Prestressed Concrete Structures

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Module - 8, Lecture - 32

Cantilever Beams

Welcome back to prestressed concrete structures. This is the first lecture on module 8, on cantilever and continuous beams.

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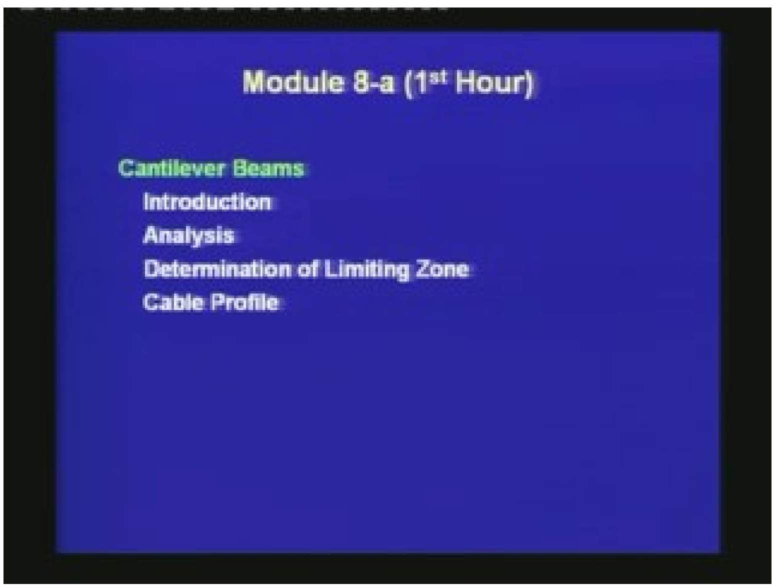

First, we shall have an introduction on cantilever beams, then we shall move on to the analysis of cantilevers, then determination of limiting zone and finally, we shall cover cable profile.

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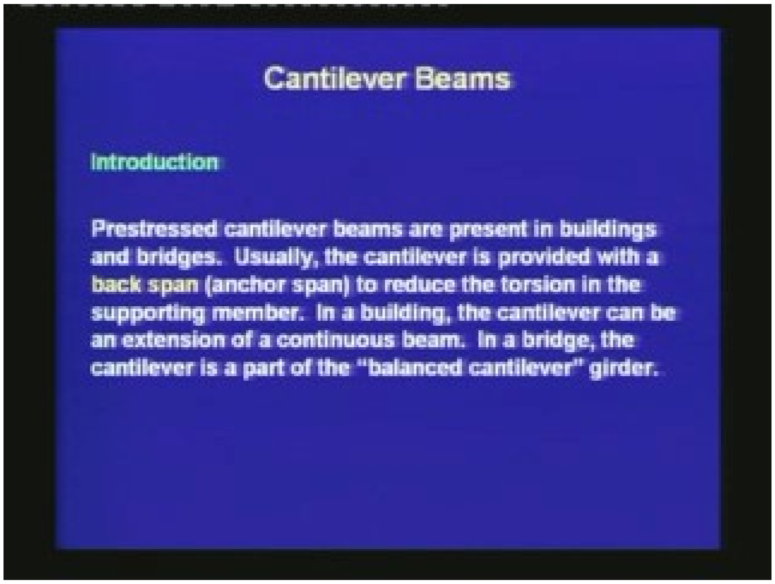

Prestressed cantilever beams are present in buildings and bridges. Usually, the cantilever is provided with a back span, which is also called anchor span, to reduce the torsion in the supporting member. In a building, the cantilever can be an extension of a continuous beam. In a bridge, the cantilever is a part of the balanced cantilever girder.

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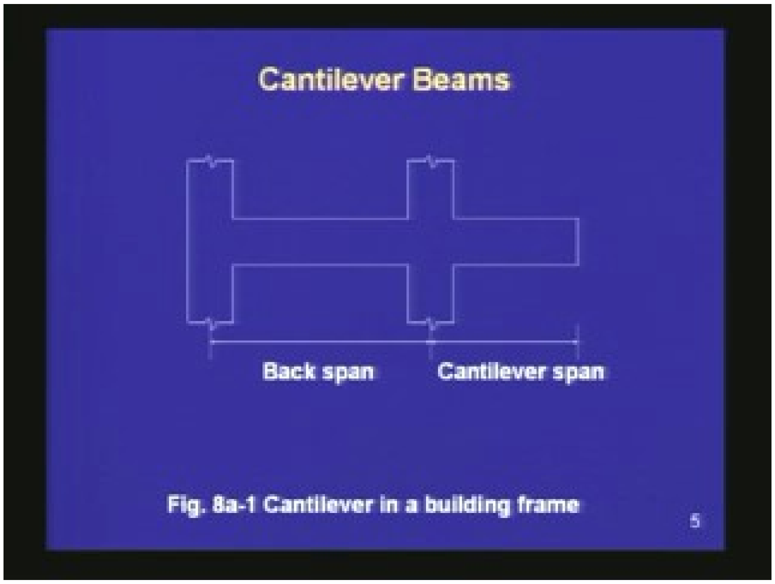

