels. The slab is supported only on the columns at the four corners. The second type is the flat slab where there are drop panels or column capitals. The third type is a two-way slab with beams, where the beams are highlighted by the dashed lines. The arrows are spanning in both the directions to represent that it is the two-way slabs, spanning in both the orthogonal directions and hence they are to be designed for both the directions. L represents the length, which is the larger horizontal dimension and B represents the breadth, which is the shorter horizontal dimension.

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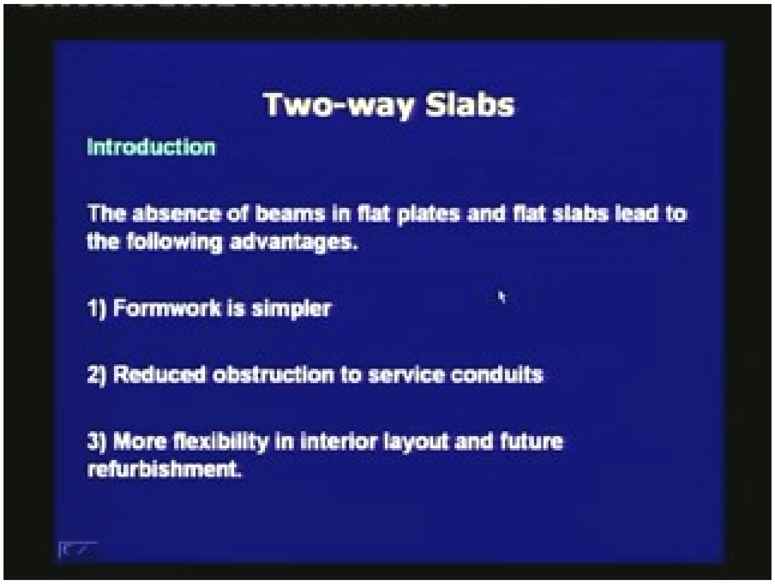

The absence of beams in flat slabs and flat plates lead to the following advantages. First, formwork is simpler. Second is reduced obstruction to services. In present day buildings, there are heating, ventilation and air-conditioning ducts and for convenience, to avoid any obstruction for the ducts, flat slab or flat plate construction is adopted. The third advantage is more flexibility in interior layout and future refurbishment. That means, once the building has been made then, the client has his/her own choice to layout the interior of the space. Now, this is more convenient if the building is made up of flat slabs or flat plates.

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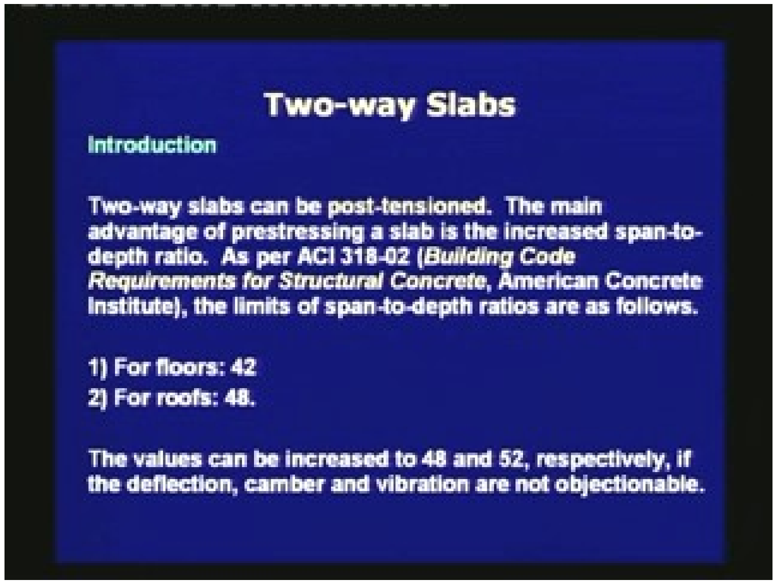

Two-way slabs can be post tensioned. The main advantage of prestressing a slab is the increased span-to-depth ratio. As per ACI 318-02, which is the building code requirements for structural concrete published by the American concrete institute, the limits of span-to-depth ratios are as follows. For floors, it is 42 and for roofs, it is 48. The values can be increased to 48 and 52 respectively, if the deflection, camber and vibration are not objectionable. Thus, we see that the main advantage of prestressing a two-way slab is to increase span-to-depth ratio. The span-to-depth ratio of a prestressed slab is much higher compared to a conventional two-way slab.

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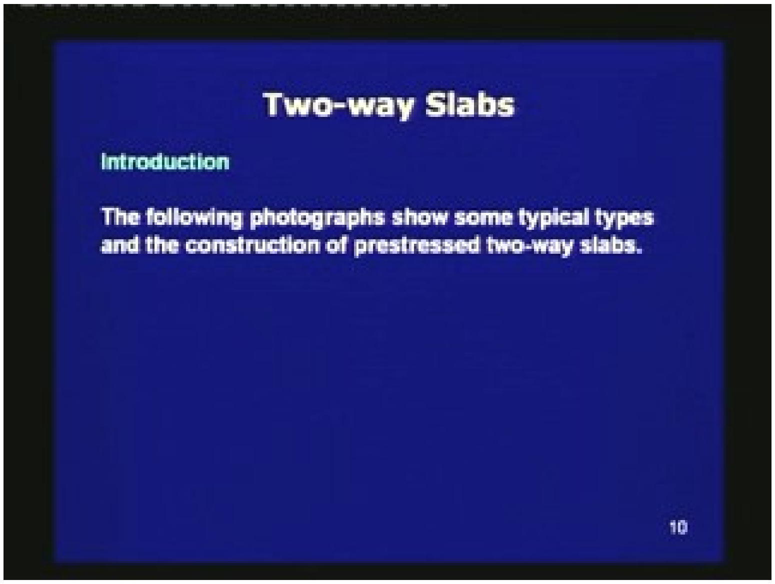

The following photographs show some typical types and the construction of the prestressed two-way slabs.

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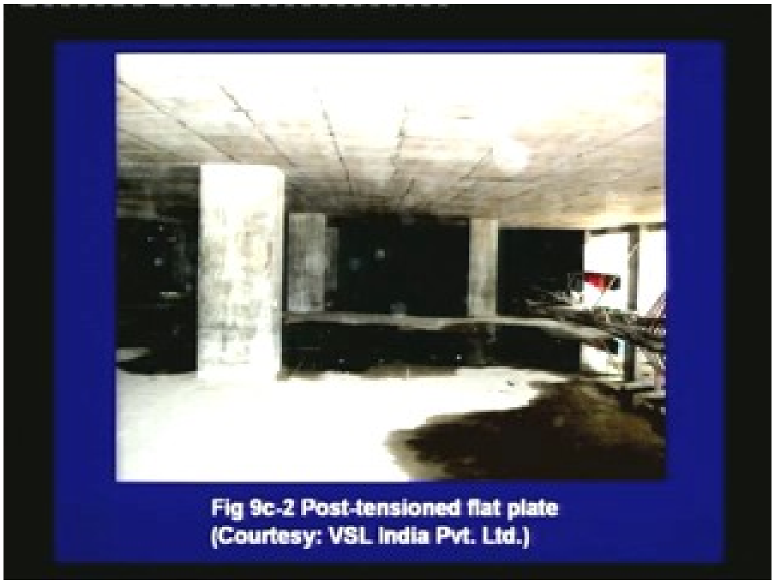

In this photograph, we can see a post tensioned flat plate after construction. We can notice that there are no column capitals or drop panels and see that the bottom of the slab is completely flat. That means this type of construction does not create any obstruction for the service conduits.

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Fig 9c-3 Post-tensioned flat slab
(Courtesy: VSL India Pvt. Ltd.)

In this figure, we can see a post tensioned flat slab where there is a drop panel. You can see that the slab has been thickened around the columns and in other columns also, you can notice the drop panel. Here also for most of the areas, the bottom surface of the slab is flat and hence, this type of construction also creates much less obstructions to the service conduits.

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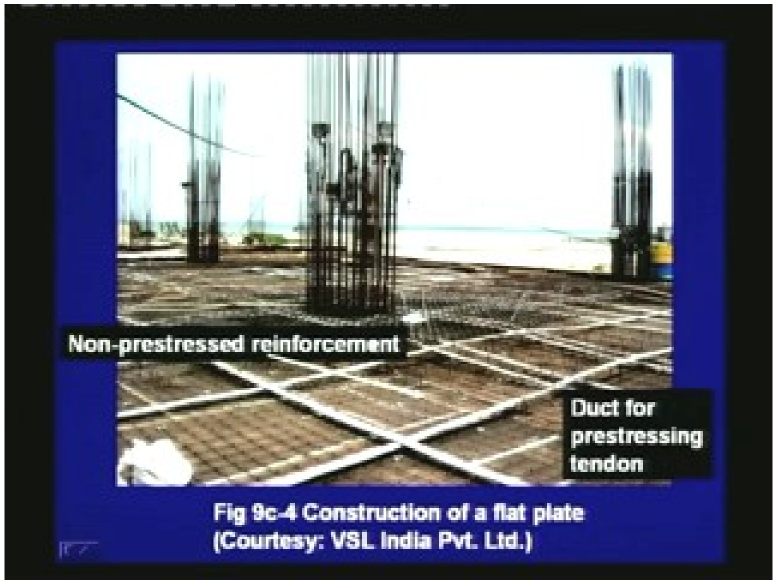

This is the construction of a prestressed flat plate. We can notice the silver ducts and the ducts for the prestressing tendency and they have been given appropriate profile for placing of the tendons which is based on the design of the flat slab. Also, you notice that non-prestressed reinforcement is provided in both the directions for the temperature and shrinkage effects. Around the columns, there are additional reinforcements to transfer the shear and also to transfer the moment from the slab to the columns. Two-way slab design needs special attention and hence it stands out as compared to a one-way slab.

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Fig 9c-5 Reinforcement at slab-to-column junction
(Courtesy: VSL India Pvt. Ltd.)

This is a closer view of the reinforcement around the slab-to-column junction. You can see that ducts have been spaced closely around the columns and also the reinforcement is much higher around the slab-to-column junction which helps to transfer the shear that generates in this junction.

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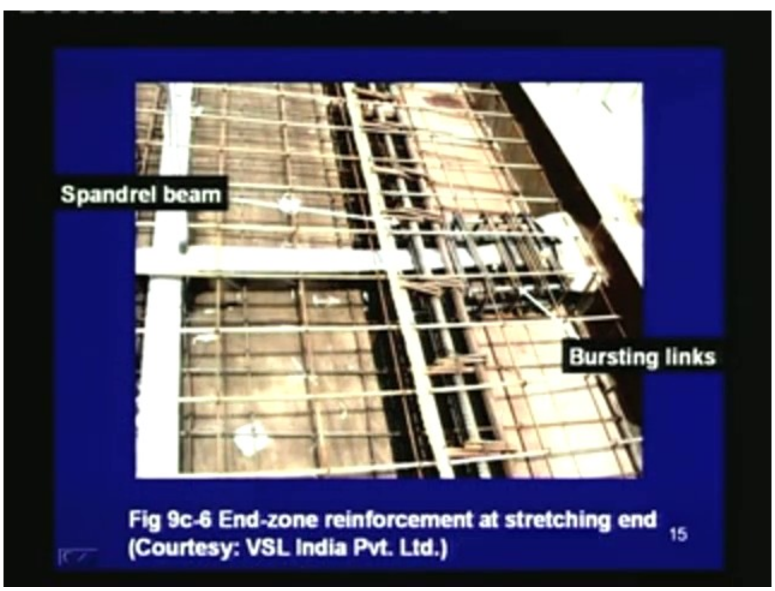

Fig 9c-6 End-zone reinforcement at stretching end
(Courtesy: VSL India Pvt. Ltd.)

