Prestressed Concrete Structures

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Module – 8 Lecture – 33
Continuous Beams (Part 1)

Welcome back to prestressed concrete structures. This is the second lecture on module eight on cantilever and continuous beams. In this lecture we shall study continuous beams.

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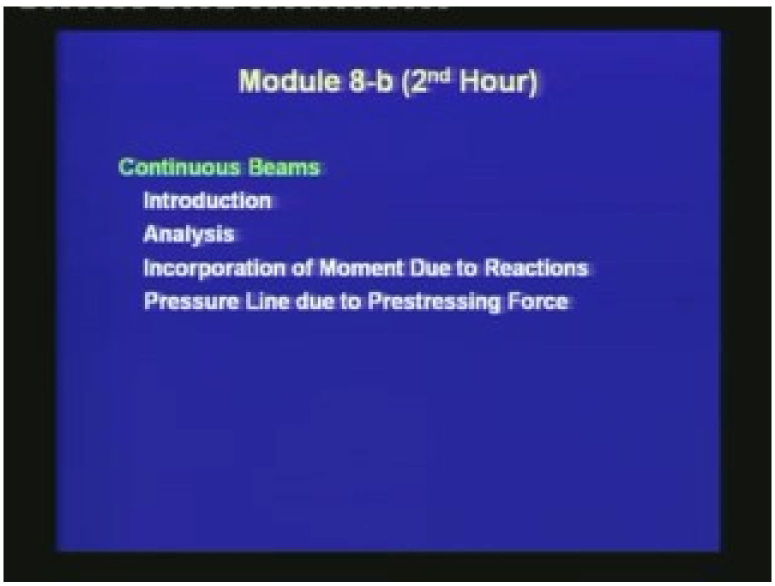

After the introduction, we shall study the analysis of continuous beams, incorporation of moment due to reactions and pressure line due to the prestressing force.

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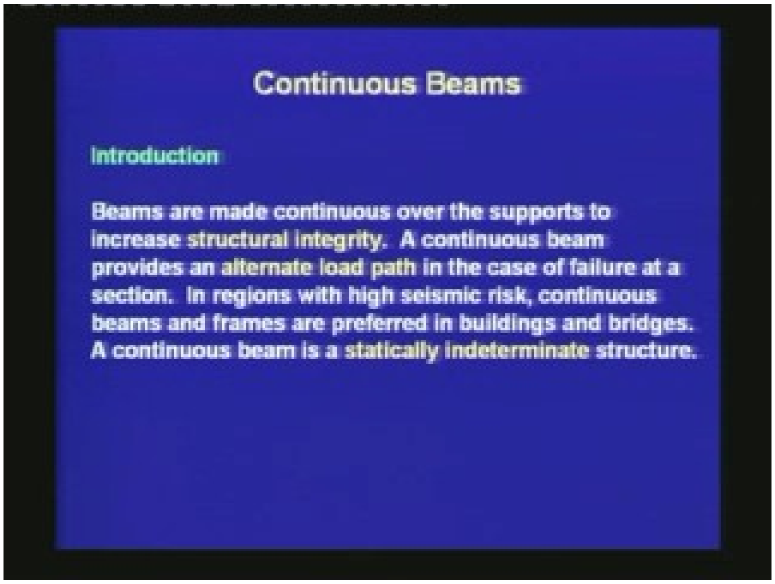

Beams are made continuous over the supports to increase structural integrity. A continuous beam provides an alternate load path in the case of failure at a section. In regions with high seismic risk, continuous beams and frames are preferred in buildings and bridges. A continuous beam is a statically indeterminate structure.

Earlier, when we talked about simply supported beams, we found that the calculation of demand moment and the demand shear is easy and it follows the conventional expression in the structural analysis text books. Continuous beams are made to increase the structural integrity, so as to have more redundancy in the structure. In case of a failure at a particular section, there will be an alternative load path for the load to go down to support of the structure. Hence, continuous beams are preferred in moment resisting frames of buildings and also in frame bridges. These are the preferred types of structure in a region with high seismicity.

The advantages of a continuous beam as compared to a simply supported beam are as follows: first, for the same span and section; vertical load capacity is more; that means, if I have the same span and same section for a continuous beam and the corresponding simply supported beam, we will find that we can apply more vertical load for a continuous beam.

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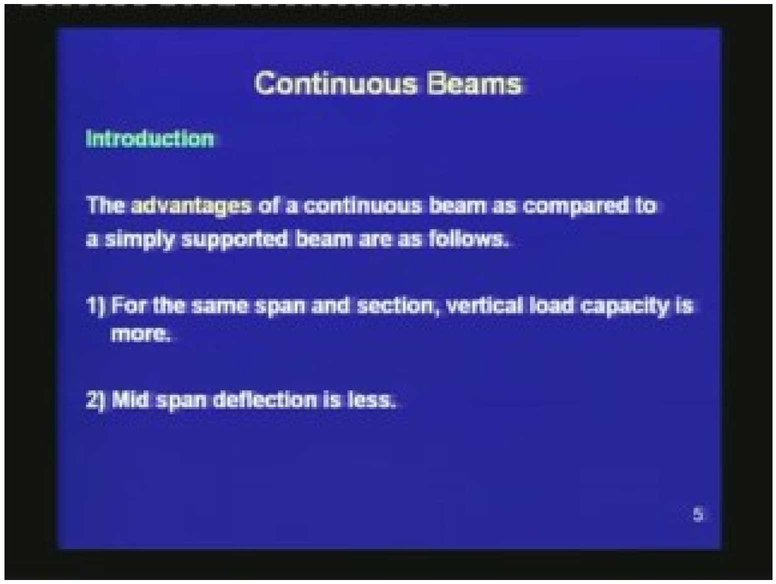

Next, the mid span deflection is less for continuous beam as compared to a simply supported beam. Since there is a strain at the two ends, the deflection at the middle is reduced in the continuous beam.

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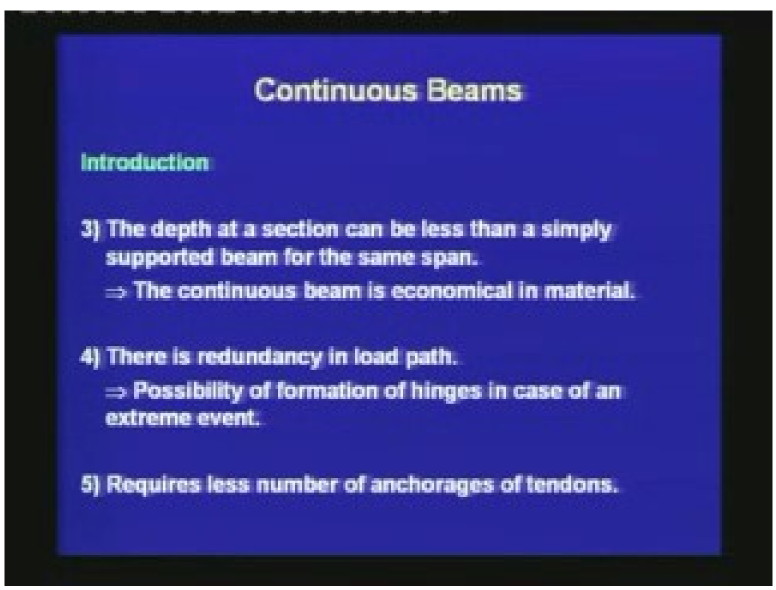

Third, the depth at a section can be less than a simply supported beam for the same span; that means, given the same span and same load, if we use a continuous beam it will have a reduced depth compared to a simply supported beam. This implies that the continuous beam is economical regarding the material.

The fourth advantage is, there is redundancy of the load path. There is possibility of formation of hinges in case of an emergency in extreme event. If one section fails, there is an alternate load path in a continuous beam and the whole beam will not come down as it happens in a simply supported beam if it fails in the middle.

The fifth advantage is that a continuous beam requires less number of anchorages of tendons. If a bridge is made of simply supported spans, then anchorage is needed at each end of each span; whereas, if bridge is made with some continuous spans, then the number of anchorages can be reduced as compared to the number in simply supported span.

There are disadvantages in continuous beams as compared to a simply supported beam. The disadvantages are as follows: first, a continuous beam is difficult to analyze and design.

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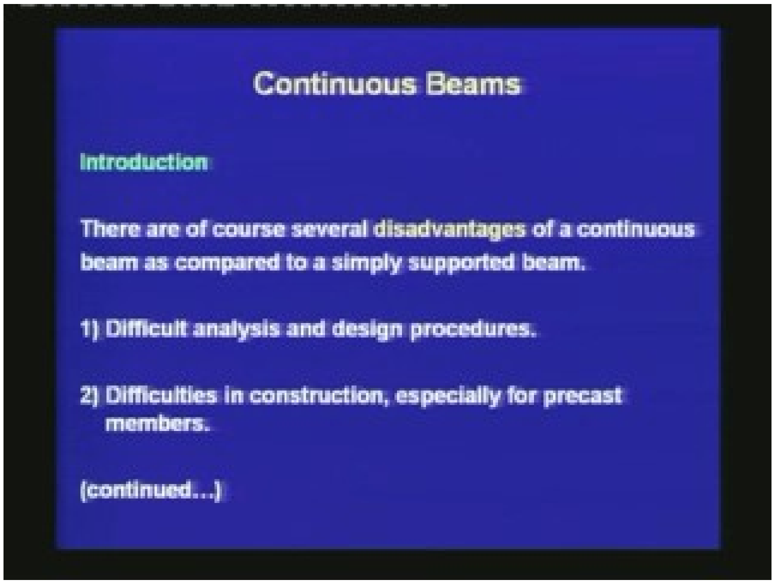

In our course, we had placed more emphasis on the simply supported beam because it is easy to understand the analysis and design principles for a simply supported beam at the beginning. For a continuous beam, the analysis procedure is more rigorous as compared to a simply supported beam. The second disadvantage is there are difficulties in construction especially for precast members.

If the members become huge, then the simply supported span can be precast somewhere else and brought in the place and rested on the supports. But, if it is a continuous beam then, it becomes difficult to introduce continuity in the precast member. This is one difficulty for which many times in bridges, simply supported spans are preferred.

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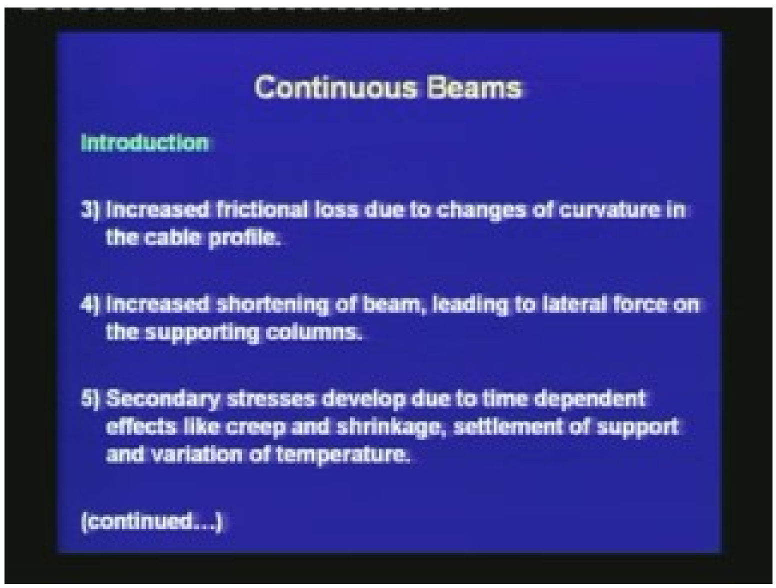

The third difficulty is increased frictional loss due to changes of curvature in the cable profile. If you have a cable running through the spans and the supports, there will be more changes in curvature and this will have increased frictional loss throughout the length of the beam. That is why at times the tendons are interrupted so as to reduce the frictional loss.

The fourth disadvantage is increased shortening of beam leading to lateral force on the supporting columns. As the beam shortens due to the effect of creep and shrinkage, this will induce some forces in the support and these forces need to be considered during the design of the support of the beams.

The fifth disadvantage is secondary stresses develop due to time dependent effects like creep and shrinkage, settlement of support and variation of temperature. The continuous beam is a statically indeterminate structure and hence it is subjected to secondary stresses which come due to the moment, creep and shrinkage, variation of temperature and settlement of the support. These effects have stresses in the structure; whereas, a simply supported beam is a statically determinate system. This does not have secondary stresses due to the settlement of the supports or due to the change in temperature.

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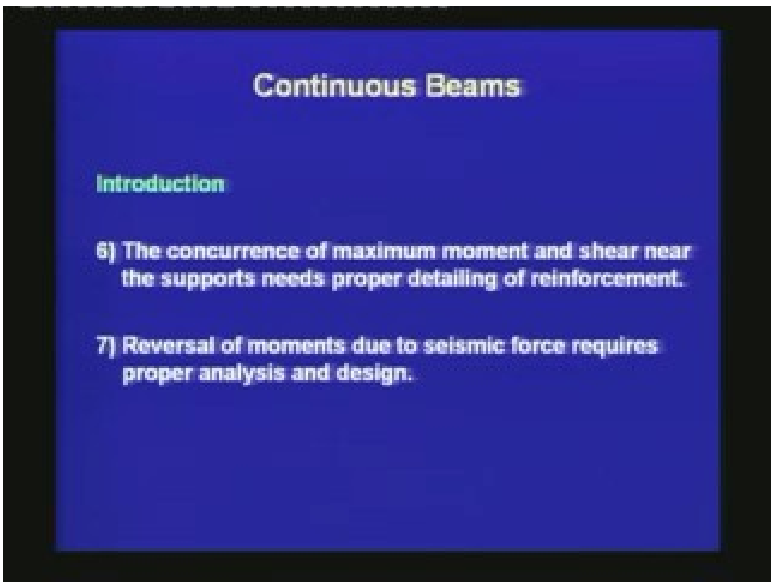

The sixth disadvantage is the concurrence of maximum moment and shear near the supports needs proper detailing of reinforcement. For a simply supported beam, the maximum moment usually occurs around the middle of the span and the maximum shear occurs near the end supports. Thus, the zone of maximum moment and the zone of maximum shear are different; whereas in a cantilever beam, which we have seen last time and in a continuous beam, the zone of maximum moment and the zone of maximum shear are both close to the support. Hence, the designing of the section and the detailing of the reinforcement near the support needs special attention in a cantilever and a continuous beam.