We shall see that the cantilever is a part of a beam; in a building it can have a back span; like in this figure, the cantilever on the right has a back span which reduces the moment in the supporting column. Also, the cantilever can be a part of a continuous beam; that means, usually we do not find cantilever by itself, we find either with a back span or as a part of a cantilever.

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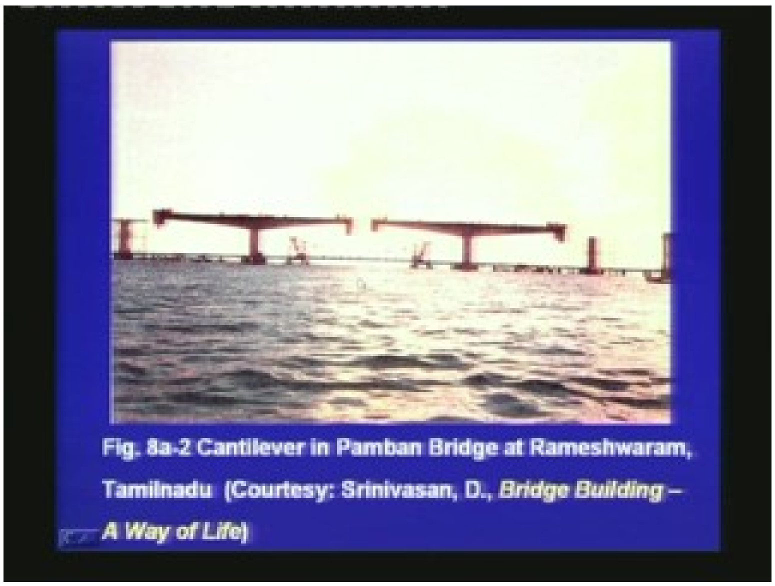

Fig. 8a-2 Cantilever in Pamban Bridge at Rameshwaram, Tamilnadu (Courtesy: Srinivasan, D., *Bridge Building – A Way of Life*)

A cantilever is a part of a balanced cantilever girder. This is the photograph during the construction of the Pamban bridge at Rameshwaram in Tamilnadu. Here you can see that on each pier there is a balanced cantilever girder, in the sense that there is a cantilever on each side, so that the eccentric load on the pier is reduced. The cantilever on the two sides is made in a progressive fashion, so as to have minimal effect of the eccentric load on the piers.

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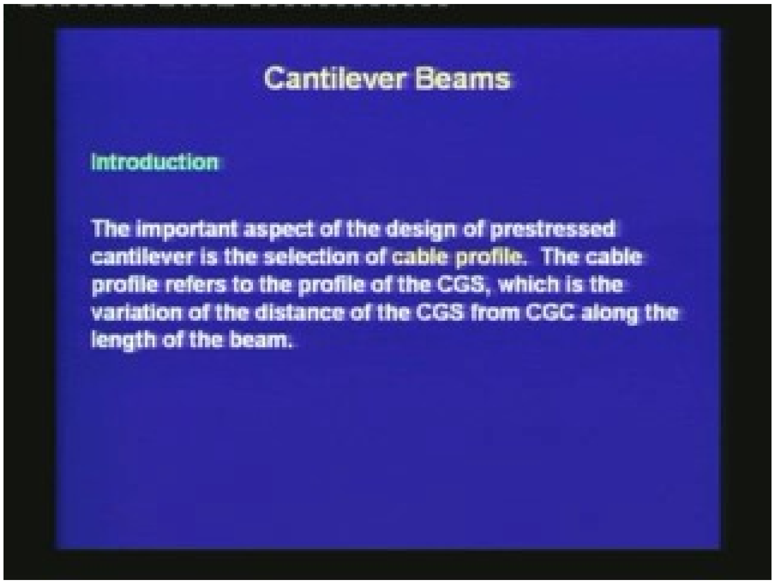

The important aspect of the design of prestressed cantilever is the selection of cable profile. The cable profile refers to the profile of the CGS, which is the variation of the distance of the CGS from CGC along the length of the beam. When we discussed simply supported beams, we had talked about a cable profile; that means, once we have done the analysis for the critical section, we find out the eccentricity at the critical section and then we determine the cable profile along the length of the beam.

