Prestressed Concrete Structures

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Lecture - 34

Continuous Beams (Part 2)

Welcome back to prestressed concrete structures. This is the third lecture in module 8 on cantilever and continuous beams. In this lecture we shall continue with the continuous beams. First, we shall study about concordant cable profile; then we shall discuss about the different cable profiles in a continuous beam; and we shall move on to partially continuous beams, analysis at ultimate limit state and moment redistribution.

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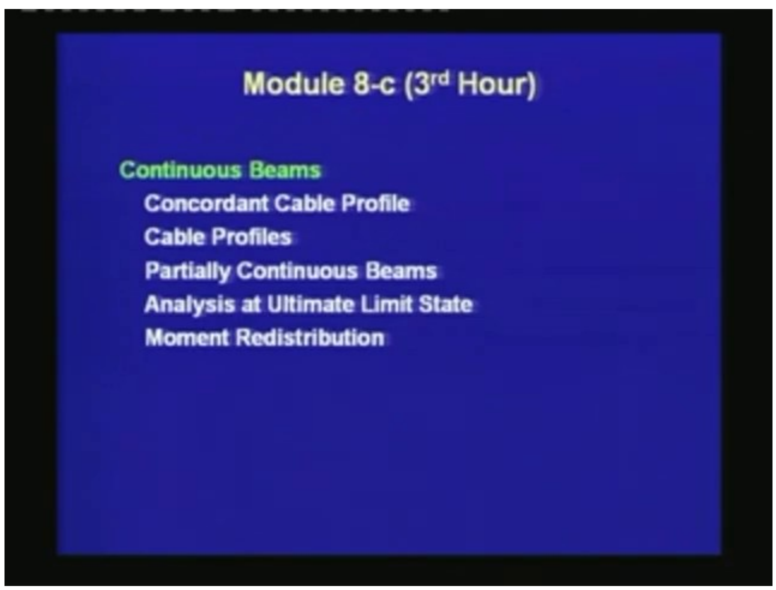

First, we shall discuss about concordant cable profile.

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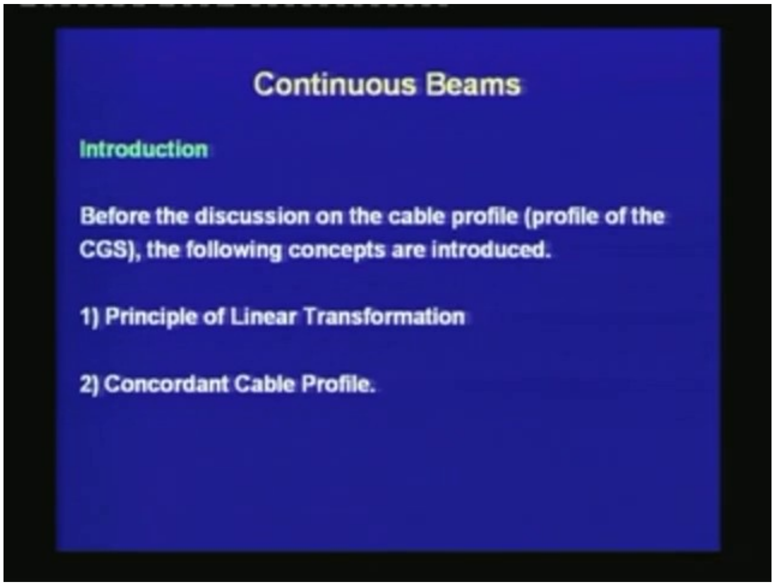

Before the discussion on the cable profile, which actually means profile of the CGS the following concepts are introduced: first, principle of linear transformation and second the concordant cable profile.

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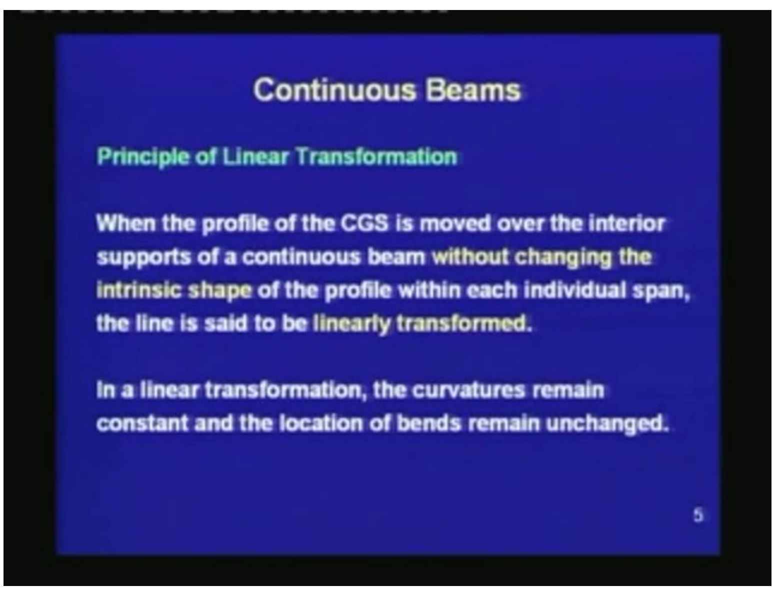

When the profile of the CGS is moved over the interior supports of continuous beams without changing the intrinsic shape of the profile within each individual span the line is said to be linearly transformed. In a linear transformation the curvatures remain constant and the location of bends remains unchanged.

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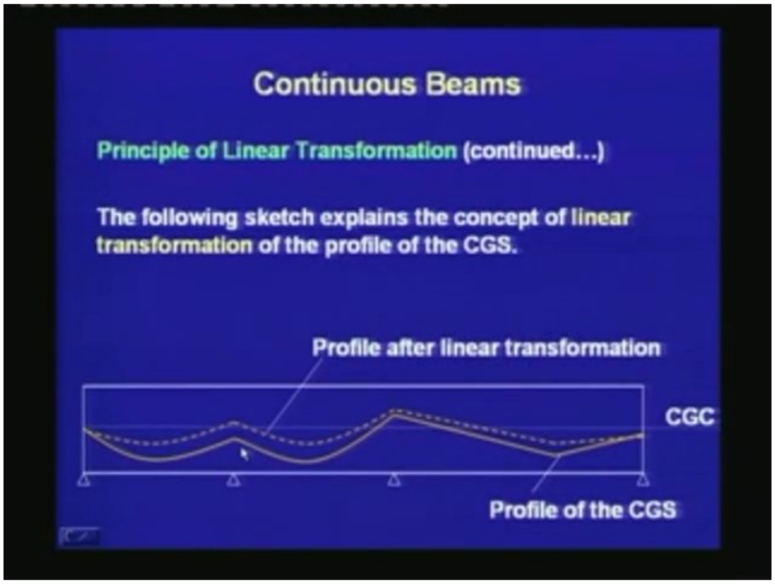

Let us try to understand this by the help of a sketch. In this continuous beam the solid line is an original cable profile and then we are shifting the cable profile such that we are maintaining the intrinsic shape in each span and the location of the bends are also kept same. If I shift the cable accordingly then this shift is called the linear transformation. Thus, the dotted line in this beam is a profile after linear transformation from the original profile of the CGS. The linear transformation cannot involve the movement of the CGS at the ends of a beam or at the support of a cantilever.

When we are shifting the profile we have to make sure that we cannot change the location of the CGS at the ends that means for this beam the left end and the right end positions of the CGS are fixed. In linear transformations those positions cannot be changed. It is only the position within the span that can be changed and the location of the bends should also be kept constant.

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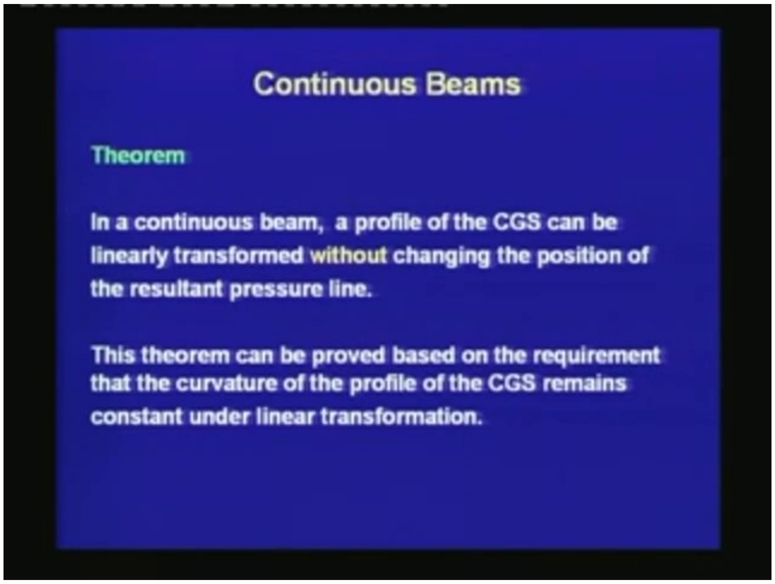

There is a theorem related with the linear transformation. In a continuous beam, a profile of the CGS can be linearly transformed without changing the position of the resultant pressure line. To repeat, if we are changing the profile by linear transformation the pressure line due to the prestressing force does not change. This theorem can be proved based on the requirement that the curvature of the profile of the CGS remains constant under linear transformation.

Since a prerequisite of a linear transformation is that the curvature in each within individual span should remain the same. This can prove that the location of the pressure line will not change after the linear transformation of the profile of the CGS.

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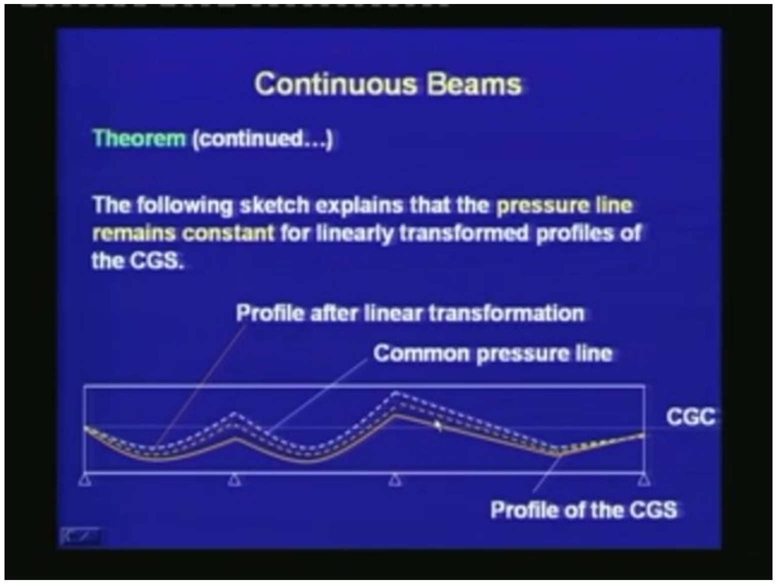

This sketch helps us to understand that the pressure line remains constant for linearly transformed profiles of the CGS. The solid line is the original profile and the dashed orange line is a profile after linear transformation. For both these profiles the pressure line is same. Thus, the dashed line is the common pressure line for the profiles which are linearly transformed.

Remember that the pressure line does not depend on the magnitude of the prestressing force. Thus, even if the prestressing force drops down from the value at transferred to the value at service due to the long term losses the pressure lines remains same.

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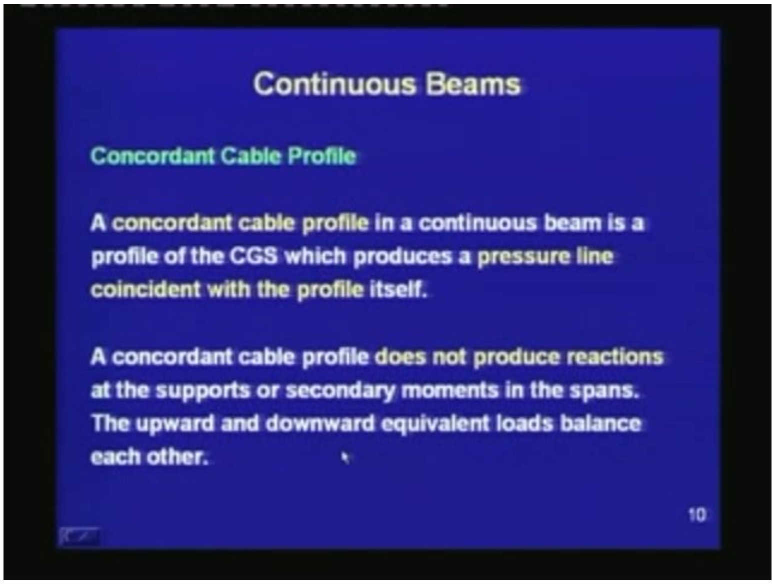

Next, we are coming to the definition of a concordant cable profile. A concordant cable profile in a continuous beam is a profile of the CGS which produces a pressure line coincident with the profile itself. Thus, if we are placing a cable profile at the location of the pressure line then it is called a concordant cable. That means, for a concordant cable the pressure line due to the prestressing force is same as the profile of the CGS.

A concordant cable profile does not produce reactions at the supports or secondary moments in the spans. The upward and downward equivalent loads balance each other. This is one property of the concordant profile that it does not produce reactions in the supports, it is a self-equilibrating system and the upward and downward equivalent loads balance each other. Since there is no reaction at the supports there is no secondary movement due to the intermediate reactions is generated within the continuous beam. The following sketch shows a concordant cable profile which is coincident with the pressure line.