This photograph shows the end-zone reinforcement at the anchorage end of the prestressing tendons. Bursting links are provided to resist the stress concentration that is generated after the prestressing operation. Also note the spandrel beam which runs along the edge of the slab and this spandrel beam is designed for torsion. Hence, the links of the stirrups that are provided in the span-beam are closed. That means there is a complete circulatory for torsion in the spandrel beams.

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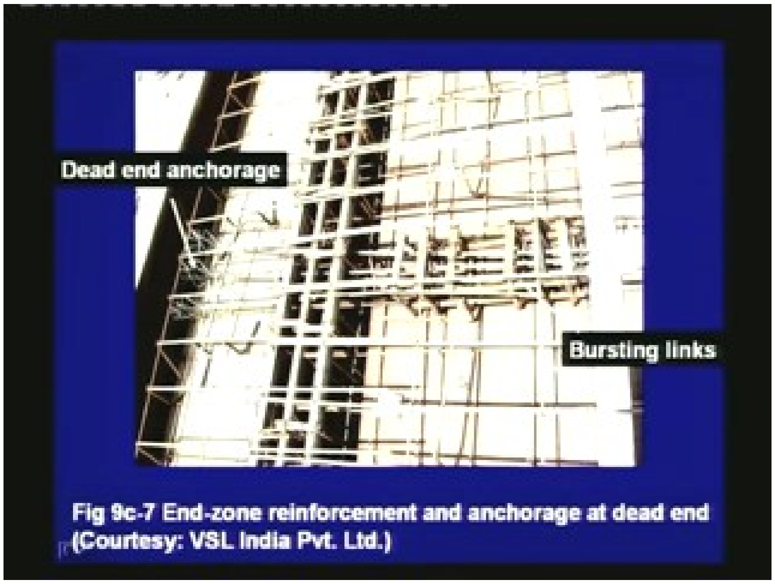

Fig 9c-7 End-zone reinforcement and anchorage at dead end
(Courtesy: VSL India Pvt. Ltd.)

This is the detailing of the other end, which is the dead end and see how the tendons has been enlarged to create the anchorage at dead end. Here also you can notice the spandrel beam with closed stirrups and also you can notice the end-zone reinforcement to check the bursting due to the concentration of stresses.

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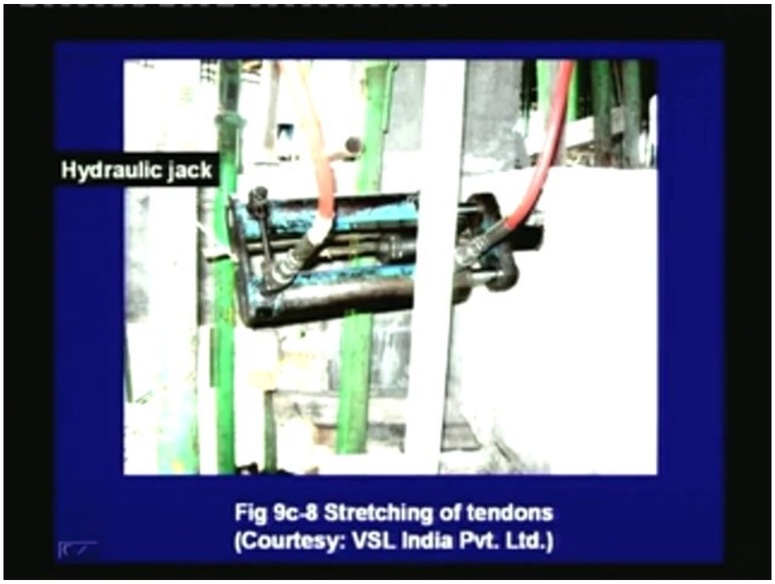

Fig 9c-8 Stretching of tendons
(Courtesy: VSL India Pvt. Ltd.)

Once the concrete is caused and it attains a desired strength then, the slab is post-tensioned by hydraulic jacks. In this photograph, you can see that the hydraulic jack has been placed at the side of the slab and the tendons are being stretched by this jack to the required amount of prestressing force

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Fig 9c-9 Anchorage block at stretching end
(Courtesy: VSL India Pvt. Ltd.)

After the stretching of the tendons, you can notice that they have anchored in the anchorage block. You can see the wedges that are holding the tendons in place and this anchorage block is transferring the prestress from the tendons to the concrete.

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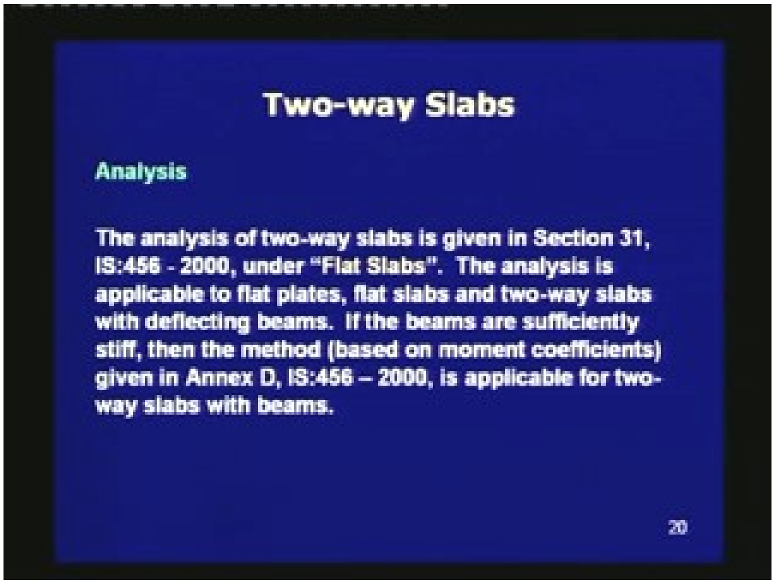

Next we are moving on to the analysis of two-way slabs. The analysis of two-way slabs is given in section 31, IS: 456-200, under "Flat Slabs". The analysis is applicable to flat plates, flat slabs and two-way slabs with deflecting beams. If the beams are sufficiently stiff, then the method which is based on moment coefficients given in Annex D of IS: 456-2000, is applicable for two-way slabs with beams. Thus if it is a flat plate or a flat slab, then we follow section 31 of IS: 456-2000. But if we have a two-way slab with beams, then we have to make a decision based on the stiffness of the beams. If the beams are not stiff enough and if the beams are also considered to be deflecting then we should adopt section 31, which is similar to flat slabs. If the beams are sufficiently stiff then we can neglect their deflection in the analysis of slabs and then we can adopt the method based on coefficients which is given Annex D of IS: 456-2000.

In this lecture, we shall cover the method for flat slabs and flat plates, and for two-way slabs, deflecting a beam which is given in section 31 of IS: 456-2000.

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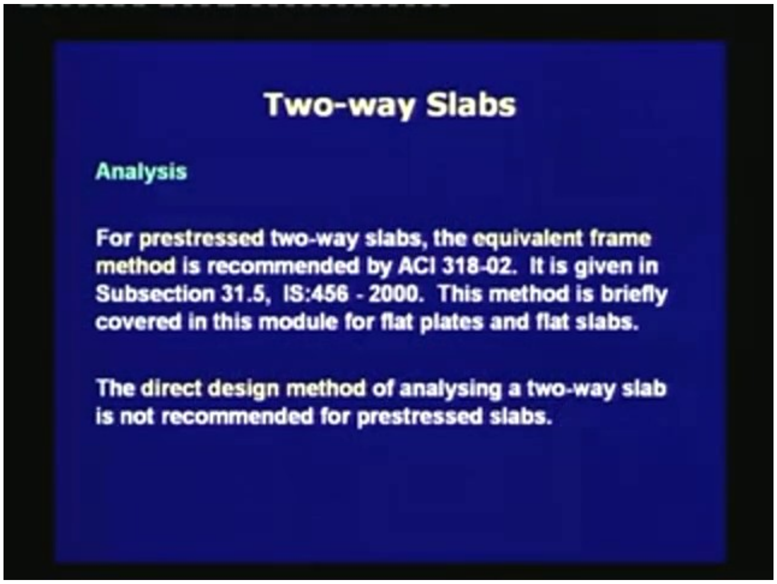

For prestressed two-way slabs, the equivalent frame method is recommended by ACI 318-02. It is given in section 31.5 of IS: 456-2000. This method is briefly covered in this module for flat plates and flat slabs. The direct design method of analyzing a two-way slab is not recommended for prestressed slabs. In section 31 of IS: 456-2000, there are 2 methods given. One is the direct design method, which is the simpler method and it is applicable for a reinforced two-way slabs if certain conditions are satisfied. For prestressed two-way slabs the second method, which is called the equivalent frames method is recommended. This recommendation comes from the building code requirement published by the American concrete institute and it is denoted as ACI 318-02.

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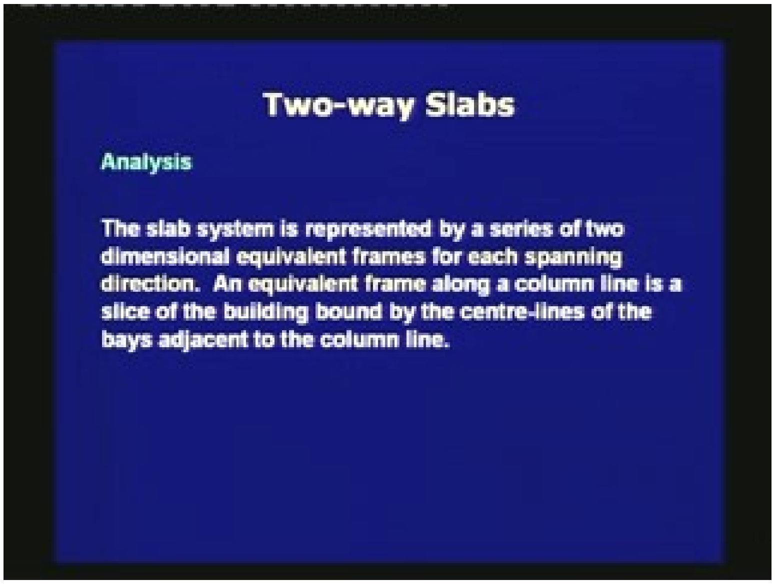

The slab system is represented by a series of 2 dimensional equivalent frames for each spanning direction. An equivalent frames along a column line is a slice of the building bound by the centre lines of the bays adjacent to the column line. As the name suggests, the equivalent frame method is based on defining equivalent frame in each orthogonal direction. Depending upon the number of types of frame in each direction, the building is analyzed for each frame and then the reinforcement is decided based on the analysis of each frame.

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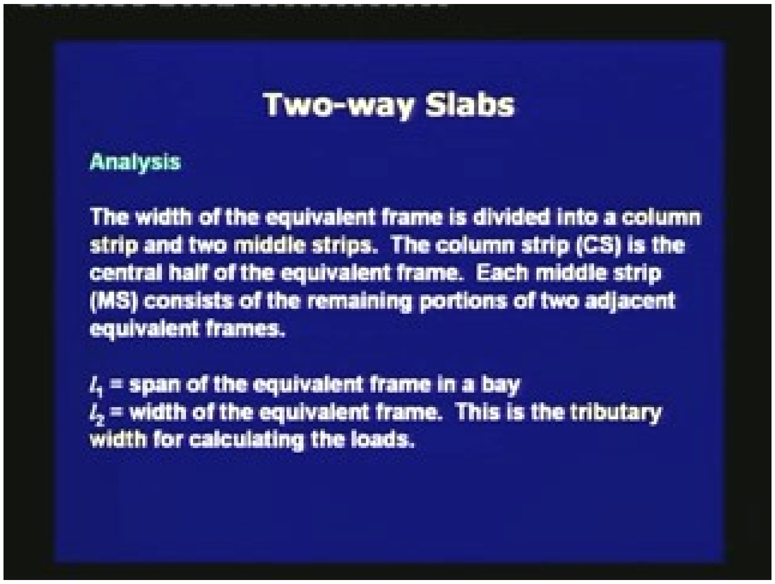

The width of the equivalent frame is divided into column strip and two middle strips. The column strip, which we denote as CS, is the central half of the equivalent frame. Each middle strip, which we shall denote as MS, consists of the remaining portions of two adjacent equivalent frames. In the following figure, we shall use l_1 as a span of the equivalent frame in a bay; l_2 as the width of the equivalent frame. This is the tributary width for calculating the loads.