The essential difference between the analysis of a simply supported beam and a cantilever is in the selection of the cable profile and for a cantilever it is different than that of a simply supported beam.

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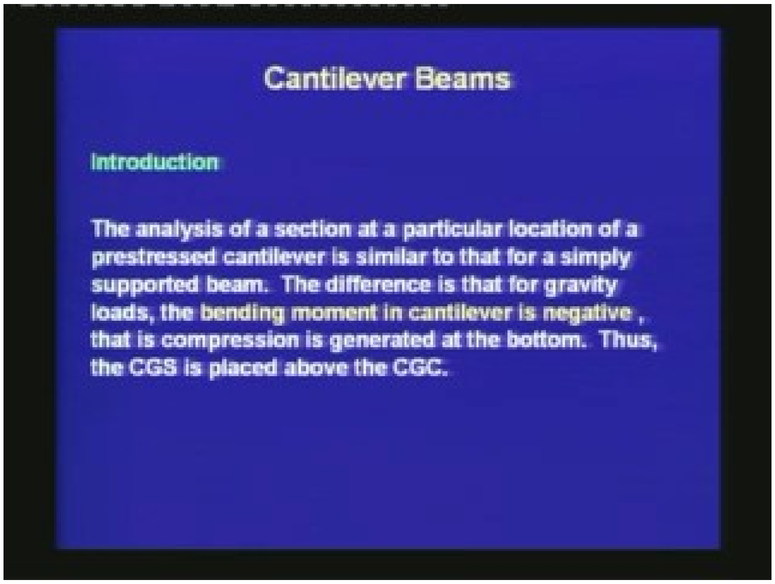

The analysis of a section at a particular location of a prestressed cantilever is similar to that of a simply supported beam. The difference is that for gravity loads, the bending moment in cantilever is negative, that is, compression is generated at the bottom of the beam. Thus, the CGS is placed above the CGC for that particular location.

Whatever we have studied for a simply supported beam, the analysis or the design is applicable for a cantilever beam as well. But the difference is that for a simply supported beam the moment is always positive; that means, compression is created at the top due to bending. For cantilever beams due to gravity loads the moment is negative; that means, compression is created at the bottom and tension is created at the top. The equations that we have used for a simply supported beam can be used for a cantilever, provided you take account of the sign of the moment and then we place the CGS above the CGC, because the moment is negative. This is the essential difference in the design of a cantilever beam with respect to that of a simply supported beam.

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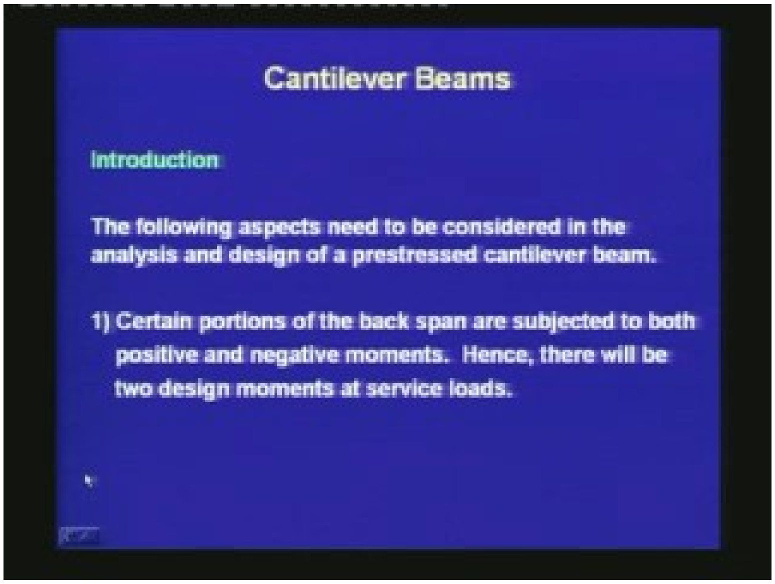

The following aspects need to be considered in the analysis and design of a prestressed cantilever beam. First, certain portions of the back span are subjected to both positive and negative moments. Hence, there will be two design moments at service loads. In a simply supported beam, we had seen that throughout the length of the beam, the beam is always subjected to a positive moment due to gravity loads. But in a back span of a cantilever beam, the same section can be subjected to a positive moment or a negative moment depending on the loading condition. Thus, we have two values of design moments for most of the back span under service loads.