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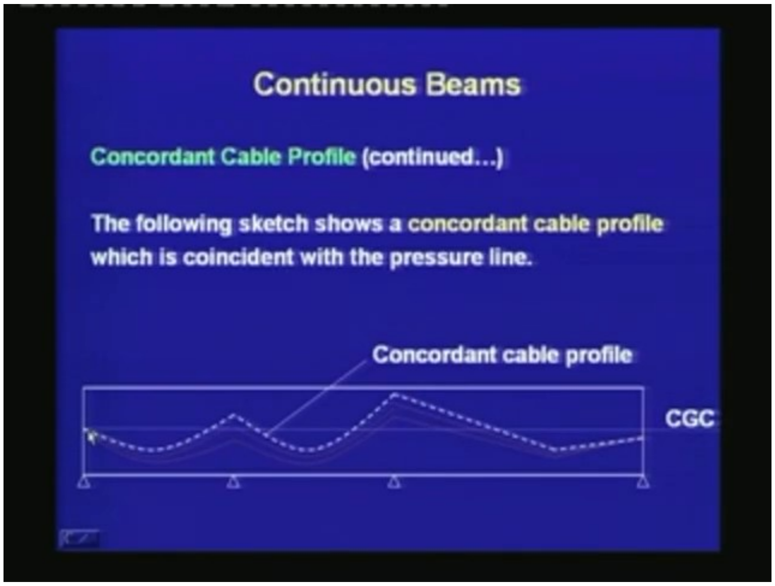

Thus, if we shift the profile such that the cable is placed along the pressure line then it is a concordant profile because our profile after linear transformation does not lead to a shift of a pressure line. Hence, if we place the CGS at the pressure line itself after linear transformation then the final pressure line will be same as the profile of the CGS hence that profile is the concordant cable profile.

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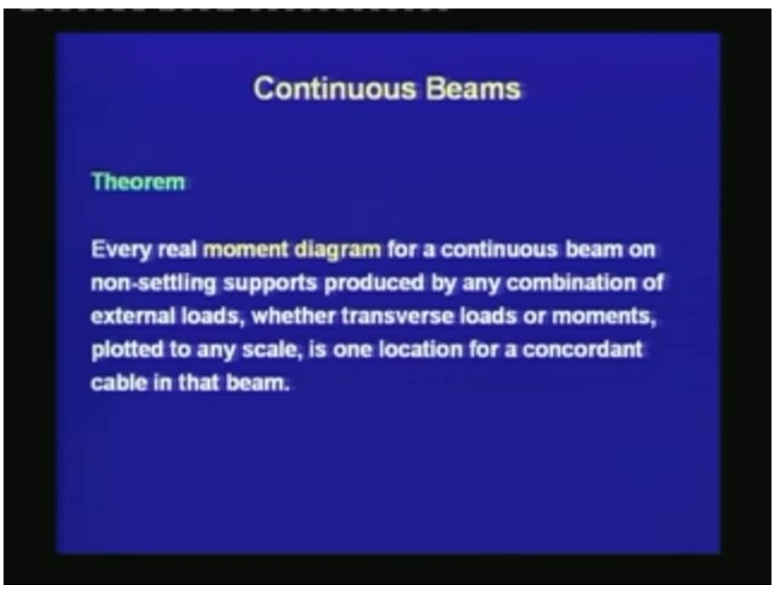

There is one theorem for concordant cable profile. Every real moment diagram for a continuous beam on non-settling supports produced by any combination of external loads whether transverse loads or moments plotted to any scale, is one location for a concordant cable in that beam. This is the important theorem to get the concordant cable profile. What it says is that, if we have a moment diagram due to a certain set of external loads and if we plot the moment diagram to a certain scale then that diagram will be a concordant profile for that particular continuous beam.

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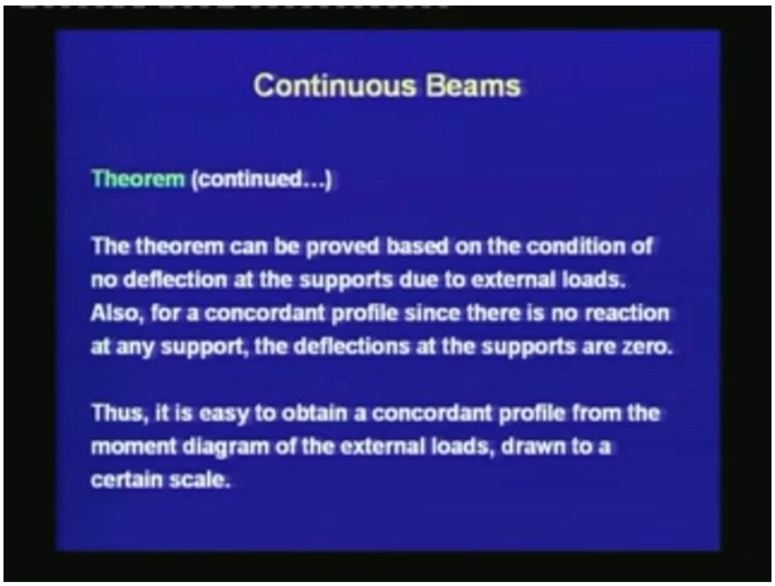

The theorem can be proved based on the condition of no deflection at the supports due to external loads. Also, for a concordant profile since there is no reaction at any support, the deflections at the supports are zero. The proof can be understood by the simple concept that when we are developing a moment diagram due to an external load we are not considering any deflection at the supports. For the concordant profile since there is no reaction at the supports there will not be any deflection.

Thus, the condition of a moment diagram is same as the criteria of the concordant profile and hence the moment diagram can be a concordant profile for that particular continuous beam. Thus, it is easy to obtain a concordant profile from the moment diagram of the external loads drawn to a certain scale. Once we know the moment diagram we can plot to a suitable scale to develop a concordant profile for the beam.

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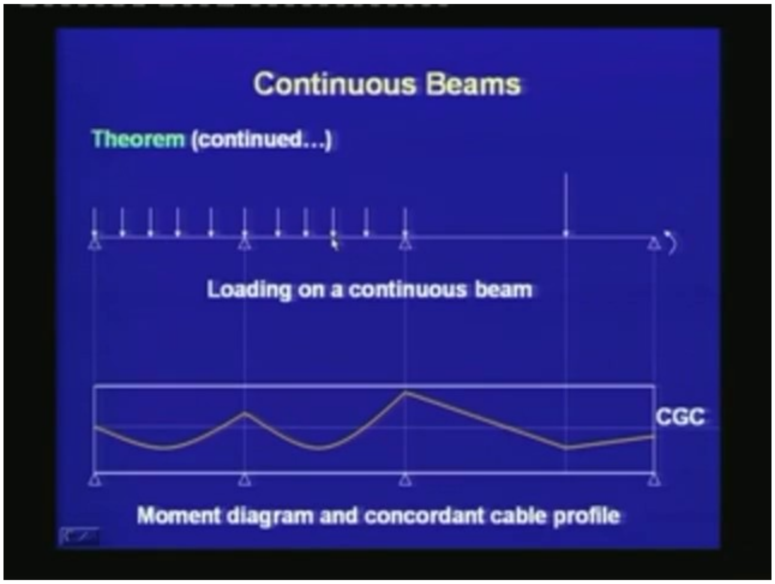

This sketch shows a certain external load for a continuous beam. First, we are drawing the moment diagram which we can evaluate by the moment distribution method or a computer analysis. Once we have drawn the moment diagram, this diagram is a concordant profile for the particular continuous beam. The computation of the concordant profile helps in the layout of the cable profile. The cable profile need not be designed to be a concordant profile. It should be such that the stresses in concrete at transfer and at service are within the allowable values. If a concordant profile is selected then the calculations become easier.

The pressure line due to prestress coincides with profile. The shift of the pressure line due to external loads is measured from the profile itself. When we are trying to layout a cable profile we have to keep in mind that it need not be a concordant profile, what we have to satisfy is that the stresses in the concrete at the extreme phases should be within the allowable values. But the use of a concordant profile helps us in the computations. If we select the concordant profile, since it does not create any reaction at the supports and no secondary moment is generated within the spans, the pressure line due to the prestressing force is in the profile itself.

The shift of the pressure line due to the external loads can be calculated from the cable profile, the way we have done for a simply supported beam. In fact for a simply supported beam, the cable profile is always a concordant profile, where the pressure line coincides with cable profile in absence of any external loads.

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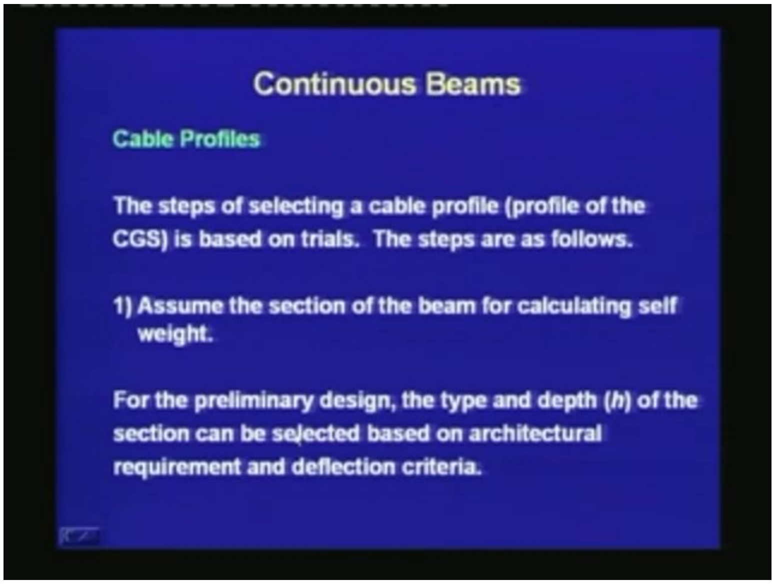

Next, we are moving on to the discussion on cable profiles of continuous beams. The steps of selecting a cable profile or the profile of the CGS is based on trials. The steps are as follows: first, assume the section of the beam for calculating self-weight. For the preliminary design, the type and depth, which is represented as h of the section can be selected based on architectural requirement and deflection criteria. That is, the first step is to assume a type of a section and a certain depth. The type of the section can be based on the application. Earlier, we had studied regarding the choice of different types of sections depending on the application and depth of the section depends on architectural requirement or it can be determined based on the deflection criteria. If we satisfy the conditions for deflection criteria then we need not have to check deflections later on.

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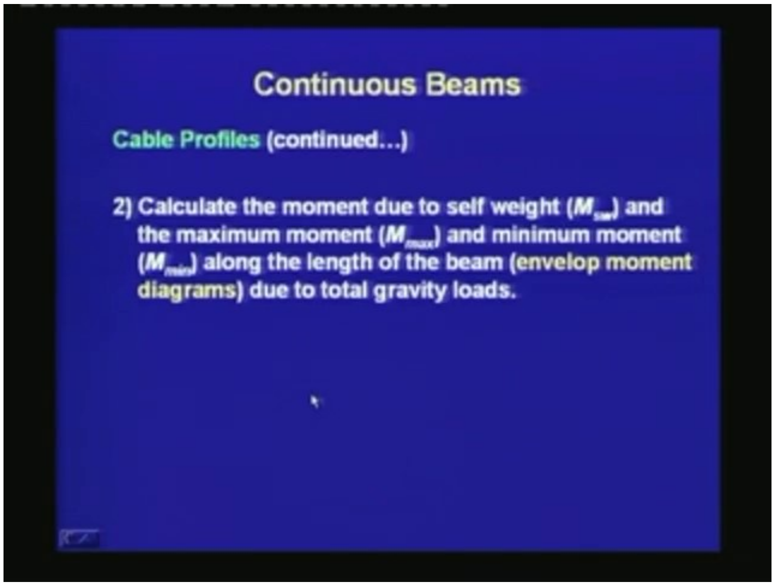

The second step is to calculate the moment due to self-weight which is represented as M_{sw} and the maximum moment M_{max} and minimum moment M_{min} along the length of the beam due to total gravity loads. Thus, the maximum moment and a minimum moment are available from the envelop moment diagrams and these moment diagrams are due to the external loads only which of course includes the self-weight of the beam.

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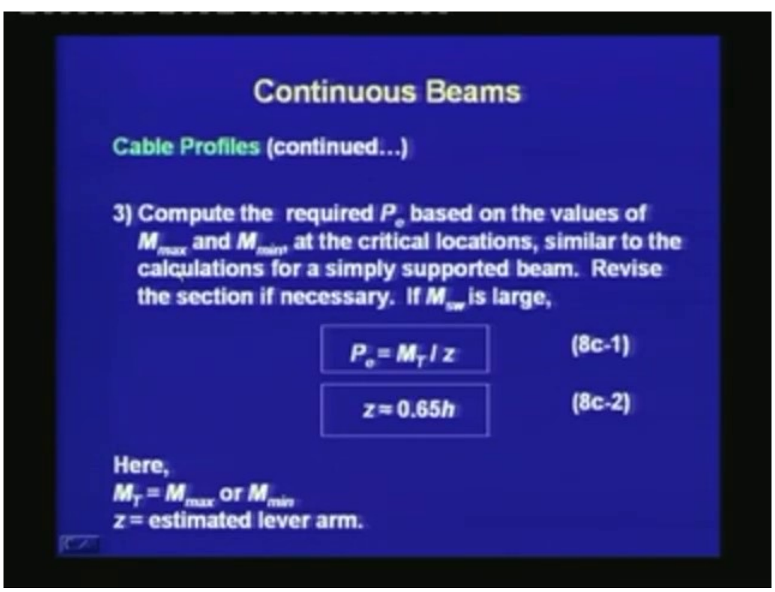

The third step is to compute the required effective prestress P_e based on the values of M_{max} and M_{min} at the critical locations, similarly to the calculations for a simply supported beam. Revise the section if necessary. What we do for a continuous beam is that, we select certain critical sections. One section in the span and two sections near the face of the supports are the critical sections for continuous beams.

The calculations of the effective prestress and the location of the CGS are very similar to the calculations of a simply supported beam. That means given a section, the steps are same as simply supported beam. If the self-weight is large then the estimate of the effective prestress is $P_e = M_T$ divided by Z where Z is roughly equal to 65% of the total depth which is represented as h. Here, M_T is either of M_{max} or M_{min} depending on which is the critical section we have selected and Z is an estimated lever arm.

