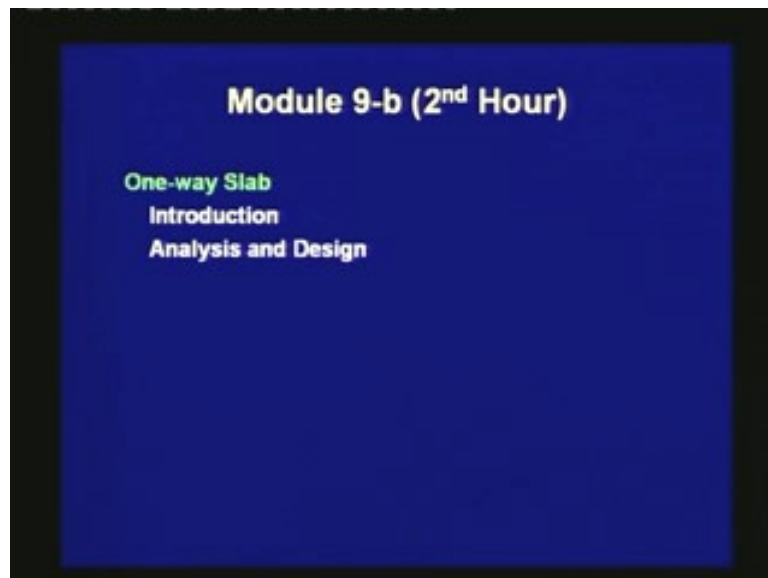


Prestressed Concrete Structures
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Lecture 36
One -Way Slabs

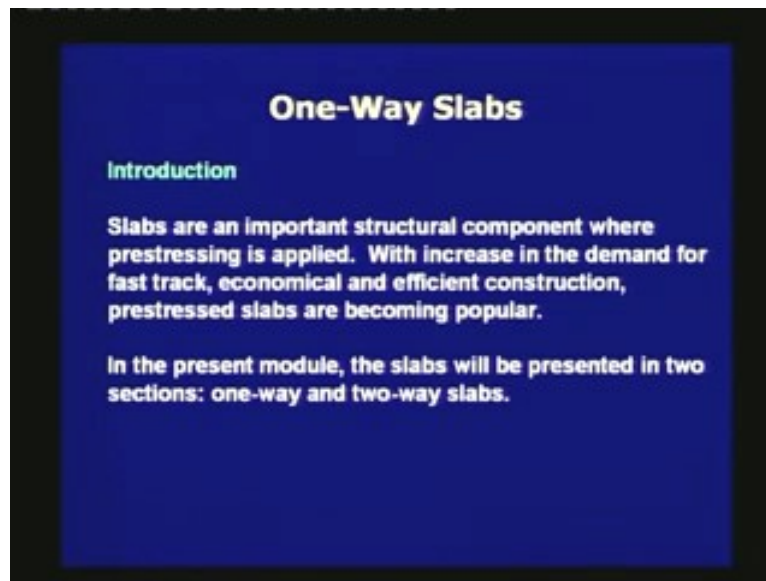
Welcome back to prestressed concrete structures. This is the second lecture of module nine. In this lecture, we shall study one-way slabs.

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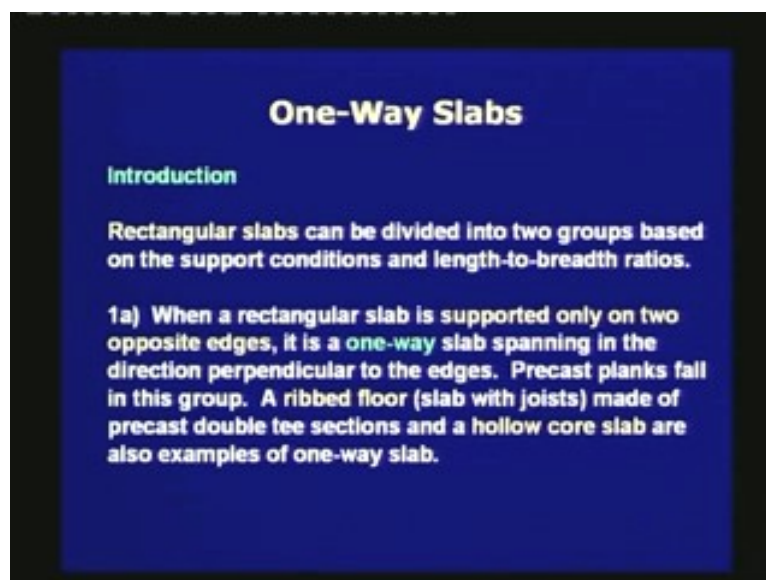
First, we shall learn about the different types of slabs and then we will move on to the analysis and design of one-way slabs.

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Slabs are an important structural component where prestressing is applied. With increase in demand for fast track, economical and efficient construction, prestressed slabs are becoming popular. In the present module, the slabs will be presented in two sections: one-way and two-way slabs.

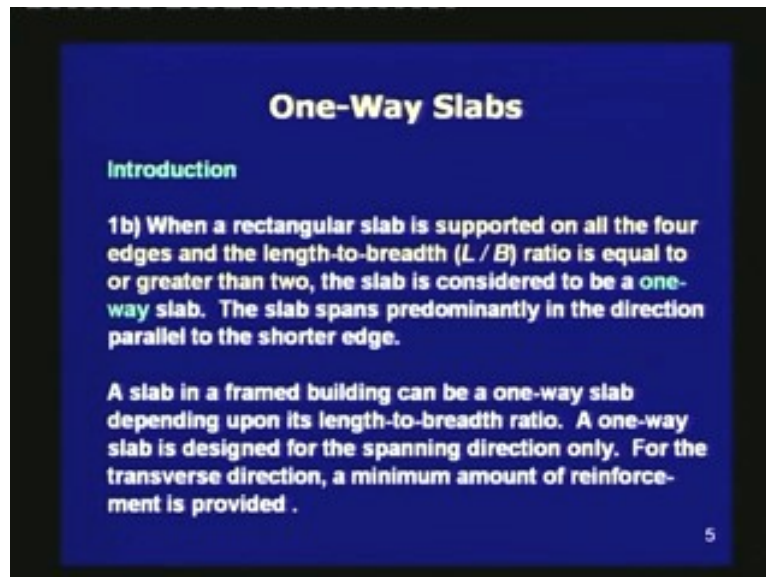
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Rectangular slabs can be divided into two groups based on the support conditions and length-to-breadth ratios.

In the first type, when a slab is supported on two opposite edges, then it is a one-way slab spanning in the direction perpendicular to the edges. Precast planks fall in this group. A ribbed floor, which means slabs with joists, consisting of double tee sections and a hollow coarse slab are also examples of one-way slab spanning in the direction.

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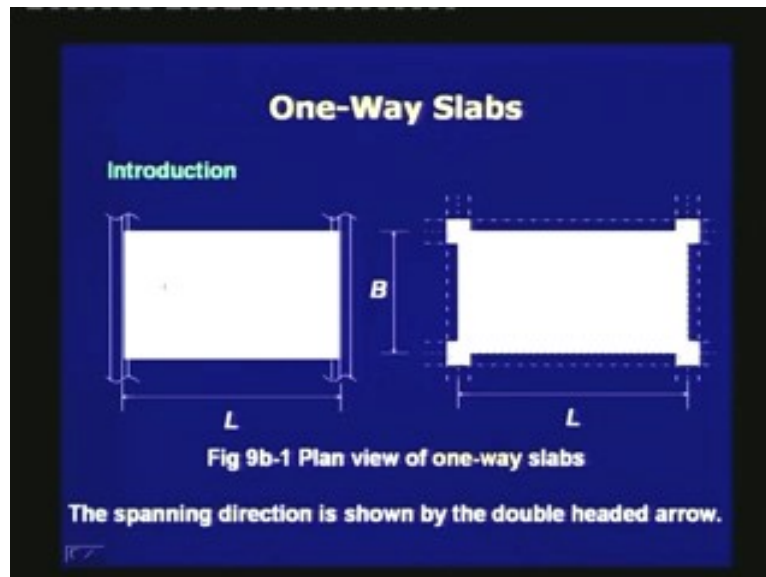
In the second type, when a rectangular slab is supported on all the four edges and the length-to-breadth ratio, which we shall denote as L by B , is equal to or greater than two, the slab is considered to be a one-way slab. The slab spans predominantly in the direction parallel to the shorter edge.

The first type of one-way slab is supported only on the two opposite edges. In the second type of one-way slab, they are supported on all the four edges. **but the length-to-breadth ratio, where** Length is the larger horizontal dimension and breadth is the smaller horizontal dimension. When this length-to-breadth ratio is greater than or equal to two, then also the slab is considered to be a one-way slab and the slab spans in the shorter direction.

A slab in a framed building can be a one-way slab depending upon its length-to-breadth ratio. A one-way slab is designed for the spanning direction only. For a transverse direction, a minimum amount of reinforcement is provided. Thus, given a building plan, first you have to determine whether a slab is one-way or not depending upon the length-to-breadth ratio and if they are supported on all the four sides. If we

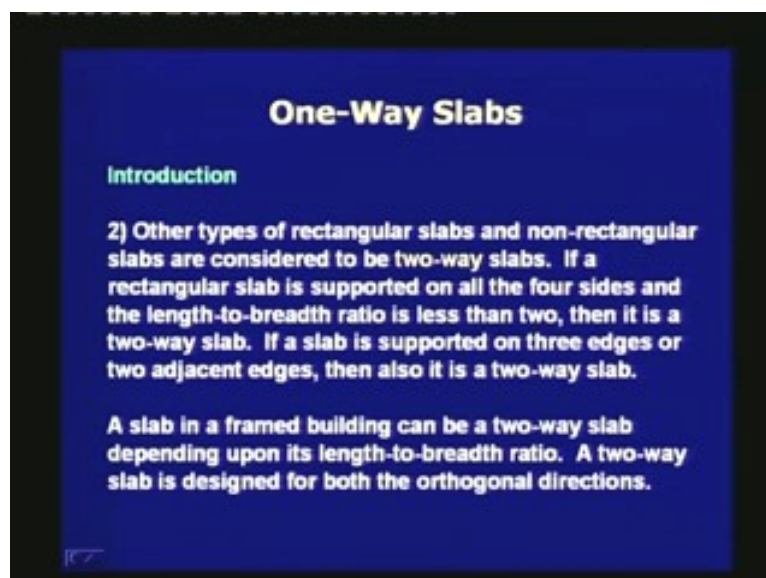
determine a slab is a one-way slab based on the criteria that the length-to-breadth ratio is greater than or equal to two then, we design the slab only for the spanning direction. For the transverse direction, we provide only minimum amount of reinforcement.

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In this figure, on the left side we have a slab which is spanning from left to right because it is supported only on the two edges; whereas, for the slab on the right side it is supported on all the four edges and here the length is greater than or equal to the breadth and the slab is spanning in the shorter direction parallel to B .

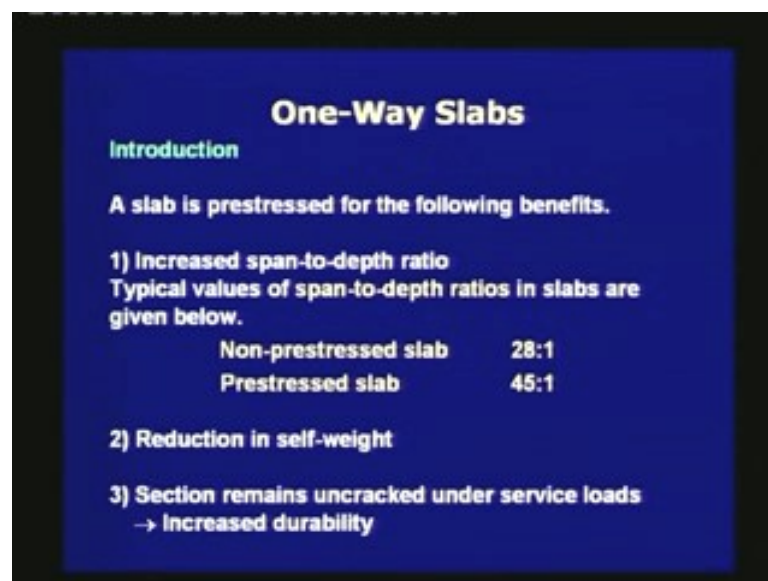
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Other types of rectangular slabs and non-rectangular slabs are considered to be two-way slabs. If a rectangular slab is supported on all the four sides and the length-to-breadth ratio is less than two, then it is a two-way slab. If a slab is supported on three edges or two adjacent edges, then also it is a two-way slab. A slab in a framed building can be a two-way slab depending upon its length-to-breadth ratio. A two-way slab is designed for both the orthogonal directions.