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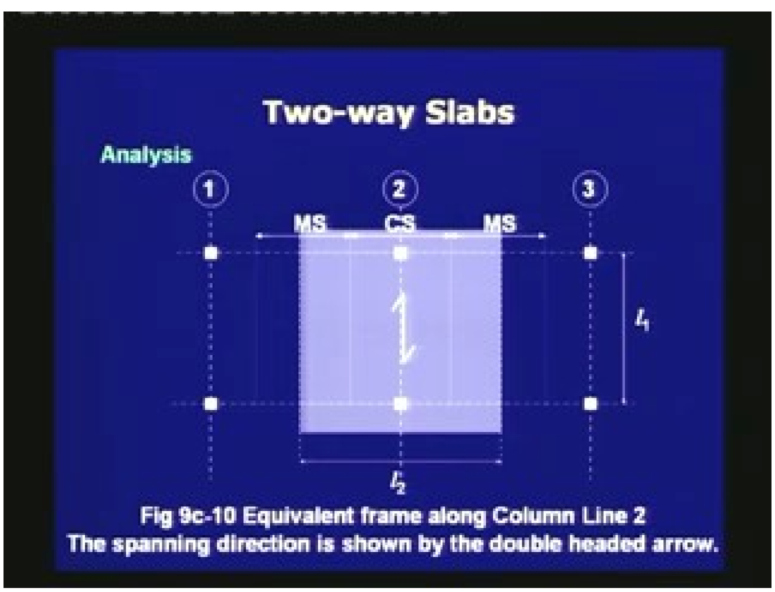

Let us try to understand the concept of equivalent frame by the sketch. As I have told before, the equivalent frame is defined in each orthogonal direction. That means in this figure, once we see the plan we identify a column line; say, in this case we have identified the column line 2 and for the column line 2, we are defining an equivalent frame which is spanning in the direction of the column line. The span in a certain bay is denoted as l_1 and the width of the equivalent frame is denoted as l_2. The width runs from the central line of one bay to the central line of another bay. The shaded part is the equivalent frame along column line 2. The equivalent frame is further divided into a column strip, which is the central half of the equivalent frame around the column line and the two portions which lies outside the column strip, are referred to as middle strips.

Note that each middle strip consists of two portions from two adjacent frames, that is, the middle strip on the left has one part from the equivalent frame along column line 1 and it has another part from the equivalent frame along column line 2. This is the first way to define an equivalent frame, given the building plan.

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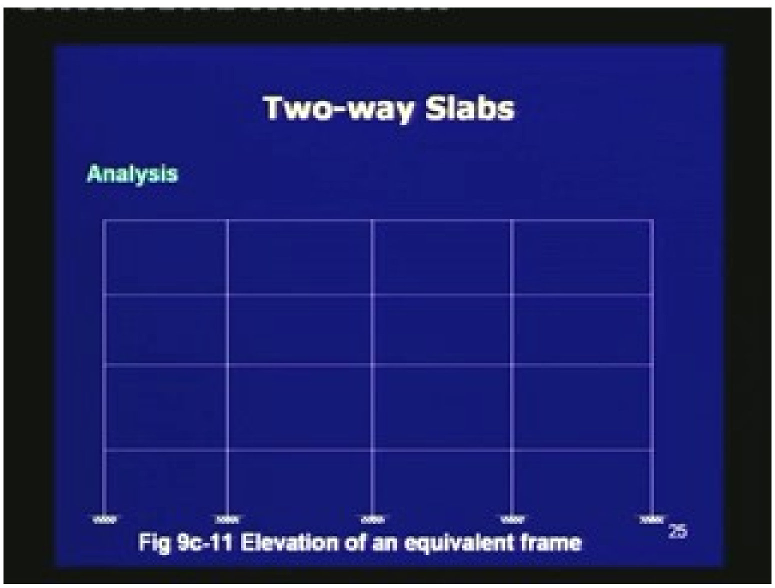

Fig 9c-11 Elevation of an equivalent frame

Once we have defined the equivalent frame in the plan, we also see the equivalent frame in the elevation. The elevation depends on the number of storeys and it also depends on whether there are any bricks in the columns or not. Now, given this equivalent frame, we start the analysis.

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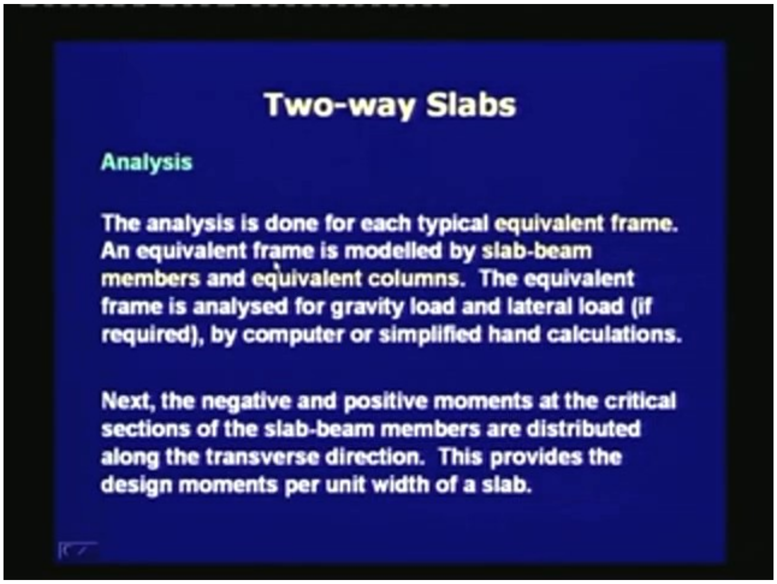

The analysis is done for each typical equivalent frame. An equivalent frame is modelled by slab-beam members and equivalent columns. The equivalent frame is analyzed for gravity load and lateral load, if required, by computer or simplified hand calculations. Next, the negative and positive moments at the critical sections of the slab-beam members are distributed along the transverse direction. This provides the design moments per unit width of the slab. The procedure is, once we have defined the equivalent frame, we analyze the equivalent frame for the gravity load which is dead load, live load or if there is no load at the roof; we may also analyze lateral loads like wind or earthquake. After the analysis is done we find out the moment in the critical section, we find out the moments in the span, we find out the moments in the support. Once we find out the moments we distribute the moments throughout the width of the equivalent frame from which we calculate the moment per unit width of the slab.

While building the equivalent frame we need to define the slab beam members which are the horizontal members and the equivalent columns which are the vertical members. If the analysis is restricted to gravity loads only, then a simplified method of analysis can be followed.

In this sketch, we find that the columns are assumed to be fixed to the remote ends.

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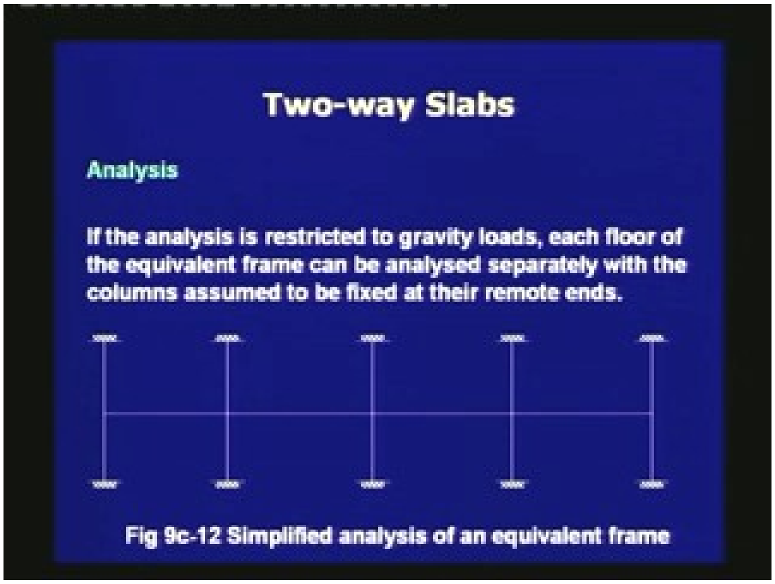

That means, for a particular flow level we can analyze that by itself and what we assume is that the columns at the far ends from that floor are fixed; that means we are not considering any influence from the loads of other floors on the moments of this floor. Once we analyze a typical floor like this, it gives the result for most of our floors because usually the loading is similar for many floors and hence our calculation gets much simpler as compared to analyzing the complete frame.

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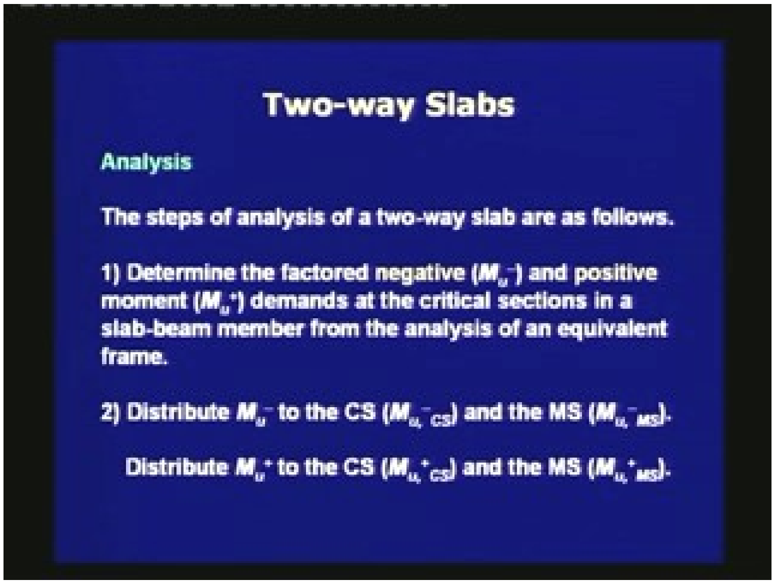

The steps for analysis of a two slabs are as follows. First, under the loads, determine the factored negative and positive moment demands at the critical section in a slab-beam member from the analysis of an equivalent frame. The negative moment will be denoted as M_u minus and the positive moment denoted as M_u plus. Thus, once we have analyzed an equivalent frame, we get these two moments. Next, distribute M_u minus to the column strip and the middle strip. The column strip moment will be denoted as M_u minus CS and the middle strip moment will be denoted as M_u minus MS. Similar, to M_u minus, we also distribute a positive moment M_u plus to the column strip and the middle strip and those are denoted as M_u plus CS and M_u plus MS, respectively.