The seventh disadvantage is that reversal of moments due to seismic forces requires proper analysis and design. I had mentioned earlier that in high seismic regions, continuous beams are preferred because they have redundancy within the structure. But if we are designing a continuous beam for seismic forces, then there are chances of reversal of moments and in such a situation it has to be accounted for in the analysis and design.

To illustrate the examples of continuous beam, this is a sketch of a continuous beam in a building.

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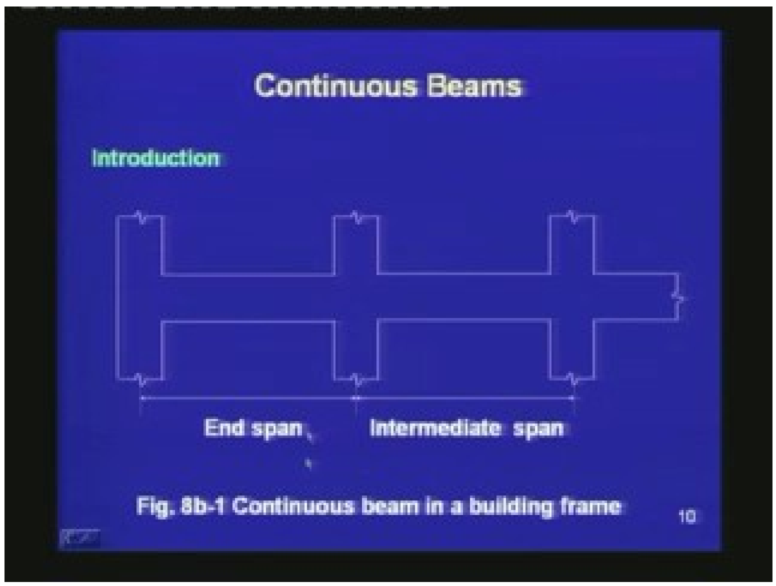

At the end we have one span which we call the end span and then the subsequent spans are called intermediate spans. The end span usually has higher positive moment in the span, as compared to the intermediate spans. Hence, the moment value of the end span is usually different from the other intermediate spans. Observe that the columns can be assumed to be beam supports because the moments on the two sides of the column can balance each other. Hence, a beam in a frame is idealized as a continuous beam with beam supports.

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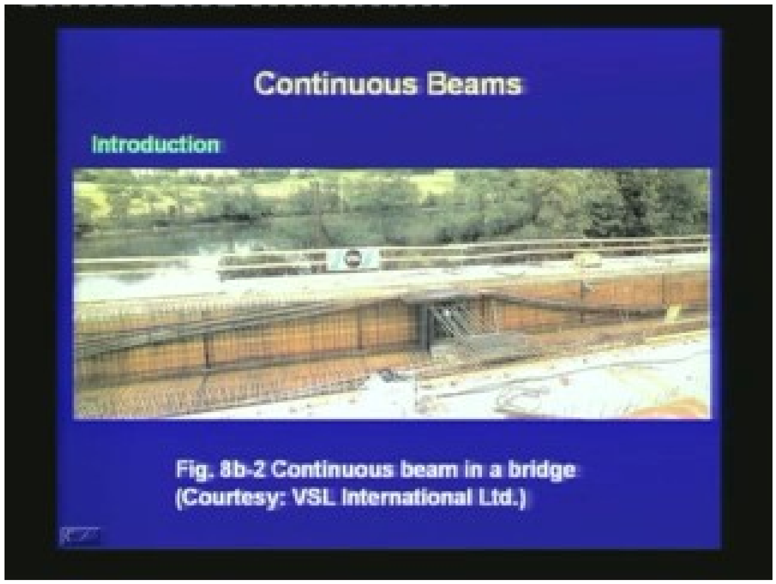

This is a photograph of a continuous beam in a bridge and here you observe that at the support region, the tendons have been brought up and again after the support region they go down. In a continuous beam, the cable profile is very important. Hence, attention should be given to have appropriate cable profile in the span region as well as in the support region. Next, we shall study the analysis of continuous beams.

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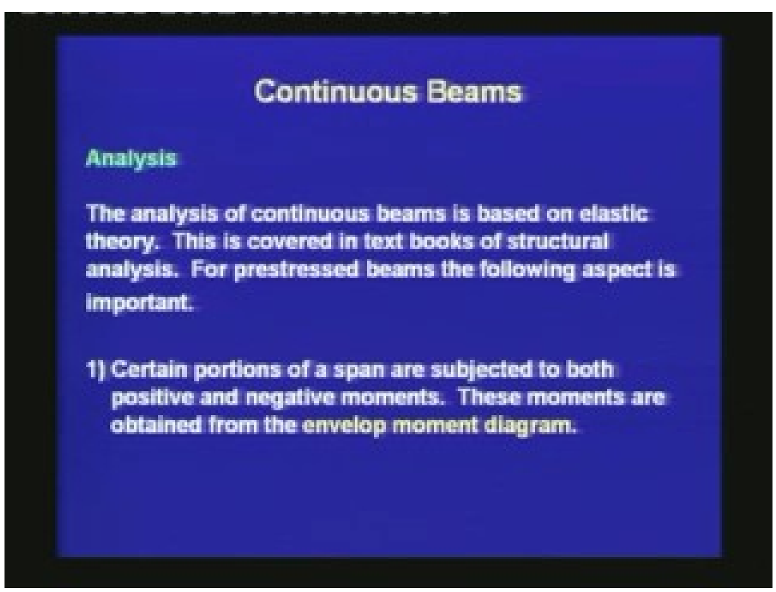

The analysis of continuous beams is based on elastic theory. This is covered in text books of structural analysis. For prestressed beams the following aspect is important. First, certain portions of a span are subjected to both positive and negative moments. These moments are obtained from the envelop moment diagram. For a simply supported beam, the span is usually under positive moment unless the effect of the upward thrust due to the prestressing force is high, but due to the external load, the moment is positive. But in a continuous beam, there are both positive and negative moments due to the external loads and in certain locations it can have both positive and negative moments depending upon the load combinations. Hence, this has to be appropriately considered in the analysis and the design moments are usually calculated by the help of the envelop moment diagram. This concept will shall see later.

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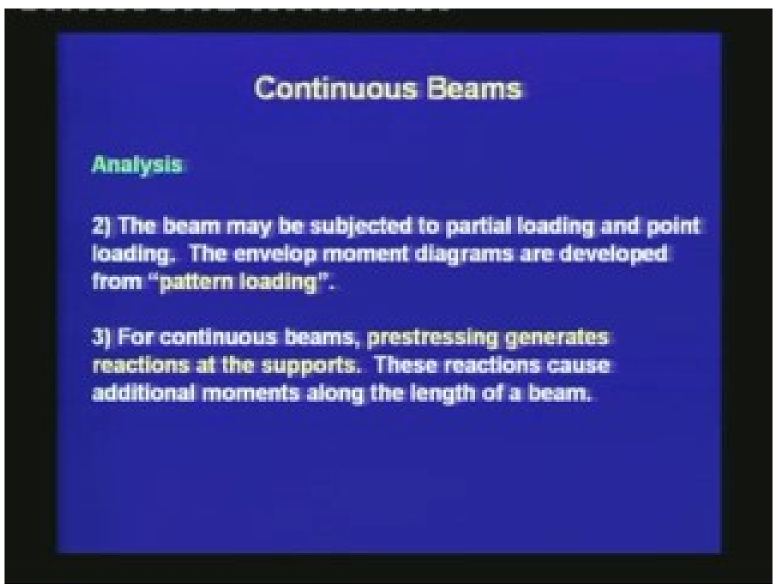

The second aspect in the analysis of a continuous beam is the beam may be subjected to partial loading and point loading. The envelop moment diagrams are developed from "pattern loading". For a simply supported beam, a live load for a certain length of the span does not give the critical condition. The critical condition is when live load is throughout the length of the span. But for a continuous beam the critical case need not be for the live load throughout the span. The critical case may be when the live load occurs in patches along the length of the beam. To consider the critical effect, a concept of pattern loading is considered in the analysis of continuous beams and again we shall come back to the pattern loading in a later stage.

Third, for continuous beams, prestressing generates reactions at the supports. These reactions cause additional moments along the length of a beam. For a simply supported beam, the concrete and steel form a self-equilibrating system. The prestressing force does not generate any reaction at the supports. But for a continuous beam, since the cable profile changes, somewhere it is down and somewhere it is up, due to that a prestressing force generates reactions at the supports. These reactions create additional moment along the length of the beam. Hence, in the analysis of a continuous prestressed beam, the effect of the moment due to the prestressing force should include the moment due to the reactions that is generated due to the prestressing force.

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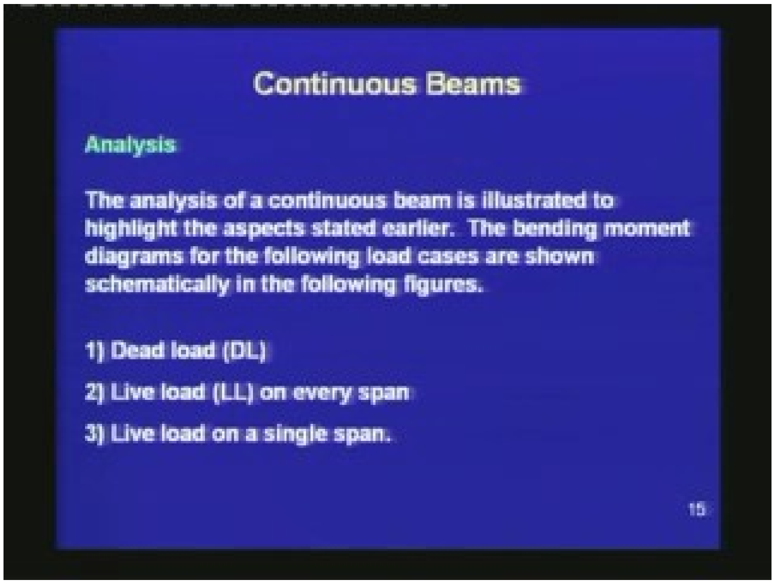

With these aspects, we are moving on to the analysis in a more subjective way. The analysis of a continuous beam is illustrated to highlight the aspects stated earlier. The bending moment diagrams for the following load cases are shown schematically in the following figures. First, we shall observe the bending moment due to dead load; next, we shall observe the bending moment due to live load on every span and third, we shall observe the bending moment due to live load on a single span. Once we understand the bending moments due to the individual load cases then, we will be able to understand the bending moment due to the load combinations because under different load combinations, the bending moments are different and from these bending moment diagrams finally we develop the envelop moment diagram.

This figure shows the dead load along the length of the beam.

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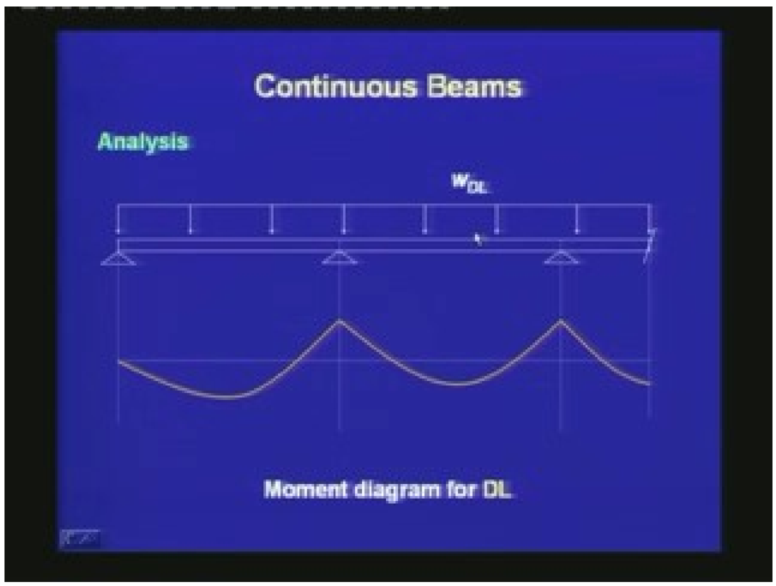

If the beam has uniform cross section, then the dead load will be uniform and for that dead load it generates a moment profile something like this. It has a positive moment. That means it creates compression at the top in the span region and it generates negative moment in the support regions; that means it creates tension at the top. Note that, for the end span, the support moment at the end support is zero, for an ideal pin condition. The positive moment in a span is substantially high as compared to the intermediate spans; whereas in an intermediate span, the negative moments are higher than the positive moments. These aspects should be clearly understood before the design of a continuous beam.

Next, we are seeing the bending moment diagram due to the live load.