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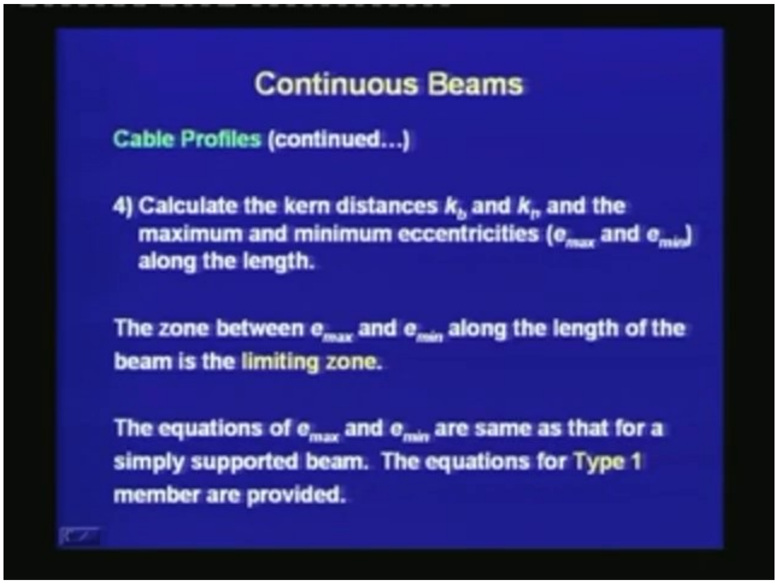

The fourth step is to calculate the kern distances K_b and K_p and the maximum and minimum eccentricities e_{max} and e_{min} along the length. Thus, once we know the section and we know the prestressing force, we can also have an estimate of a prestressing force at transfer and from this we can calculate the maximum and minimum eccentricities, similar to the calculations for a simply supported beam. The zone between e_{max} and e_{min} along the length of beam is the limiting zone for the placement of the CGS. The equations of e_{max} and e_{min} are same as that for a simply supported beam. The equations for type 1 member are provided.

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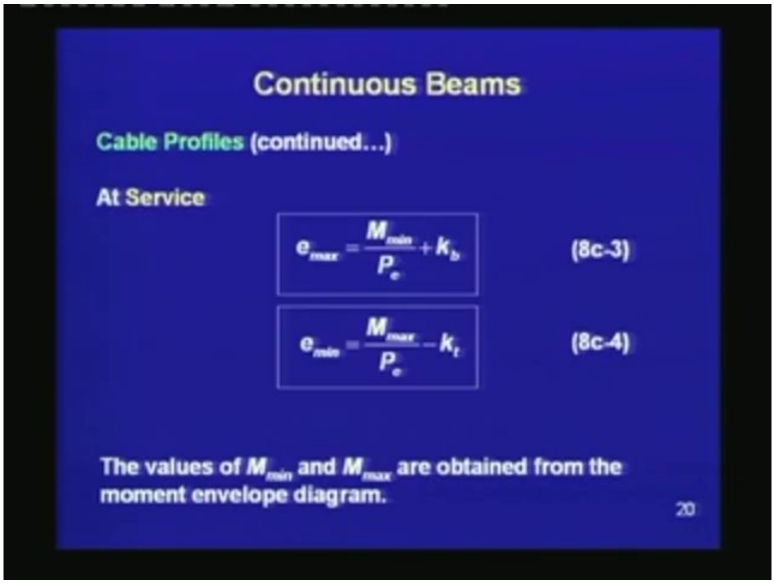

At service, e_{max} equal to M_{min} divided by the effective prestress P_e plus the bottom kern distance K_b. e_{min} is equal to M_{max} divided by P_e minus the top kern distance K_t. Remember that the values of M_{min} and M_{max} are obtained from the envelop moment diagram and P_e has been estimated based on the approximate relationship given earlier.

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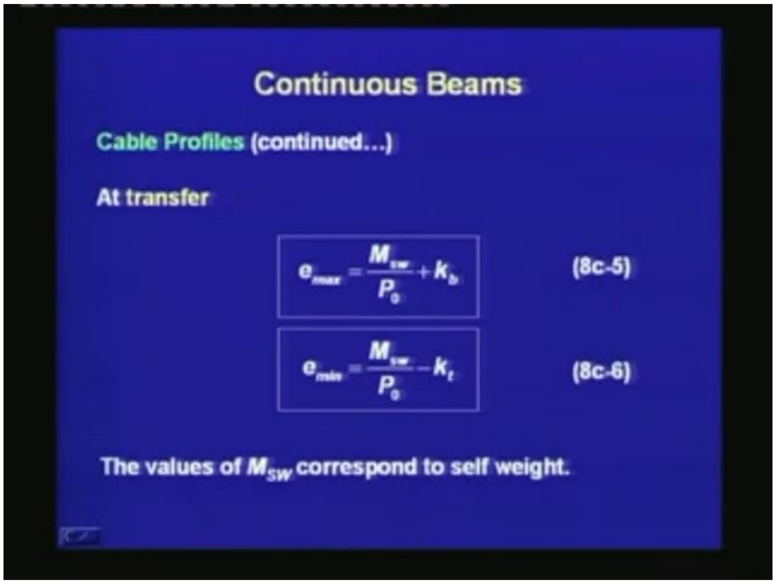

We calculate another set of e_{max} and emin for transfer. At transfer, e_{max} = M_{sw} divided by P_0 + $K_{b.}$ e_{min} = M_{sw} divided by P_0 – $K_{t.}$ The values for M_0 for the different locations correspond to the self-weight, and P_0 is estimated from the effective prestress $P_{e.}$

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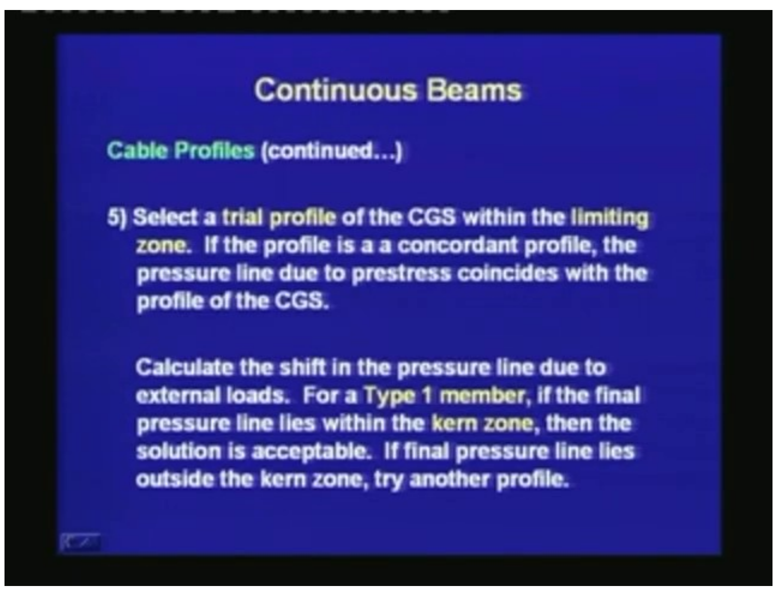

Once we know the limiting zone which is the zone in between the e_{max} and e_{min}, the fifth step is to select a trial profile of the CGS within the limiting zone. If the profile is a concordant profile, the pressure line due to prestress coincides with the profile of the CGS. Calculate the shift in the pressure line due to external loads. For a type 1 member, if the final pressure line lies within the kern zone, then the solution is acceptable. If final pressure line lies outside the kern zone, try another profile.

The fifth step means that once we know the limiting zone we are having a trial cable profile. If we have a profile which is a concordant profile based on the external moment diagram then the pressure line due to the prestressing force is same as the cable profile. Hence, any shift of the cable profile due to external loads can be calculated from the profile itself. Now, once we are able to calculate the shift of the pressure line to external loads, we need to make sure that it lies within the zone, so that we satisfy the allowable stresses in the member. For type 1 member for which we cannot have kern such stress at all in the section, the pressure line after we have the external loads should lie within the kern zone. If the pressure line due to the external loads is falling outside the kern zone, then we have to revise our cable profile.

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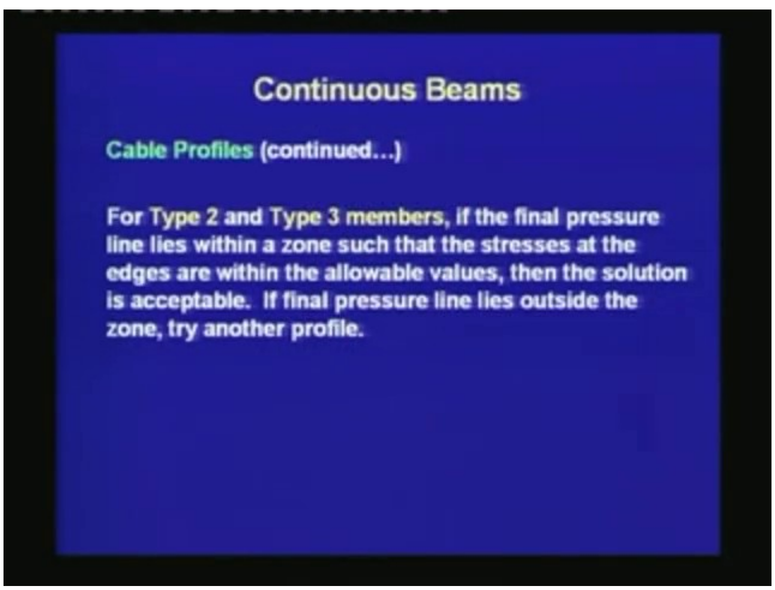

Similarly for type 2 and type 3 members, if the final pressure line lies within a zone such that the stresses at the edges are within the allowable values, then the solution is acceptable. If final pressure line lies outside the zone, try another profile. Let us recollect, for type 2 and type 3 members we allow tensile stresses in the prestress member. Hence, the pressure lines due to the external loads can lie outside the kern zone, but still it should be limited within a zone such that the stresses in the concrete at the ages are within the allowable values. If it is satisfied and our trial cable profile is fine; if the pressure line due to the external loads lies outside the particular zone which creates tensile stresses exceeding the allowable values then we need to change the cable profile.

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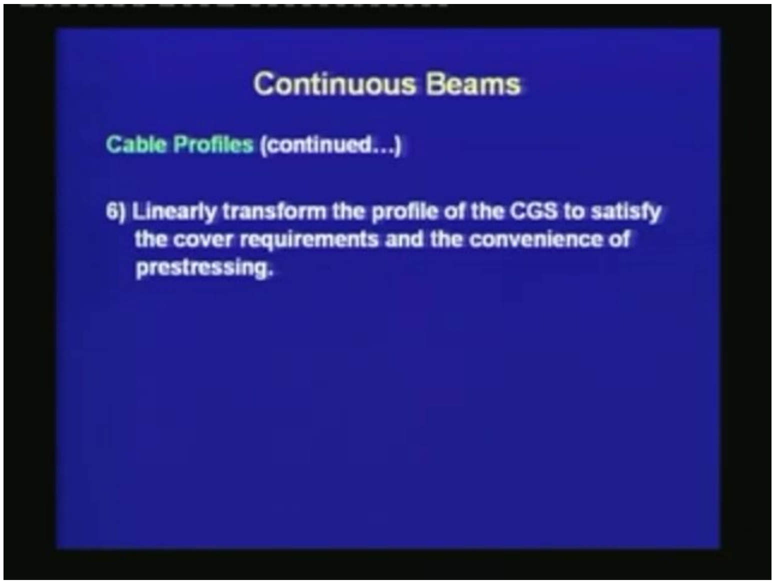

The sixth step is linearly transforming the profile of the CGS to satisfy the cover requirements and the convenience of prestressing. As I said before, that cable profile cannot have a short kink at the supports; it is adjusted such that the tendon can be placed conveniently within the beam and tension can be applied without much loss in the prestressing force. Also we need to make sure that we are satisfying the cover requirements. Hence, the trial profile needs to be checked and if required needs to be adjusted such that we satisfy the cover requirements, and as well we think of the convenience of the placement of the tendon and prestressing the tendon.

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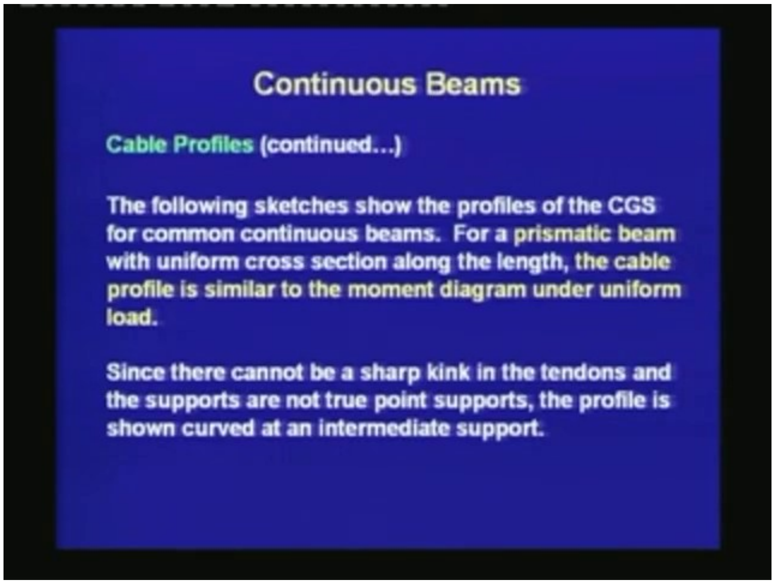

The following sketches show the profiles of the CGS for common continuous beams. For a prismatic beam with uniform cross-section along the length, the cable profile is similar to the moment diagram under uniform load. Since there cannot be a sharp kink in the tendons at the supports, the supports are not true point supports; the profile is shown curved at an intermediate support.