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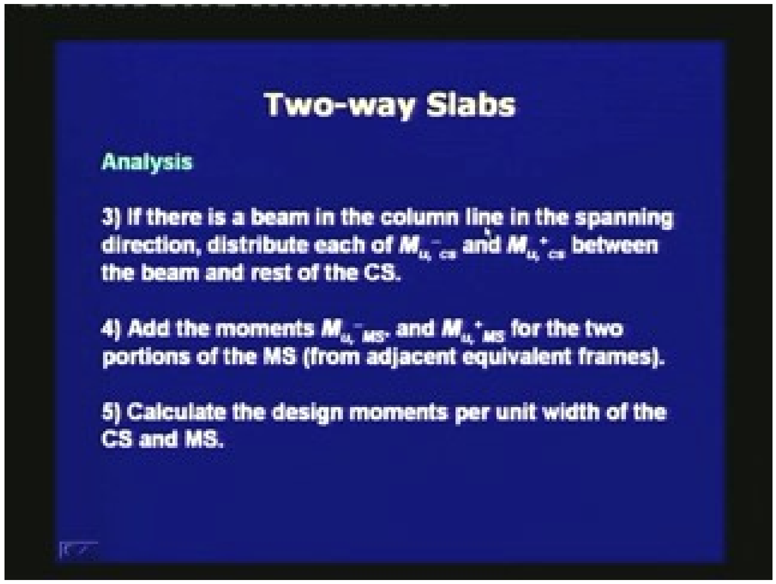

Third if there is a beam in the column line in the spanning direction, distribute each of M_u minus CS and M_u plus CS between the beam and the rest of the column strip. This is the special case. If we have beams along the spanning direction for two-way slabs with deflecting beams, in that case, the column strip moment is divided into the portion that goes to the beam in the spanning direction and the portion which goes to the rest of the column strip. The fourth step is to add the moments M_u minus MS and M_u plus MS for the two portions of the middle strip which are from adjacent equivalent frames. We have calculated the middle strip moments for one equivalent frame. We can calculate the middle strip moment from the adjacent equivalent frame and since the middle-strip consists of two portions from the two equivalent frames, we add these two moments and get the total moment over the middle strip.

Fifth, calculate the design moments per unit width of the column strip and middle strip. Thus, once we get the total moments acting in a column-strip and the middle-strip and given their width, we can find out what is the moment per unit width of the column strip and middle strip and based on the moment per unit width, we can design the reinforcement.

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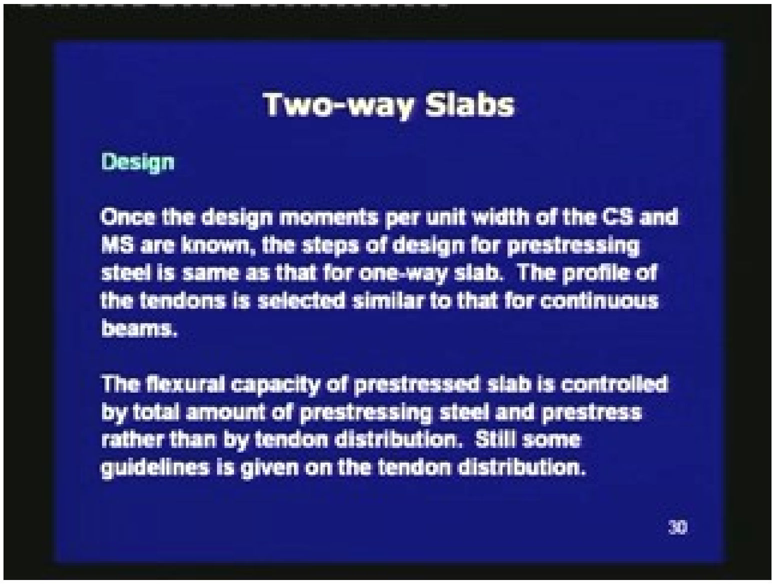

Once the design moments per unit width of the column strip and middle strip are known the steps of design for prestressing steel is same as that for one-way slab. The profile of the tendons is selected similar to that for continuous beams. We had studied how to design the prestressing steel for a one-way slab. For a two-way slab once we know the design moments per unit width, then the design is similar to that of one-way slab and the selection of the cable profile is similar to continuous beams that we had studied earlier.

The flexural capacity of prestressed slab is controlled by total amount of prestressing steel and prestress rather than by tendon distribution. Still some guidelines are given on the tendon distribution. It has been found that the flexural capacity of a slab is governed by the total amount of the prestressing tendons and the prestressing force. That means it is independent of the distribution of the prestressing tendons. Hence, there is flexibility in placing the prestressing tendons but still there are some guidelines to distribute the tendons.

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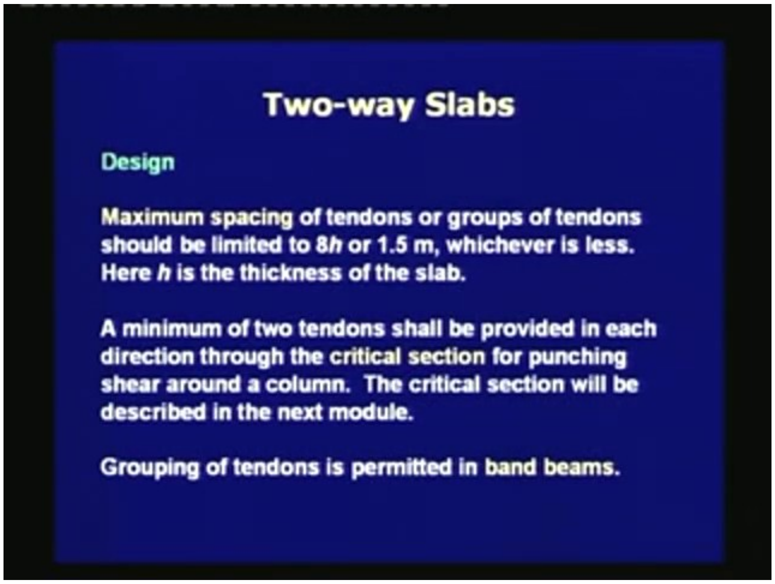

The maximum spacing of tendons or groups of tendons should be limited to 8 times h or 1.5 meters, whichever is less. Here, h is the thickness of the slab. Thus, the maximum spacing of tendons or groups of tendons should be 8 times the thickness of the slab or 1.5 meter, whichever is less. A minimum of 2 tendons shall be provided in each direction, through the critical section for punching shear around the column. The critical section shall be described in the next module. We had seen that around the column that area is subjected to higher reinforcement and it is recommended that at least two groups of tendons should pass through the critical section around the slab to column junction. The definition of critical section will be covered in the next continuing module. Grouping of tendons if permitted in band beams; that is the purpose having a wide beam is to accommodate the tendons in its width.

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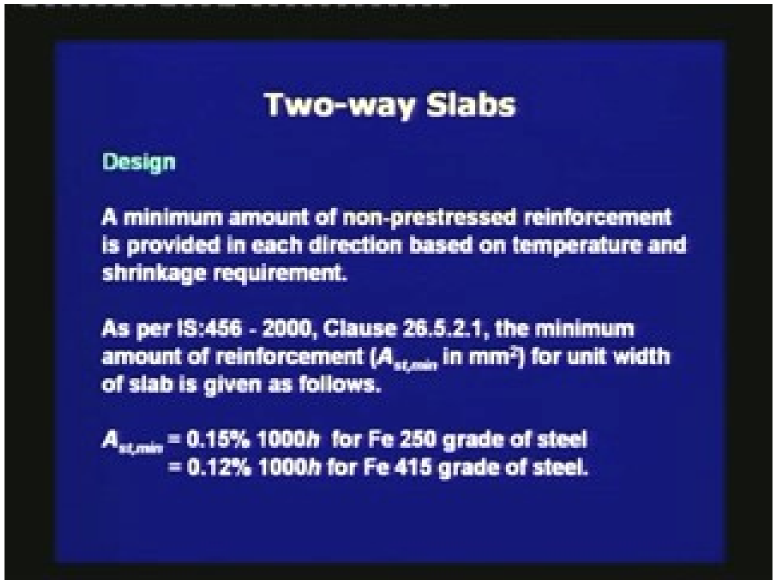

A minimum amount of non-prestressed reinforcement is provided in each direction based on temperature and shrinkage requirement. As per IS:456-2000, clause 26.5.2.1, the minimum amount of reinforcement which is $A_{st,min}$ in millimeter square, for unit width of slab is given as follows. $A_{st,min}$ is equal to 0.15% times 1000 times h, for Fe 250 grade of steel and $A_{st,min}$ is equal to 0.12% times 1000 times h, for Fe 415 grade of steel. Here, we are considering a unit width of the slab in each direction and hence the breadth is equal to 1000 millimeters and the reinforcement is based on the total cross-section of the unit width of the slab.

Next, we are describing features in modelling and analysis of an equivalent frame.

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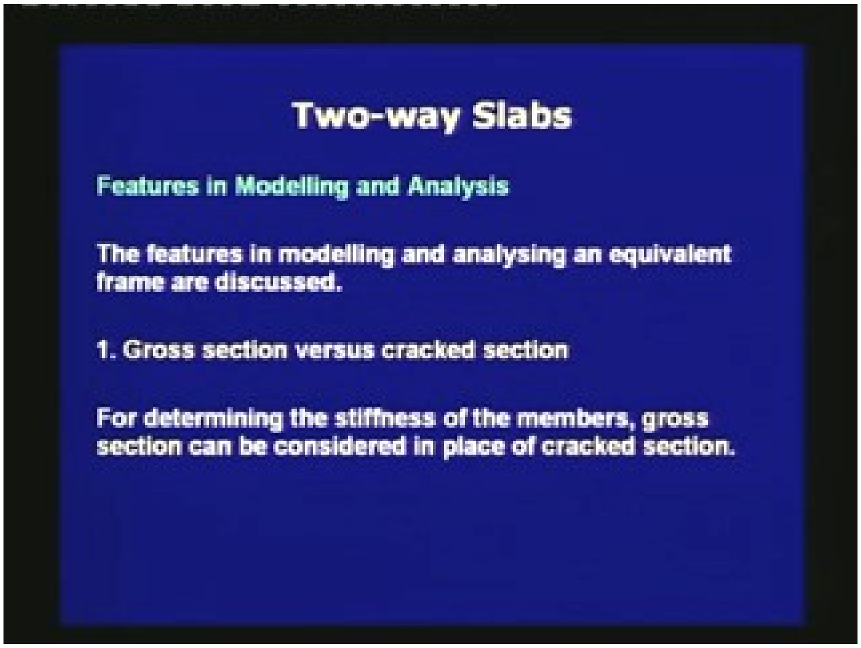

First is the gross section versus cracked section. The difficulty in analyzing a reinforced concrete structure is that the sections may become cracked if the load exceeds the service loads. The chances of cracking are reduced when the slabs are prestressed. The code allows us to use the gross cross-section instead of the cracked section which is difficult to find out. Thus for determining the stiffness of the members, gross section can be considered in place of cracked section.

When we are developing a model in computer, we have to assign the stiffness of the slab beam members and the column members. We can assign the gross sectional properties and if we are consistent throughout the building then, the forces do not make any difference if you have assigned cracked section properties. Since, the gross cross-section properties are easier to calculate, the code allows us to use the gross section properties.

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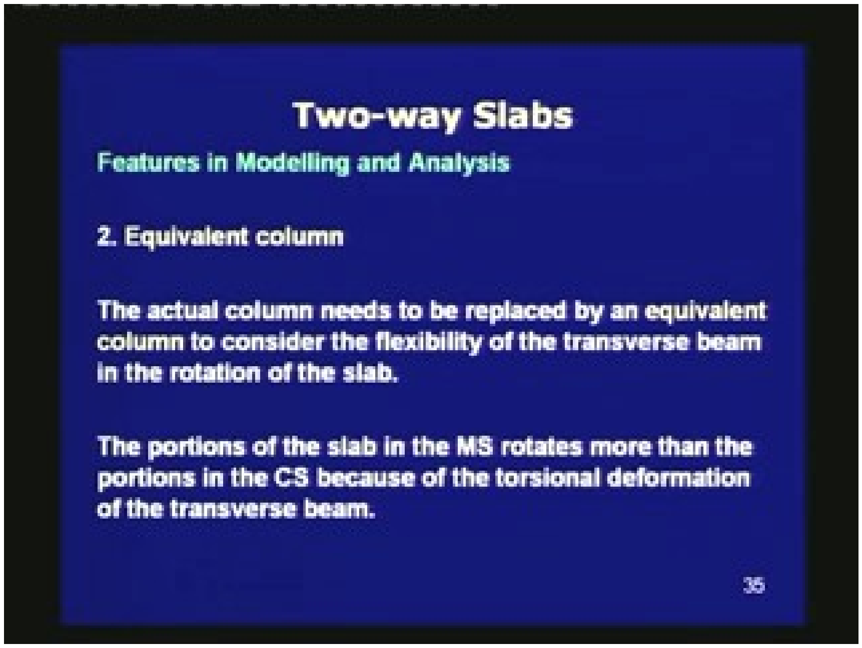

Second is the concept of equivalent column. The actual column needs to be replaced by an equivalent column to consider the flexibility of the transverse beam in the rotation of the slab. The portions of the slab in the middle strip rotate more than the portions in the column strip because of the torsional deformation of the transverse beam.