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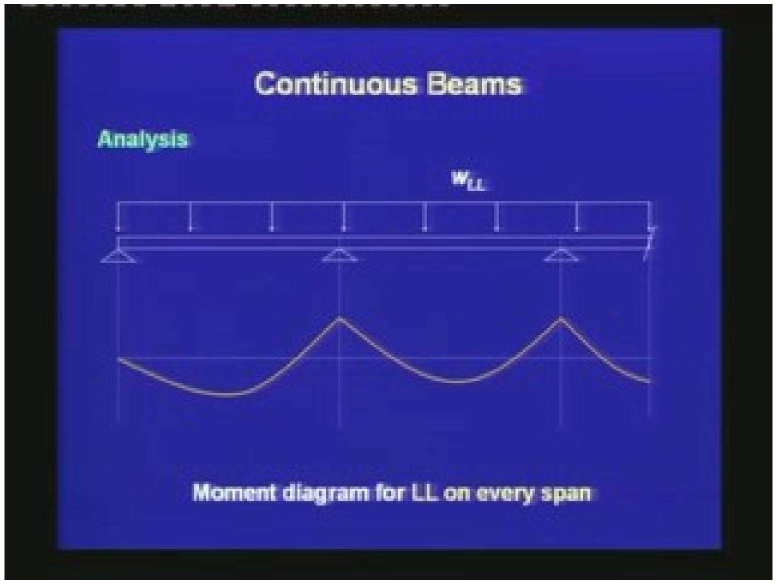

If the live load is uniform in all the spans, then the bending moment diagram is similar to that due to the dead load. It creates a positive moment in the end span, which is higher than the positive moment in the intermediate span and for the intermediate spans, the values of the negative moment is higher than that of the positive moment.

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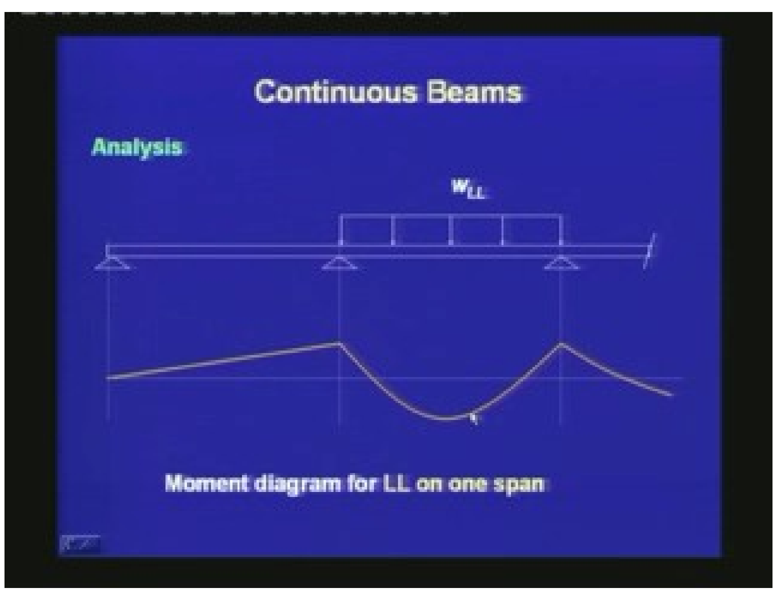

The third case what we are seeing is, due to the live load in one intermediate span. Here, we have not placed any live load at the end span and for this the moment diagram is that throughout the end span the moment is negative. This is unlike the bending moment diagrams due to the uniform live load, where it has created positive moment in the end span; but if there is live load only in one span, then it can create negative moments throughout the spans in the end span region. This type of conditions have to be considered in developing the envelop moment diagram.

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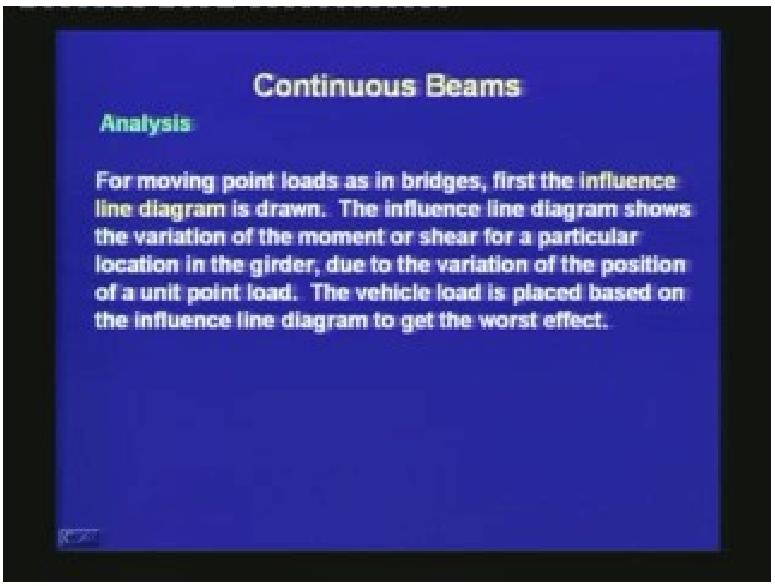

For moving point loads, as in bridges, first the influence line diagram is drawn. The influence line diagram shows the variation of the moment or shear for a particular location in the girder, due to the variation of the position of a unit point load. The vehicle load is placed based on the influence line diagram to get the worst effect. Thus, if we are analyzing a continuous beam for a moving point load, then first we develop an influence line diagram. The influence line diagram shows the variation of the moment at a particular location due to the variation of the placing of the point load. Once we develop the influence line diagram, then the point load, say due to a vehicle in the bridge, is placed in such a way so as to get the worst effect along the continuous beam. Hence, the use of influence line diagram is extremely important in case of a moving point load.

In order to consider the maximum effect, a uniformly distributed load is also placed in patches based on the influence line diagram for a moving point load.

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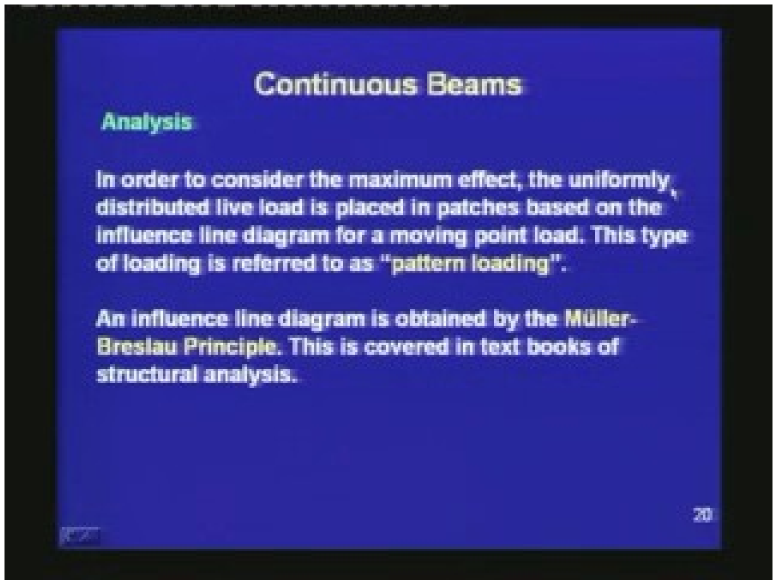

Thus, the influence line diagram is used not only for a moving point load, but is also used to have a critical placement of the live load, which can be static, but it is not uniform throughout the length of the beam. The influence line concept helps us to place the live load in patches for the worst condition. This type of patch loading is termed as "pattern loading". Thus, the live load in a building or in a bridge is not just placed uniformly, it is also placed in patches and this type of loading is called "pattern loading".

An influence line diagram is obtained by the Muller Breslau principle. This is covered in test books of structural analysis. Thus, if you have to get the influence line diagram, then we can use the Muller Breslau principle to develop the influence line diagram. Based on the influence line diagram, we can place the live load in patches to generate the pattern loading, which may give the critical loading condition.

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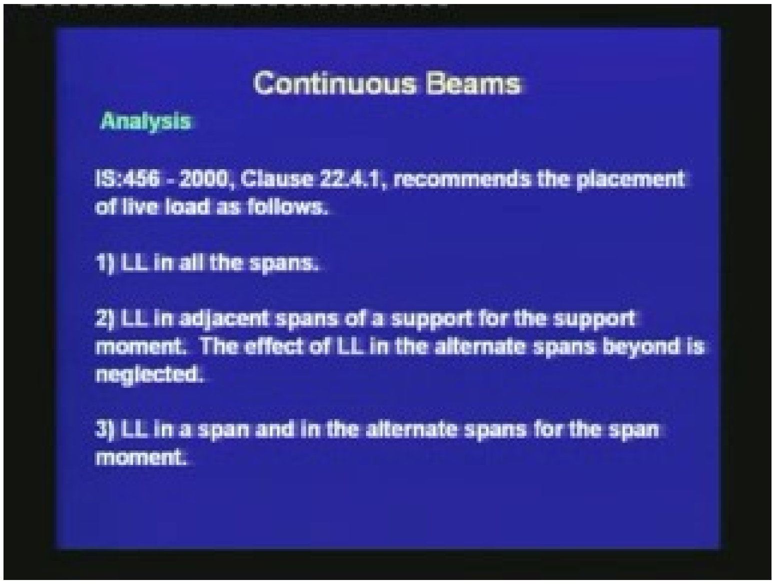

IS: 456 - 2000 clause 22.4.1, recommends the placement of live load as follows. There are three load cases for the live load that has to be considered. First is, we should place the live load in all the spans, this is the most basic case. Second is the live load in adjacent spans of a support for the support moment. The effect of live load in the alternate spans beyond is neglected. It has been observed from influence line diagram that the maximum support moment comes when the live is placed just in the adjacent spans next to the two supports. If we are not placing any load in alternate spans, then it creates maximum effect of the live load.

The code says, in order to get the support moment for the most critical case, we should place the live load in the adjacent spans and the rest of the beam need not have the live load. This loading neglects the effect of the live load, which if placed in the alternate spans may have more moment in the support, but their effect is negligible. Hence, the code just recommends placing the live loads only in the adjacent spans next to the support.

The third load case is the live load should be placed in a span and in the alternate spans for the span moment. To get the most critical case for the span moment, we should place the live load in the span as well as in the alternate spans beyond that. This will give the critical positive moment in the span. From all these load cases, we develop the envelop moment diagram.

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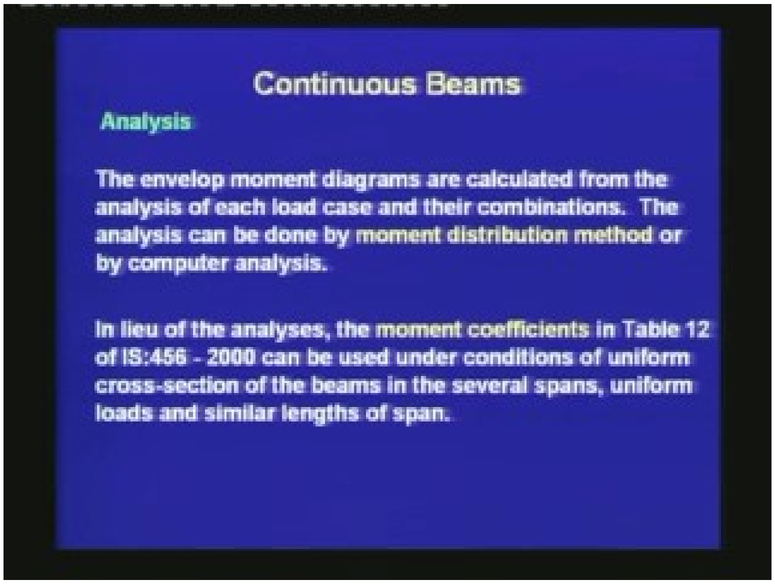

The envelop moment diagrams are calculated from the analysis of each load case and their combinations. The analysis can be done by a moment distribution method or by computer analysis. In lieu of the analysis, the moment coefficients in table 12 of IS: 456 - 2000 can be used under conditions of uniform cross section of the beams in the several spans, uniform loads and similar lengths of span. Thus, the method of analyzing a continuous beam is the moment distribution method or by a computer analysis. When we analyze the several load cases, one for the dead load, another for the uniform live load and others for the pattern loading, from this we get the moments at the spans and at the supports and then we combine the moments based on the load combinations.

If we have to avoid a rigorous structure analysis, then the code IS: 456 allows us to use moment coefficients, provided we satisfy certain conditions. If the beam is of uniform cross section or the spans are more or less of similar length and the loading is more or less uniform throughout the length of the beam, then the moment coefficients can be used which simplifies the calculations of the design moments at the span and the support regions. From the design moments, we can develop the envelop moment diagrams.

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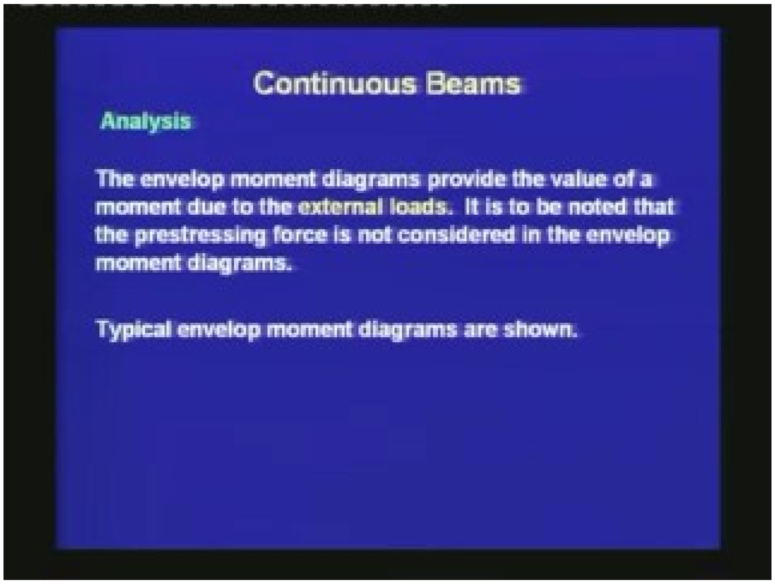

The envelop moment diagrams provide the value of a moment due to the external loads. It is to be noted that the prestressing force is not considered in the envelop moment diagrams. When we are talking of the structure analysis due to the external loads, we are not considering the effect of the prestressing force. The envelop moment diagrams gives the moment just due to the external loads. The effect of prestressing force has not been considered yet. Typical envelop moment diagrams are shown in the following sketches.