The types of slab which do not fall under the category of one-way slabs come under two-way slabs. For rectangular slabs, if the length-to-breadth ratio is less than two, when all the sides are supported, then it is a two-way slab and for a two-way slab, it has to be designed for both the orthogonal directions. Again, given a building plan, first we have to identify whether a slab is a two-way slab or not and then we have to proceed as per the two-way slab design. In this module, we shall focus only on the one-way slab analysis and design.

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One-Way Slabs

Introduction

A slab is prestressed for the following benefits.

- 1) Increased span-to-depth ratio
Typical values of span-to-depth ratios in slabs are given below.

Non-prestressed slab	28:1
Prestressed slab	45:1

- 2) Reduction in self-weight
- 3) Section remains uncracked under service loads
→ Increased durability

A slab is prestressed for the following benefits: first, it is the increased span-to-depth ratio. Typical values of span to depth ratios in slabs are given below. For non-prestressed slabs, the span to depth ratio is 28: 1; for the prestressed slab, it is 45: 1. Here, you can see that a prestressed slab can have larger span compared to its depth, and this is a big benefit when we are looking for large column free space. Thus, one very important aspect of prestressing a slab is to increase the span to depth ratio.

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One-Way Slabs

Introduction

A slab is prestressed for the following benefits.

1) **Increased span-to-depth ratio**
Typical values of span-to-depth ratios in slabs are given below.

Non-prestressed slab	28:1
Prestressed slab	45:1

2) **Reduction in self-weight**

3) **Section remains uncracked under service loads**
→ Increased durability

The second benefit is the reduction in self weight. Once the self weight is reduced, the design of the supporting beams or columns also leads to smaller sections; the building mass is reduced. It leads to lower seismic forces and we get a tangible benefit from the prestressing of the slabs. The third benefit is that the sections remain uncracked under service loads and this leads to increased durability. Durability is of concern wherever we have exposure conditions worse than moderate and if the slab is uncracked, then we have increased durability.

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One-Way Slabs

Introduction

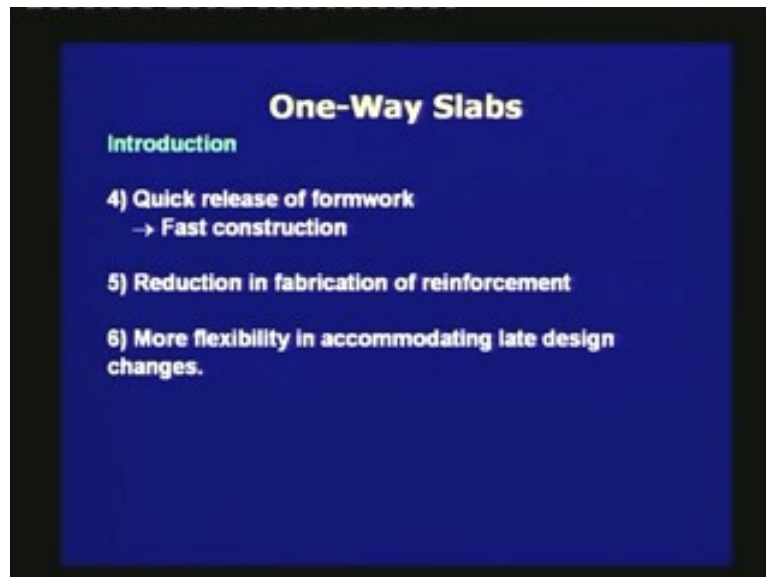
4) **Quick release of formwork**
→ Fast construction

5) **Reduction in fabrication of reinforcement**

6) **More flexibility in accommodating late design changes.**

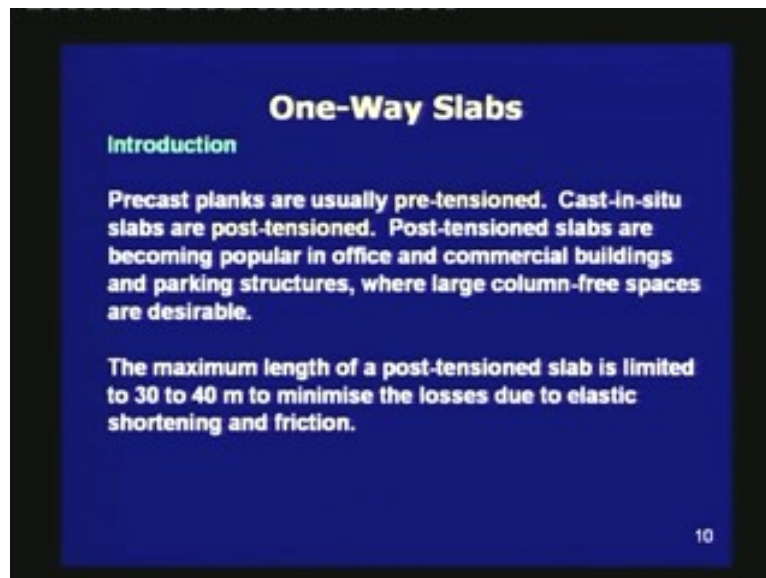
The fourth benefit is a quick release of formwork. Once a slab is prestressed, the formwork supporting the grill concrete can be removed quicker because, the concrete can sustain its weight due to the effect of prestressing. Hence, this leads to a fast track construction.

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The fifth benefit is the reduction in fabrication of reinforcement. In conventional slabs, the reinforcement is by ordinary steel; it needs more time to place and tight the reinforcing parts. Whereas in prestressing slabs, the prestressing strands are either placed in a precast yard or the post-tensioning is done in strands which are placed in ducts and both these operations are faster than the conventional fabrication of the reinforcement cage. The sixth benefit is more flexibility in accommodating late design changes. Thus, once a design has been made for a particular utilization of an area and if the client needs some change in the utilization of the floor space, the column free space has greater flexibility to accommodate that change. Although, we have the benefits of this prestressing of slabs, we have to be careful that the analysis, design and construction has to be done carefully and the quality of construction has to be strictly adhered too.

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One-Way Slabs

Introduction

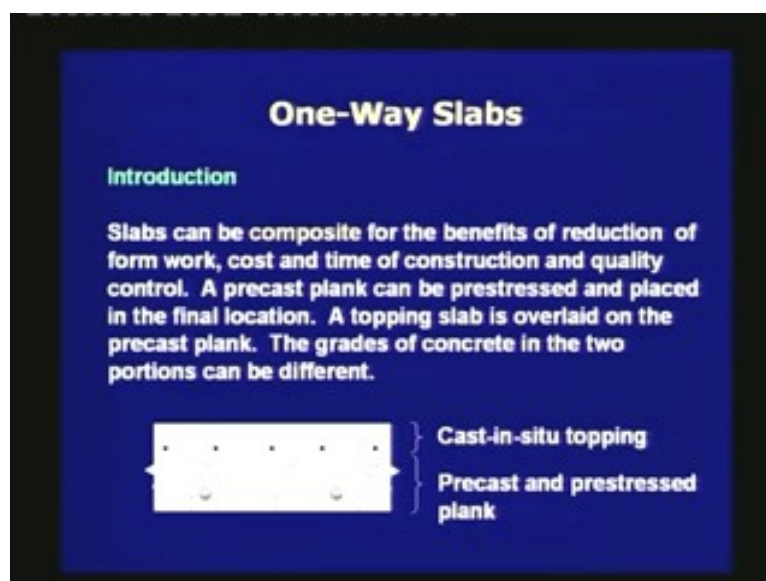
Precast planks are usually pre-tensioned. Cast-in-situ slabs are post-tensioned. Post-tensioned slabs are becoming popular in office and commercial buildings and parking structures, where large column-free spaces are desirable.

The maximum length of a post-tensioned slab is limited to 30 to 40 m to minimise the losses due to elastic shortening and friction.

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Precast planks are usually pre-tensioned. They are manufactured in a yard under controlled environment, whereas cast-in-situ slabs are usually post tensioned. Post-tensioned slabs are becoming popular in office, commercial buildings and parking structures, where large column-free spaces are desirable. The maximum length of a post-tensioned slab is limited to 30 to 40 meters to minimize the losses due to elastic shortening and friction. Although, theoretically we can go for even larger spans, but since you have elastic shortening of the slab and there is loss due to friction, usually in the post-tensioned slabs the spans are limited to 30 to 40 meters.


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One-Way Slabs

Introduction

Slabs can be composite for the benefits of reduction of form work, cost and time of construction and quality control. A precast plank can be prestressed and placed in the final location. A topping slab is overlaid on the precast plank. The grades of concrete in the two portions can be different.



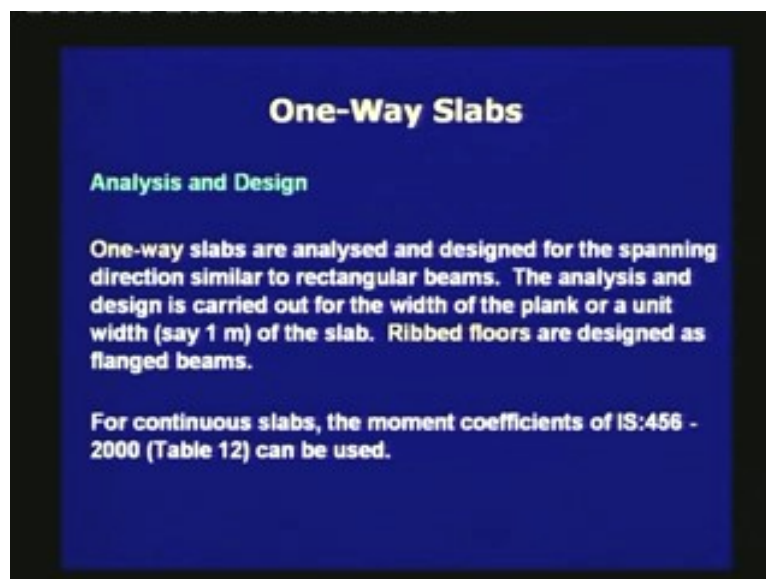
Cast-in-situ topping

Precast and prestressed plank

Slabs can also be composite for the benefits of reduction of form work, cost and time of construction; the quality control is also better in a composite slab. A precast plank can be prestressed and placed in the final location. A topping slab is overlaid on the precast plank. The grades of concrete in the two portions can be different. This figure shows a precast plank, but the bottom part has been precast and prestressed in the yard. These are the prestressing ducts and the top part is a cast-in-situ topping, where there has been some reinforcement to consider the effects of temperature and shrinkage; the combination of these two is called a composite slab. That means the bottom part is precast and prestressed whereas the top part is cast-in-situ.

Next, we move on to the analysis and design of one-way slabs. One-way slabs are analyzed and designed for the spanning direction similar to rectangular beams. The analysis and design is carried out for the width of the plank or a unit width, say 1 meter, of the slab. Ribbed floors are designed as flanged beams. Whatever we have studied for the design of beams, they are also applicable for the design of one-way slabs. The slabs are designed based on their width, if it is a precast plank or it will be designed for a unit width, say 1 meter and the analysis and design procedure for one-way beam is applied for the one-way slabs. If one-way slab is the ribbed floor, then we have to apply the analysis and design procedure for a flanged section.

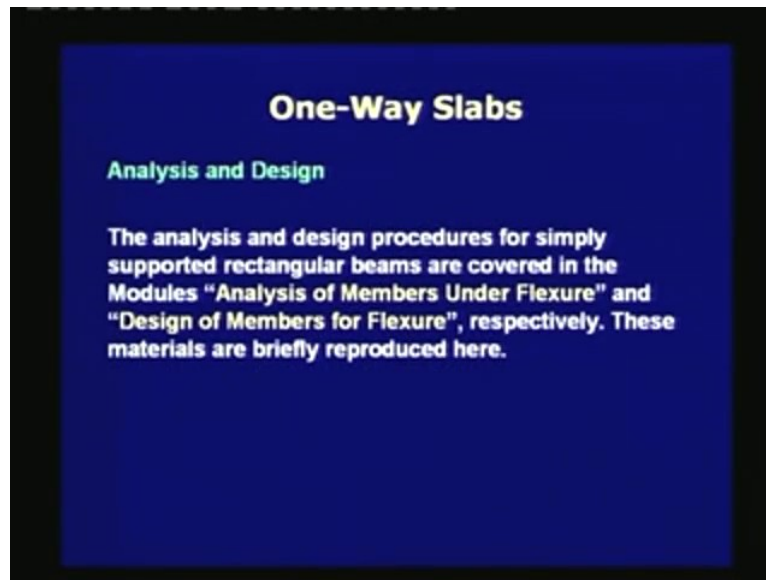
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For continuous slabs, the moment coefficients of IS: 456-2000, which are given in table 12, can be used. That means, in the conventional analysis, if it is a simply

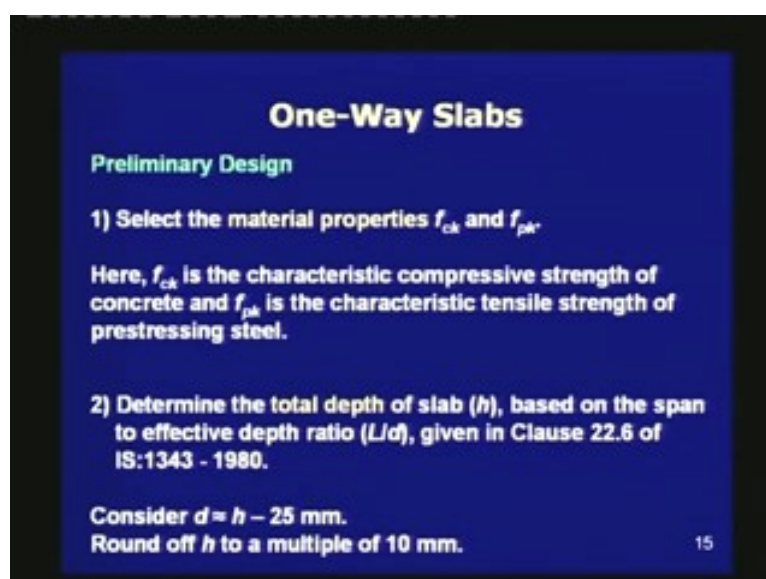
supported slab, you use the conventional structural analysis formulae, but, if it is a continuous slab, then we can use moment coefficients that is provided in IS: 456-2000. We need not go for a rigorous analysis.