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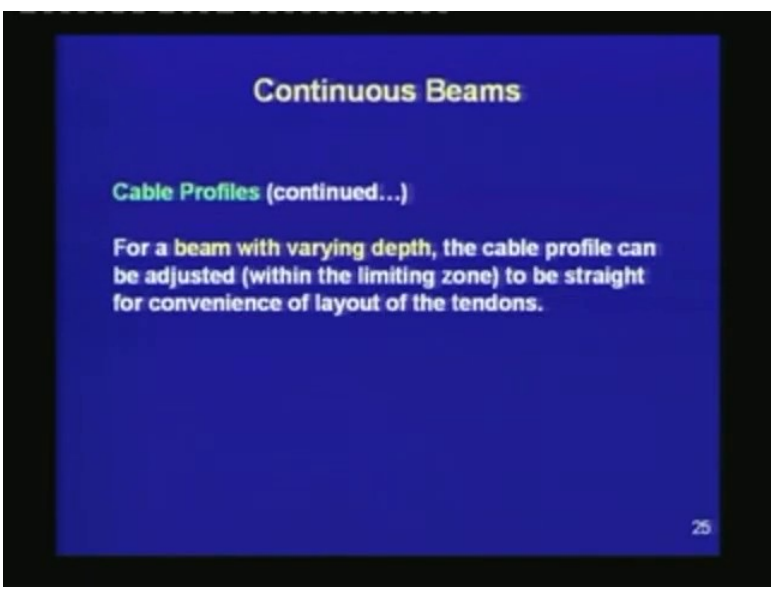

For a beam with varying depth the cable profile can be adjusted within the limiting zone to be straight for convenience of layout of the tendons.

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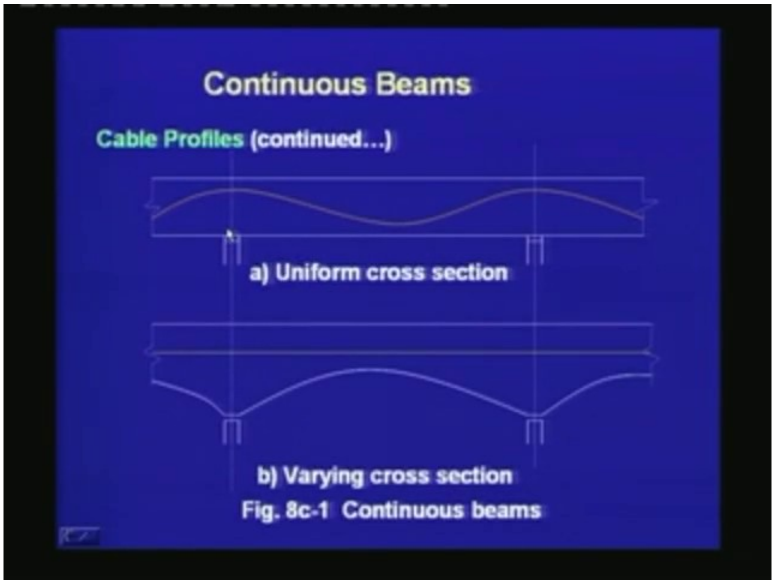

In the top sketch, the beam is a prismatic beam of uniform cross-section. Note that the cable profile is similar to the moment diagram and in uniform loads. But as we cannot have short kinks in the support region, we are having small curvatures near the support region. And for a beam with a varying depth, we adjust the cable profile so as to reduce the curvature and to reduce the friction losses.

In the bottom sketch you see that the tendon has been selected to the straight it lies within the limiting zone and it has been made straight for the convenience of the prestressing operation. The option number c is a combination of options a and b where we are changing the section and also we are varying the depth of the CGS along the length of the beam.

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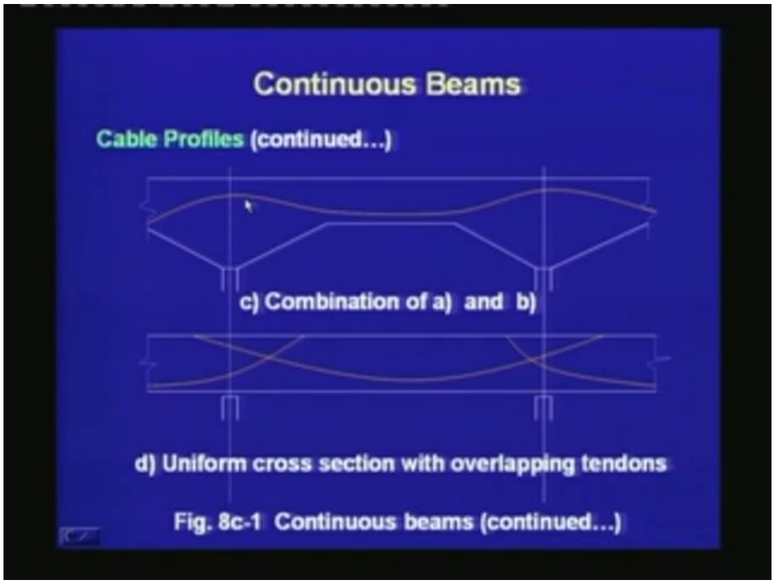

The fourth type is a uniform cross-section with overlapping tendons. Note that the tendons are anchored at the top of the beam and with this, each individual tendon has only a single curvature and it reduces the friction in the tendon. Thus, the different innovative cable profiles are possible, which are selected based on frictional losses based on the external loads; based on the section that we are selecting for the beam.

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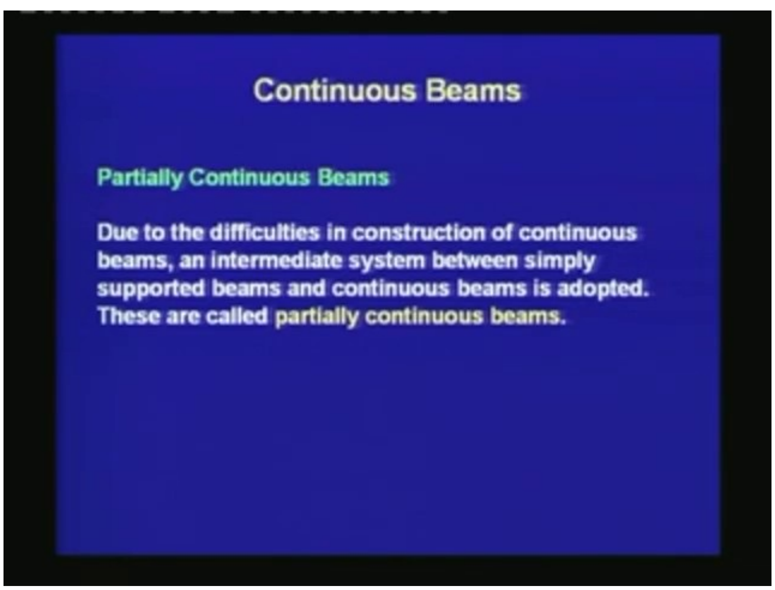

Next, we shall discuss about partially continuous beams. Due to the difficulties in construction of continuous beams, an intermediate system between simply supported beams and continuous beams is adopted. These are called partially continuous beams. As I had said earlier there are disadvantages in the construction of continuous beam and hence, true continuous beam may not be possible in large applications. In that case, continuity is introduced and these type of beams are partially continuous beams, which are in intermediate form of a simply supported beam and a truly continuous beam.

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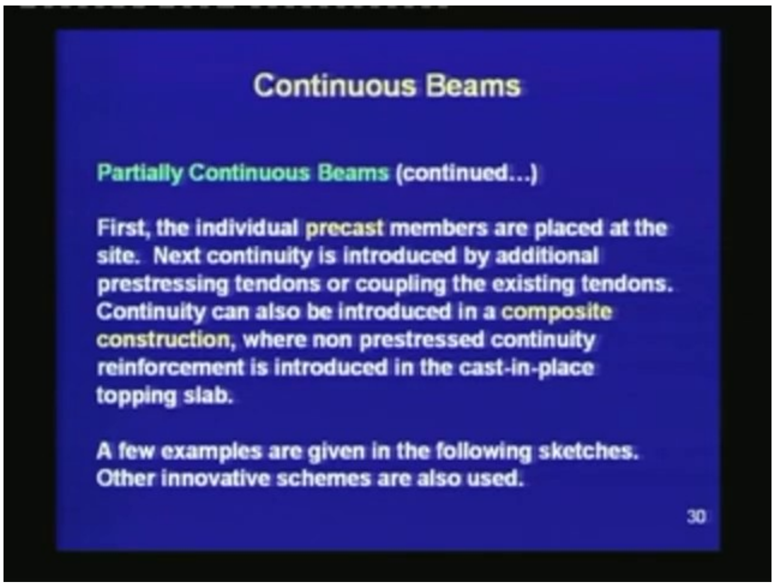

First, the individual precast members are placed at the site. Next, continuity is introduced by additional prestressing tendons or coupling the existing tendons. Continuity can also be introduced in a composite construction, where non-prestress continuity reinforcement is introduced in the cast-in-place topping slab. A few examples are given in the following sketches. Other innovative schemes are also used.

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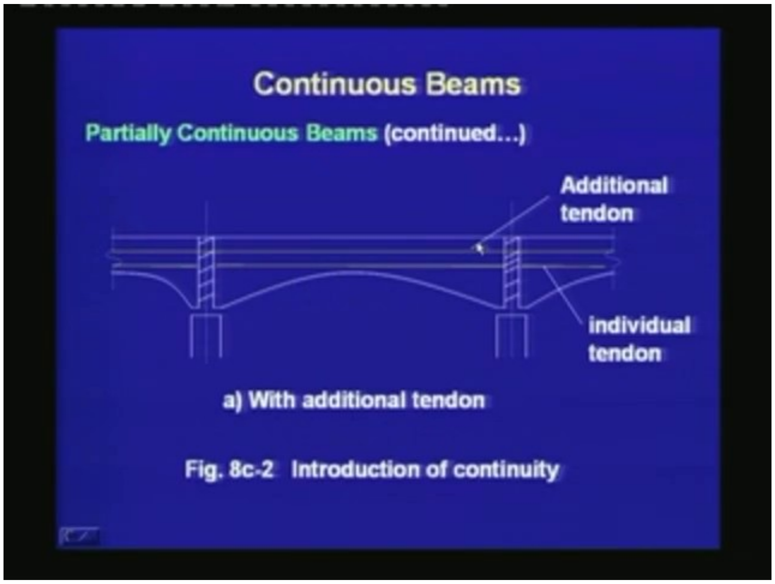

In this sketch, the precast members have been brought to the site and they have been placed on the pillars. The yellow line shows the cable for each individual member. Once, the member has been placed we can place an additional tendon which will tie the simply supported spans to introduce continuity for all the spans.

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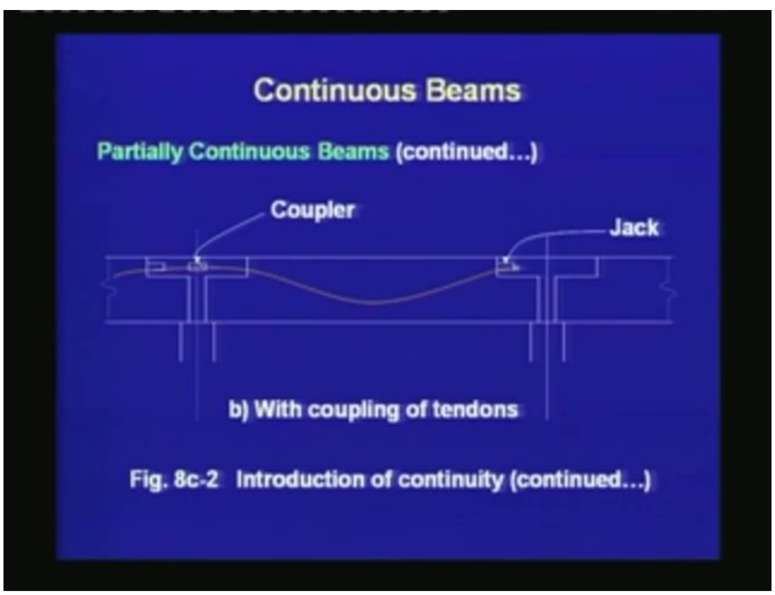

A second option is that we have the prestressed members placed on the pillars and then we couple the tendons of the adjacent members. In this sketch, you can see that the tendons have been connected by a coupler. We are also stretching the tendons by a jack in order to couple with the adjacent member. The coupling of the tendons is helping to introduce continuity in the members. We can also have a composite construction where a topping concrete is placed inside and in the casting place topping we are introducing non-prestress reinforcement such that they have some continuity over the supports.

This is a common form of construction because it is easy to perform. The precast members are made in the yard; they are placed; they are brought to the side and then a topping slab is laid over precasting members. The details of such construction will be covered in module on composite members.

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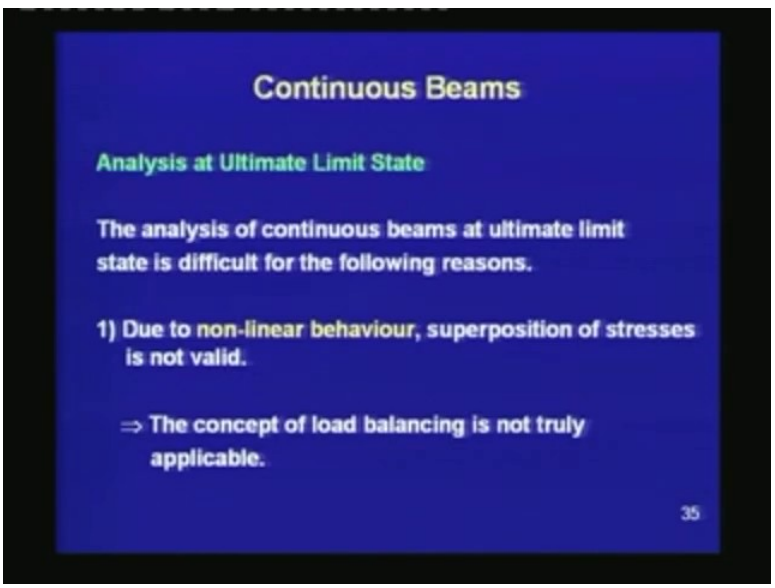

Next, we are discussing about the analysis of continuous beams at ultimate limit state. The analysis of continuous beams at ultimate limit state is difficult for the following reasons: due to non-linear behaviour, superposition of stresses is not valid. The concept of load balancing is not truly applicable. At ultimate state both the concrete and the steel will enter into the non-linear region hence the principle of superposition is not truly applicable. We use the principle of superposition in the load balancing concept that the upward thrust balances part of the downward load but this super position is not applicable in the true sense at the ultimate limit state.