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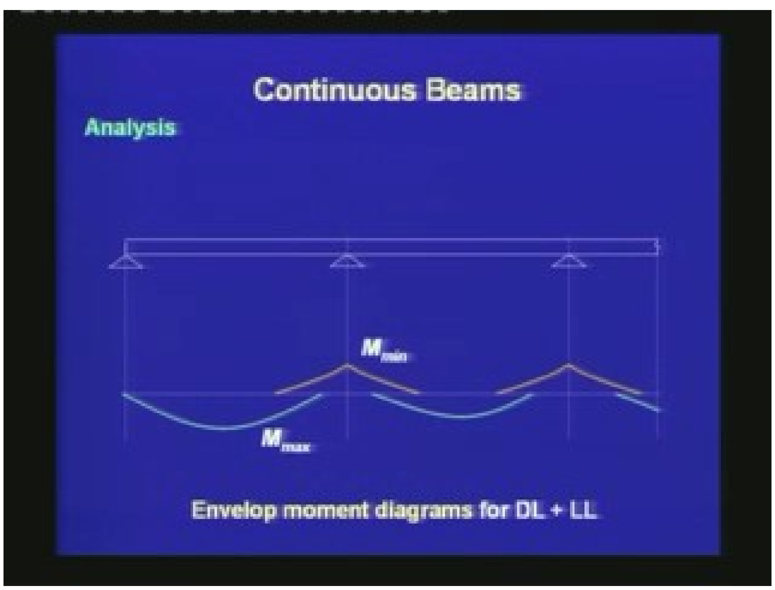

In this figure, we see that for the end span, the green line gives the maximum positive moment throughout the span and in the intermediate spans again a green line gives the maximum positive moment. Near the supports, the orange line gives the maximum values of the negative moments. We are denoting M_{max} and M_{min} for the maximum values of positive and negative moments in an algebraic sense. This is a typical envelop moment diagram for a continuous beam. Note that in certain regions, we can have both a positive moment and a negative moment depending upon the loading condition. In the envelop moment diagram, M_{max} and M_{min} represent the highest and lowest values of the moments at a section respectively.

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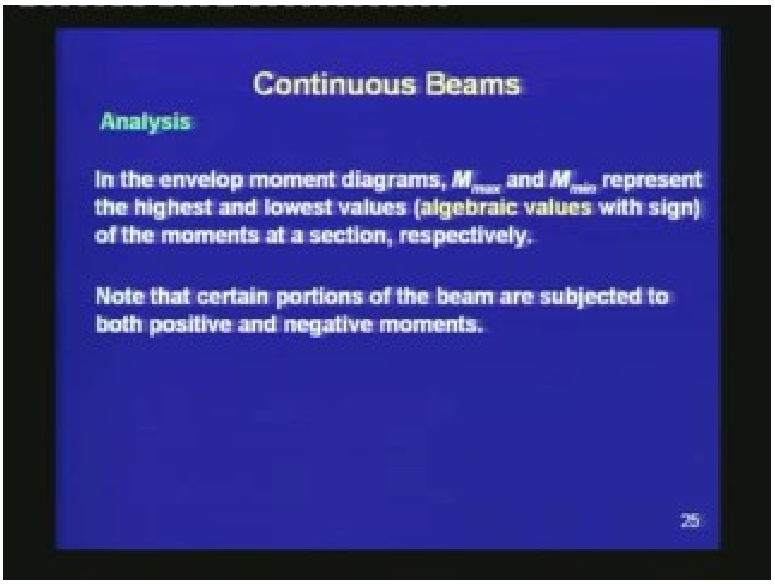

These are algebraic values with sign. That means when we are saying a maximum moment and a minimum moment, we are not just talking of the numerical values but we are considering their signs. M_{max} is considered to be a positive moment, M_{min} is considered to be the negative moment with a maximum numerical value. Note that, certain portions of the beam are subjected to both the positive and negative moments. The next important aspect is the incorporation of moment due to the reactions.

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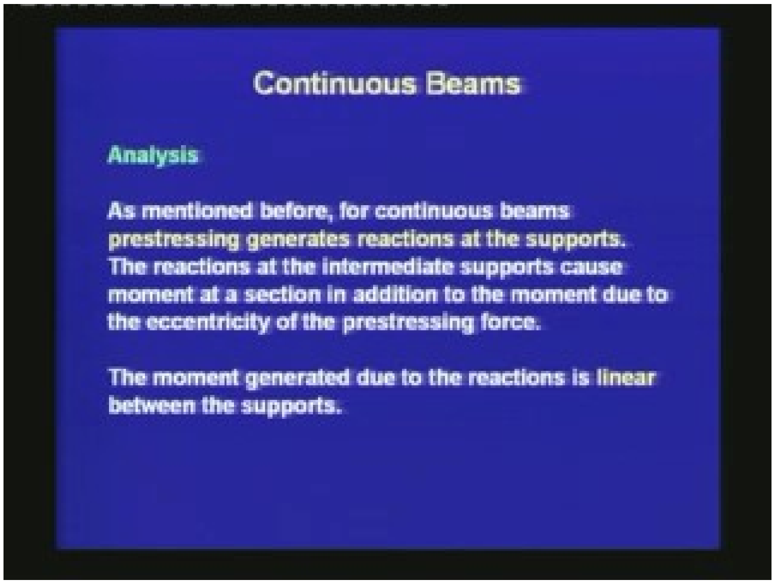

As mentioned before, for continuous beams prestressing generates reactions at the supports. The reactions at the intermediate supports cause moments at a section, in addition to the moment due to the eccentricity of the prestressing force. This is an important aspect of a prestressed continuous beam that when the prestressed is applied in the beam, it generates reactions in the supports, which is unlike a simply supported prestressed beam. The reactions have additional moment in the spans, which is in addition to the moment that comes to the eccentricity of the prestressing force. These moments need to be considered in the analysis of a prestressed continuous beam.

The moment generated due to the reactions is linear between the supports. Since the reactions are point loads separated by a certain distance, the moment due to the reactions is a linear variation from one support to another support.

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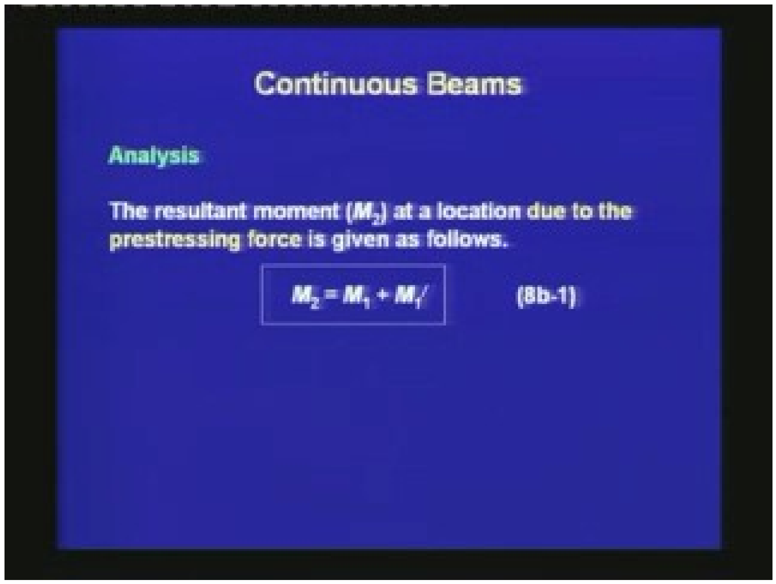

The resultant moment at a location due to the prestressing force is given as follows: M_2 is equal to M_1 plus M_1 prime. In this equation, M_1 is the moment due to prestressing force without the intermediate supports which is given as P_e times e.

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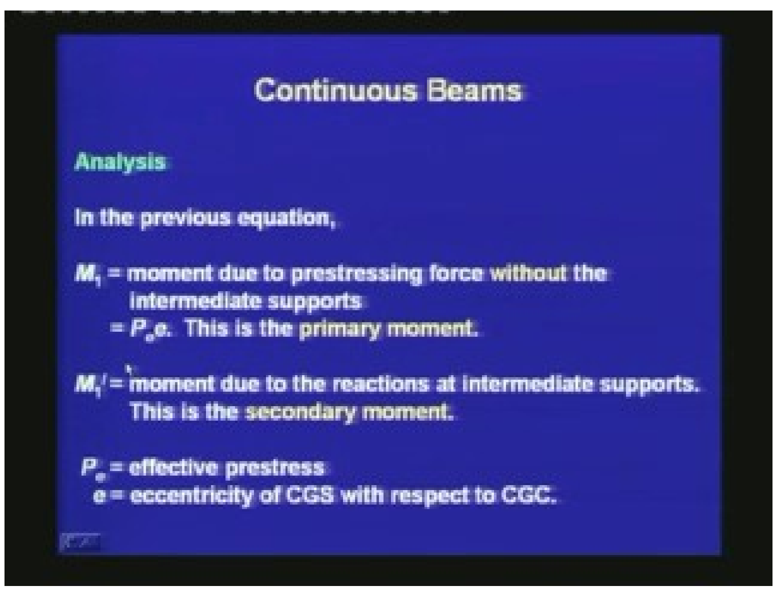

This is the primary moment due to the prestressing force. Since the CGS is at eccentricity to the CGC, the prestressing itself generates some moment in the section. The second component, M_1 prime is the moment due to the reactions at intermediate supports and this is the secondary moment which is generated due to the prestressing force. P_e is the effective prestress and e is the eccentricity of CGS with respect to CGC. Thus, to summarize the total moment due to the prestressing force M_2 is equal to M_1 plus M_1 prime.

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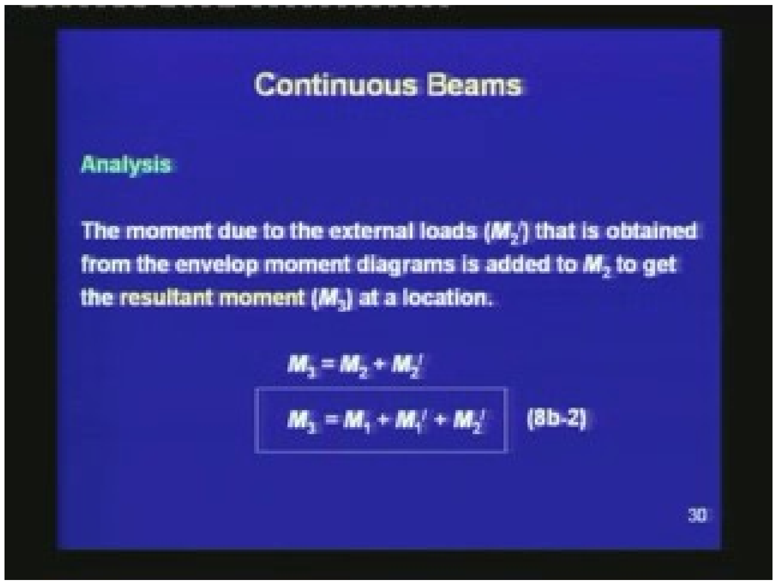

The moment due to the external loads which will be denoted as M_2 prime is obtained from the envelop moment diagrams and is added to M_2 to get the resultant moment M_3 at a location. Thus, the final resultant moment M_3 is the moment due to the prestressing force which is M_2 plus the moment due to the external loads which is obtained from the envelop moment diagrams and that is denoted as M_2 prime. Thus M_3 is equal to M_2 plus M_2 prime and since M_2 is equal to the moment due to the prestressing force alone plus the moment due to the reactions, that is why, M_3 is equal to M_1 plus M_1 prime plus M_2 prime. To summarize the final resultant moment in a continuous beam under the service loads consist of three components: one due to the prestressing force eccentricity of the strands, second due to the reactions that is generated due to the prestressing force and third due to the external load that comes to the beam during the service life.

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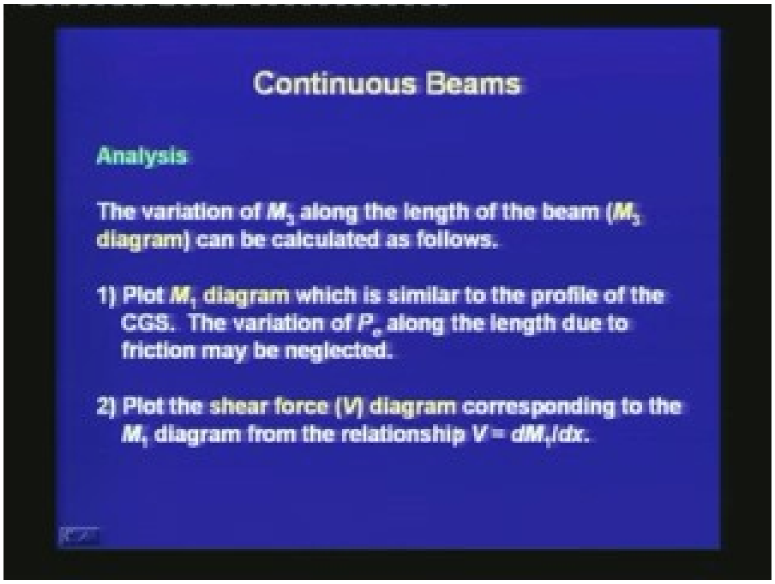

The variation of M_3 along the length of the beam can be calculated as follows. The next task in hand is to develop the resultant moment diagram which is denoted as the M_3 diagram. This diagram can be calculated as follows: the first step is, plot the M_1 diagram which is similar to the profile of the CGS. The variation of P_e along the length due to friction may be neglected. When the prestress is applied due to the eccentricity of the CGS with respect to the CGC a moment is developed, that moment is the M_1 moment. The M_1 diagram is similar to the profile of the CGS and at any point M_1 can be calculated as the prestress times the eccentricity of the CGS with respect to CGC. The first step is to develop the M_1 diagram from the cable profile and the value of the prestressing force.

The second step is to plot the shear force diagram corresponding to M_1 diagram from the relationship; V is equal to dM_1 divided by dx. At this stage, we do not know the reactions due to the prestressing force. In order to calculate the reactions, we are going through the principles of structural analysis. We are first plotting the shear force diagram, that corresponds to the M_1 diagram and this shear force can be calculated from the expression, V is equal to dM_1 divided by dx. This is the relationship between the shear generated due to the moment from the prestressing force.