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The analysis and design procedures for simply supported rectangular beams are covered in the modules "Analysis of members under Flexure" and "Design of members of Flexures", respectively. These materials are briefly reproduced here. Earlier, in module three and module four, we have studied the analysis and design procedures of beams in detail. Here, we are revising them briefly

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First, we do a preliminary design. In the preliminary design, first the material properties are selected. That means f_{ck} , which is the characteristic compressive strength of concrete and f_{pk} , which is the characteristic tensile strength of the prestressing steel are selected. Before starting a project, the weight of concrete and the type of prestressing steel is fixed during the conceptual process and from that, we know the values of f_{ck} and f_{pk} .

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One-Way Slabs

Preliminary Design

1) Select the material properties f_{ck} and f_{pk} .

Here, f_{ck} is the characteristic compressive strength of concrete and f_{pk} is the characteristic tensile strength of prestressing steel.

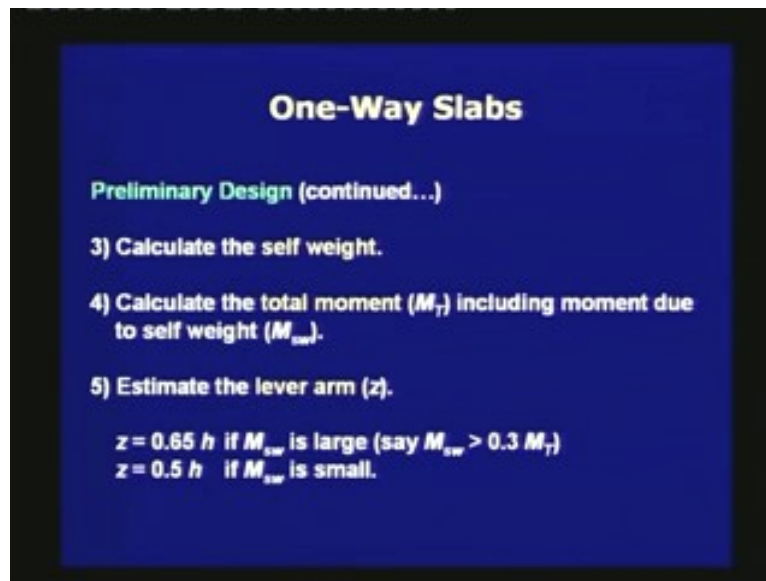
2) Determine the total depth of slab (h), based on the span to effective depth ratio (L/d), given in Clause 22.6 of IS:1343 - 1980.

Consider $d = h - 25$ mm.
Round off h to a multiple of 10 mm.

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Second, determine the total depth of the slab which we shall denote as 'h' based on the span to effective depth ratio given in clause 22.6 of IS 1343-1980. The span to effective depth ratio will be denoted as L by d and this ratio is limited to avoid deflection computations. If we fix our total depth, satisfying the span to depth ratio, then we do not have to check the deflections of the slab if the load is conventional. In order to relate the effective depth with the total depth, we can use an approximate estimate that d is about h minus 25 millimeters. Once we select d , based on the l by d ratio, we can select h and then we have to round off h to a multiple of 10 millimeters. Thus, the total depth of the slab is determined based on the deflection requirement.

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One-Way Slabs

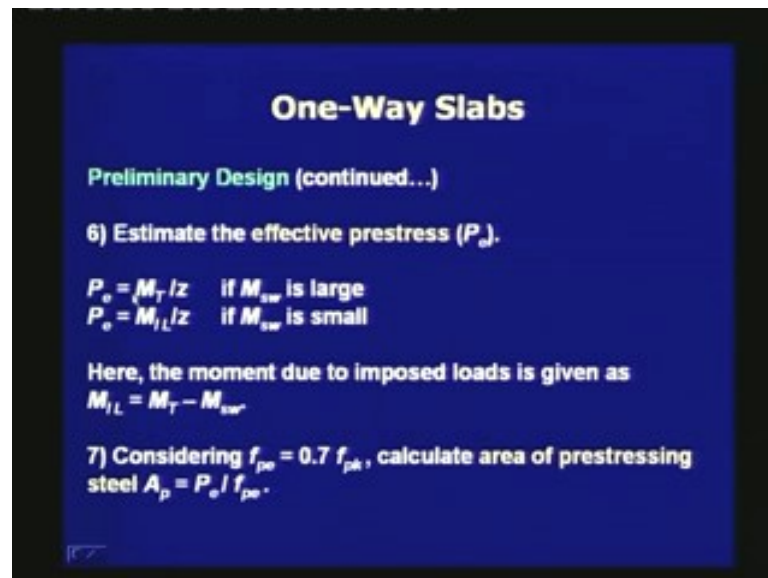
Preliminary Design (continued...)

- 3) Calculate the self weight.
- 4) Calculate the total moment (M_T) including moment due to self weight (M_{sw}).
- 5) Estimate the lever arm (z).

$z = 0.65 h$ if M_{sw} is large (say $M_{sw} > 0.3 M_T$)
 $z = 0.5 h$ if M_{sw} is small.

Next, we will calculate the self-weight of the slab. Once the depth is known, the weight per unit width of the slab can be determined. Then, we calculate the total moment, which we are representing as M_T including moment due to self weight, which we are representing as M_{sw} . Once, we are able to calculate the self-weight, we add that to the other dead load moment and live load moment to get the total moment M_T . After this, we estimate the lever arm which we denote as z . z is equal to 65% of the total depth, which is $0.65 h$, if the self-weight moment is large; say the self-weight moment is greater than 30% of the total moment. If the self-weight moment is small, then we can estimate z to be about 50% of the total depth that is z is equal to $0.5h$. These are the rough estimates of the lever arm by which the compressive force will move up from the CGS under service conditions.

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One-Way Slabs

Preliminary Design (continued...)

6) Estimate the effective prestress (P_e).

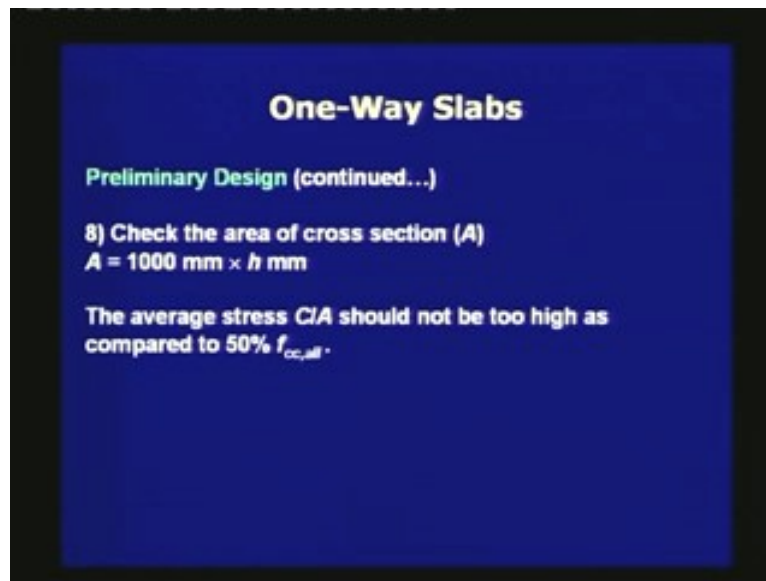
$P_e = M_T / z$ if M_{sw} is large
 $P_e = M_{IL} / z$ if M_{sw} is small

Here, the moment due to imposed loads is given as
 $M_{IL} = M_T - M_{sw}$

7) Considering $f_{pe} = 0.7 f_{pk}$, calculate area of prestressing steel $A_p = P_e / f_{pe}$.

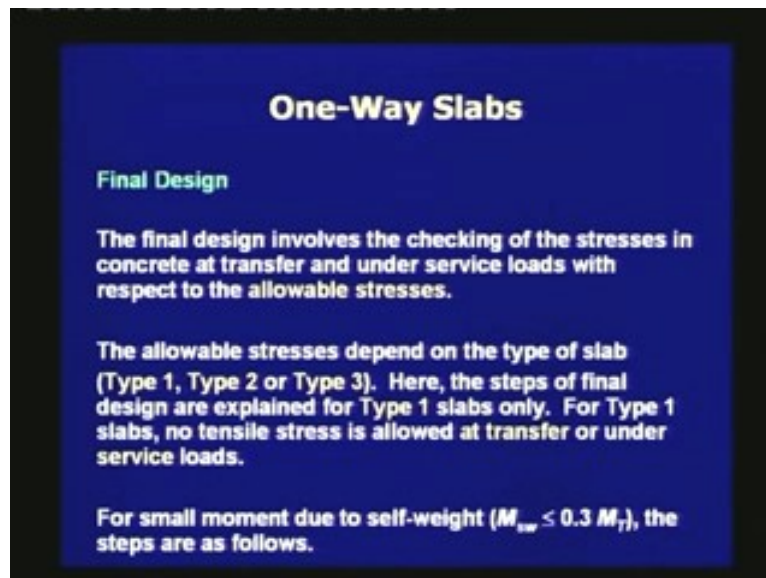
The sixth step is to estimate the effective prestress, which we shall denote as P_e . P_e is equal to the total moment, M_T divided by z , if the self weight moment is large. Or else, P_e is equal to the moment due to imposed loads which we are denoting as M_{IL} divided by z , if M_{sw} is small. These are some rough guidelines to estimate the prestressing force, given the moment and the estimate of the lever arm. Here, the moment due to imposed loads is given as M_{IL} is equal to M_T minus M_{sw} . That is, it is the moment due to the super imposed dead load and the live load. How are we calculating it? We are subtracting the self-weight moment from the total moment and we are saying that is the moment due to imposed loads. In the seventh step, we are considering that the effective prestress is about 70% of the characteristic strength of the prestressing steel. From that, we are calculating the area of the prestressing steel: A_p is equal to P_e divided by f_{pe} .

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Thus, first we have estimated the effective prestress. We have estimated the effective prestressing in tendon. From there, we are having an estimate of the amount of the prestressing steel for the chosen width of the slab. In the eighth step, check the area of cross section A , where A is equal to 1000 times h ; 1000 is the width of a 1 meter wide slab and h is the total depth. The average stress C divided by A should not be too high as compared to 50% of $f_{cc,allowable}$; C is equal to the effective prestress, P_e . Thus, we are saying that the average prestress over the cross-section which is given as C by A is equal to P_e by A , should not be larger than 50% of the allowable compressive stress for the concrete in the slab. With these steps, we have found a preliminary design of the unit width of the slab. We move on to the final design, where we are checking the stresses in the slab under the service load conditions.

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The final design involves the checking of the stresses in concrete as transfer and under service loads with respect to the allowable stresses. The allowable stresses depend on the type of slab: Type 1, Type 2 or Type 3. Here, the steps of final design are explained for Type 1 slabs only. For Type 1 slabs, no tensile stress is allowed at transfer or under service loads. Thus, the steps of the final design primarily consist of checking the stresses in the sections and at transfer and under service conditions. The allowable stresses depend on the type of prestressed member that we are designing for. Earlier, we had known that the prestressed members are divided into three types: Type 1, where we do not allow any tensile stress either at transfer or during service; Type 2, where we allow tensile stresses but we do not allow cracking and Type 3, where we allow cracking, but the crack width is limited.

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One-Way Slabs

Final Design

The final design involves the checking of the stresses in concrete at transfer and under service loads with respect to the allowable stresses.

The allowable stresses depend on the type of slab (Type 1, Type 2 or Type 3). Here, the steps of final design are explained for Type 1 slabs only. For Type 1 slabs, no tensile stress is allowed at transfer or under service loads.

For small moment due to self-weight ($M_{sw} \leq 0.3 M_T$), the steps are as follows.