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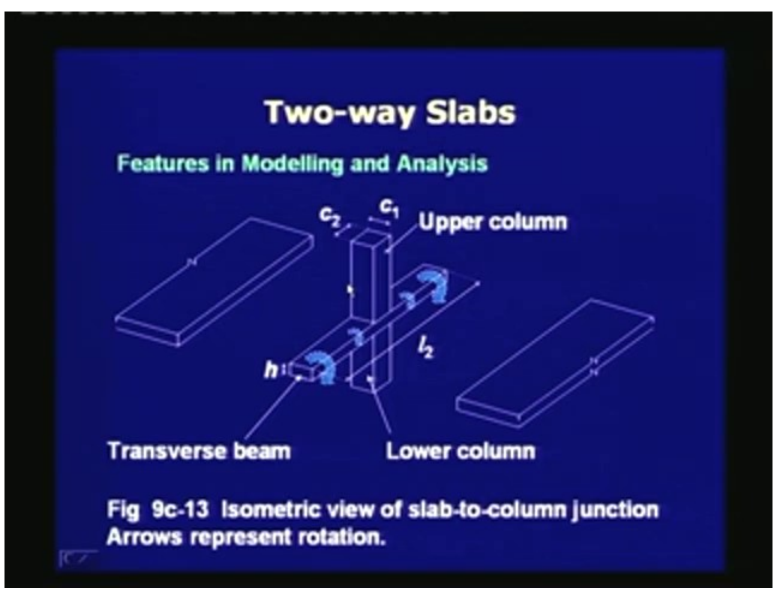

Let us understand this by a sketch. This is a sketch of a slab-to-column junction and we have split to show the behaviour across the width of equivalent frame. The arrows represent the rotation in the slab. If you are closer to the columns, then the rotation is less. As we are moving out from the column, the rotation increases. Thus, the flexibility in the slab is more away from the columns than closer to the columns.

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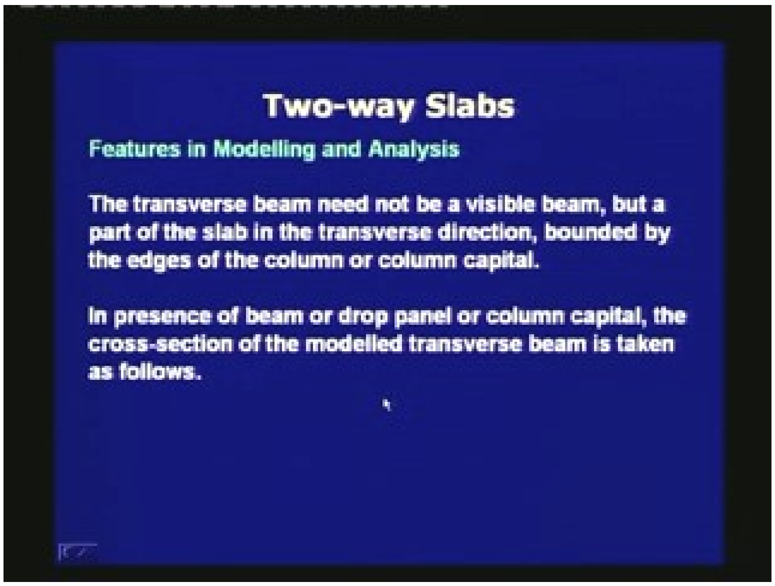

To account of this increased flexibility, as we move out from the column an equivalent column is defined in place of the actual column. The transverse beam need not be a visible beam, but a part of the slab in the transverse direction bounded by the edges of the column or column capital. In presence of beam or drop panel or column capital, the cross-section of the modelled transverse beam is taken as follows.

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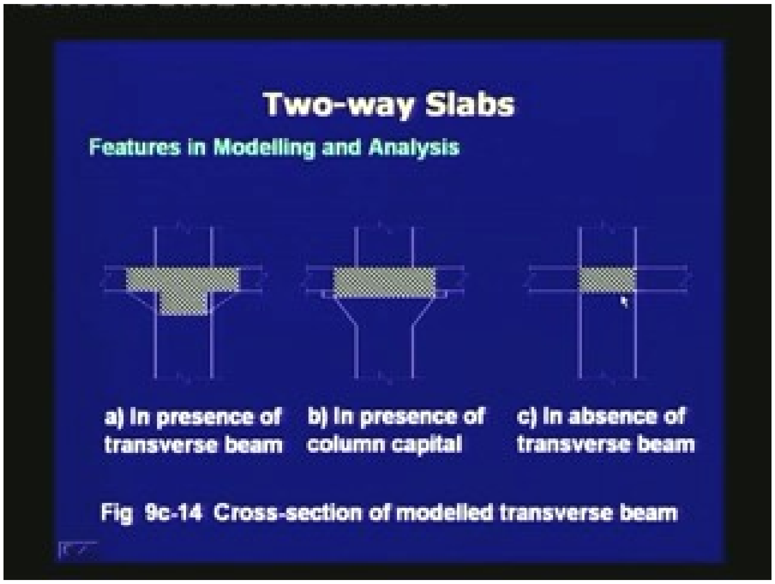

In this sketch, we are seeing the cross-section of the transverse beam that is considered. As I said, the transverse beam need not be a visible beam. It can be a part of the slab which is bound by the column. In this figure, in the right hand side there is no visible beam, but the modelled transverse beam is the part of the slab which is bound between the column faces. In the middle one, there is a column capital and a drop panel and we see that the part of the drop panel which is bound within the column capital is selected as the transverse beam. In the first figure, there is a visible transverse beam and if there is a transverse beam then we also consider a part of the slab to be a part of the transverse beam; this part of the slab is determined by a 45 degree projection of this outstanding part of the transverse beam. Thus once we are able to identify the transverse beam, we can find out the properties of the equivalent column.

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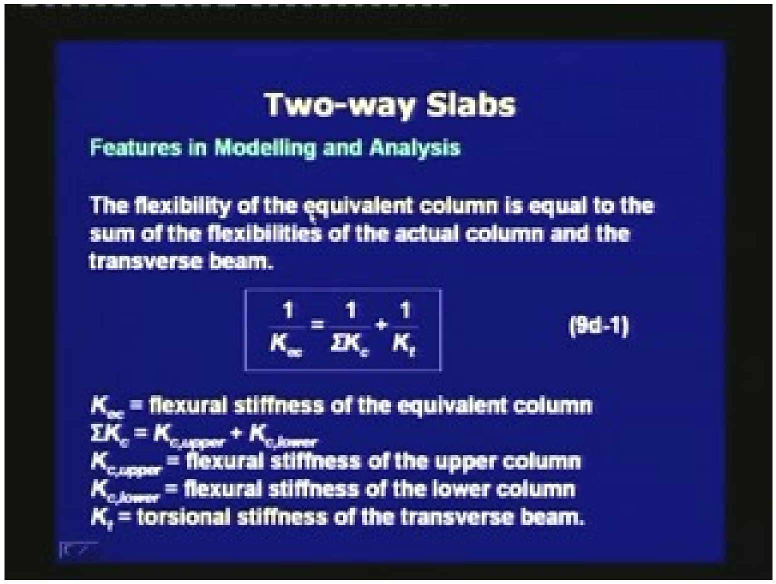

The flexibility of the equivalent column is equal to the sum of the flexibilities of the actual column and the transverse beam. This is the basic idea of defining the equivalent column that we are adding up the flexibilities of the column and the transverse beam in the rotation of the slab to column junction. 1 by K_{ec} is equal to 1 by sigma K_c plus 1 by K_t. Here, we are adding the flexibilities to consider the increased rotation of the slab as we move away from the column. K_{ec} is equal to the flexural stiffness of the equivalent column, sigma K_c equal to $K_{c,upper}$ plus $K_{c,lower}$, where $K_{c,upper}$ is the flexural stiffness of the upper column, $K_{c,lower}$ is the flexural stiffness of the lower column and K_t is the torsional stiffness of the transverse beam. Thus, what we see in this expression is in the adding up of flexibilities which is the inverse of stiffness. K_c is the flexural stiffness of a column and K_t is the torsional stiffness of the transverse beam. When we appropriately add the flexibility, we get the flexibility of the equivalent column from which we calculate the stiffness of the equivalent column.

An approximate expression for the flexural stiffness of a column K_c is given as follows.

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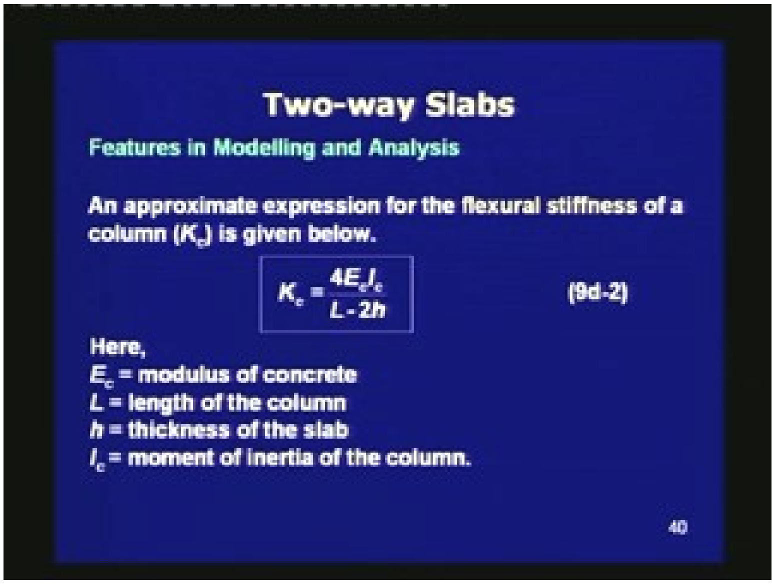

K_c is equal to 4 E_c times I_c divided by L minus 2 h. This is very close to the expression that we learnt from the slope deflection methods in basic structure analysis. E_c is the modulus of the concrete, L is the length of the column, h is the thickness of the slab and I_c is the moment of the inertia of the column. Thus, we have an expression of the flexural stiffness of a column which we can apply to the upper column and the lower column.

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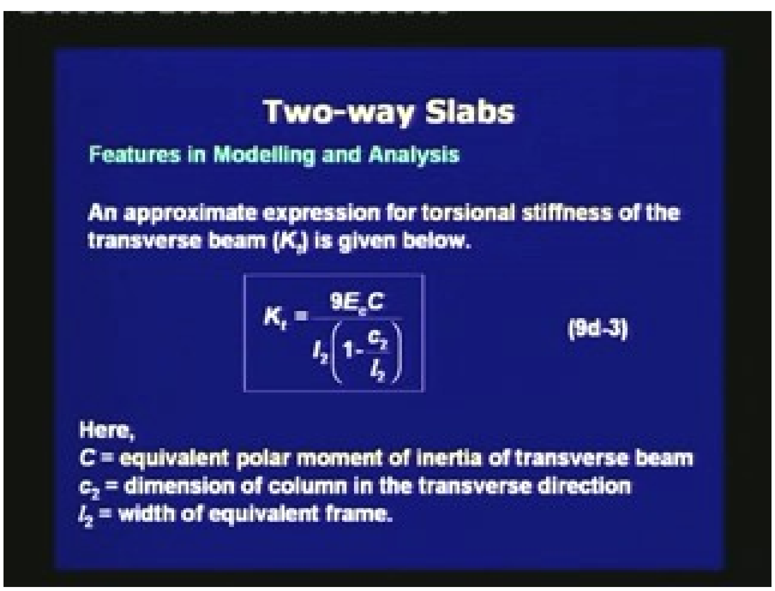

An approximate expression for the torsional stiffness of the transverse beam, K_t is given below. K_t is equal to 9 times E_c times C divided by I_2 times, within bracket, 1 minus C_2 divided by l_2. Here, C is the equivalent polar moment of inertia of transverse beam, C_2 is the dimension of the column in the transverse direction, l_2 is the width of the equivalent frame.