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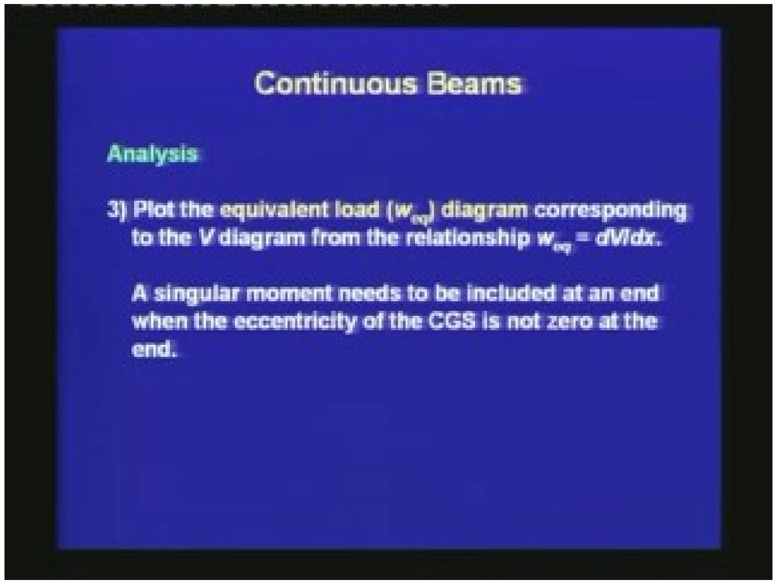

The third step is to plot the equivalent load or w_{eq} diagram, corresponding to the shear diagram from the relationship w_{eq} is equal to dV divided by dx. Thus, from the M_1 diagram we are calculating the shear force diagram; from the shear force diagram we are calculating the equivalent load diagram.

We had talked about this equivalent load diagram for simply supported beams. For a simply supported beam, the equivalent load is always upwards and we say that it is the upward load which balances part of the external load. For a continuous beam, the equivalent load is not always upwards, at the support regions the equivalent load can be downwards. Hence, instead of saying as upward load, we are saying that we need to plot the equivalent load diagram and this is obtained from the relationship that w_{eq} is equal to dV divided by dx. This also comes from the principle of structural analysis.

Before we go to the fourth step, another important point is, a singular moment needs to be included at an end when the eccentricity of the CGS is not zero at the end. If this CGS has an eccentricity at the end, then we need to add an equivalent moment at the end.

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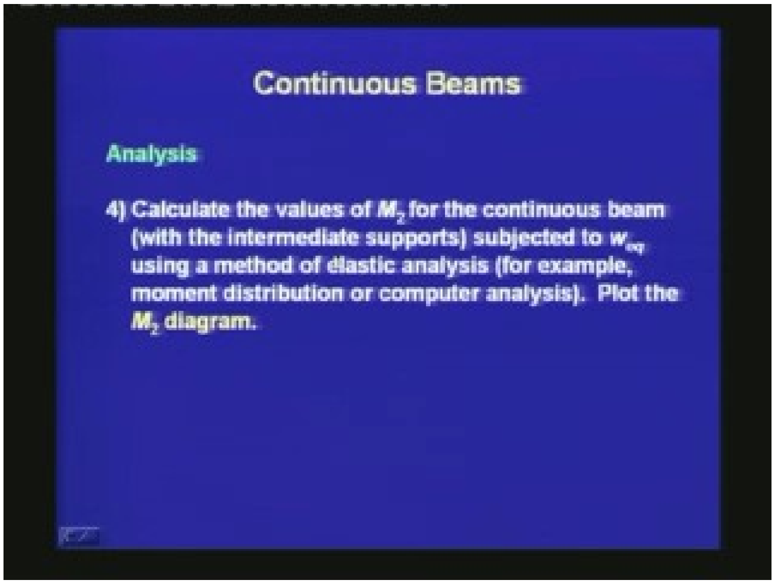

The fourth step is to calculate the values of M_2 for the continuous beam with the intermediate supports subjected to w_{eq} using a method of elastic analysis. For example, moment distribution or computer analysis. Plot the M_2 diagram. Once we have the equivalent load known, we can analyze the continuous beam to develop the M_2 diagram which is the resultant moment due to the prestressing force. Once we know the M_2 values along the length of the beam, we are able to plot the M_2 diagram.

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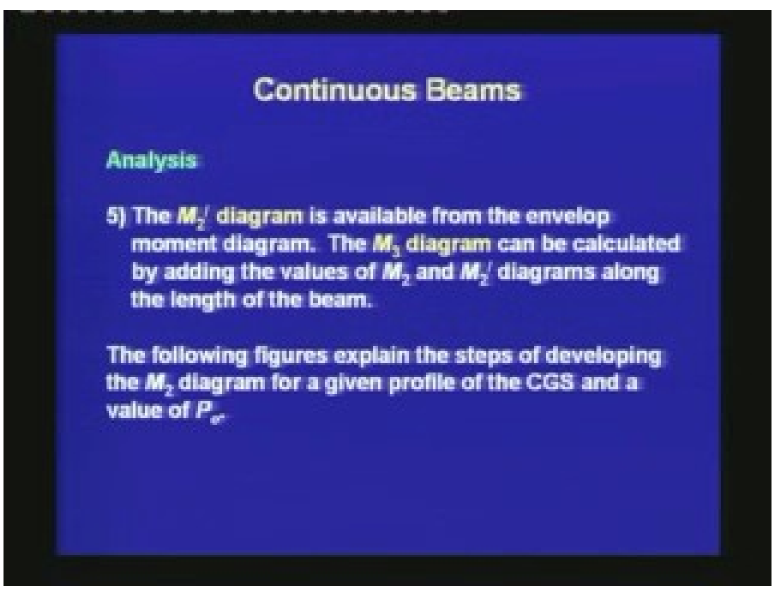

The fifth step is the M_2 prime diagram is available from the envelop moment diagram. Then the M_3 diagram can be calculated by adding the values of M_2 and M_2 prime diagrams along the length of the beam. We had earlier seen how to calculate the envelop moment diagrams. Once we add the ordinates of the envelop moment diagrams to the ordinates of the M_2 diagram, then we get the final resultant moment due to the prestressing force and the external loads and that is called the M_3 diagram.

The following figures explain the steps of developing the M_2 diagram for a given profile of the CGS and a value of P_e. Thus, before we can develop M_2 diagram, we need to have a cable profile and the value of the prestressing force.

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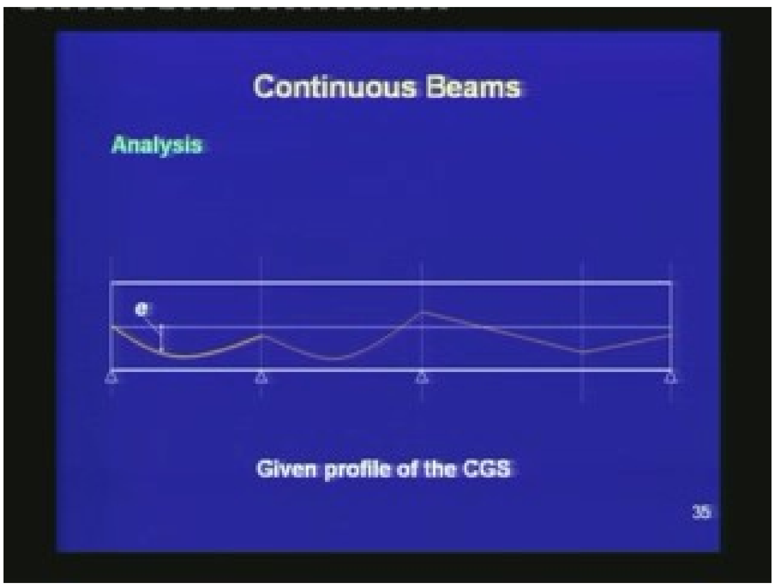

This is a continuous beam, where the vertical scale has been enlarged to show the clarity of the profile of the CGS. The CGS is zero eccentricity at the end and then it goes down at the end span, it comes up near the first intermediate support and again it goes down. Then for the second intermediate support, it goes above the CGC and then it comes down linearly below the CGC and again it goes up and it has a certain eccentricity at the end. This is just a trial 'C' profile of the CGS, which will show the different aspects of developing the M_2 diagram from this profile of the CGS. Once we have the profile of the CGS, we can develop first the M_1 diagram.

The M_1 diagram is very similar to the profile of the CGS.

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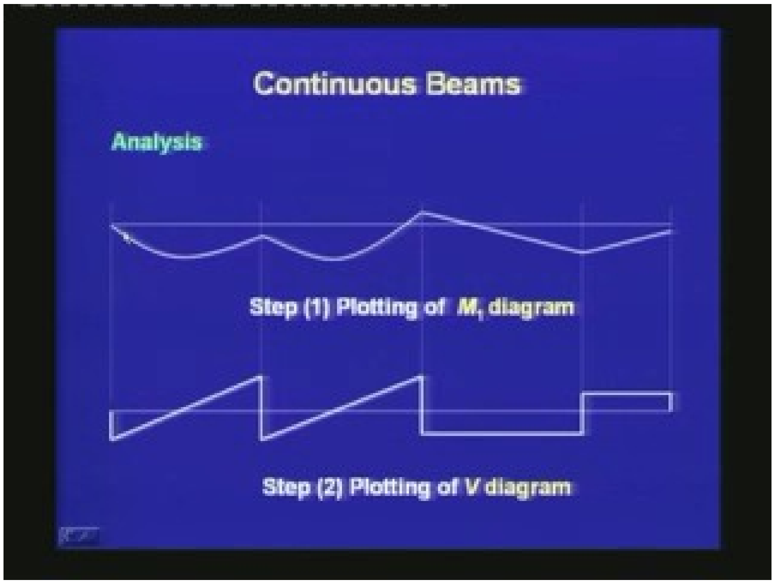

It follows the same characteristics. It goes down in the end span, again it comes up near the end supports and again it goes down at the intermediate span and then it goes above the reference line at the support and it follows the chain of the profile of the CGS. This M_1 diagram is calculated from the relationship, M_1 is equal to P_e times the eccentricity. From the M_1 diagram, we are calculating the V diagram. The V diagram is obtained from the relationship, V is equal to dM_1 divided by dx. Once we take the differentiation of this curve, we can plot the V diagram and the V diagram will look something like this. In the first end span, we will have the V diagram linearly increasing when the profile is parabolic, then there is a jump at the support and again in the intermediate span it is linearly increasing. We have another jump in the second intermediate support. For a linearly varying M_1 diagram, we have a constant V diagram and then it has a jump in the support and again it is constant at the other end span.

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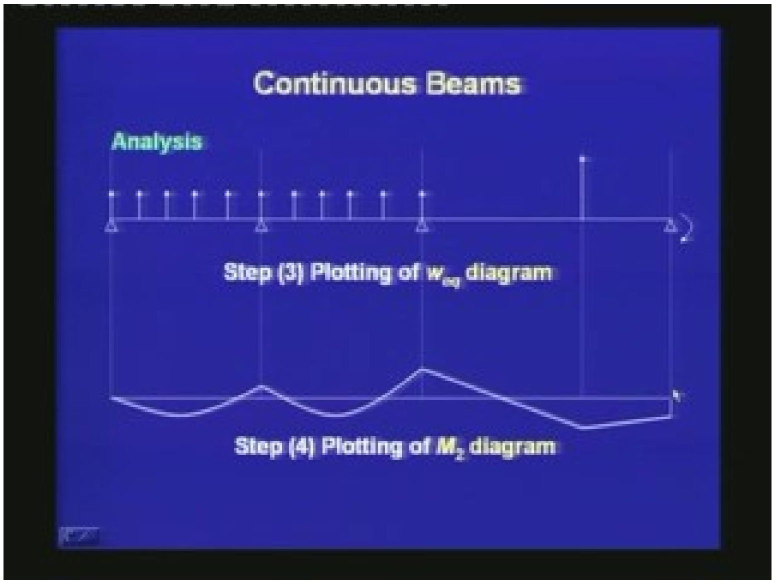

From the V diagram, we can calculate the w_{eq} diagram from the relationship w_{eq} is equal to dV divided d_X. We have an upward load in the spans. Note that, the downward loads at supports need not be considered because they directly transmit through the supports to the ground. Hence, we are showing only the loads that come in the spans and not the point loads which come at the supports. From the w_{eq} diagram, we can get the M_2 diagram by a structure analysis and the M_2 diagram looks similar to the M_1 diagram. In this example, it has a parabolic variation in M span, another parabolic variation in the first intermediate span, then it has a linear variation in the second intermediate span and a linear variation in the other end span. This is the M_2 diagram that is the resultant moment due to the prestressing force variation along the length of the beam.

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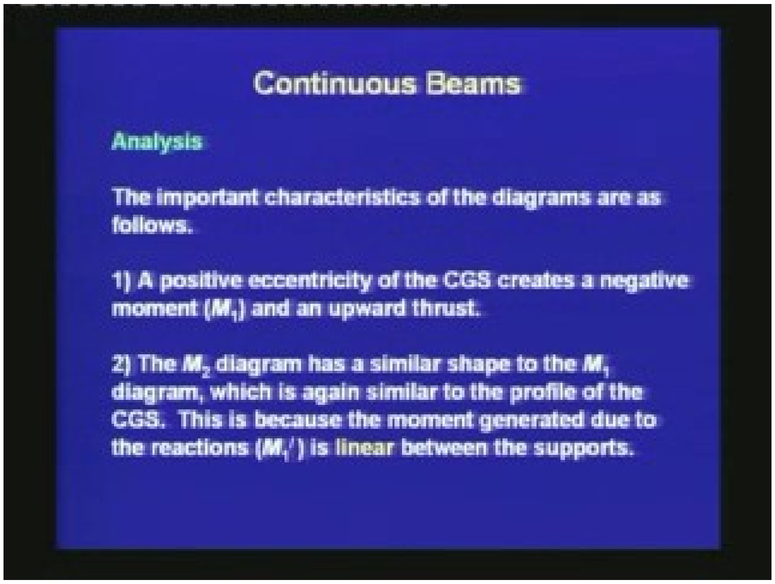

The important characteristics of the diagrams are as follows. A positive eccentricity of the CGS creates a negative moment M_1 and an upward thrust. This we had seen from a simply supported beam that when the CGS is below the CGC, that means, when it has a positive eccentricity, the moment M_1 is negative, the beam tends to hog up and the thrust or the equivalent load is upwards. This is the first check to make sure that the calculations of the diagrams make sense, given the eccentricity of the cable profile.