In this section, we are studying only the design of Type 1 slabs. The design of Type 2 and Type 3 are very similar to the design of Type 1 slabs, the difference being we have to incorporate the allowable tensile stress in the expressions. This was covered in detail in the analysis and design of beams. For small moments due to self-weight, say, when M_{sw} is less than 30% of the total moment, the steps are as follows:

(Refer Slide Time: 24:21)

One-Way Slabs

Final Design (continued...)

1) Calculate eccentricity (e) to locate the centroid of the prestressing steel (CGS).

The lowest permissible location of the compression (C) due to self-weight is at the bottom kern point (at a depth k_b below CGC) to avoid tensile stress at the top.

The design procedure based on the extreme location of C gives an economical section.

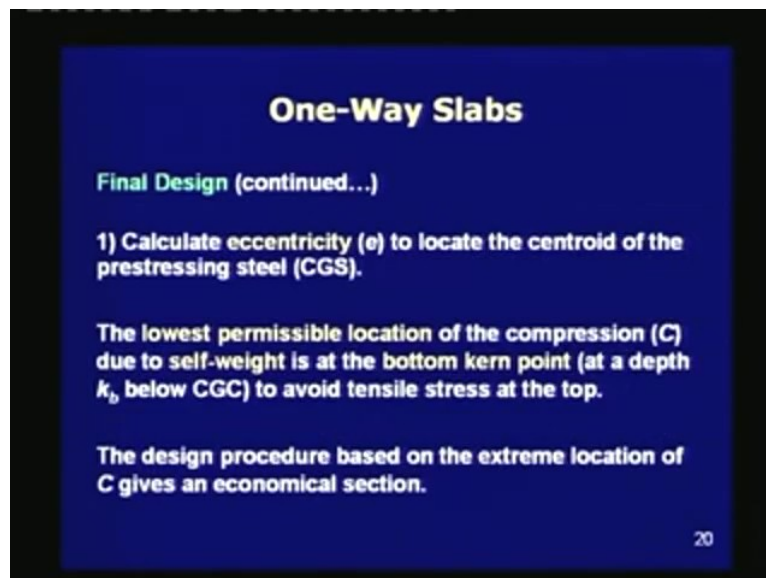
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First, calculate the eccentricity e , to locate the centroid of the prestressing steel. Here, we are doing an accurate calculation and we are trying to locate the centroid of the prestressing steel with respect to the centroid of the concrete, the CGC. This distance

between the CGS and the CGC is denoted as the eccentricity e . The lowest permissible location of the compression due to self-weight is at the bottom kern point to avoid tensile stress at the top. Earlier, we have studied about these kern locations; the kern points are those that if the compression is at those points, then there will not be any tension in the section.

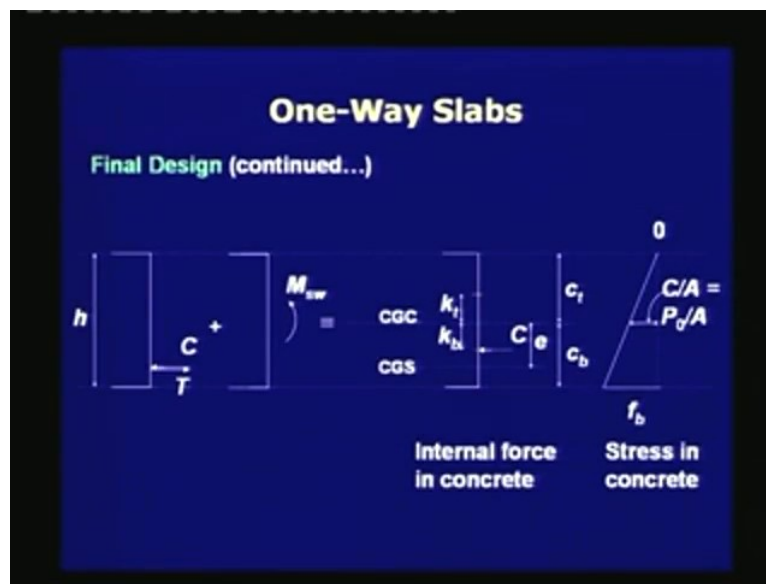
Thus when the slab is under minimum load that is when it is only under its self-weight, the lowest possible position of compression is the bottom kern point. The distance of the bottom kern point from the CGC is denoted as k_b . If we base our design based on this bottom most position of C , then we get an economical section.

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The design procedure based on the extreme location of C gives an economical section.

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In this figure, we are studying the stress conditions; h is the total depth of the slab. First, at prestressing, the compression is at the same location of the CGS. Then, as the self-weight acts, the compression moves up from the CGS. It has to be within the kern zone to avoid any tensile stress in the section. The lowest possible location of C , that means, the minimum that C has to travel up from the location of CGS is up to the bottom kern point which is at a distance k_b from the CGC. Here, e is the distance between the CGC and the CGS. This figure shows the internal force in concrete due to the self-weight of the slab and the stress condition is when C is located at the bottom kern point, the stress at the top is zero. The stress at mid-height, which is the average stress, is given as C by A equal to P_0 at transfer divided by A and the stress at the bottom is denoted as f_b . This is the stress diagram for the depth of the slab due to its own weight.

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One-Way Slabs
Final Design (continued...)

From the stress profiles, the following equation can be derived.

$$e = \frac{M_{sw}}{P_0} + k_b \quad (9b-1)$$

The magnitude of C or T is equal to P_0 , the prestress at transfer after initial losses.

From the stress profiles, the following equations can be derived: e is equal to M_{sw} divided by P_0 plus k_b , the derivation of this expression was shown earlier. We are recollecting the expression of e , which gives us the location of C at the bottom kern point and this is a function of the self-weight of the moment, the prestressing force at transfer P_0 and the distance of the bottom kern point from CGC. The magnitude of C or T is equal to P_0 , the prestress at transfer after initial losses. Thus, by this equation, we are able to determine the lowest possible location of the centroid and this will lead to an economical section.

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One-Way Slabs
Final Design (continued...)

The value of P_0 can be estimated as follows.

$$P_0 = 0.9 P_i \quad \text{for pre-tensioned slab}$$
$$= P_i \quad \text{for post-tensioned slab}$$

Here, P_i is the initial applied prestress.

$$P_i = (0.8 f_{pk}) A_p \quad (9b-2)$$

The permissible prestress in the tendon is $0.8 f_{pk}$.

The value of P_0 can be estimated as follows: P_0 is equal to 0.9 P_i . That means 90% of the initial prestress for pre-tensioned slab and it can be 100% of the initial prestress for the post tensioned slab. In the pre-tensioned slab, due to elastic shortening P_0 is lower than P_i . Here, P_i is the initial applied prestress and the maximum value of P_i is equal to 80% of the characteristic strength times the estimated area of the prestressing steel A_p . The permissible prestress in the tendon is $0.8 f_{pk}$ and this is used to find out the maximum initial prestress that we can apply on the estimated amount of prestressing steel. Once, we have determined the eccentricity, next we are recomputing the effective prestress P_e and the area of prestressing steel A_p . This is based on the stress diagram under service conditions. Under service conditions, due to the total moment the maximum distance C can traverse is up to the top kern point. At this location, the stress at the bottom is zero, the stress at the top is f_t and again the average stress at mid height is C by A is equal to P_e by A . This is the stress profile under service conditions.

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One-Way Slabs

Final Design (continued...)

From the previous figure, the shift of C due to the total moment gives an expression of P_e .

$$P_e = \frac{M_T}{e + k_T} \quad (9b-3)$$

For solid rectangular slab, $k_b = k_t = h / 6$.

25

Based on this stress profile, the shift of C due to the total moment gives an expression of P_e . The derivation of the equation was shown earlier and again here it is recollected. From that stress profile, we get P_e is equal to the total moment divided by e plus k_t , where e plus k_t is the distance by which C has traversed from CGS to the top kern point. For solid rectangular slabs, k_t equal to k_b equal to h by 6. Thus, the top and bottom kern points are located at one sixth of h from the CGC.

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One-Way Slabs

Final Design (continued...)

Considering $f_{pe} = 0.7f_{pk}$, the area of prestressing steel is recomputed as follows.

$$A_p = P_e / f_{pe} \quad (9b-4)$$

The number of tendons and their spacing is determined based on A_p .

The value of P_0 is updated.

Once, we have calculated P_e , next we are recomputing the amount of prestressing steel. Considering again, the effective prestress f_{pe} is equal to 70% of the characteristic strength f_{pk} , the area of prestressing steel is recomputed as follows: A_p is equal to P_e divided by f_{pe} . The number of tendons and the spacing is determined based on A_p . Given the value of P_e and f_{pk} , we are calculating the amount of prestressing steel that we need for the unit width or whatever width we have selected for the slab. Once we calculate A_p , we can distribute that in the tendons with suitable spacing.

With this, we know the amount of prestressing force under service conditions and you can update the value of prestressing force at transfer. Once we have updated P_0 , we can recompute e again with the updated values of A_p and P_0 . Thus, you have to appreciate that the design is sequential process; that you compute some variables based on the estimated quantities, again you recompute the estimated variables and converge to a result which is suitable regarding the stresses and the lay out of the prestressing tendons.

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One-Way Slabs

Final Design (continued...)

3) Recompute e with the updated values of A_p and P_0 .

If the variation of e from the previous value is large, another cycle of computation of the prestressing variables can be undertaken.

For large M_{sw} , if e violates the cover requirements, e is determined based on cover.

If the variation of e from the previous value is large, another cycle of the computations of the prestressing variables can be undertaken. For large self-weight moment M_{sw} , if e violates the cover requirements, e is determined based on the cover. The expression of e that you have written, was determined based on the force conditions, but, if the self weight moment is high, then e may come out to be large and we may violate the cover requirements for the prestressing tendons. In that case, e is selected based on the cover requirements and then, we check the amount of the prestressing steel and prestressing force.

(Refer Slide Time: 34:35)

One-Way Slabs

Final Design (continued...)

4) Check the compressive stresses in concrete

At transfer

$$f_b = -\frac{P_0}{A} \frac{h}{c_t} = -\frac{2P_0}{A} \quad (9b-5)$$

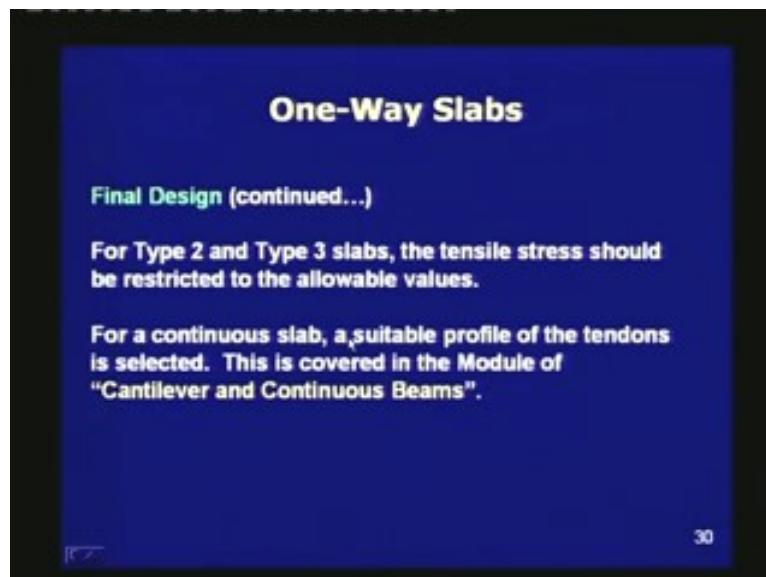
The stress at the bottom f_b should be less than $f_{cc,all}$, where $f_{cc,all}$ is the allowable compressive stress in concrete at transfer.

$$|f_b| \leq f_{cc,all}$$

In the fourth step, we check the compressive stresses in concrete. At transfer, the stress at the bottom, f_p is equal to minus P_0 by A , which is the average stress times h divided by c_t . This is equal to minus $2 P_0$ by A , because h is equal to two times of C_t . Thus we are able to calculate the stress at the bottom based on the design, prestress at transfer and the properties of the section. The stress at the bottom f_p , should be less than the allowable compressive stress $f_{cc,allowable}$. This can be expressed as an equation that the magnitude of f_p should be lower than or equal to $f_{cc,allowable}$.

Next, we are checking the compressive stress in concrete at service conditions. The expression at the stress in the top is similar that f_t is equal to minus P_e by A , which is the average prestress times h by C_b . This relationship is again coming from the triangular stress distribution and this is equal to minus $2 P_e$ by A , because h is equal to two times of C_b . The stress at the top f_t should be less than $f_{cc,allowable}$, where $f_{cc,allowable}$ is the allowable compressive stress in concrete at service. Here also, we are expressing this relationship as the magnitude of f_p should be less than $f_{cc,allowable}$. Note that the allowable compressive stress at transfer and the allowable compressive stress at service are different.