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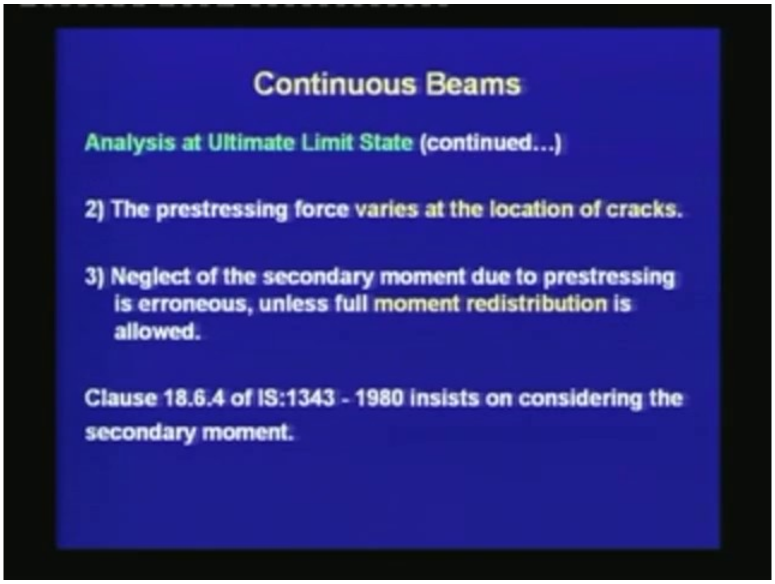

Second, the prestressing force varies at the location of cracks. As I earlier said, before a crack occurs in a member, the prestress is more or less uniform throughout the length of the member. But once there is a crack and then if the steel heals at the crack then the prestressing force is substantially different around the cracks than in the other parts of the beam. Thus, we cannot assume that the prestressing force is uniform throughout the length of the member.

The third difficulty is the neglect of the secondary moment due to prestressing is erroneous, unless full moment redistribution is allowed. If we are not having a concordant cable then there will be secondary moment due to prestressing force and neglecting that is erroneous. Of course, we can take advantage of moment redistribution which we are discussing next. In this case, we may neglect the secondary moment. Clause 18.6.4 of IS: 1340-1980 insists on considering the secondary moment.

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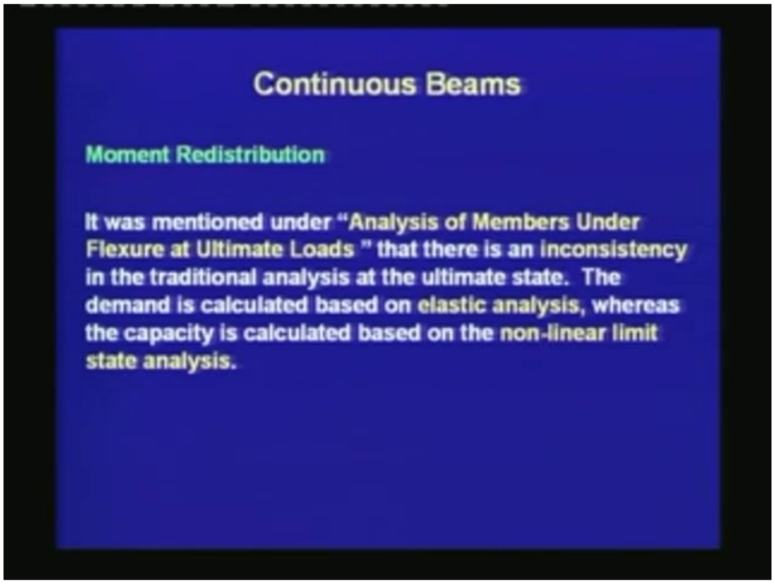

Next, we are discussing the moment redistribution. It was mentioned under analysis of members under flexure at ultimate loads, that there is an inconsistency in the traditional analysis at the ultimate state. The demand is calculated based on elastic analysis; whereas the capacity is calculated based on the non-linear limit state analysis. Thus, the inconsistency in the analysis for an ultimate limit state is that the demand we are finding out by an elastic analysis, where as the capacity we are calculating based on a non-linear elastic analysis.

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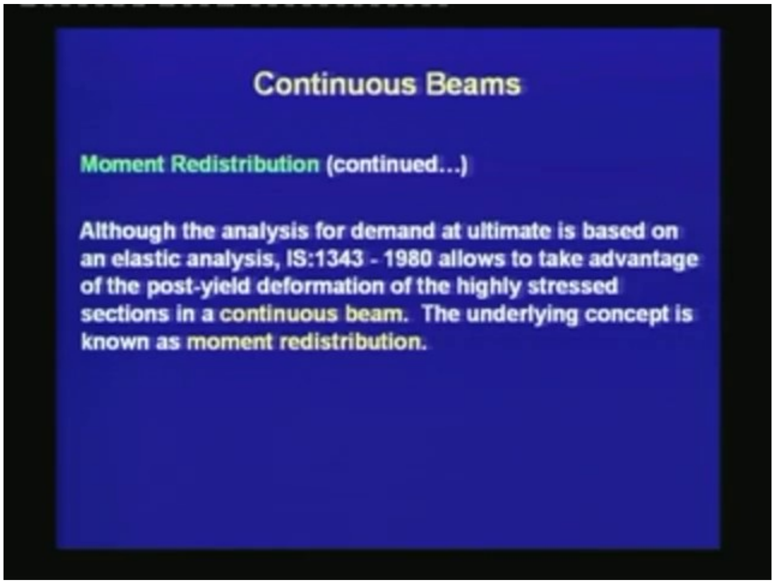

Although the analysis for demands at ultimate is based on an elastic analysis, IS: 1343-1980 allows taking advantage of the post-yield deformation of the highly stressed sections in a continuous beam. The underlying concept is known as moment redistribution. Although we are finding out the demand based on elastic analysis, but the code allows us to take advantage to allow us take advantage of the non-linearity which is further amplified once there is yielding of the steel at certain locations of the beam. But if we consider this yielding and moment redistribution then, we can economise on the sections.

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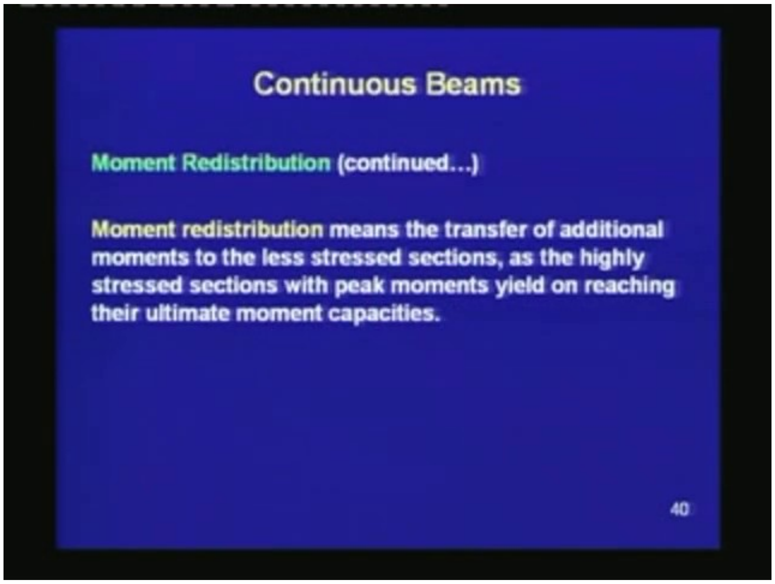

Moment redistribution means the transfer of additional moments to the less stressed sections as the highly stressed sections with peak moments yield on reaching their ultimate moment capacities. As I had said earlier, a continuous beam is a statically indeterminate structure. That means, if one particular section starts to have yielding in the steel there will be other sections which will be able to pick up the additional load.

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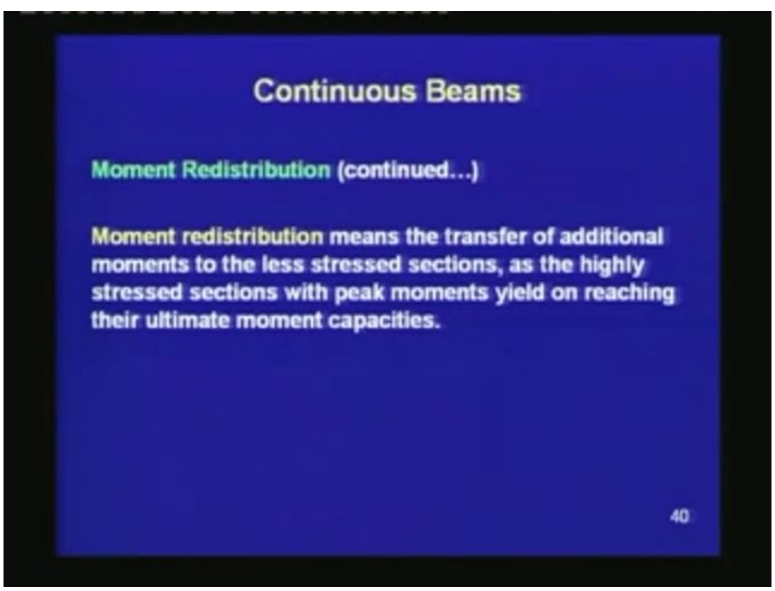

Moment redistribution means that additional load is assigned to the less stressed regions once the higher stress region starts to end.

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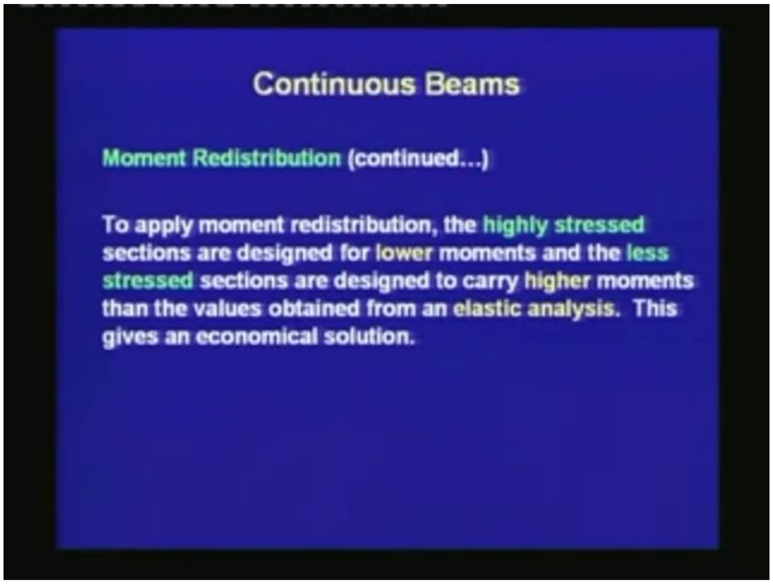

To apply moment redistribution the highly stressed sections are designed for lower moments and the less stressed sections are designed to carry higher moments than the values obtained from an elastic analysis. This gives an economical solution. What we are doing by considering moment redistribution is that, the highly stressed sections are designed for lower moments than we calculate based on the elastic analysis and the less stressed sections are designed to carry higher moments than the values obtained from the elastic analysis. This balances the amount of steel in the different section and this leads to an economical solution.

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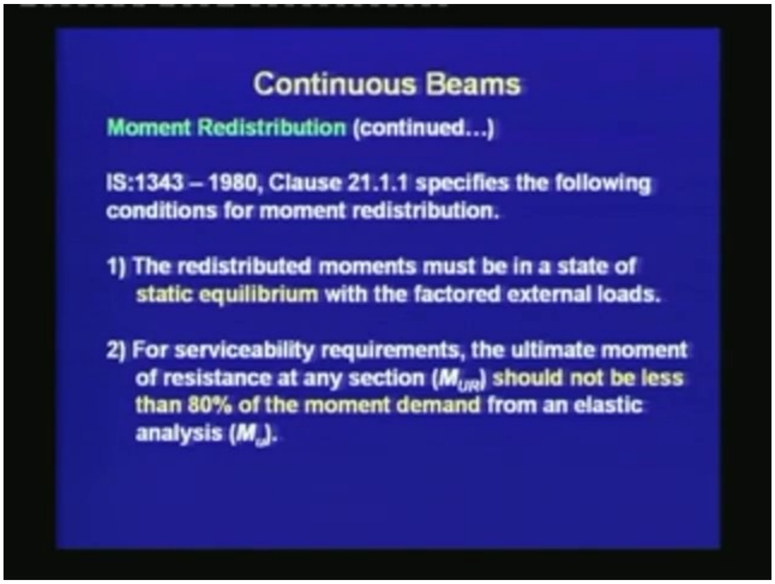

The IS: 1343-1980, clause 21.1.1 specifies the following conditions for moment redistribution. First, the redistributed moments must be in a state of static equilibrium with the factored external loads. That means, even if we alter the design moments or sections from the values based on elastic analysis we need to maintain a static equilibrium. That means in some regions we are designing for a lower moment but in other regions we are designing for a higher moment maintaining the requirement of static equilibrium. This is the first condition we need to satisfy if we are taking advantage of moment redistribution.

The second condition is for serviceability requirements, the ultimate moment of resistance at any section M_{ur} should not be less than 80% of the moment demand from a elastic analysis M_u. The moment redistribution is to a limited extent, we cannot go on changing the design moment too much from the values from the elastic analysis. We can have a moment of resistance M_{ur}, which is not less than 80% of the demand based on elastic analysis.