The second characteristic of the M_2 diagram is the M_2 diagram has a similar shape to the M_1 diagram which is again similar to the profile of the CGS. This is because the moment generated due to the reactions M_1 prime is linear between the supports. An important characteristic of the M_2 diagram is that the variation of the M_2 diagram is similar to the variation of the M_1 diagram, which is again similar to the variation of the cable profile. The reason is that the moments generated due to the reactions which is denoted as M_1 prime is linear between the supports and hence the cable profile, the M_1 diagram and the M_2 diagram are of similar shape.

Next, we are studying how to develop the pressure line due to the prestressing force.

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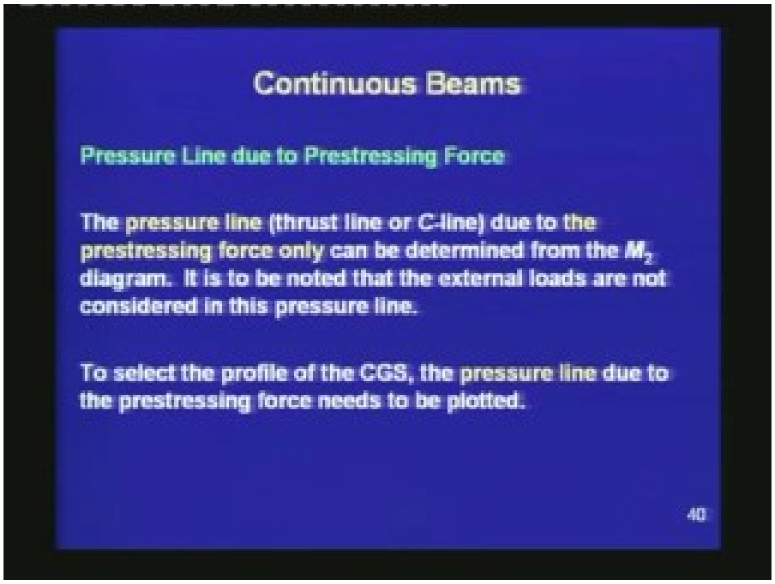

The pressure line which is also known as thrust line or C line due to the prestressing force only, can be determined from the M_2 diagram. It is to be noted that the external loads are not considered in this pressure line. To select the profile of the CGS, the pressure line due to the prestressing force needs to be plotted. The calculation of the pressure line helps us to select an appropriate cable profile in a continuous beam and the pressure line can be calculated from the M_2 diagram that we have just covered and note that here the pressure line is due to the prestressing force only. We are not considering the effect of the external load in the pressure line. The calculation of pressure line from the M_2 diagram is based on the following expression e_c is equal to M_2 divided by P_e.

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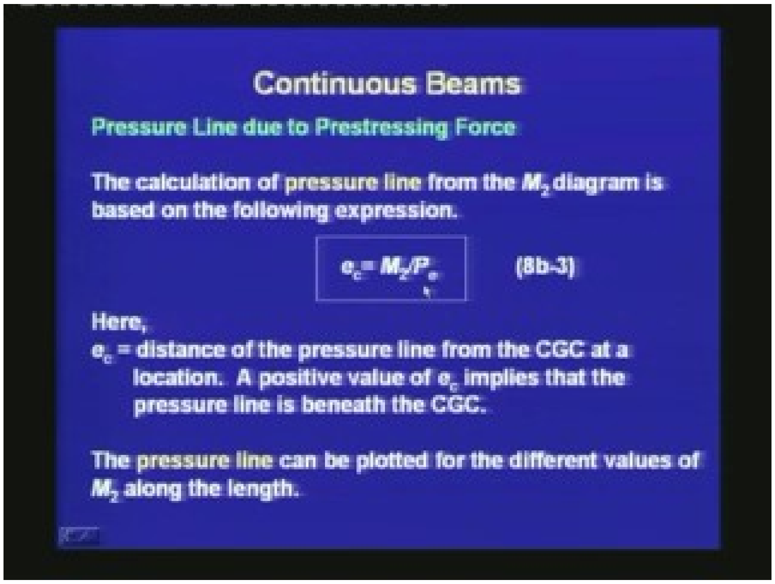

Here, e_c is the distance of the pressure line from the CGC at a location. That means e_c is the eccentricity of the compressive force in the concrete and it is measured from the CGC of the section. A positive value of e_c implies that the pressure line is beneath the CGC. Similar to the eccentricity of the tendon, a positive value of e_c implies that the C is below the CGC and a negative value of e_c implies the location of C is above the CGC. The pressure line can be plotted for the different values of M_2 along the length. That means once we have the M_2 diagram, we can use the expression e_c is equal to M_2 divided by P_e to calculate the pressure line throughout the length of the beam.

The following sketch shows the pressure line for a given profile of the CGS.

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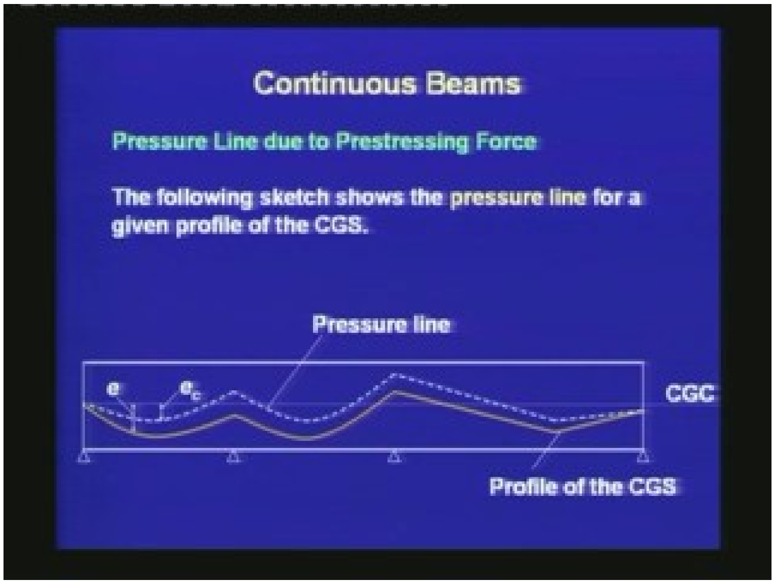

The orange line is the profile of the CGS and the dashed line is the pressure line. Observe that the pressure line has a similar variation as the profile of the CGS. Thus, due to the prestressing force the compression shifts up from the CGS to the pressure line. The important characteristics of the pressure line are as follows.

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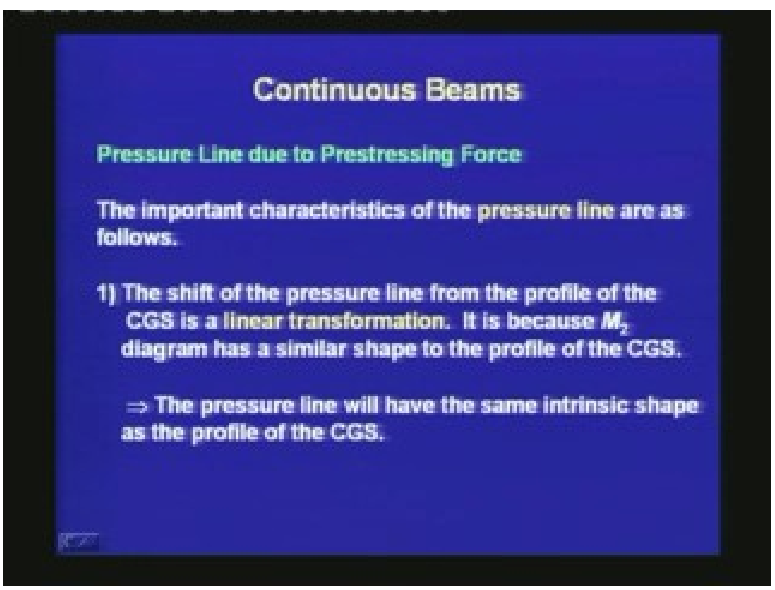

The shift of the pressure line from the profile of the CGS is a linear transformation. It is because M_2 diagram has a similar shape to the profile of the CGS. We had earlier seen that the M_2 diagram has a similar variation as the profile of the CGS and since the pressure line is calculated from the relationship e_c is equal to M_2 divided by P_e, the pressure line has a similar shape as that of the M_2 diagram. This leads to an important corollary, that the pressure line will have the same intrinsic shape as the profile of the CGS. Thus, we can do a back check that the pressure line should be of the same shape as that of CGS.

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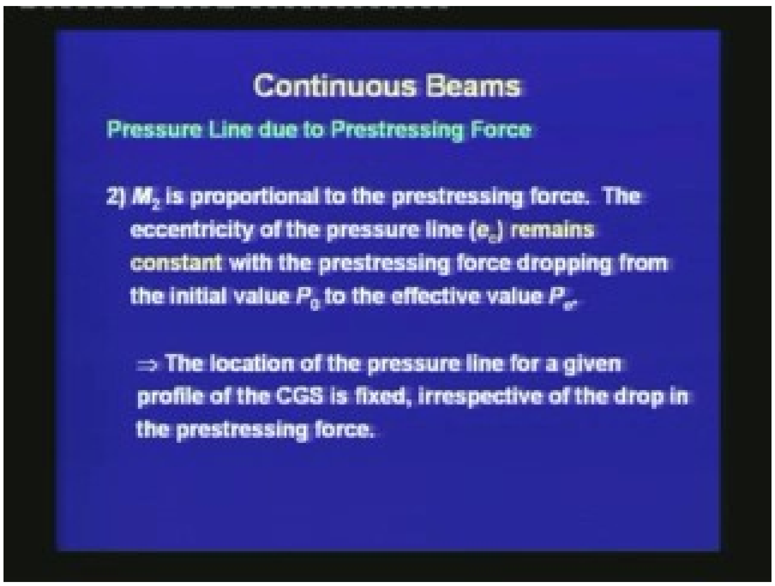

The second important point is M_2 is proportional to the prestressing force. The eccentricity of the pressure line, which is denoted as e_c, remains constant with the prestressing force dropping from the initial value P_0 to the effective value P_e. Even if the prestressing force changes with time, the eccentricity of the compression which is e_c, stays constant. It is only M_2 which changes with the drop in the prestressing force. This gives us a very important conclusion that the location of the pressure line for a given profile of the CGS is fixed irrespective of the drop in the prestressing force. Whatever is the prestressing force the pressure line is constant for a given profile of the CGS.

Let us understand the development of the pressure line by an example.

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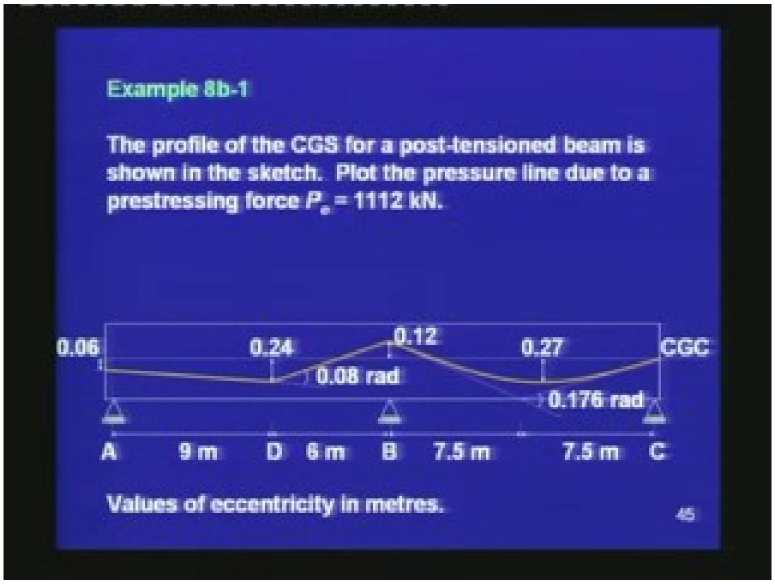

The profile of the CGS for a post-tensioned beam is shown in the sketch. Plot the pressure line, due to a prestressing force P_e is equal to 1112 Kilonewtons. For these two span beams, the CGS has an eccentricity of 0.06 meters at the end. Then it drops down linearly to an eccentricity of 0.24 meters and again the CGS goes up linearly to an eccentricity of 0.12 meters above CGC. The change in the slope is 0.08 radians. In the second span, the profile is parabolic with an eccentricity of 0.27 meters in the span and no eccentricity at the end. The change in the slope of the CGS in the second span is 0.176 radians. The first span is 15 meters, which is divided into 9 meters and 6 meters and the second span is also 15 meters which is divided into 7.5 meters and 7.5 meters at the locations of the maximum eccentricity.