(Refer Slide Time: 37:05)

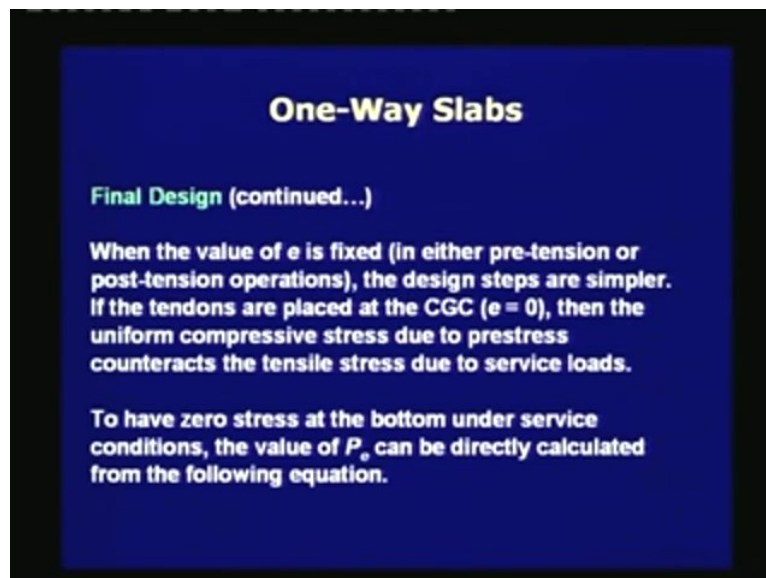


For Type 2 and Type 3 members, the tensile stress should be restricted to the allowable values. So when we show this design for Type 1 members, the allowable tensile stress was zero. If we do the design for the Type 2 and Type 3 members, then the allowable tensile stress is as per the values given in the curve.

For a continuous slab, a suitable profile of the tendons is selected; this is covered in the module of cantilever and continuous beams. So, the value of eccentricity that we have found is at the critical location. For a simply supported beam, the critical location is at the middle, we can have a parabolic profile or if the depth is small, then we can have a straight profile with constant e .

For a continuous slab, the prestressing tendon goes above CGC at the support locations and then we need to determine several values of the eccentricity, one at the span and another at the support. This procedure was shown in the module of the continuous beams. For continuous beams or slabs then, important design element is selecting an appropriate profile of the tendons.

(Refer Slide Time: 38:50)



When the value of e is fixed, say in either pre-tension or post-tension operations, the design steps are simpler. If the tendons are placed at the CGC, say for which e is equal to zero, then the uniform compressive stress due to prestress counteracts the tensile stress due to service loads. Sometimes, the eccentricity is fixed based on the construction requirement. If the eccentricity is zero, that means CGS lies at the CGC, in that case there is a uniform compression in the slab and this uniform compression counteracts the tensile stress that is generated due to the bending. To have zero stress at the bottom under service conditions, the value of P_e can be directly calculated from the following equation:

(Refer Slide Time: 39:45)

One-Way Slabs

Final Design (continued...)

$$\frac{P_e}{A} = \frac{M_T}{Z_b}$$

or, $P_e = A \frac{M_T}{Z_b}$ (9b-7)

Z_b is the section modulus. The above expression is same as $P_e = M_T / k_t$, which is Eq. (9b-3), with $e = 0$. The stresses at transfer can be checked with an estimate of P_0 from P_e .

P_e by A , which is the uniform compressive stress is equal to M_T divided by Z_b , which is the tensile stress generated due to bending under service conditions. From this, we directly get the value of the prestressing force, P_e is equal to A times M_T divided by Z_b , and this expression is a simple expression which we can use if the eccentricity is zero. This is based on the concept that the uniform compressive stress balances the tensile stress under service conditions due to flexure.

(Refer Slide Time: 40:32)

One-Way Slabs

Final Design (continued...)

$$\frac{P_e}{A} = \frac{M_T}{Z_b}$$

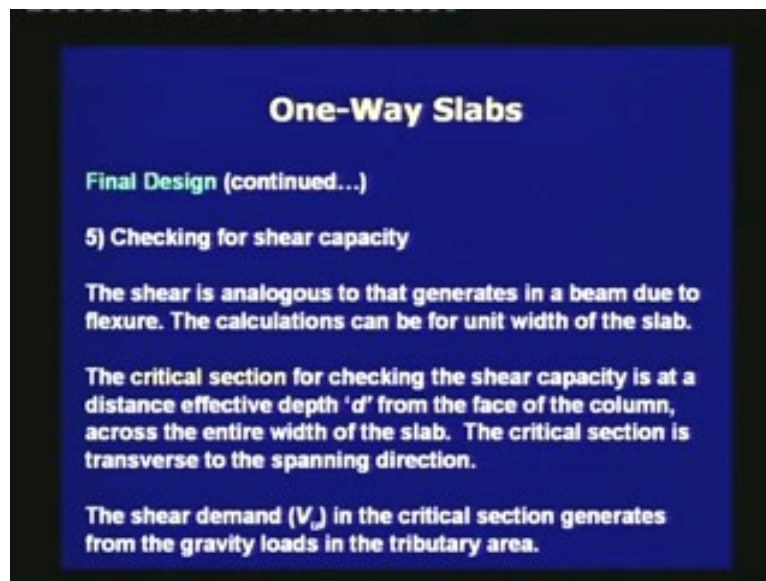
or, $P_e = A \frac{M_T}{Z_b}$ (9b-7)

Z_b is the section modulus. The above expression is same as $P_e = M_T / k_t$, which is Eq. (9b-3), with $e = 0$. The stresses at transfer can be checked with an estimate of P_0 from P_e .

Z_b is the section modulus. The above expression is same as P_e equal to M_T divided by k_t which is equation 9b-3 with e equal to 0.

Earlier we have seen one expression, where P_e is equal to M_T divided by e plus k_t . In that equation if you substitute e equal to 0, then we get P_e is equal to M divided by k_t and this expression is same as P_e equal to A times M_T divided by z_b . Thus both these expressions are same, but the one given here is a convenient expression when the eccentricity is zero. The stresses at transfer can be checked with an estimate of P_0 from P_e . That means, once we have calculated P_e , we can estimate P_0 from P_e and based on P_0 we can calculate the stresses at transfer.

(Refer Slide Time: 41:49)



The fifth step is to check the shear capacity. The shear is analogous to that generates in a beam due to flexure. The calculations can be per unit weight of the slab. The critical section for checking the shear capacity is at a distance effective depth d , from the face of the column or the supporting beam across the entire width of the slab. The critical section is transverse to the spanning direction. Thus, for one-way slabs the checking for shear is similar to beams, where the critical section is considered to be at a distance effective depth from the face of the support and the critical section is perpendicular to the spanning direction. The shear demand V_u in the critical section generates from the gravity loads in the tributary area. That means once we know the tributary area for the slab, you can calculate the shear demand that comes in the critical section.

(Refer Slide Time: 43:05)

One-Way Slabs

Final Design (continued...)

For adequate shear capacity

$$V_{ur} \geq V_u$$

Here,

$V_{ur} = V_c$, the shear capacity of uncracked concrete of unit width of slab

The expression of V_c is given in the Module of "Analysis and Design for Shear and Torsion". If this is not satisfied, it is preferred to increase the depth of the slab to avoid shear reinforcement.

For adequate shear capacity the resistance V_{ur} should be greater than V_u , where V_{ur} is equal to V_c , the shear capacity of uncracked concrete of unit width of the slab. We do not usually provide shear reinforcement in slabs. Thus, the shear capacity is equal to the shear capacity of the concrete for the uncracked section. The expression of V_c is given in the module of analysis and design for shear and torsion. If this is not satisfied, it is preferred to increase the depth of the slab to avoid shear reinforcement. Finally, we provide transverse reinforcement based on temperature and shrinkage.

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One-Way Slabs

Final Design (continued...)

6) Provide transverse reinforcement based on temperature and shrinkage.

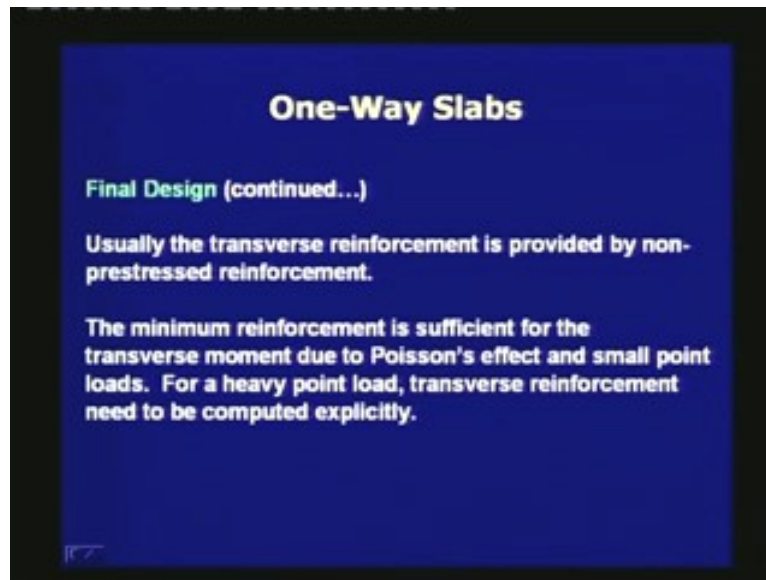
As per IS:456 - 2000, Clause 26.5.2.1, the minimum amount of transverse reinforcement ($A_{st,min}$ in mm^2) for unit width of slab is given as follows.

$$A_{st,min} = 0.15\% 1000h \text{ for Fe 250 grade of steel}$$
$$= 0.12\% 1000h \text{ for Fe 415 grade of steel.}$$

35

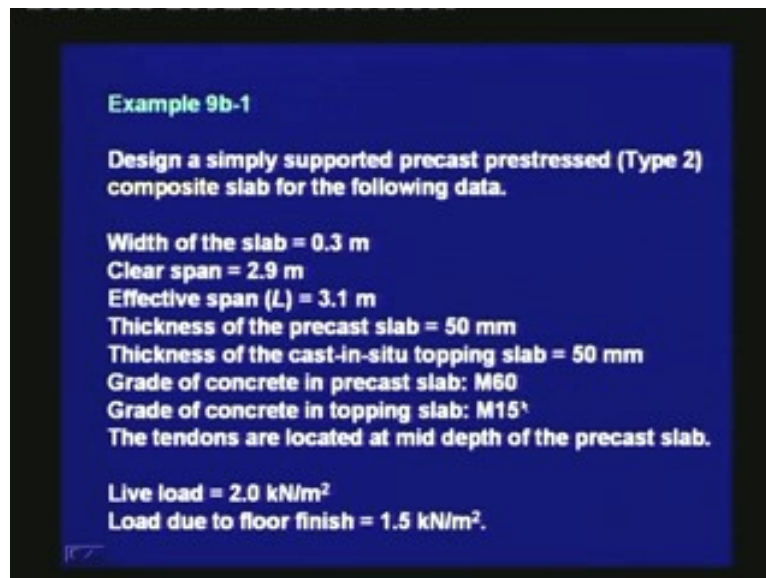
As per IS: 456-2000, clause 26.5.2.1, the minimum amount of transverse reinforcement $A_{st,minimum}$ in millimeter square per unit width of slab is given as follows: $A_{st,minimum}$ equal to 0.15% of the total section, which is 1000 times h for Fe 250 grade of steel and $A_{st,minimum}$ equal to 0.12% of 1000 h for Fe 415 grade of steel. Usually, the transverse reinforcement is provided by non-prestressed reinforcement.

(Refer Slide Time: 44:35)



A minimum reinforcement is sufficient for the transverse moment due to Poisson's effect and small point loads. For heavy point load, transverse reinforcement needs to be computed explicitly. Thus, whatever transverse reinforcement we provide is adequate for any transverse bending due to the Poisson's effect or due to small point loads. If there is a large point load then we have to explicitly calculate the transverse reinforcement.

(Refer Slide Time: 45:08)



Example 9b-1

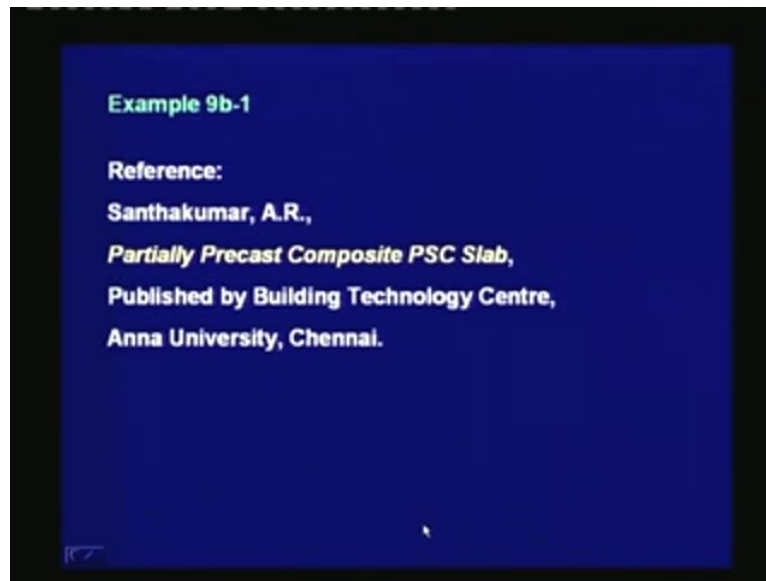
Design a simply supported precast prestressed (Type 2) composite slab for the following data.