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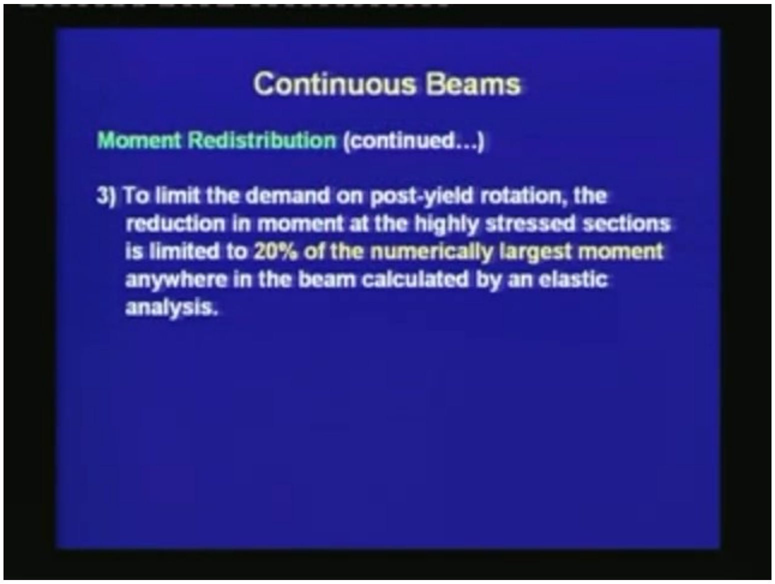

The third requirement is to limit the demand on post-yield rotation; the reduction in moment at the highly stressed sections is limited to 20% of the numerically largest moment anywhere in the beam calculated by an elastic analysis. The moment redistribution realize on the yielding of the steel and the formation of the plastic hedge at the highly stressed section. In order to reduce the amount of rotation that is required, we are limiting the moment redistribution, such that the moment in the highly stressed sections cannot be reduced more than 20% of the numerically largest moment anywhere along the length of the beam.

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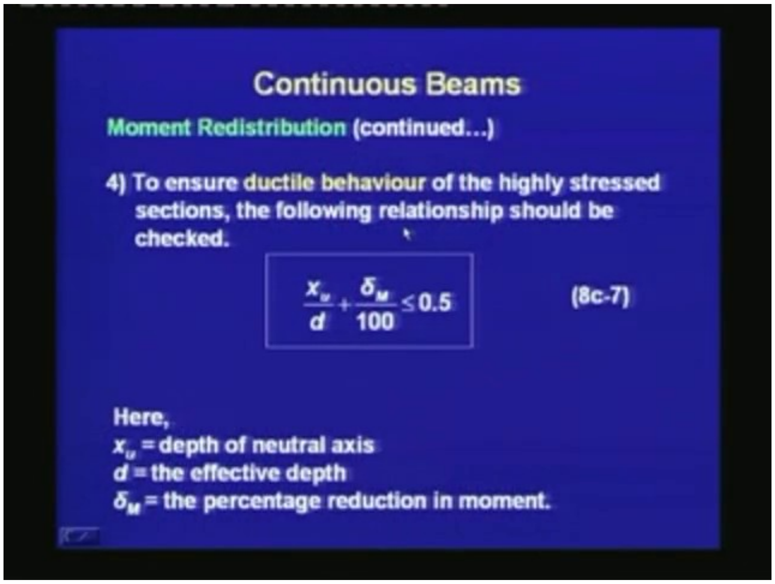

The fourth condition is to ensure the ductile behaviour of the highly stressed sections; the following relationship should be checked, X_u divided by d plus $delta_M$ divided by 100 should be less than 0.5. Here, X_u is the depth of neutral axis; d is the effective depth; d_M is percentage reduction in moment. This equation ensures that the depth of the neutral axis is limited to a certain value which depends on the change in the moment that we are incorporating in our design. Thus, it ensures a ductile behaviour after the design of the beam considering moment redistribution.

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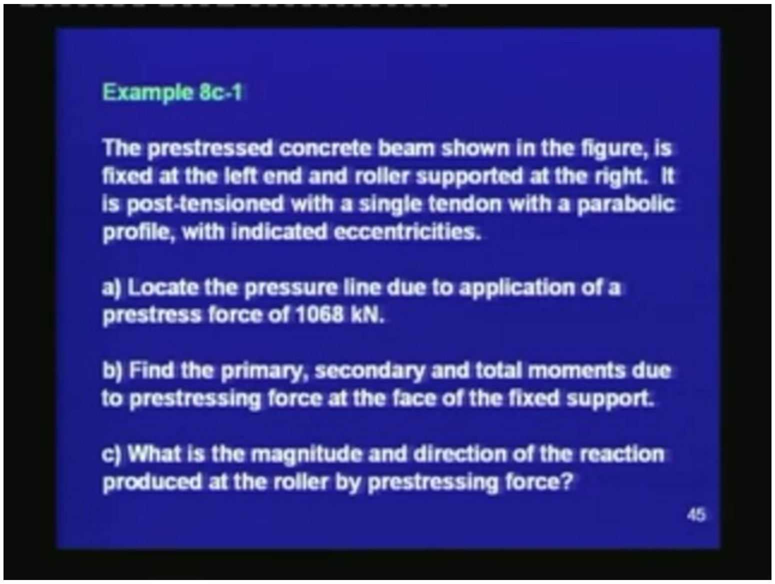

Let us now understand the design principle of the cable profile in a continuous beam with the help of a simple example. The prestressed concrete beam shown in the figure is fixed at the left end and a roller supported at the right. The figure will be coming in the next slide. It is post-tensioned with a single tendon with a parabolic profile with indicated eccentricities. The problem at hand is to locate the pressure line due to application at prestress force of 1068 Kilonewtons. Second part is, to find the primary; secondary and total moments due to prestressing force at the face of the fixed support. The third part is what is the magnitude and direction of the reaction produced at the roller by prestressing force. That means we are not assuming that the profile is a concordant profile.

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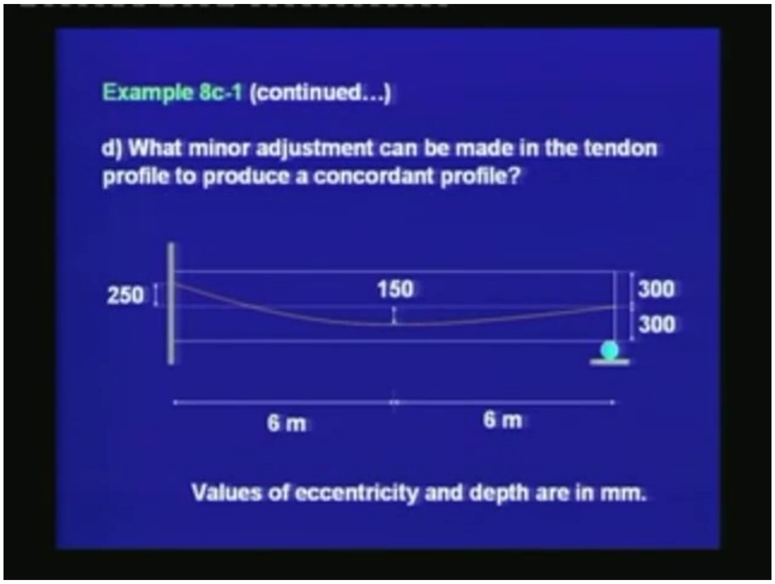

The fourth part is what minor adjustment can be made in the tendon profile to produce a concordant profile? This is the prop cantilever which is fixed at one end and the roller supported at on the other end. The cable profile is given, which is 250 mm above the CGC at the left end, then at midway it is 150 mm below the CGC and on the right side it is the CGS coincides with the CGC. The total depth of the beam is 600 mm and the total spam of the beam is 12 mm.

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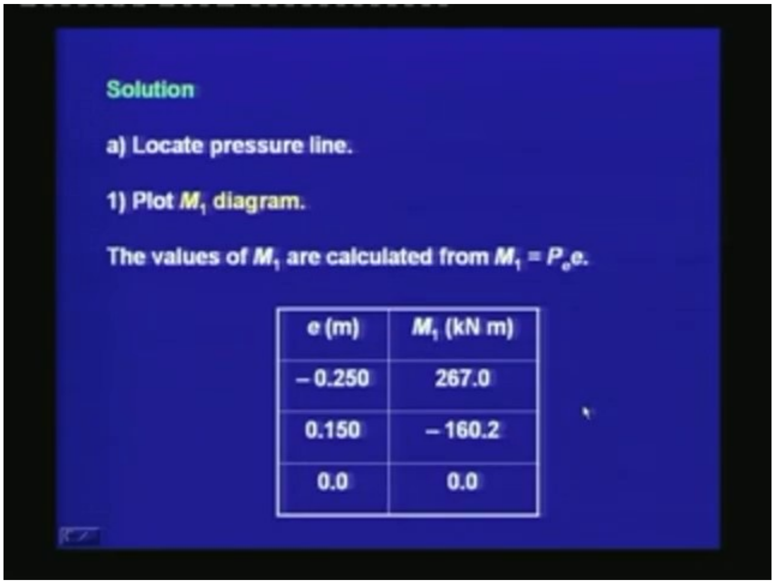

e (m)	M_1 (kN m)
– 0.250	267.0
0.150	– 160.2
0.0	0.0

The first part is to locate the pressure line for the given prestressing force. Remember that the pressure line does not actually depend on the prestressing force. Here we need a value of the prestressing force to get the pressure line, but if we do the calculations for a different prestressing force we shall still get the same location of the pressure line.

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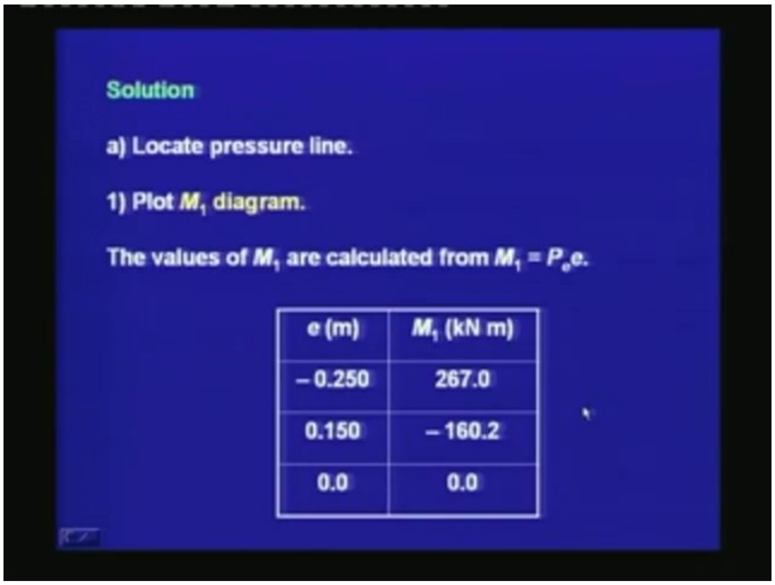

The first step is to plot the M_1 diagram which is the moment due to the prestressing force without considering the effect of the reactions. M_1 is calculated from the relationship $M_1 = P_e$ (e). Now, e three values are given – 0.250 meters on the left end 0.150 m at the middle and 0 at the right end. Corresponding to three values of e, we multiply this by the prestressing force of 1068 Kilonewtons and we get the values of M_1 = 267 – 160.2 and 0 Kilonewton meters. Note that, the sign of M_1 is opposite to that of the eccentricity.

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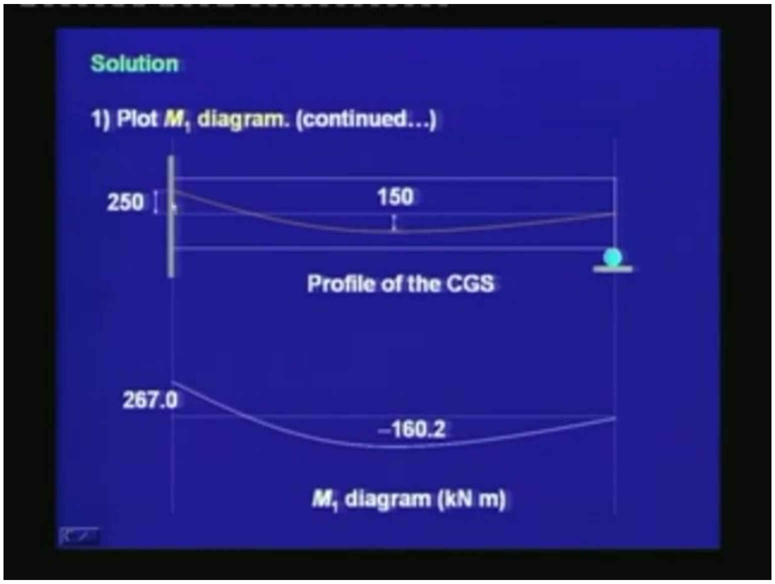

If the eccentricity is negative, that means the CGS is located above the CGC then M_1 is positive; whereas if the CGS is below the CGC which is a positive eccentricity the moment is negative. Hence, we can observe that the M_1 diagram is of the same shape as that of the profile of the cable.

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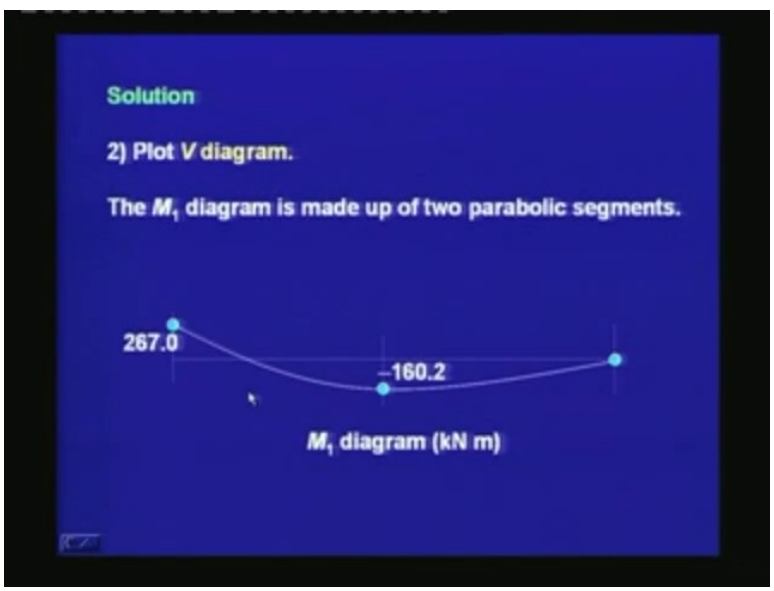

The second step is to plot the shear diagram which is generated due to the prestressing force. In order to calculate the shear diagram we need an expression of the M_1 diagram; for that, M_1 diagram is idealized to be parabolic and the particular profile consists of two parabolas, one on the left side and one on the right side.