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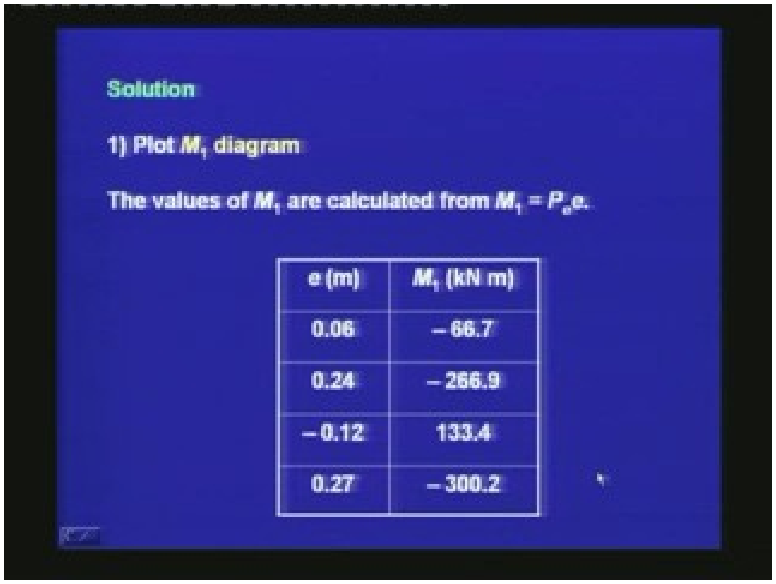

First, we are plotting the M_1 diagram. The values of M_1 are calculated from M_1 is equal to P_e times e. P_e is known. For the given values of the eccentricity, we are able to calculate M_1 and the M_1 values are of opposite sign to that of the eccentricity. If the eccentricity is positive, M_1 will be negative. If the eccentricity is negative, then M_1 will be positive.

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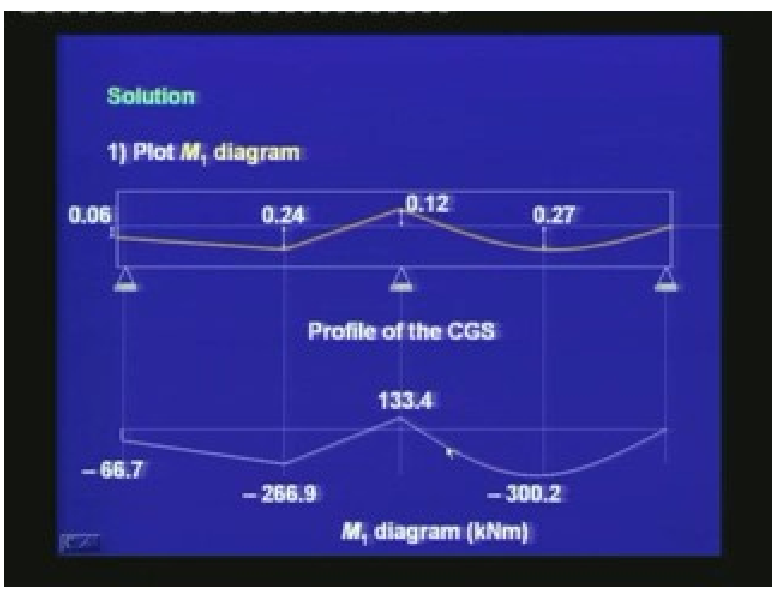

Thus, given the profile of the CGS, we are able to calculate the M_1 diagram, which has the same variation as the profile of the CGS.

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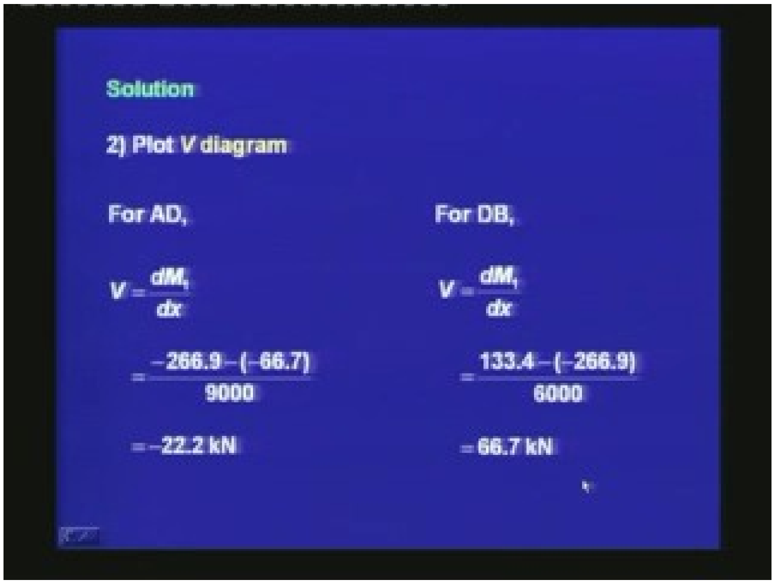

The second step is to calculate the shear diagram. For the first section AD, V is equal to dM_1 by dx, and we have the end moments, which is 266.9 and 66.7. The length of the span is 900 millimeters and we find that the shear is given as 22.2 Kilonewtons. For the section DB, V is equal to 133.4 minus of minus 266.9 divided by the span which is 600 or 6000? millimeters, which gives 66.7 Kilonewtons.

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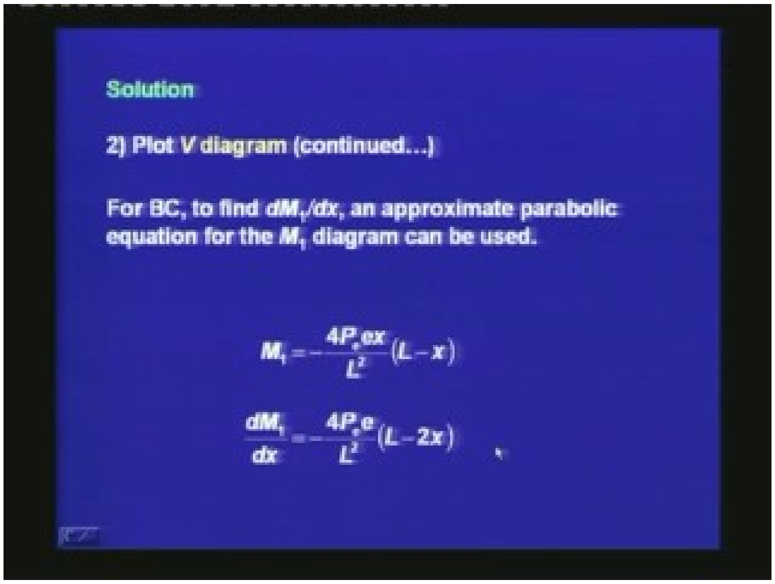

For BC, to find dM_1 by dx, an approximate parabolic equation for the M_1 diagram can be used. M_1 can be expressed as a function of P_e and e as follows. M_1 is equal to minus $4P_e$ times e times x times within bracket L minus X divided by L square, from which we can find out an expression of dM_1 by dx.

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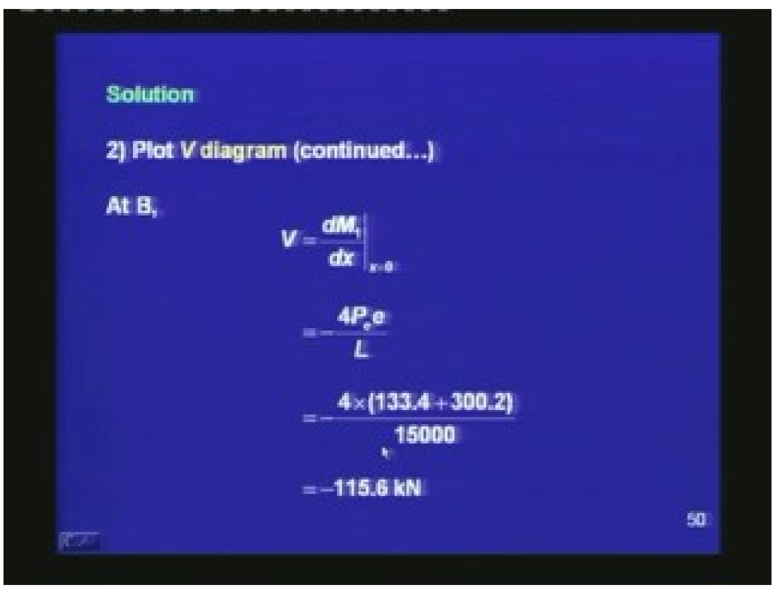

At B, dM_1 by dx is calculated for x equal to 0, which is equal to $4P_e$ times e divided by L and P_e times e is the total shift in the M_1 diagram between the support and the span, which is given as 133.4 plus 300.2 divided by the length which is 15000 millimeters, gives us a shear of -115.6 Kilonewtons.

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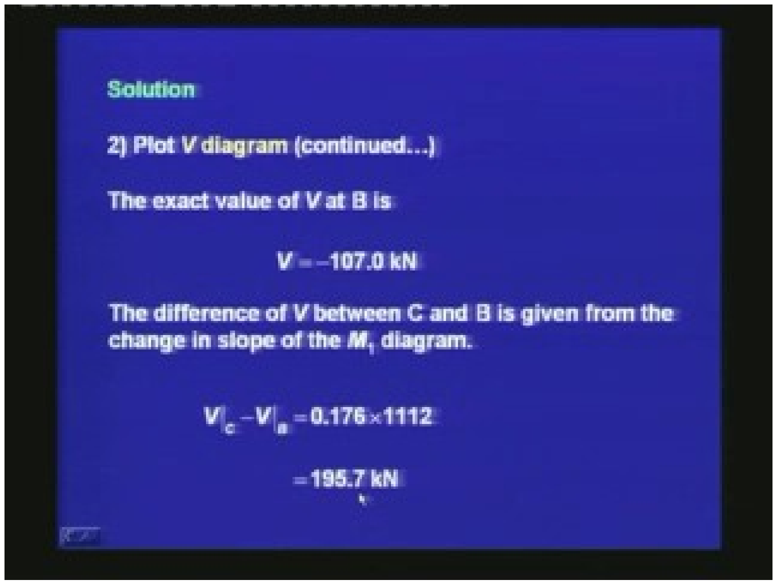

The exact value of B which can be calculated by considering an exact equation of the parabola is given as -107 Kilonewtons. Thus we observe that, if we use an approximate relationship, we get a value which is close to the exact value. The difference of the shear between C and B is given from the change in slope of the M1 diagram. V_C minus V_B is equal to the change in the slope which is 0.176 radians times the prestressing force which is 1112 and this gives us a difference of shear equal to 195.7 Kilonewtons between B the intermediate support and the end support at the right.

Since we know the shear at the intermediate support, we can calculate the shear at C by the relationship, 195.7 minus 107 which give us 89 Kilonewtons at C.

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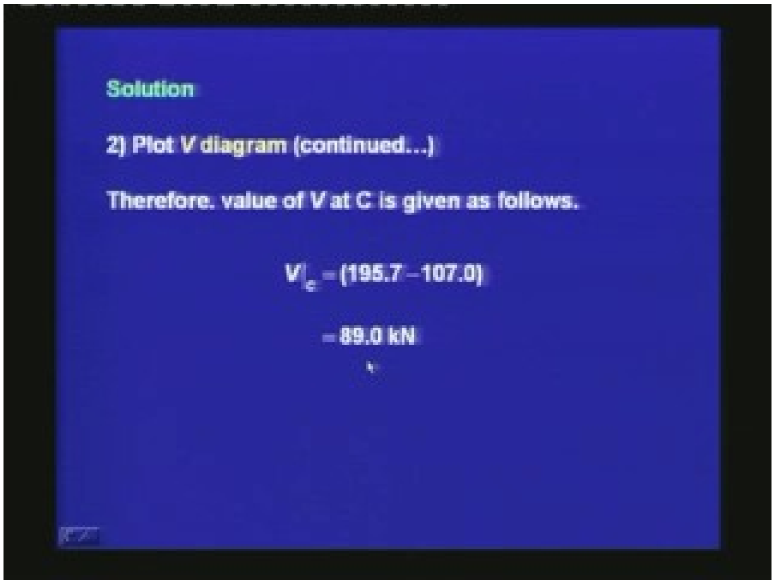

We are plotting the shear diagram and it is constant in AD, then DB, and it is linearly varying between B and C.

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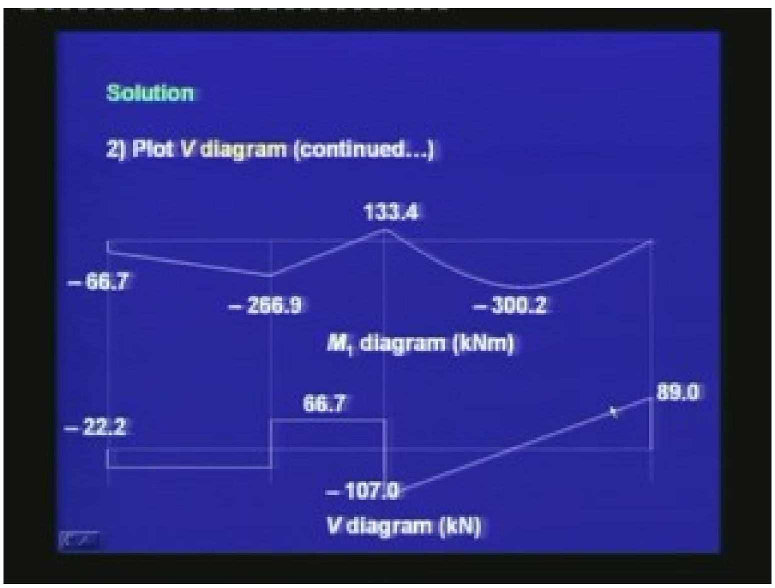

Then we are plotting the equivalent diagram. We are including a moment of 66.7 Kilonewton meters at A.