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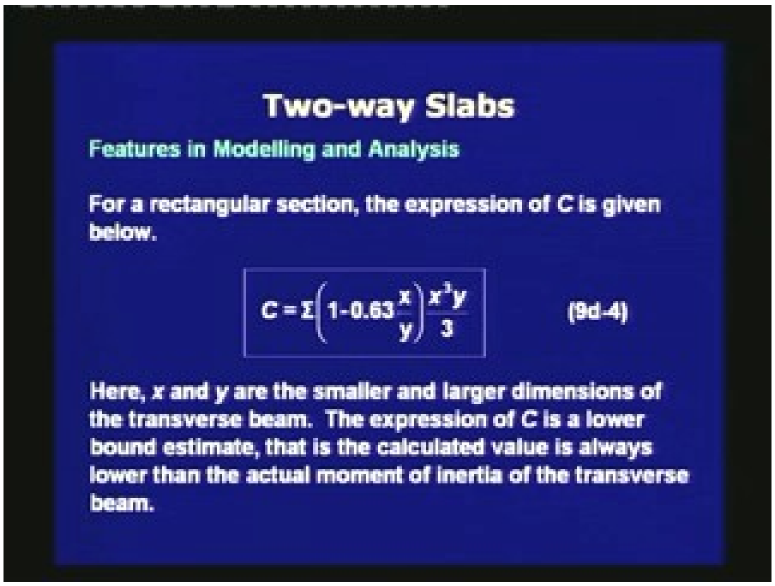

For a rectangular section the expression of C is given below. Note that C is the quantity, which is analogues to the polar moment of inertia for a circular section. For a rectangular section such a quantity is difficult to define exactly and an approximate expression is given as follows. C is equal to the summation of, within the bracket, 1 minus 0.63 times x divided by y times x cube times y divided by 3. The summation is taken over the component rectangles of a compound section which we shall come later; but let us first understand what is x and y.

Here, x and y are the smaller and larger dimensions of the transverse beam for a particular rectangle. The expression of C is a lower bound estimate, that is, the calculated value is always lower than the actual moment of inertia of the transverse beam. Again as I said, this is an approximate expression of a quantity which is equivalent to the polar moment of inertia for a circular section.

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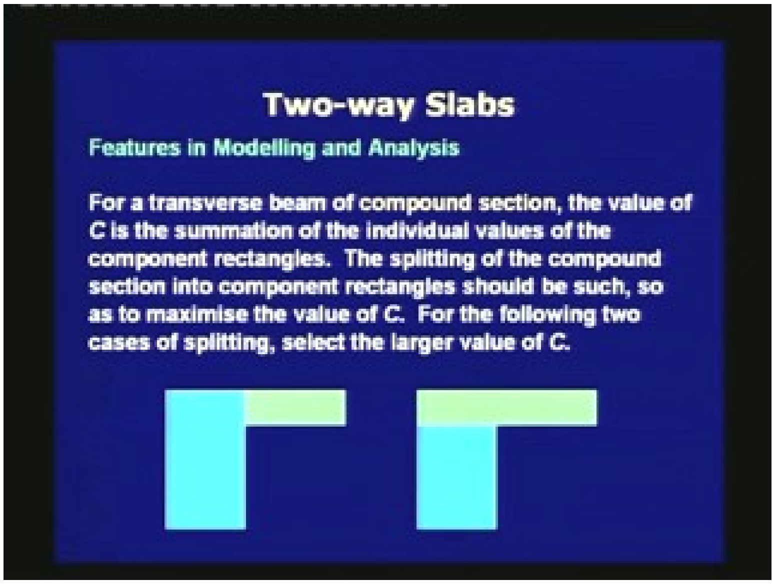

When we have a compound section, we make a summation over the individual rectangles to find out the value of C. The splitting of the compound section into component rectangles should be such so as to maximize the value of C. As I said that C is anyhow lower bound; that means, the actual value cannot be lower than the value calculated based on the individual rectangles. Hence, we can divide a compound section in such a way that we can get the maximum value of C, which anyhow will be lower than the true value. For the following two cases of splitting, select the larger value of C. Thus, if we have an L shaped transverse beam then we can either split it into two rectangles as shown on the left or we can split into two rectangles which is on the right.

Now for each rectangle, we have to identify x and y which are the smaller and larger dimensions, respectively; place those values in the expression, take the summation and calculate the value of C. Once we have got the value of C for both the schemes of splitting, we pick up the value which is higher. Thus, once we calculate C and the other geometric properties we can find out the transverse beam stiffness, the torsion stiffness, which is denoted as K_t.

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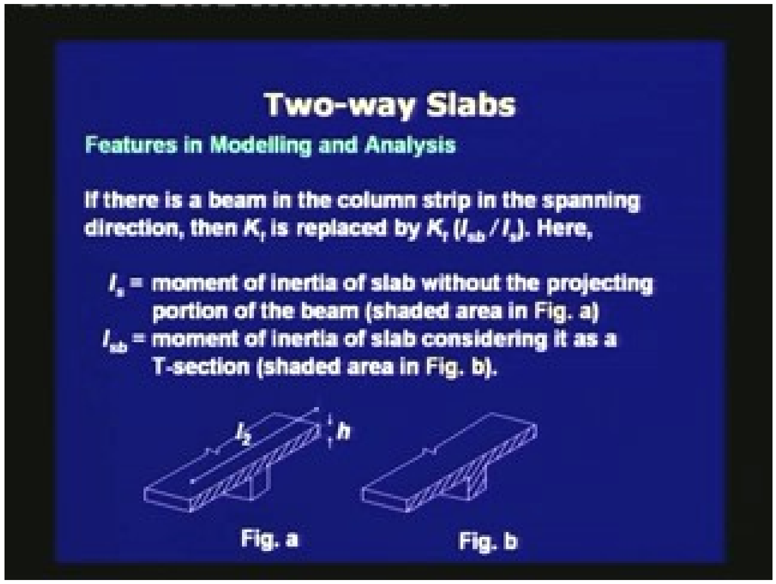

If there is a beam in the column strip in the spanning direction, then K_t is replaced by K_t times I_{sb} divided by I_s. Here, I_s is the moment of inertia of slab without the projecting portion of the beam as shown in the shaded area in figure a. That is, in this figure whatever has been shaded in the cross-section of the slab beam member without the projecting beam is considered to calculate I_s and I_{se} is the moment of inertia of slab considering it as a T-section; shaded area in figure b and I_{sb} considers the full cross-section of the slab beam member. Thus, if there are beams in between the column lines which may be present in a two-way slab with deflecting beams, in that case we reduce K_t to consider the flexibility of the beam also and it is changed proportionately.

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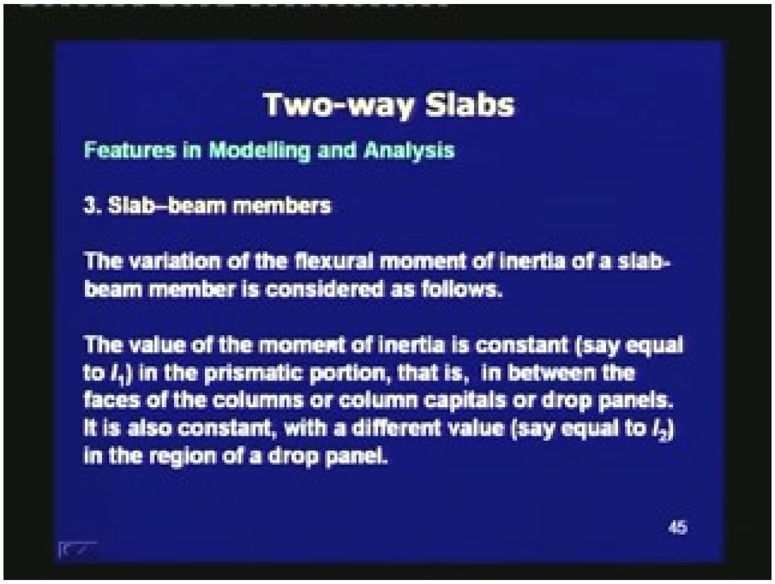

Third, we are defining the slab beam members. The variation of the flexural moment of inertia of a slab beam member is considered as follows. The value of the moment of inertia is constant, say equal to I_1, in the prismatic portion; that is, in between the faces of the columns or column capitals or drop panels. It is also constant, with a different value in the region of a drop panel. A slab beam member is the horizontal member; its flexural stiffness is different in the span as compared to the values in the drop panel.

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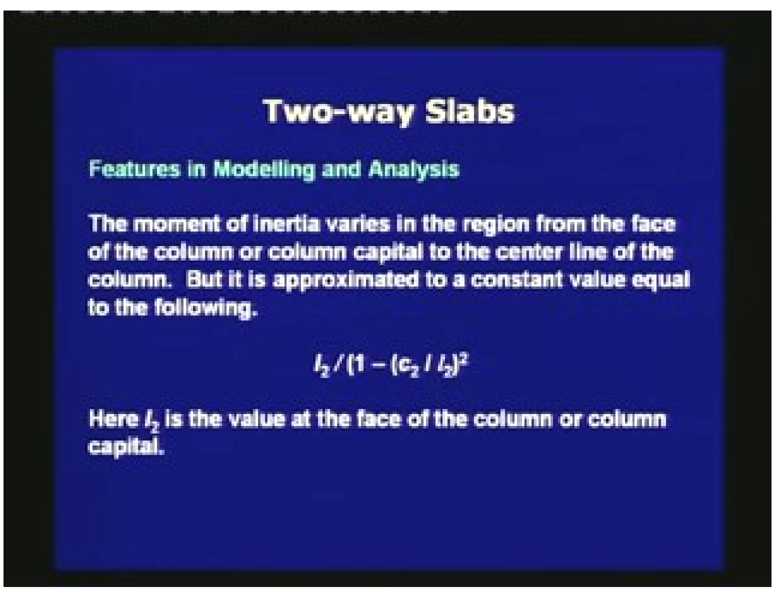

What values to be picked up is recommended by the code. Since the moment of inertia varies in the region from the face of the column or column capital to the center line of the column, it is approximated to a constant value equal to the following. I_2 divided 1 minus C_2, divided by l_2 the whole squared. Here, I_2 is the value at the face of the column or column capital.