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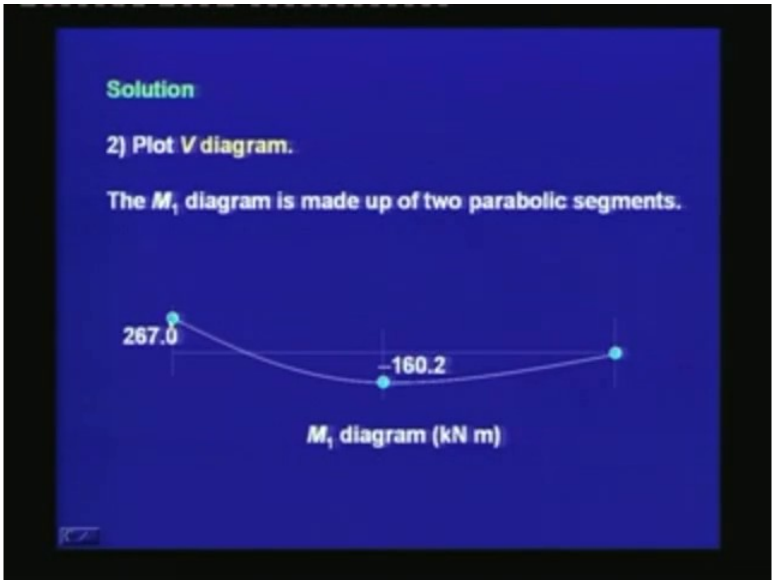

In this sketch you can see that the parabola on left has a total dip of 267 + 160.2, whereas the parabola on the right has a dip of 160.2 from the right hand. We shall use these values of the dip to calculate the equations of the profile of the M_1diagram.

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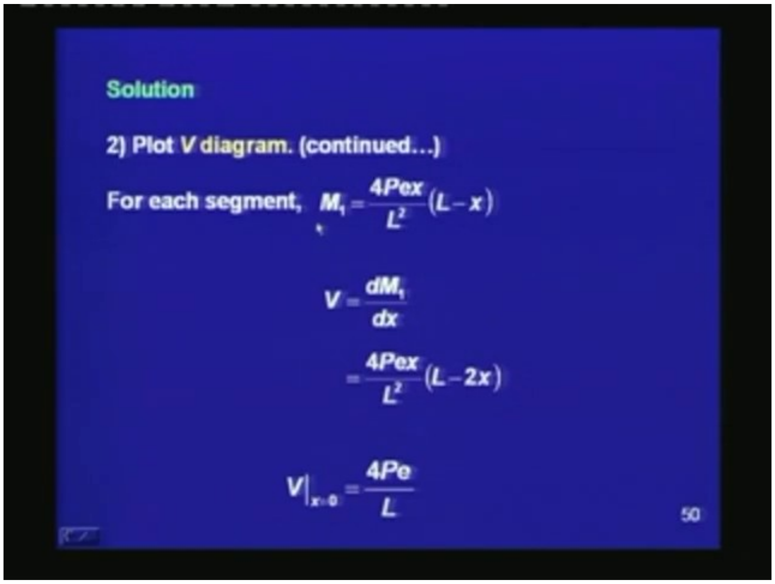

For each segment, M_1can be written as 4Pex divided by L square (L – x). This is a parabolic equation to fit the M_1 diagram. The shear is given as dM_1 by dx from principles of structural analysis. Thus, V = 4Pex divided by L square (L – 2x). The shear at one end at x = 0 is given as 4Pe divided by L. This is the end which is the origin of equation that has been fitted for the moment diagram.

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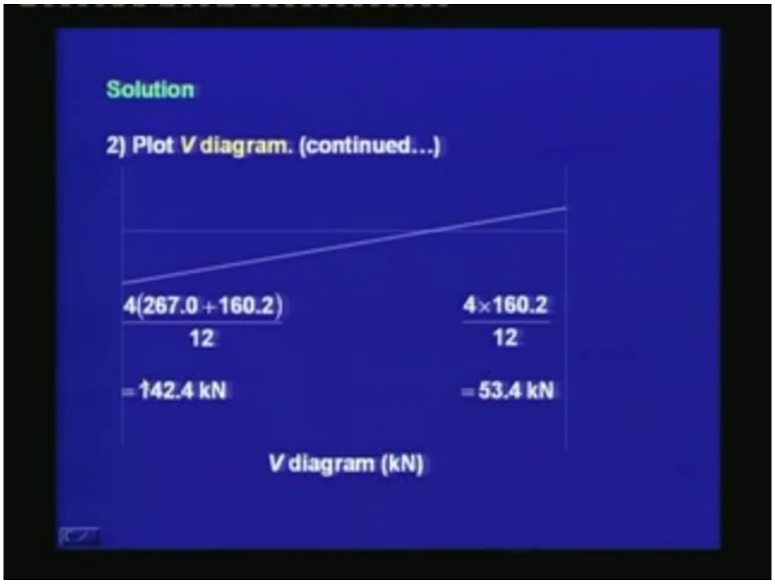

Once we have the expression of the shear we can calculate the values of the shear at the two ends because for the left curve we are selecting the origin as the left end; for the right curve we are selecting the origin as the right end. The values of shear is $4P_e$ where P_e is the total change in the moment diagram within that particular segment and the total change is 267 + 160.2 and then whole divided by the length gives us 142.4 Kilonewtons.

Note that this length is not the length of the half the segment of the parabola, but this length is the total length of the parabola which is 12 mm. Similarly, on the right side the value is 4 times the dip which is 160.2, divided by the length of the parabola which is 12 which gives us 53.4 Kilonewtons. Thus, the V diagram is a linearly varying diagram for a parabolic profile with these two values at the two ends; the left hand side has 142 4; and the right hand side it is 53.4.

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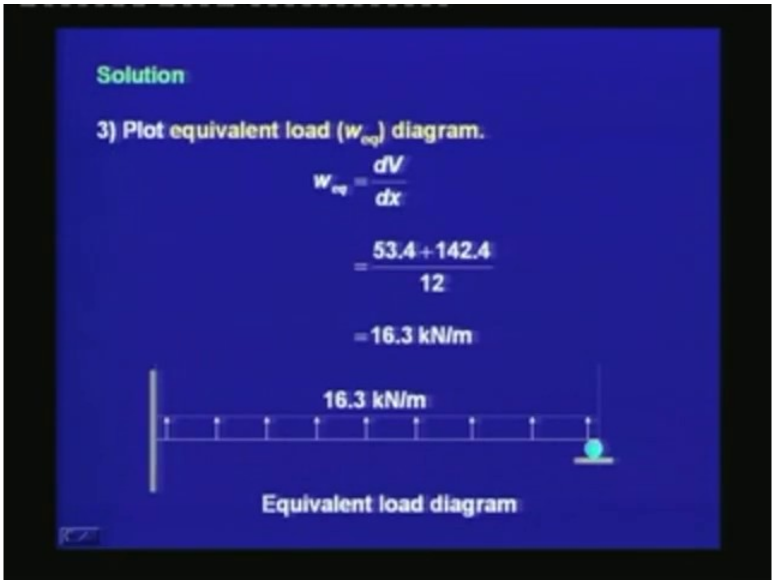

The third step is to get the pressure line to plot the equivalent load diagram. The equivalent load is given as dV by dx and since, d varies linearly W equivalent will be a constant value throughout the length of the beam, that is a parabolic profile will create a uniform thrust upwards, W equivalent is given as the total change in the shear which is 53.4 + 142.4 divided by the length 12 which is equal to 16.3 Kilonewtons per meter. Thus, the equivalent due to the prestressing force is a uniform load 16.3 Kilonewtons per meter which acts upwards. From the equivalent load diagram we are calculating the M_2 diagram which is the resultant moment diagram due to the prestressing force.

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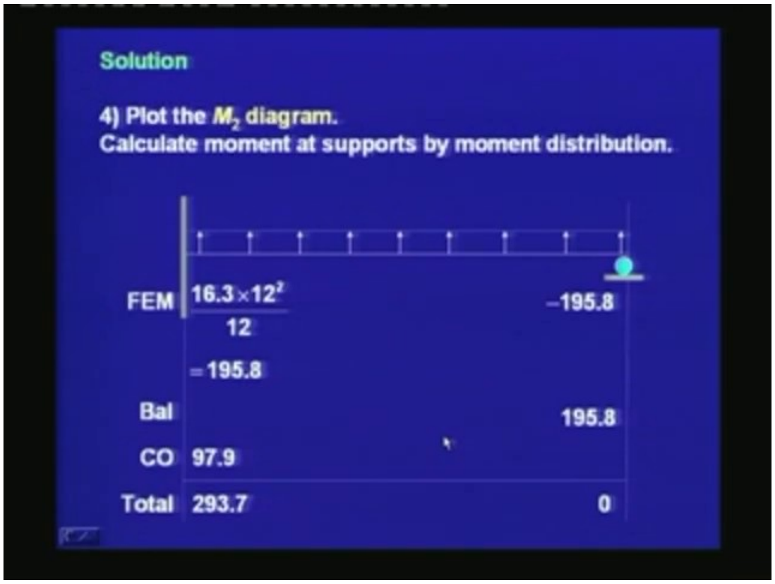

Now, the forth the calculations we are using is the moment distribution method, we know the loading condition in the beam due to the prestressing force from that we are calculating the fix end moments which is WL square divided by 12, which is 195.8 Kilonewton meters, then we are balancing the moment on the right side we are placing a moment of 195.8 which leads us to carry over moment of 97.9 Kilonewton meters on the left side and we are calculating the total moment as 293.7 Kilonewton meters at the left side and on the right side there is no moment since it is a roller support.

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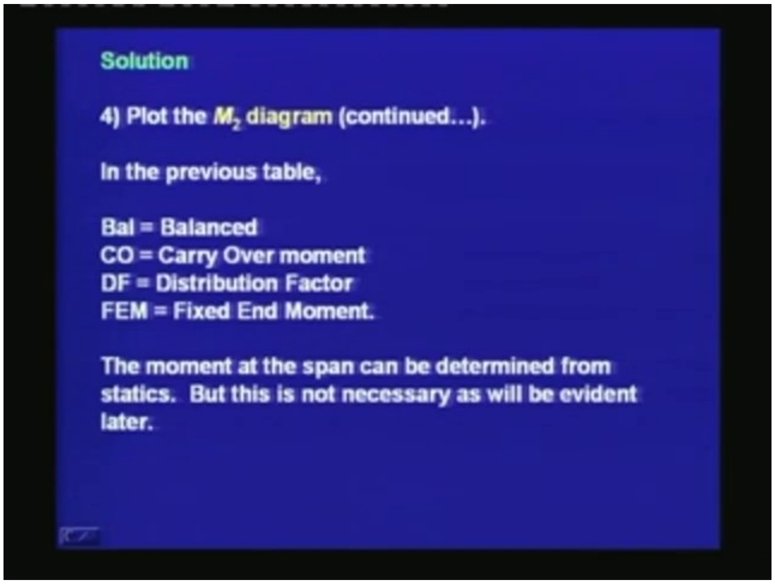

In the previous table, Bal stands for balanced, CO stands for carry over moment, DF is the distribution factor and FEM is the fixed end moment. The moment at the span can be determined from statics. But this is not necessary as will be evident later. We are calculating the moments only at the ends. We need not calculate the moment at the span because we use the concept of the linear shift of the pressure line to calculate the location of the pressure line in the span.

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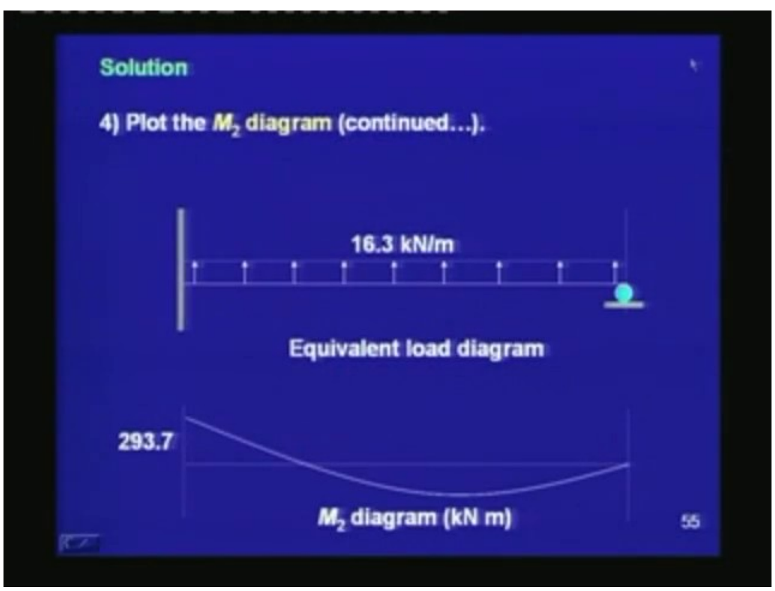

Now the M_2 diagram appears similar to the cable profile with the value of 293.7 Kilonewton meters at the left. There is another value at the span, which we are not calculating and the value is 0 at the right end. This is the resultant moment diagram due to the prestressing force.