Width of the slab = 0.3 m
Clear span = 2.9 m
Effective span (L) = 3.1 m
Thickness of the precast slab = 50 mm
Thickness of the cast-in-situ topping slab = 50 mm
Grade of concrete in precast slab: M60
Grade of concrete in topping slab: M15
The tendons are located at mid depth of the precast slab.

Live load = 2.0 kN/m²
Load due to floor finish = 1.5 kN/m².

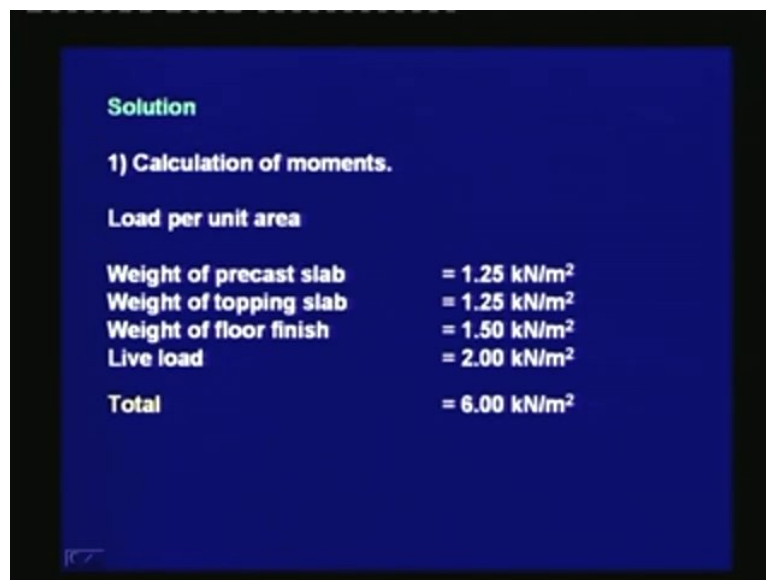
Next, we are learning this principle from a design example. Design a simply supported precast prestressed, Type 2, composite slab for the following data: width of the slab is 0.3 meters; clear span is equal to 2.9 meters; effective span l is equal to 3.1 meter; thickness of the precast slab is 50 millimeters; thickness of the cast-in slab is 50 millimeters; the grade of concrete in the precast slab is M60 and the grade of concrete in the topping slab is M15. Note that the two grades of concrete are different. The topping slab is a linear concrete as compared to the precast portion. The tendons are located at the mid depth of the precast slab. The live load is 2 Kilonewton per meter square and the load due to floor finish is 1.5 Kilonewton per meter square.

(Refer Slide Time: 46:14)



This problem has been taken from a guide book titled Partially Precast Composite PSC Slab, authored by Professor A. R. Santhakumar and it has been published by the building technology center of Anna University, Chennai.

(Refer Slide Time: 46:31)



We are calculating the moments. The load per unit area: first, we are calculating the weight of the precast slab. Next, we are calculating the weight of the topping slab. Then, we are calculating the weight of the floor finish and finally we are adding the live load. This gives us a total load of 6 Kilonewton per meter square of the slab.

(Refer Slide Time: 46:59)

Solution

1) Calculation of moments (continued...).

Total moment (M_T) along the width of the slab

$$\frac{wBL^2}{8} = \frac{6 \times 0.3 \times 3.1^2}{8}$$
$$= 2.16 \text{ kNm}$$

40

The total moment M_T along the width of the slab is given as the weight times the width times the span square divided by 8. This is a simple expression for a simply supported slab and substituting the values, we get M_T equal to 2.16 Kilonewton meters.

(Refer Slide Time: 47:26)

Solution

1) Calculation of moments (continued...).

The individual moments are calculated based on the proportionality of the loads.

M_{sw} = moment due to self weight of precast slab
 $= 2.16 \times (1.25 / 6.00) = 0.45 \text{ kNm}$

M_{top} = moment due to weight of topping slab
 $= 2.16 \times (1.25 / 6.00) = 0.45 \text{ kNm}$

M_{fin} = moment due to weight of floor finish
 $= 2.16 \times (1.50 / 6.00) = 0.54 \text{ kNm}$

M_{LL} = moment due to live load
 $= 2.16 \times (2.00 / 6.00) = 0.72 \text{ kNm}$

The individual moments are calculated based on the proportionality of the loads. We can calculate M_{sw} , which is the moment due to self-weight of the precast slab, which comes out to be 0.45 Kilonewton meters. M_{top} , the moment due to the weight of topping slab comes out to be again to 0.45 Kilonewton meters. M_{finish} , which is the

moment due to the weight of the floor finish is equal to 0.54 Kilonewton meters and M_{LL} , moment due to live load is coming out to be 0.72 Kilonewton meters. Once, we have calculated M_T , we can quickly calculate the individual moments by taking the load proportionality.

(Refer Slide Time: 48:11)

Solution

2) Calculation of geometric properties.

Precast section

Area

$$A_1 = 300 \times 50 = 15000 \text{ mm}^2$$

Moment of inertia

$$I_1 = \frac{1}{12} \times 300 \times 50^3$$
$$= 3125000 \text{ mm}^4$$

300mm

50mm

Next, we are calculating the geometric properties. For the precast section, the cross-section area is 300 millimeters times the depth 50 millimeters which is 15,000 millimeters square. The moment of inertia, I_1 is equal to 1 by 12 times B times depth cube which comes out to be 3125000 millimeter to the power 4.

(Refer Slide Time: 48:40)

Solution

2) Calculation of geometric properties.

Precast section
Distance to the extreme fibres

$$c_b = c_t = \frac{50}{2}$$
$$= 25 \text{ mm}$$

Section moduli

$$Z_b = Z_t = \frac{3125000}{25}$$
$$= 125,000 \text{ mm}^3$$

Diagram: A rectangular section with a width of 300mm and a height of 50mm.

We are calculating the distance to the extreme fibers, c_b and c_t which are both 25 millimeters. The section modulus can be calculated as I_1 divided by c_b or c_t and both of them are equal to 125,000 millimeter cube.

(Refer Slide Time: 49:02)

Solution

2) Calculation of geometric properties (continued...).

Composite section

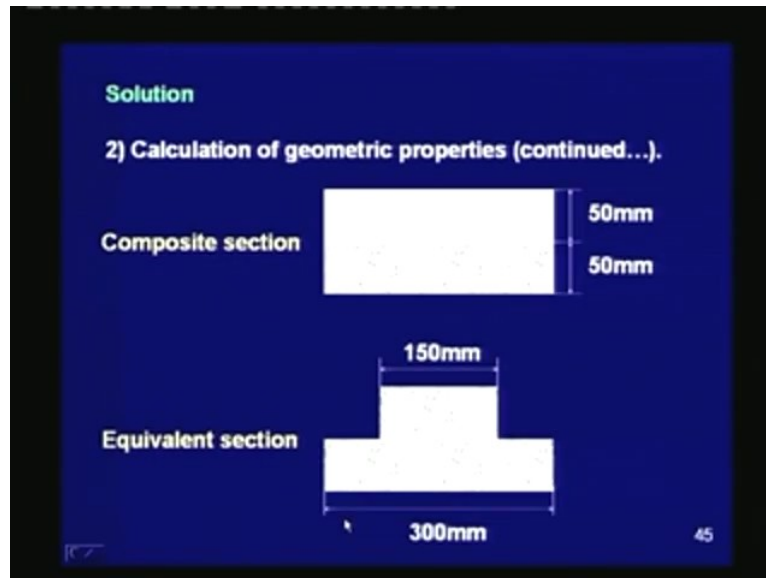
Since the grades of concrete are different for the precast-prestressed (PP) and cast-in-situ (CIS) portions, an equivalent (transformed) area is calculated. The CIS portion is assigned a reduced width based on the equivalent area factor (modular ratio).

Equivalent area factor = Modulus of CIS / Modulus of PP
= $\sqrt{\text{Grade of CIS} / \text{Grade of PP}}$
= $\sqrt{15/60}$
= 0.5

For the composite section, since the grade of concrete are different for the precast-prestressed and cast-in-situ (CIS) portions, an equivalent or transformed area, is calculated. The CIS portion is assigned a reduced width based on the equivalent area

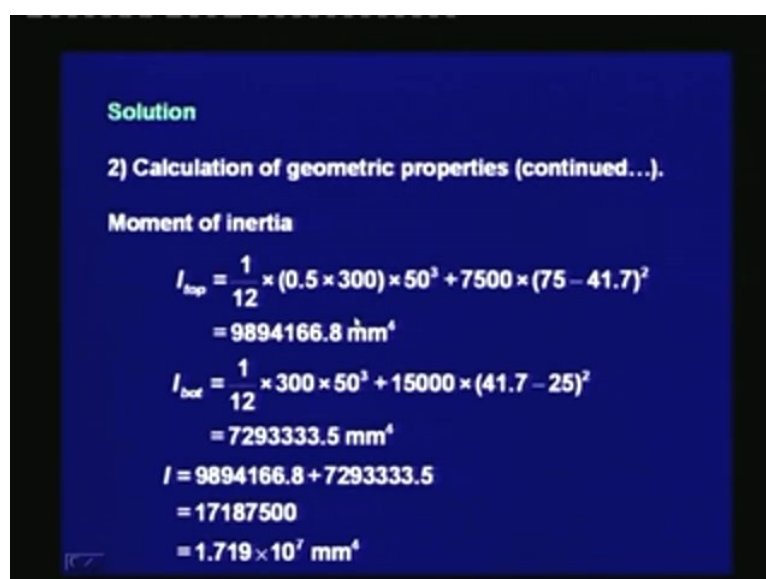
factor which is also called the modular ratio. The equivalent area factor is determined based on the grades of concrete and we get a value of 0.5.

(Refer Slide Time: 49:33)



Thus, the composite section is being transformed to an equivalent section with a reduced width of the topping slab which is 50% of its original value. For the composite transformed section, we are finding out the centroid, calculating the area of the top, the area of the bottom and the total area, from which we are locating the centroid which is at a distance of 41.7 millimeters from the bottom. This is the CGC of the equivalent transformed section.

(Refer Slide Time: 50:12)



We are calculating the moment of inertia of this equivalent section. First, we are calculating the I_{top} of the top part for the axis passing through the CGC. We are calculating I_{bottom} of the bottom part and then we are adding these two values to get the total I for the composite section.

(Refer Slide Time: 50:38)

Solution

2) Calculation of geometric properties (continued...).

Distance to the extreme fibres

$$y_b = 41.7 \text{ mm}$$

$$y_t = 58.3 \text{ mm}$$

Section moduli

$$Z_b = 17.19 \times 10^6 / 41.7$$

$$= 412527 \text{ mm}^3$$

$$Z_t = 17.19 \times 10^6 / 58.3$$

$$= 294703 \text{ mm}^3$$

Distance of the extreme fibers is given as 41.7 and 58.3 and the section modulus for the composite transformed section can be calculated by I divided by the respective y values which gives the section moduli.

(Refer Slide Time: 51:01)

Solution

3) Calculation of prestress

The tendons are located at the mid depth of the precast slab. Hence, $e = 0$ for the precast slab. The value of P_o is calculated directly from the following stress profiles.

Section

P_o

$M_{SW} + M_{top}$

$M_{fin} + M_{LL}$

Stress profiles

We are calculating the prestress. Since the tendons are located at the mid depth of the precast slab, e is 0 for the precast slab. The value of P_e is calculated directly from the following stress profiles. This P_e generates a uniform compressive stress. Then, the self weight and the topping load generates a stress profile like this and finally, we have the finished loads and the live load moment which is acting over the full composite section. From these stress profiles, you can say, to avoid tensile stress at the bottom under service conditions, the resultant stress is equated to zero.

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Solution

3) Calculation of prestress (continued...)

To avoid tensile stress at the bottom under service conditions, the resultant stress is equated to zero.