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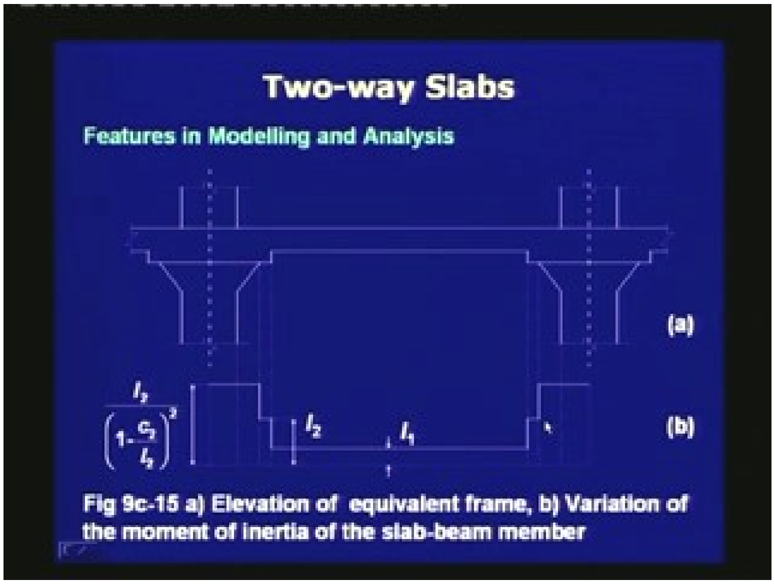

Let us understand the recommendation by the help of a sketch. The top figure is the elevation of an equivalent frame at a particular flow level within a bay and we see that the flexural moment of inertia is varying along the span of the slab beam member. In the middle region, it is constant to a value of I_1; near the drop panels it has a value of I_2 and from the face of the column capital to the center line of the column it varies. The code says that in this region from the phase of the column capital to the center of the column, we can consider flexural moment of inertia equal to I_2 divided by 1 minus C_2 by l_2 whole squared. Thus, for this particular case, you have 3 values of the moment of inertia for the slab beam member: I_1, I_2 and finally, a value which is from the face of the column capital to the center line of the column.

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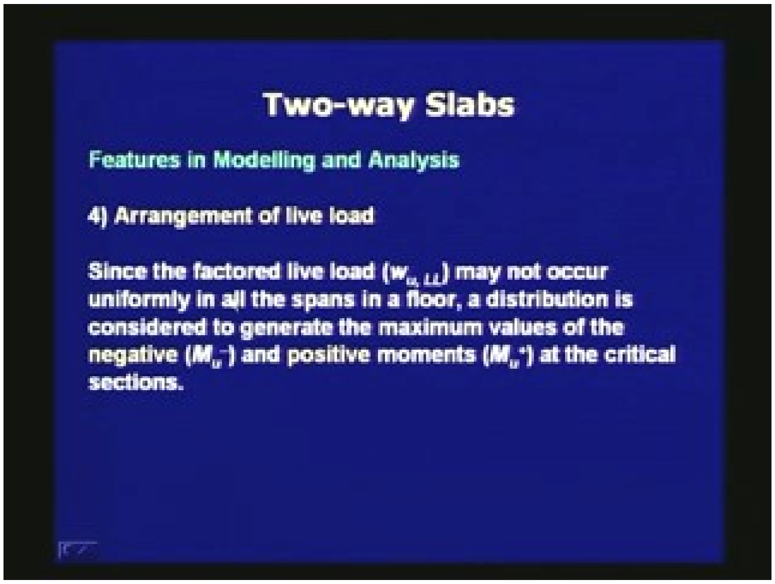

The fourth point is the arrangement of live load. Since the factored live load $W_{u,,LL}$ may not occur uniformly in all the spans in a floor, a distribution is considered to generate the maximum values of the negative and positive moments at the critical section. When we are applying the gravity load, we have to be careful that if the live load should be placed uniformly or not. The dead load can be placed as is given for the member section but the live load may not be uniform throughout the span and for the different spans in the equivalent frame.

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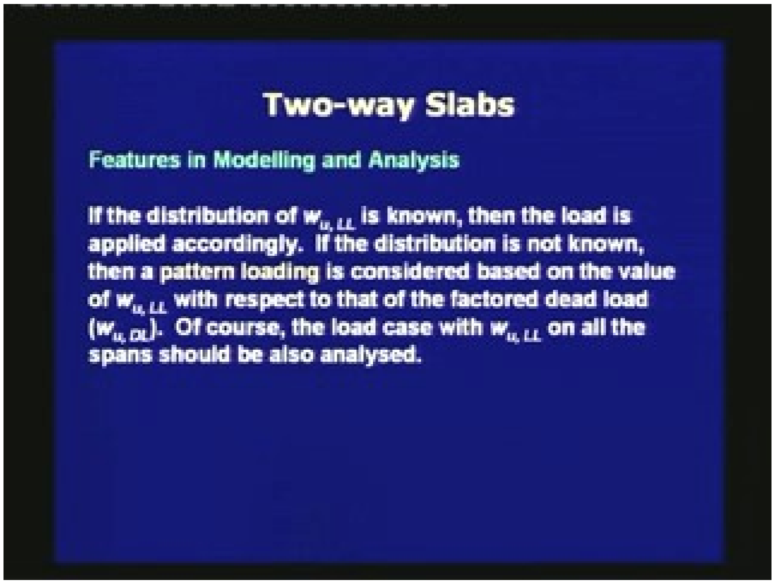

The coded recommendation is, if the distribution of $W_{u,LL}$ is known, then the load is applied accordingly. If the distribution is not known, then a pattern loading is considered based on the value of $W_{u,LL}$ with respect to that of the factored dead load $W_{u,Dl}$. The load case with $W_{u, LL}$ on all the spans should be also analyzed.

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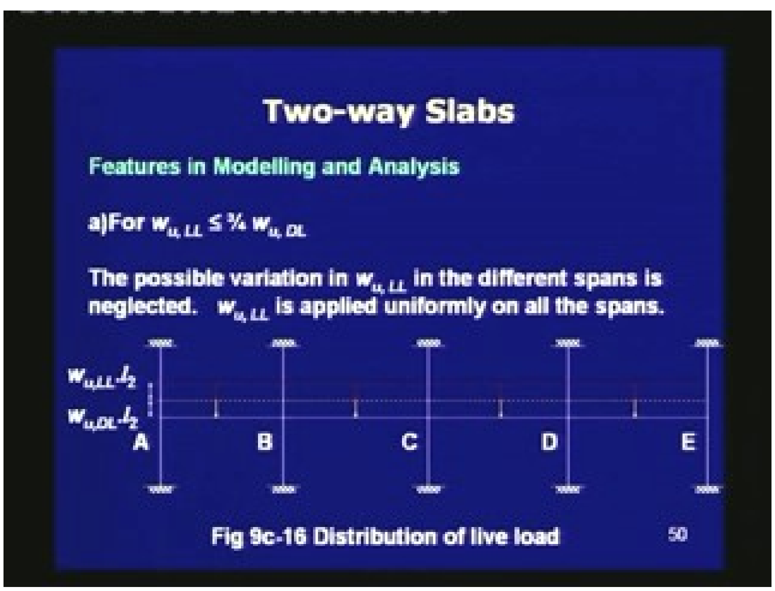

Let us understand this with the help of a sketch. For the live load to be less than three quarters of the dead load, which is the case for most of the buildings, the possible variation in $W_{u, LL}$ in the different spans is neglected. Thus, the live load can be applied uniformly only if the live load is less than three quarters of the dead load. In this sketch, you see that the live load has been applied uniformly in all the spans and we are analyzing the member with that loading scheme.

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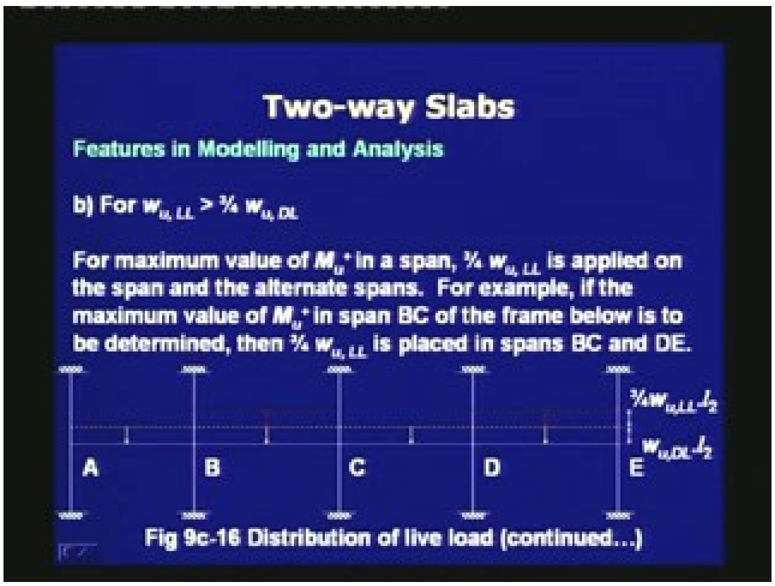

If the live load exceeds three quarters of the dead load, that means, the live load is substantially high then, for maximum moment of M_u plus in a span three quarters of $W_{u,LL}$ is applied on the span and alternate spans. For example, if the maximum value of M_u plus in span BC of the frame below is to be determined, then three fourth of $W_{u,,LL}$ is placed in spans BC and DE. That means, if we are looking for the mid-span moment of BC, we shall place the live load in BC and alternate span like DE. The value of the live load is three fourth of the live load coming in the tributary width, which is $W_{u,,LL}$ times l_2. For maximum value of M_u minus near the support, three fourth $W_{u, LL}$ is applied on the adjacent spans only. For example, if the maximum value of M_u minus near support B is to be determined, then three fourth $W_{u, LL}$ in placed in spans AB and BC. Thus, if you are looking for the negative moment at the support, we place the live load only on the adjacent spans. Thus, we have placed the live loads in AB and BC only; this may give a critical value of the negative moment at B.

We also have to analyze the case where the live load is uniformly placed in all the spans. That is, from this load conditions we find out the maximum values of the positive moments in the span and the negative moments in the support and these maximum values are called envelope values from which we design our slab.

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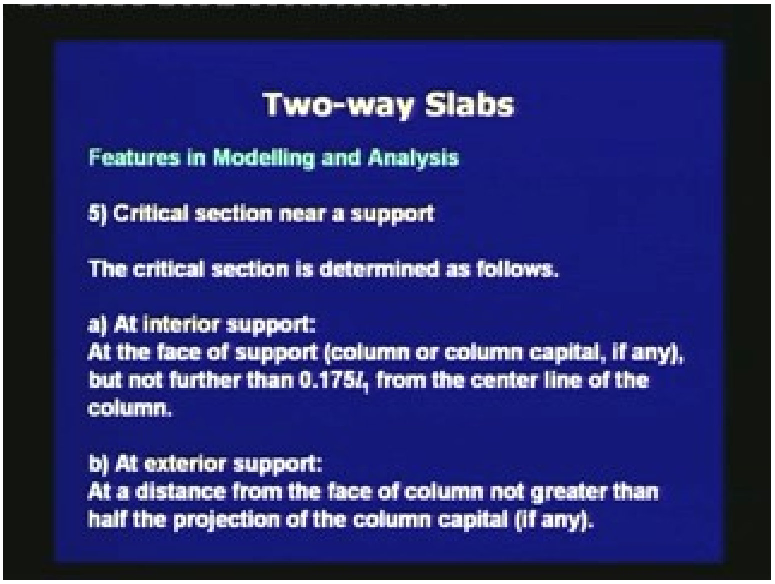

The fifth consideration is the defining the critical section near a support. The critical section is determined as follows. At an interior support, it is the face of support which is the column or column capital if any, but not further than 0.175 l_1 from the center line of the column. For an exterior support, it is at a distance from the face of column not greater than half the projection of the column capital, if any.