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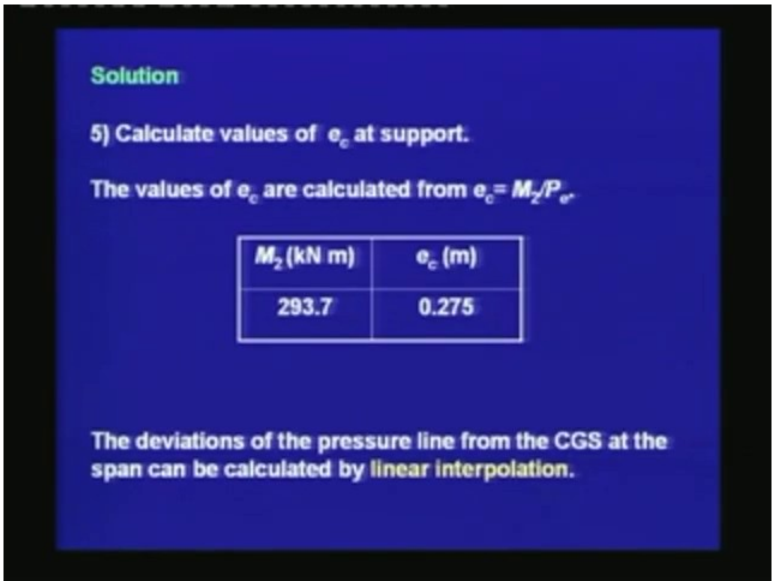

From this we are calculating the values of e_c to locate the pressure line and on the left support e_c is given as M_2 divided by P_e, M_2 is 293.7 divided by 1068 gibbs e_c equal to 0.275 meters. The deviations of the pressure line from the CGS at the span can be calculated by linear interpolation.

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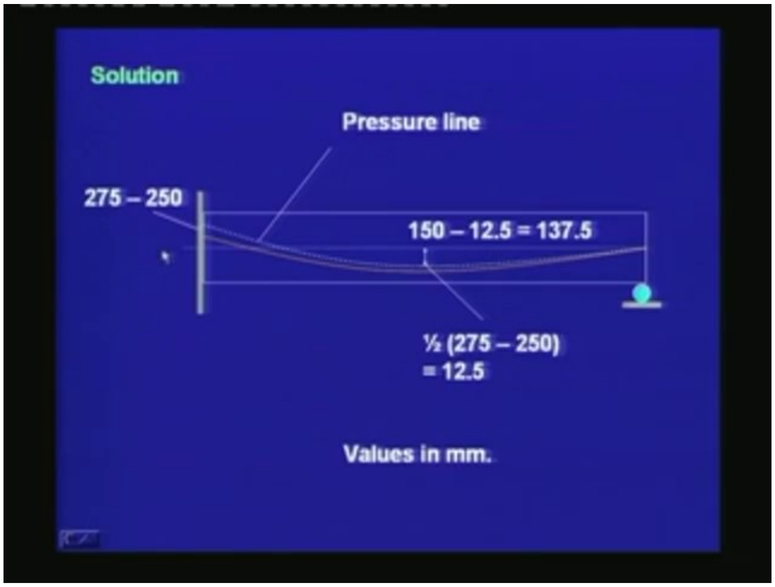

Thus, we are locating the pressure line from the e_c value at the left end, note that we had found e_c to be 275 from the CGC distance of the CGS is from the CGC is 250. Thus, the shift of the pressure line from the cable profile is 275 – 250 at the left end. On the right side, there is no shift of the pressure line; the pressure line coincides with the CGS which is again at the level of the CGC.

Now, within the span at the middle, the shift of the pressure line will be half that at the end. Thus, the shift is half of 275 – 250, which is 12.5 mm. Now we can locate the pressure line at the middle which is the distance of the CGC which is 150 minus the shift which is 12.5 gives us a value of 137.5 mm. Thus, we did not have to calculate the value of M_2 at the middle because we are able to calculate the location of the pressure line based on the concept of linear shift within the span. Once we know the pressure line we can have a concordant profile. Note that the original profile was not a concordant profile because the pressure line is shifted from the original profile.

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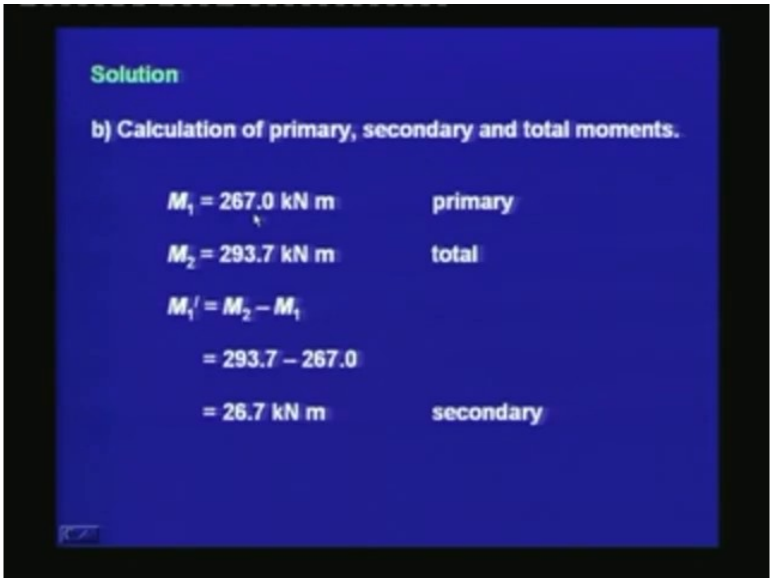

The second part of the problem is to calculate the primary, secondary and total moments due to the prestressing force. The primary moment we had find out 267.0 Kilonewton meters $M_1 = P_e$ (e). Thus, the total moment we calculated from the moment distribution method was 293.7 Kilonewton meters. Thus, the secondary moment due to the reaction is given as M_1 prime = $M_2 - M_1$ = 293.7 – 267 = 26.7 Kilonewton meters. Thus, we have this additional moment due to the reactions in this continuous beam.

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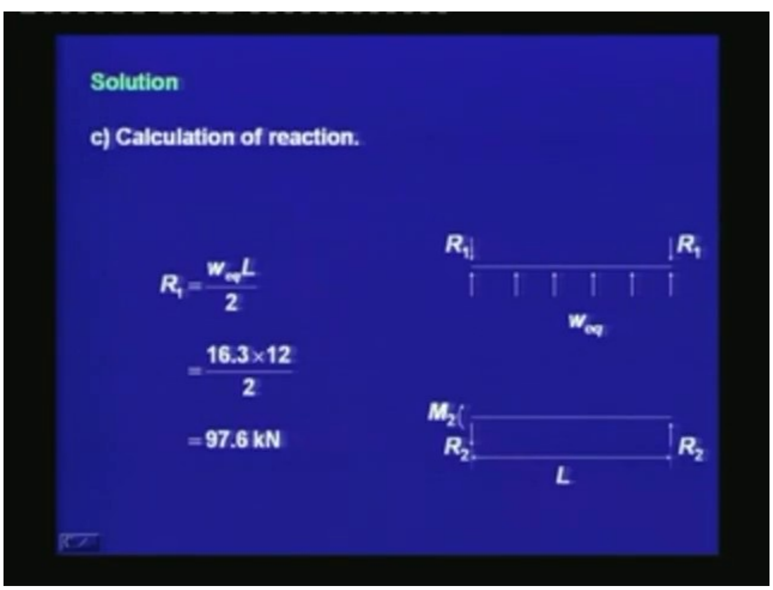

The third part of the problem is to calculate the reaction on the right end. To calculate the reactions we are using the principle of statics due to the equivalent load only the reaction on the two ends is same and R_1 is equal to W_{eq} times L divided by 2 is equal to 163 times 12 divided by 2 gives us R_1 is equal to 97.6 Kilonewtons.

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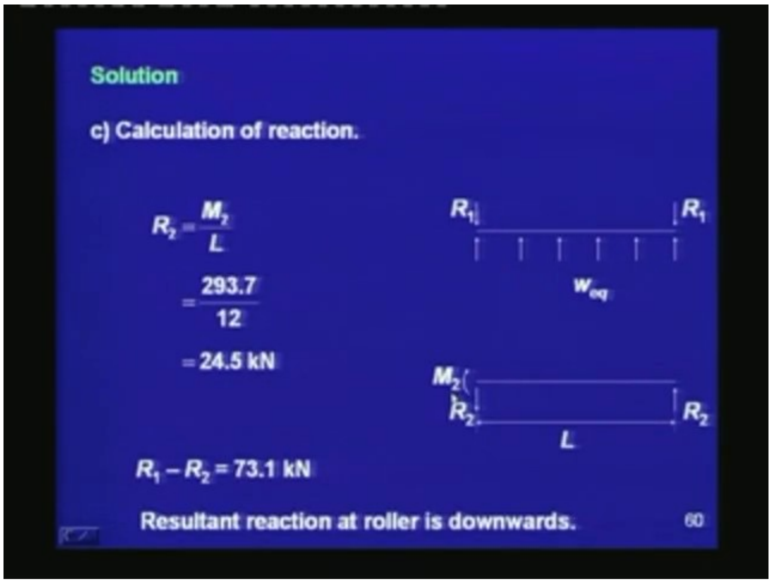

The second part is due to the moment M_2 at the end and R_2 will be given as, R_2 equal to M_2 divided by L is equal to 293.7 divided by 12 equal to 24.5 Kilonewtons. Note that the reaction in the right side due to W equivalent is downwards, because the load is acting upwards; whereas, there is reaction due to the moment M_2 is upwards. Hence, the final reaction of the right side is $R_1 - R_2 = 73.1$ Kilonewtons and this resultant reaction at the roller is downwards. Thus, we need a hold down of the right hand side when we apply the prestressing force.

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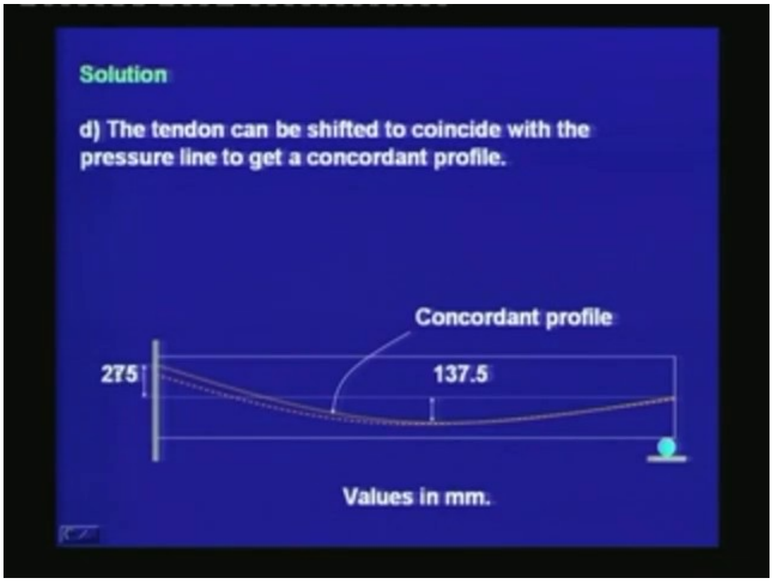

The fourth part of the problem is to find out a concordant profile. We have calculated the pressure line. And now, the tendon can be shifted to coincide with the pressure line to get a concordant profile. The pressure line is at a distance of 275 mm from the CGC.

Once we shift the CGS the pressure line, we get a concordant profile. Note that this is within the limits of the beam and hence it is a satisfactory concordant profile. And if we layout the profile along the concordant profile then the subsequent calculations become simpler. The calculation of the shift of the pressure line due to the external loads can be done based on this location of the concordant profile. Hence, the concordant profile is not a must in the design of the layout of the cable profile. But if we do a layout based on concordant profile then the calculations become simpler.

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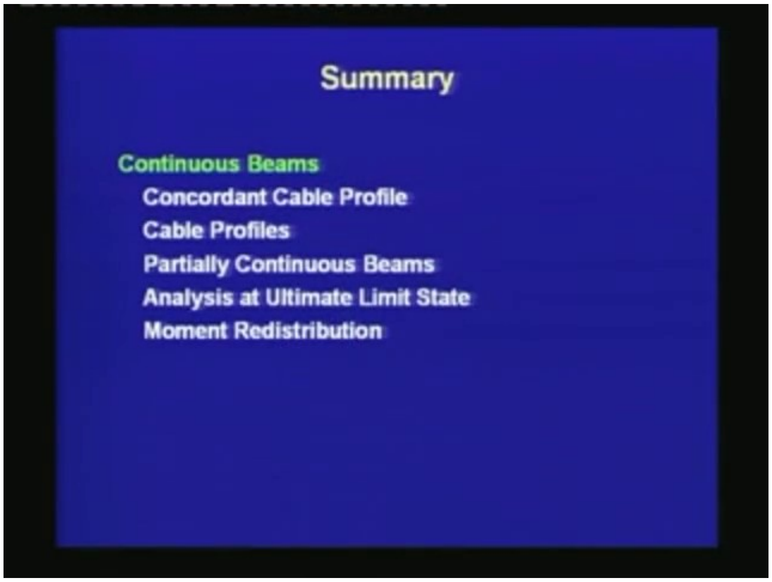

In today's lecture, we continued with the continuous beams. First, we introduced the concept of concordant profile. Under that, we talked of the principle of linear transformation that a cable profile can be shifted such that the curvature between each segment is maintained, the points of bends are maintained and the location of the CGS at the ends should remain unchanged. Under this situation the shift of the cable profile is called a linear transformation of the cable profile.

There is a theorem for this, that, for all the linearly transformed cable profiles there is only a single pressure line, that is a pressure line that does not change if the cable profile is linearly transformed. From this, we develop the profile of concordant profile that we can place a cable along the pressure line and for this profile the CGS and the location of C, the pressure line is coincident and this is the definition for concordant profile. There is a theorem for the concordant profile, that any real moment diagram in our continuous beam drawn to a particular scale can be used as a concordant profile for that particular beam. Thus, once we know the moment diagram due to the external loads we can develop a concordant profile quite easily.

Next, we went to the discussion of cable profiles. The cable profiles are selected based on the design requirements and also the convenience of prestressing operations. The cables are adjusted such that the friction losses are reduced. We should not have sharp kinks near the supports.

We saw different types of cable profiles for beams of uniform cross-section or varying cross-section and the selection is based on the application. Next, we went with the partially continuous beams; these beams are precast members placed on side and then continuity is introduced. It is in between truly simply supported and truly continuous beams; partially continuous beams are adopted because of convenience in construction.

We discussed about the analysis at ultimate limit state that there are difficulties in the analysis of ultimate limit state for a continuous beams that they have to be aware of the situations. We also discussed about moment redistribution which we can take advantage of if we have to economize the section. Finally, we discussed a problem. With this we are ending the chapter on continuous beams. Thank you