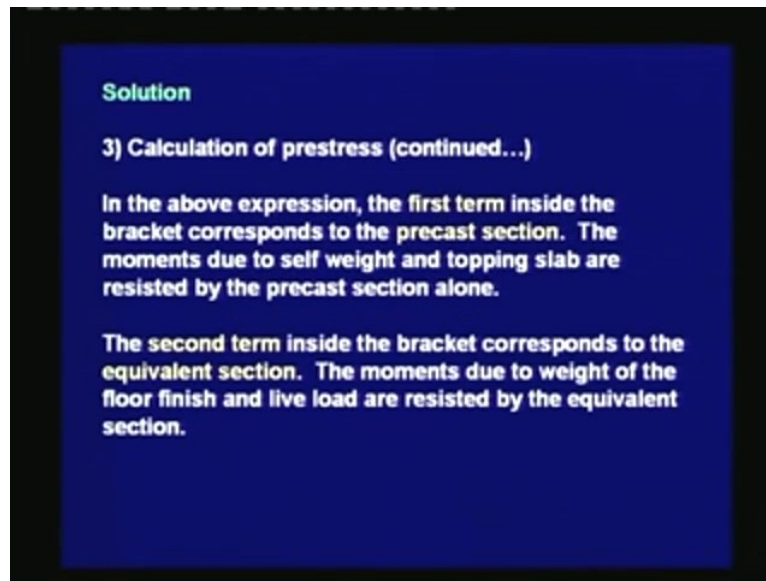
$$-\frac{P_e}{A_t} + \frac{M_{SW} + M_{top}}{Z_{top}} + \frac{M_{fin} + M_{LL}}{Z_{bot}} = 0$$

or, $P_e = A_t \left[\frac{M_{SW} + M_{top}}{Z_{top}} + \frac{M_{fin} + M_{LL}}{Z_{bot}} \right]$

50

Once, we write the expression of the resultant stress and we equate it to zero, we get an expression of the effective prestress that we need to apply at the precast plant. From this expression we can find out the value of P_e .

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Solution

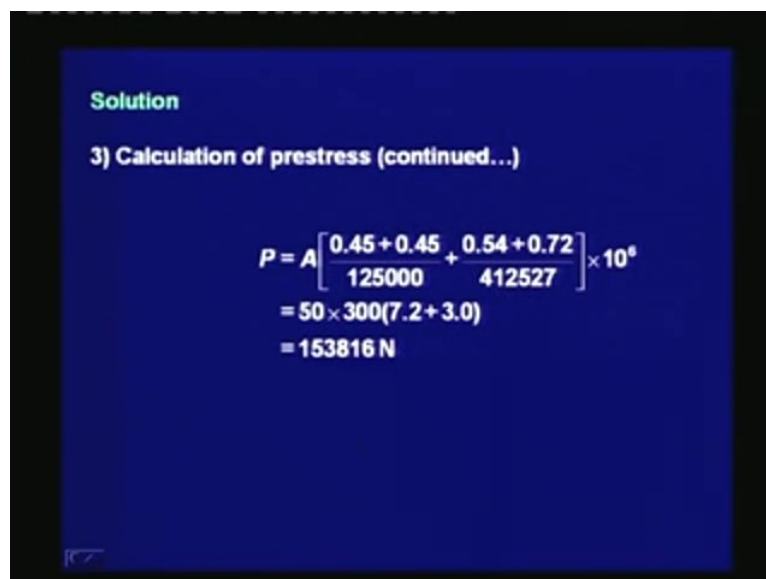
3) Calculation of prestress (continued...)

In the above expression, the first term inside the bracket corresponds to the precast section. The moments due to self weight and topping slab are resisted by the precast section alone.

The second term inside the bracket corresponds to the equivalent section. The moments due to weight of the floor finish and live load are resisted by the equivalent section.

In the above expression, the first term inside the bracket corresponds to the precast section. The moments due to self-weight and topping slab are resisted by the precast section alone. The second term inside the bracket corresponds to the equivalent section. The moments due to the weight of the floor finish and live load are resisted by the equivalent section. Thus, we are using the principles of a composite section to analyze this composite slab.

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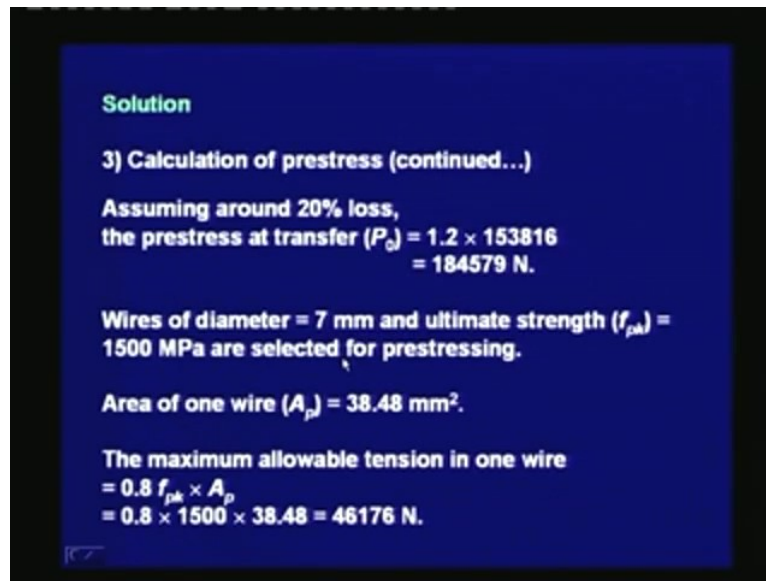
Solution

3) Calculation of prestress (continued...)

$$P = A \left[\frac{0.45 + 0.45}{125000} + \frac{0.54 + 0.72}{412527} \right] \times 10^6$$
$$= 50 \times 300 (7.2 + 3.0)$$
$$= 153816 \text{ N}$$

Substituting the values, we get the value of the prestress as 153816 Newton

(Refer Slide Time: 52:50)



Solution

3) Calculation of prestress (continued...)

Assuming around 20% loss,
the prestress at transfer (P_0) = 1.2×153816
= 184579 N.

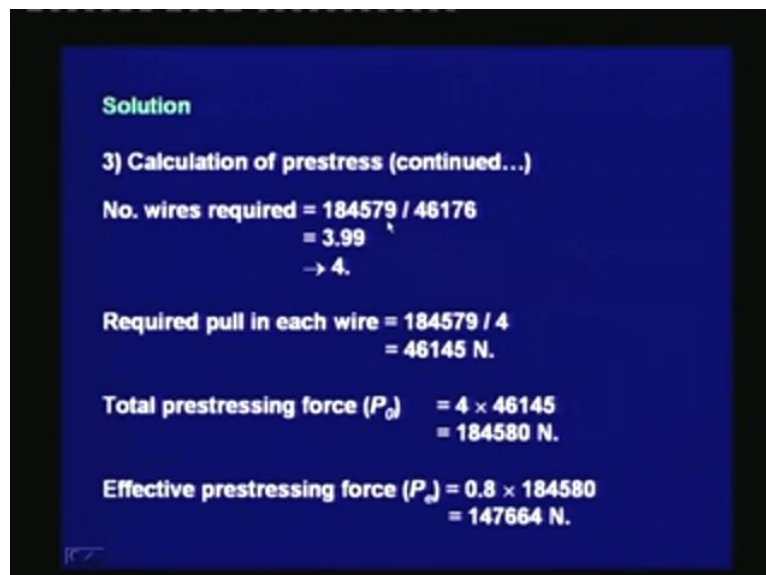
Wires of diameter = 7 mm and ultimate strength (f_{pk}) = 1500 MPa are selected for prestressing.

Area of one wire (A_p) = 38.48 mm².

The maximum allowable tension in one wire
= $0.8 f_{pk} \times A_p$
= $0.8 \times 1500 \times 38.48 = 46176$ N.

Assuming 20% loss, the prestress at transfer P_0 is almost equal to 1.2 times P_e , which is equal to 184579 Newton. Selecting wires of diameter 7 millimeters and ultimate strength f_{pk} equal to 1500 Megapascal, we are calculating the area of the prestressing steel. Area of one wire is 38.48 millimeter square and the maximum allowable tension in one wire is equal to 0.8 f_{pk} times A_p , which is 46176 Newton.

(Refer Slide Time: 53:30)



Solution

3) Calculation of prestress (continued...)

No. wires required = $184579 / 46176$
= 3.99
→ 4.

Required pull in each wire = $184579 / 4$
= 46145 N.

Total prestressing force (P_0) = 4×46145
= 184580 N.

Effective prestressing force (P_e) = 0.8×184580
= 147664 N.

Thus, the number of wires required is 184579, which is the prestressing force at transfer divided by the maximum prestressing force applied in one wire which gives us a value of 4 wires needed within 300 millimeters.

Required pull in each wire is given as the total prestressing force P_0 divided by 4 and thus the total prestressing force is equal to P_0 equal to 4 times 46145 equal to 184580 Newton. These values turn out to be close because we have got a value 4 which is very close to the calculated 3.99. The effective prestressing force is 80% of the prestressing force at transfer which comes out to be 147664 Newton.

(Refer Slide Time: 54:34)

Solution

4) Checking of stresses in concrete

At transfer

The compressive strength at 7 days (f_{ci}) = $0.7 f_{ck}$
= 0.7×60
= 42 MPa.

Allowable compressive stress ($f_{cc,all}$) = $0.44 f_{ci}$
= 0.44×42
= 18.5 MPa.

For Type 2 members, the allowable tensile stress ($f_{ct,all}$) is 3 MPa.

55

We are checking the stresses. At transfer, the compressive strength is 70% of f_{ck} which is 42 Megapascal. The allowable compressive stress is $0.44 f_{ci}$ which comes out to be 18.5 Megapascal. For Type 2 members, the allowable tensile stress is 3 Megapascal.

(Refer Slide Time: 54:56)

Solution

4) Checking of stresses in concrete (continued...)

At transfer

At the mid-span of the precast portion

$$f_c = -P_0/A_1 \pm M_{SW}/Z_1$$
$$= -12.3 \pm (0.45 \times 10^6 / 125000)$$
$$f_t = -15.9 \text{ MPa}$$
$$f_b = -8.7 \text{ MPa}$$

$\therefore |f_t| \leq f_{cc,all}$ OK

At transfer, we see that for the precast portion, the average stress is minus 12.3 Megapascal which is less than the allowable value of 18 Megapascal. Hence, it is satisfactory for the transfer load case. At mid-span of the precast section, we are including the moment due to self-weight. There also we find that the stress at the top which is of higher value 15.9 is less than the allowable value and hence this stress profile is acceptable at transfer.

(Refer Slide Time: 55:40)

Solution

4) Checking of stresses in concrete (continued...)

After casting of topping slab at 28 days

$$\text{Allowable compressive stress } (f_{cc,all}) = 0.44 f_{ck}$$
$$= 0.44 \times 60$$
$$= 26.4 \text{ MPa.}$$

The allowable tensile stress ($f_{ct,all}$) is 3 MPa.

After casting of the topping slab at 28 days, the allowable compressive stress is now 0.44 of 60 which is 26.4 Megapascal and allowable tensile stress still remains as 3 Megapascal.

(Refer Slide Time: 55:58)

Solution

4) Checking of stresses in concrete (continued...)

After casting of topping slab

At the mid-span of the precast portion

$$f_c = -P_0/A_1 \pm (M_{SW} + M_{top})/Z_1$$

$$= -12.31 \pm ((0.45 + 0.45) \times 10^6 / 125000)$$

$f_t = -19.5$ MPa

$f_b = -5.1$ MPa

$\therefore |f_t| \leq f_{cc,all}$ OK

Diagram showing stress distribution: -19.5 at top, -5.1 at bottom.

After the casting of the topping slab, we are calculating the stress conditions. At the mid span, we have the force due to the prestressing force and due to the self weight and topping slab. These values are coming out to be minus 19.5 which is the higher one and this is also less than the allowable compressive value.

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Solution

4) Checking of stresses in concrete (continued...)

At service, for precast portion

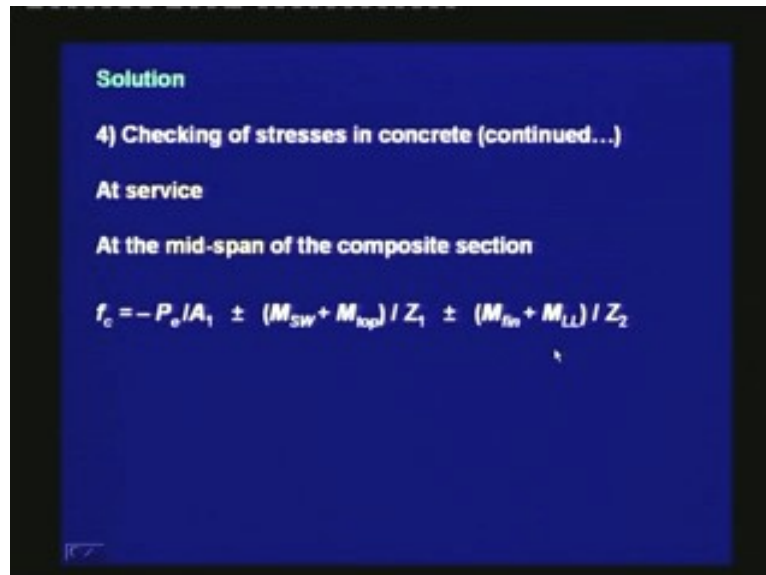
Allowable compressive stress ($f_{cc,all}$) = $0.35 f_{ck}$
 = 0.35×60
 = 21 MPa.