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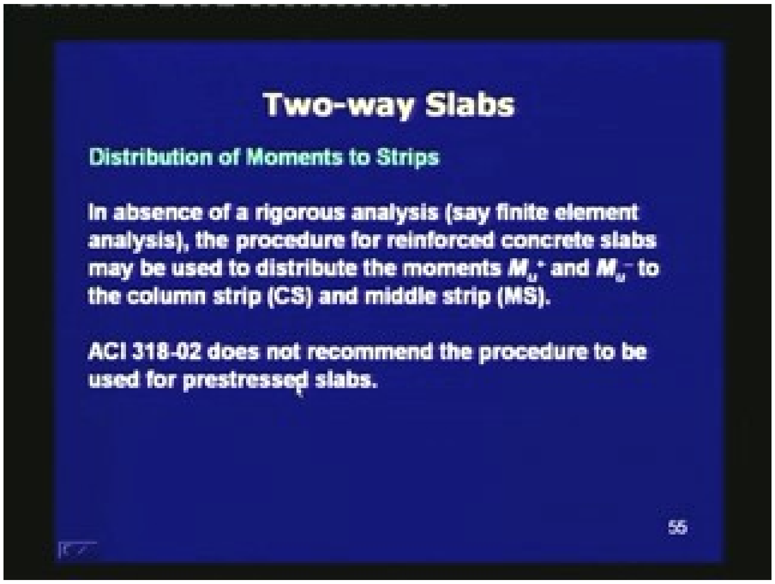

Next, we understand the distribution of moments in the two strips, column strips and middle strips. In absence of rigorous analysis, say, finite element analysis, the procedure for reinforced concrete slabs may be used to distribute the moments M_u plus and M_u minus to the column strip and middle strip. ACI 318-02 does not recommend the procedure to be used for prestressed slabs. Thus, we can use this procedure of distributing the moments with caution. It is preferred that we do a more vigorous analysis to find out the moments in the column strip and the middle strip.

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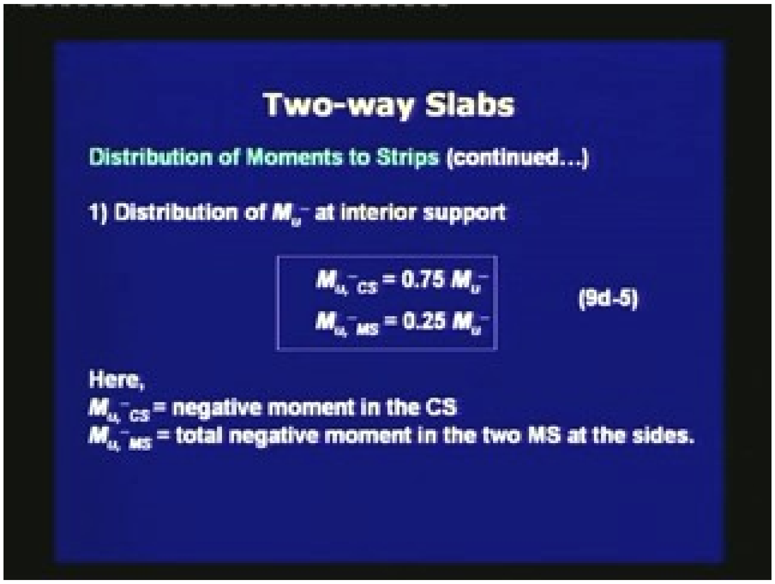

The first is that the distribution of M_u minus and interior support is as follows. For the column strip, it is 75% of M_u minus; for the middle strip on the two sides, total moment is 25% of the M_u minus. Remember that M_u minus MS is the total negative moment in the two middle strips in the 2 sides.

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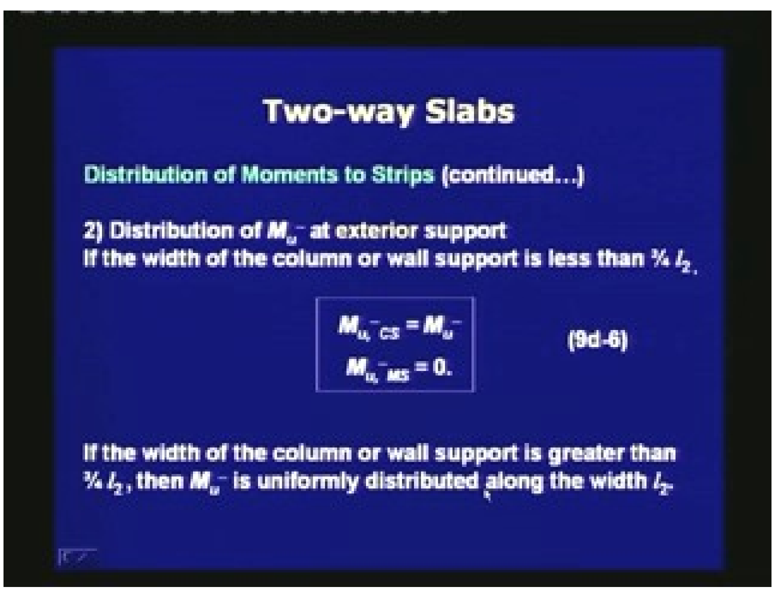

For distribution of M_u minus at exterior support, if the width of the column or wall support is less than three quarters of l_2, then M_u minus CS is the total moment at the support which is M_u minus and the moment in the middle strip is 0. If the width of the column of wall support is greater than three fourth l_2, then M_u minus is uniformly distributed along the width l_2. Thus, depending on the width of the column with respect to the width of the equivalent frame, we decide upon whether the full negative moment should come to the exterior column or not.

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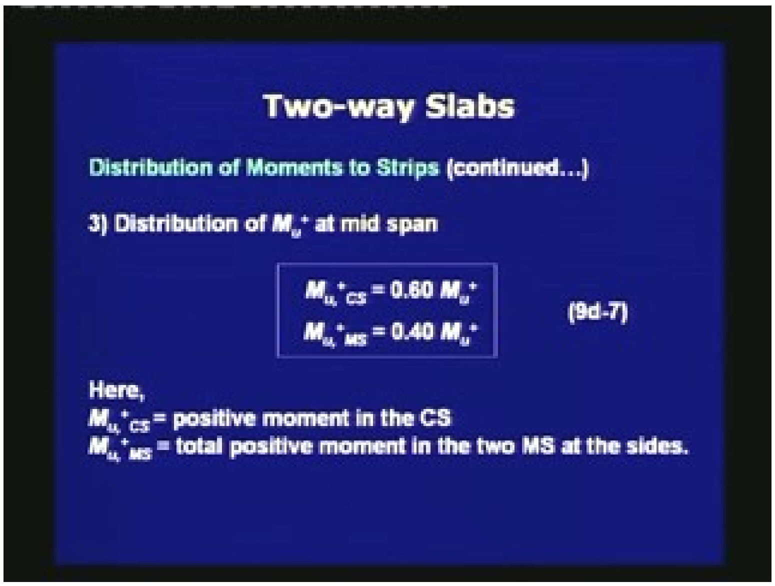

Third is a distribution of M_u plus at mid-span. The column strip gets 60% of the positive moment and the middle strip gets 40% of the positive moment. Again, note that for the middle strip, the M_u plus MS, represents the total positive moment in the two middle strip at the two sides.

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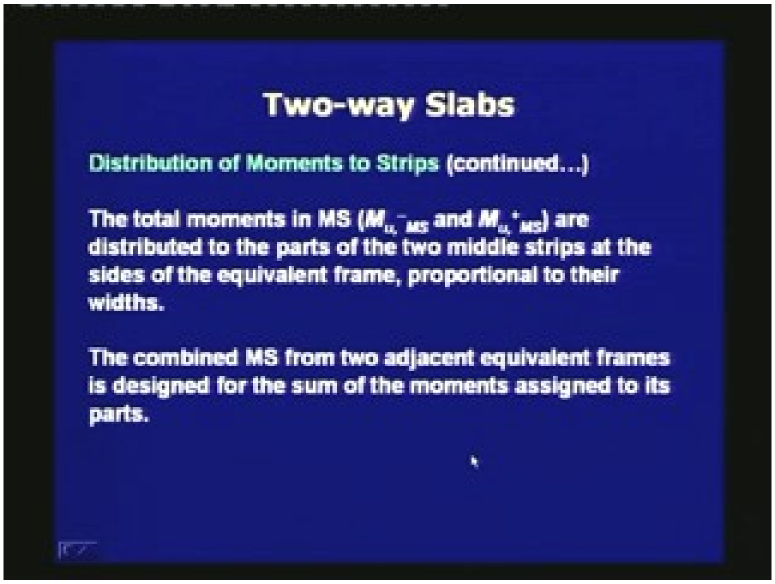

The total moments in middle strip which is M_u minus MS and Mu plus MS are distributed to the parts of the two middle strips at the two sides of the equivalent frame, proportional to their widths. The combined middle strips from two adjacent equivalent frames, is designed for the sum of the moments assigned to its parts. That means, for one particular equivalent frame the value that we have got for the middle strip, we distribute that in the two portions of the middle strips at the two sides. Once we have distributed for each middle strip, we combine the two values from one equivalent frame and the adjacent equivalent frames, to get the total moment in that particular middle strip.

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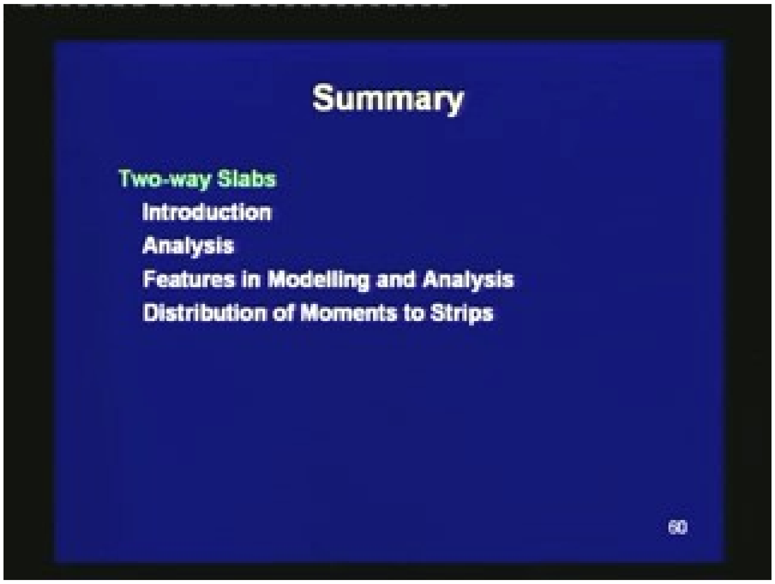

Thus, in today's lecture, we have introduced the two-way slabs. We said that a non-rectangular slab is always designed as a two-way slab and a rectangular slab, if it is supported on all the four sides then it is designed as a two-way slab depending on length-to-breadth ratio. If the length-to-breadth ratio is less than 2 then we design a rectangular slab as a two-way slab which spans in both the direction.

There can be different types of two-way slabs. The first is the flat plate, which does not have any beams or drop panels or column capitals. The second type is the flat slabs which do not have any beam but it can have drop panel or column capital and the third type is the two-way slab with beams, where beams are running in between the column lines. The flat plates can have beams at the edges which are called spandrel beams. The design of two-way slabs can be done by two methods. For flat slabs and flat plates, the method is given in section 31 of IS: 456-2000. This method is also applicable for two way slabs with beams that can deflect and there is a second method which is for the two-way slab whose beams are stiff enough and their deflection can be neglected. That method is given in Annex D of IS: 456. In this module, we discussed the section 31 of IS: 456 which is the one used conventionally for flat plates and flat slabs. Now, again in that section there are two methods given. One is the simplified method which is called direct design method. This is not suitable for prestressed slabs. The second method, which is the equivalent frame method and is based on computer analysis, is recommended for two-way prestressed slabs.

In the equivalent frame method, first we define the equivalent frames in each orthogonal direction. An equivalent frame is a slice of a building along a column line whose width spans between the central lines of the two adjacent bases. Within an equivalent frame we define a column strip and two middle strips at the sides. Once an equivalent frame is defined, we analyze the frame for gravity loads and lateral loads if any and find out the moments in the critical sections, the positive moment in the span and the negative moment in the support.

Once these critical moments are known, then we distribute that moment along the width of the equivalent frame from which we calculate the moment per unit width of the slab. Once we know this value, we can design the prestressing tendon just the way as we have done for one-way slab. In our next lecture, we shall move on to the design for shear of two-way slabs. Thank you.