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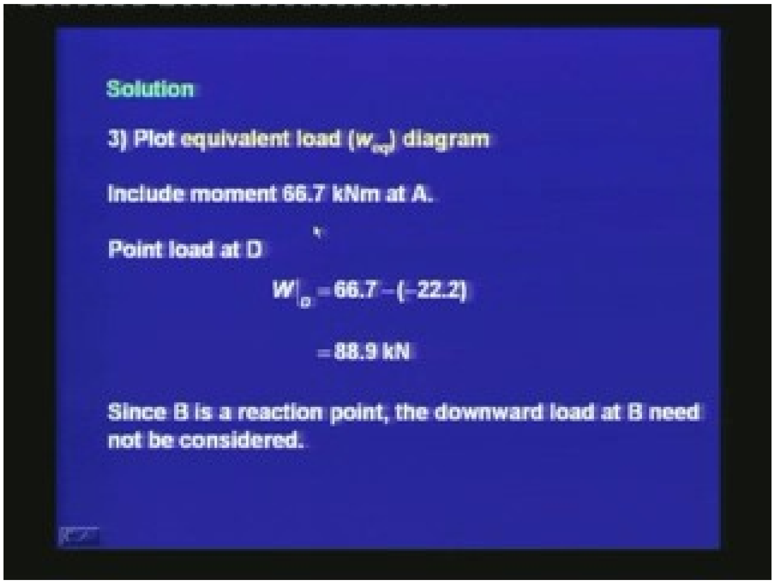

The point load at D is calculated from the change in the shear at the two ends and which gives a point load of 88.9 Kilonewtons. We are not considering an equivalent load at B, because B is a reaction point and the downward load at B need not be considered. The distributed load in the span BC is again calculated from the relationship, w_{eq} to dV divided by dx and from change in the shear diagram we are calculating w_{eq} is equal to 13.0 Kilonewtons per meter.

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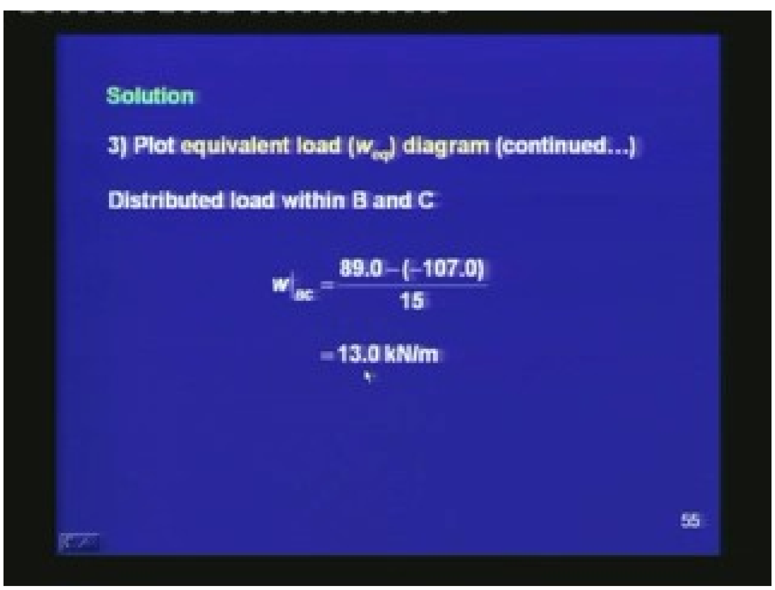

Thus, given the shear diagram, we are able to calculate the equivalent load diagram due to the prestressing force.

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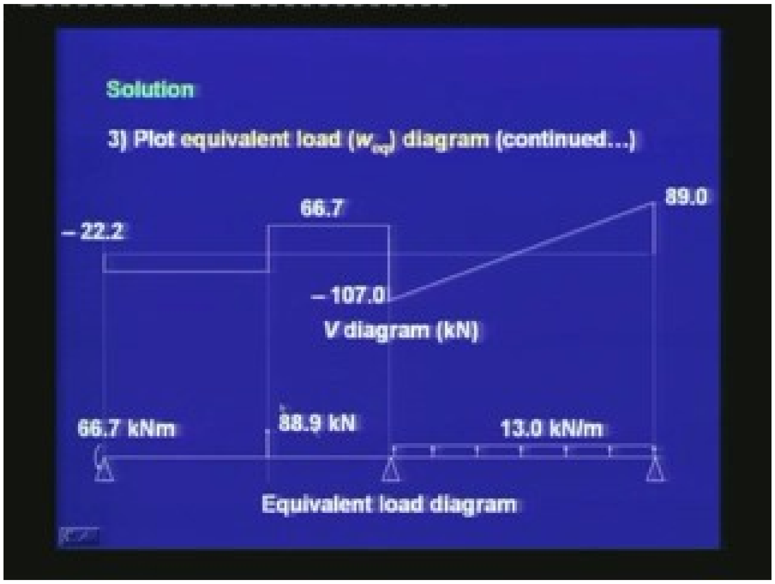

It has a point load, where the CGS has a short bend at D and it has a distributed load where the CGS has a parabolic variation in the span BC.

Finally, we are calculating M_2 diagram by the moment distribution method.

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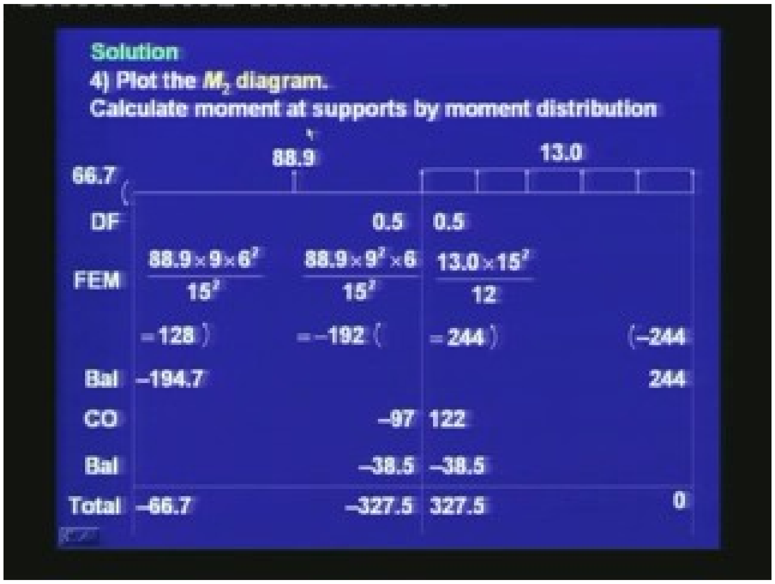

We are calculating the distribution factor, the fixed end moments for the given uniform load and then by the balance and carry over, we are calculating the total moments.

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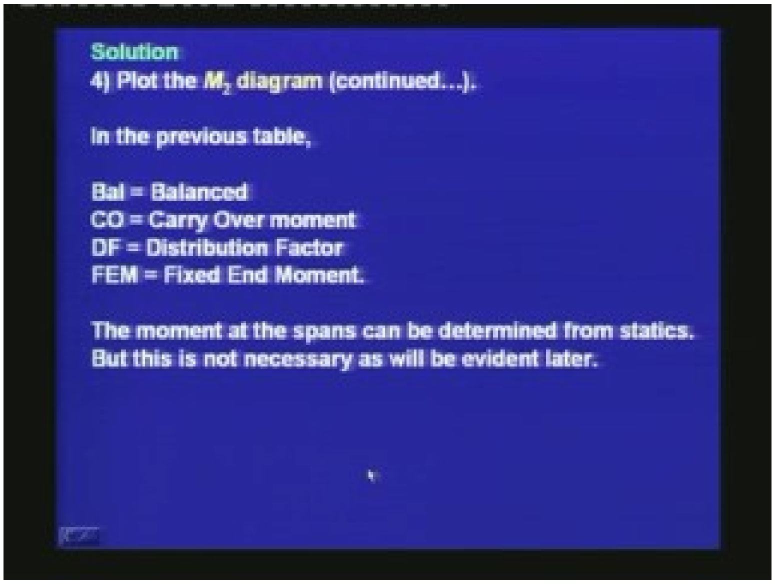

We get the M_2 diagram from the equivalent load diagram from the structural analysis.

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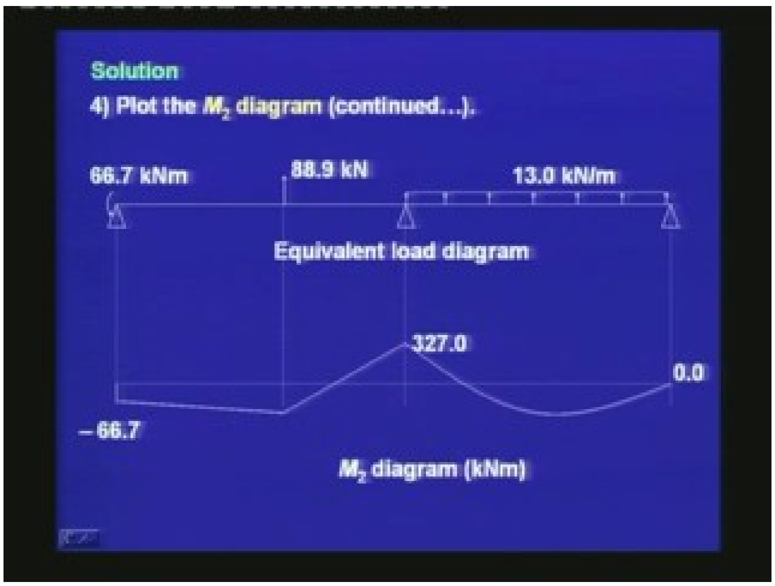

Then we are calculating the values of e_c at the supports from the relationship e_c is equal to M_2 divided P_e.

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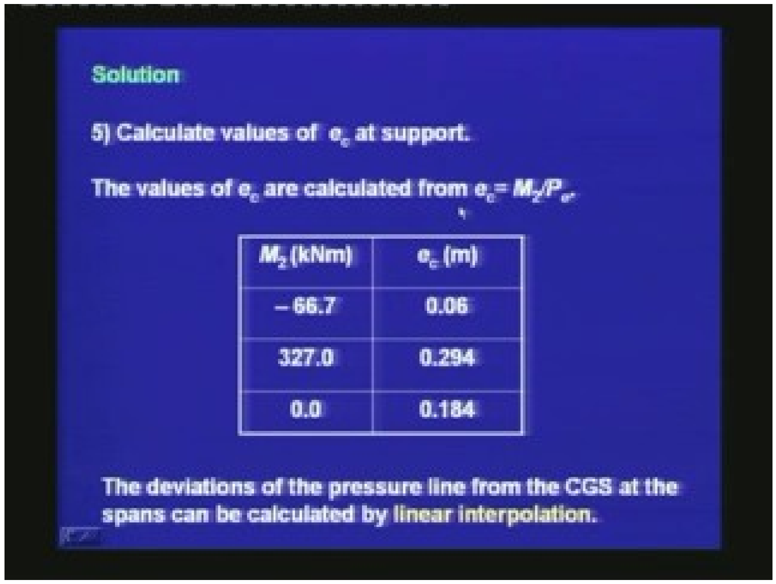

Thus, once we know e_c at the changes, we are able to plot the pressure line.

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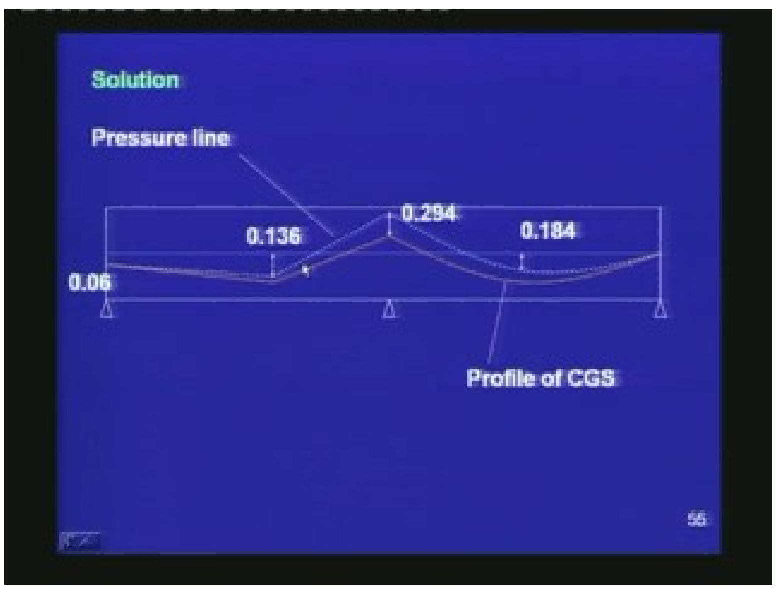

In this lecture, we covered the analysis of continuous beams.

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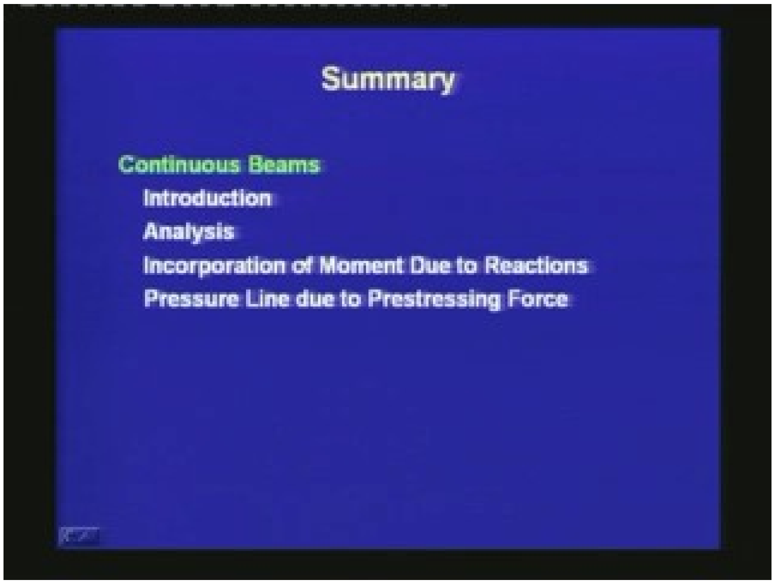

We studied the incorporation of moments due to the reactions and we also studied the development of the pressure line due to the prestressing force. Thank you.