The allowable tensile stress ($f_{ct,all}$) is 3 MPa.

60

At service conditions, for the prestressing portion the allowable compressive stress is 21. The allowable tensile stress is 3.

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Solution

4) Checking of stresses in concrete (continued...)

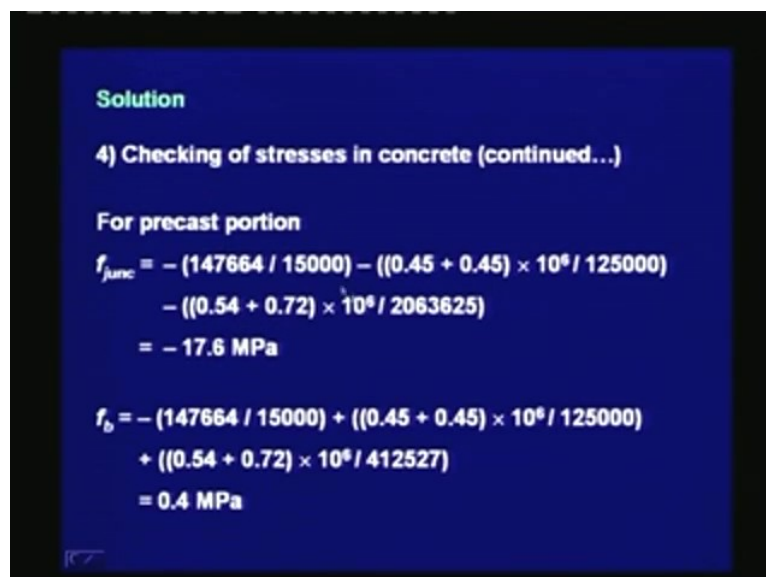
At service

At the mid-span of the composite section

$$f_c = -P_e / A_1 \pm (M_{SW} + M_{top}) / Z_1 \pm (M_{fin} + M_{LL}) / Z_2$$

The expression of the stress at the mid-span for the composite section is given by: minus P_e by A , the prestressing force, then the effect of the self-weight and the topping moment, and the effect of the finishing moment and the live load moment.

(Refer Slide Time: 56:55)



Solution

4) Checking of stresses in concrete (continued...)

For precast portion

$$f_{junc} = - (147664 / 15000) - ((0.45 + 0.45) \times 10^6 / 125000) - ((0.54 + 0.72) \times 10^6 / 2063625)$$
$$= - 17.6 \text{ MPa}$$
$$f_b = - (147664 / 15000) + ((0.45 + 0.45) \times 10^6 / 125000) + ((0.54 + 0.72) \times 10^6 / 412527)$$
$$= 0.4 \text{ MPa}$$

We are calculating the stresses at the junction. We find out it is minus 17.6 and the stress at the bottom is 0.4 Megapascal; these values are for the precast portion.

(Refer Slide Time: 57:10)

Solution

4) Checking of stresses in concrete (continued...)

For cast-in-situ portion

$$f_t = - (0.54 + 0.72) \times 10^6 / 294703$$
$$= - 4.3 \text{ MPa}$$

$$f_{\text{junc}} = - (0.54 + 0.72) \times 10^6 / 2063625$$
$$= - 0.6 \text{ MPa}$$


For the cast-in-situ portion, the stress at the top is minus 4.3 Megapascal and the stress at the junction is minus 0.6 Megapascal.

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Solution

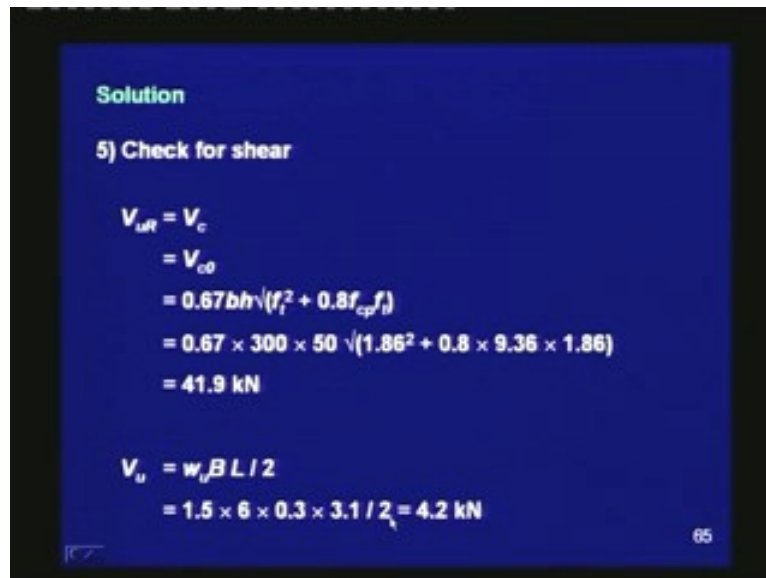
4) Checking of stresses in concrete (continued...)

The critical stresses are in the precast portion.

$$|f_{\text{junc}}| \leq f_{cc,all} \quad \text{OK}$$
$$f_b \leq f_{ct,all} \quad \text{OK}$$


Thus, we get the stress profile for the composite section and we see that, most of the stress is concentrated in the precast portion. Here also, this value minus 17.6 is less than the allowable compressive stress and hence the stress profile under service condition is acceptable.

(Refer Slide Time: 57:45)



Solution

5) Check for shear

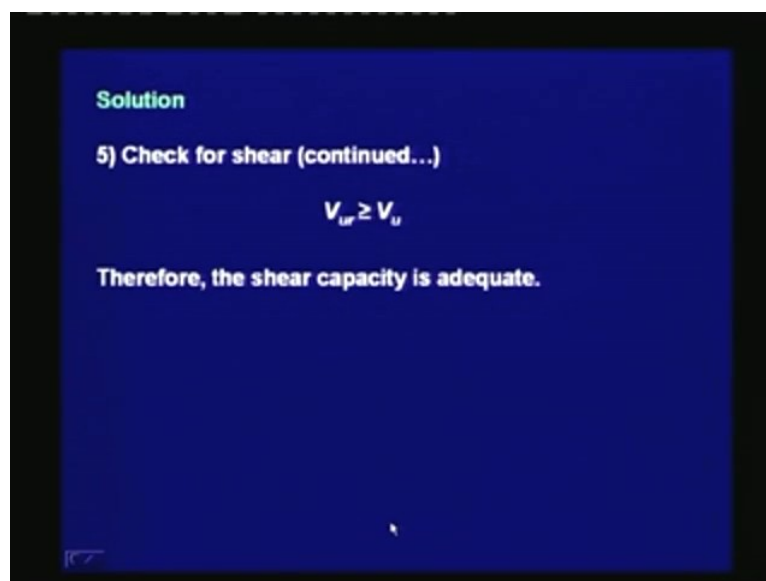
$$\begin{aligned} V_{cr} &= V_c \\ &= V_{c0} \\ &= 0.67bh\sqrt{f_t^2 + 0.8f_{cp}f_t} \\ &= 0.67 \times 300 \times 50 \sqrt{1.86^2 + 0.8 \times 9.36 \times 1.86} \\ &= 41.9 \text{ kN} \end{aligned}$$

$$\begin{aligned} V_u &= w_u B L / 2 \\ &= 1.5 \times 6 \times 0.3 \times 3.1 / 2 = 4.2 \text{ kN} \end{aligned}$$

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We are checking the shear capacity. We are first finding out the shear capacity to be 42 Kilonewton and the shear demand to be 4.2 Kilonewton.

(Refer Slide Time: 57:58)



Solution

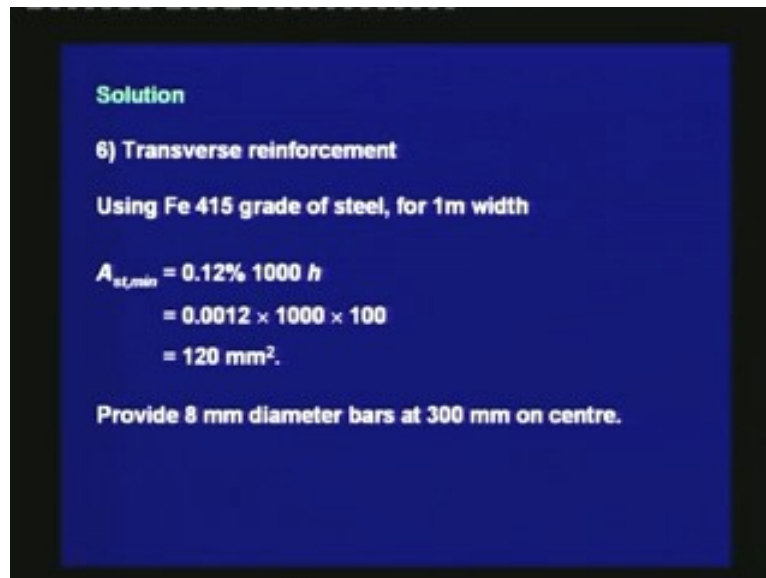
5) Check for shear (continued...)

$$V_{cr} \geq V_u$$

Therefore, the shear capacity is adequate.

Since the shear strength is greater than the demand, the shear capacity is adequate.

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Solution

6) Transverse reinforcement

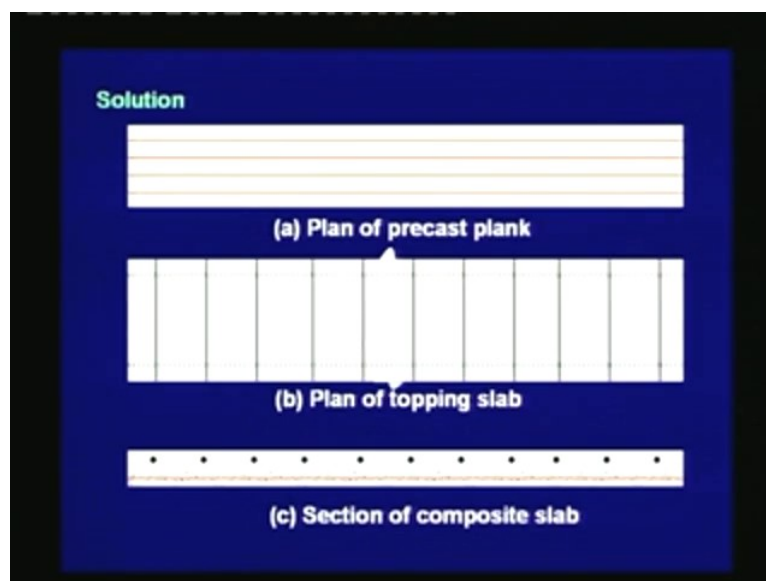
Using Fe 415 grade of steel, for 1m width

$$A_{st,min} = 0.12\% 1000 h$$
$$= 0.0012 \times 1000 \times 100$$
$$= 120 \text{ mm}^2.$$

Provide 8 mm diameter bars at 300 mm on centre.

We are providing some transverse reinforcement as per the minimum requirement that is 8 millimeter diameter at 300 millimeter on center.

(Refer Slide Time: 58:13)



Solution

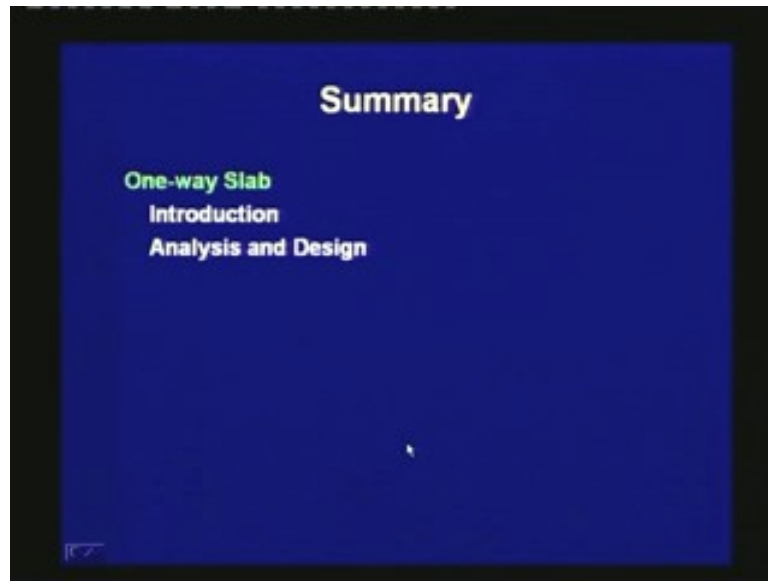
(a) Plan of precast plank

(b) Plan of topping slab

(c) Section of composite slab

Thus, this is the final design of the precast plank. We have laid the prestressing steel, the topping steel and we have cast the concrete. Finally, we get the precast plank for the design forces.

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Thus, in this module we covered one-way slabs and then we studied the analysis and design of one-way slabs. Thank you.