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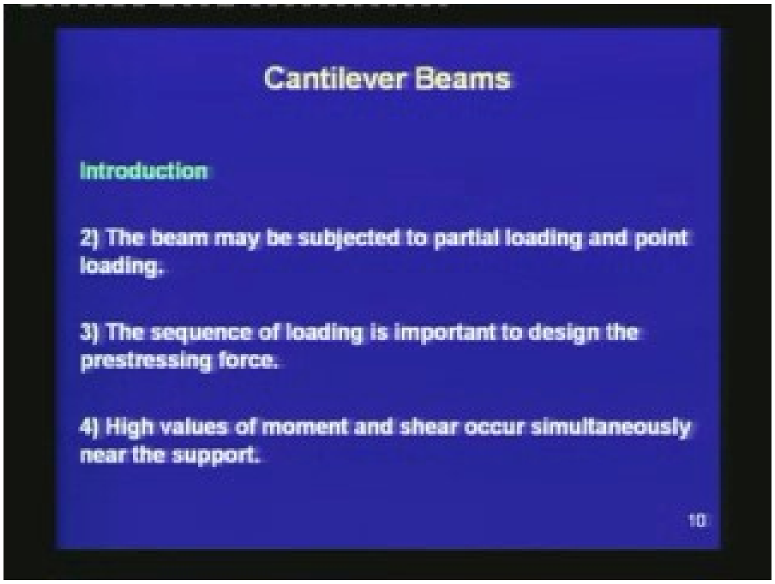

The second aspect is the beam may be subjected to partial loading and point loading. A cantilever beam, if it is subjected to a partial loading or a point loading, can generate a different type of moment condition as compared to when the load is distributed throughout. Hence, a partial loading or a point loading analysis becomes important for a cantilever.

The third important aspect is the sequence of loading is important to design the prestressing force. During construction, the sequence of loading in a cantilever is important to design the prestressing force and prestressing force can be applied in stages to take account of the sequence in the loading.

The fourth important aspect is high values of moment and shear occurs simultaneously near the support. For a simply supported beam, the mid span is usually the location of maximum flexure and a section close to the support is the critical section for shear, but in a cantilever beam, the sections near the supports can be critical both for shear and moment.

Hence, the sections near the supports have a larger depth compared to the sections away from the support. This variation of the depth along the length of the beam should also be considered in the analysis.

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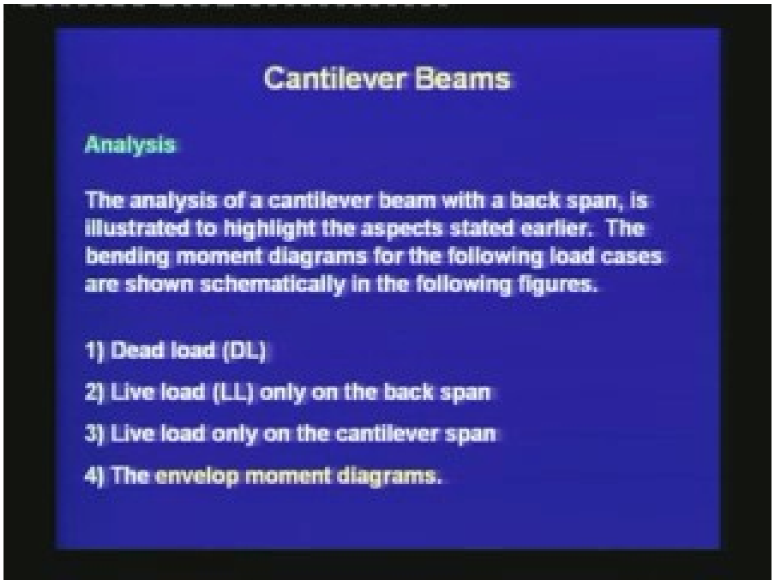

Next, we are studying the important aspects of the analysis of a cantilever beam. The analysis of a cantilever beam with a back span is illustrated to highlight the aspects stated earlier. The bending moment diagrams for the following load cases are shown schematically in the following figures. First, we shall see the bending moment due to the dead load; next, we shall see the bending moment due to live load only on the back span; third, we shall see the live load only on the cantilever span and what is the bending moment due to that and finally, we shall see the envelop moment diagrams.

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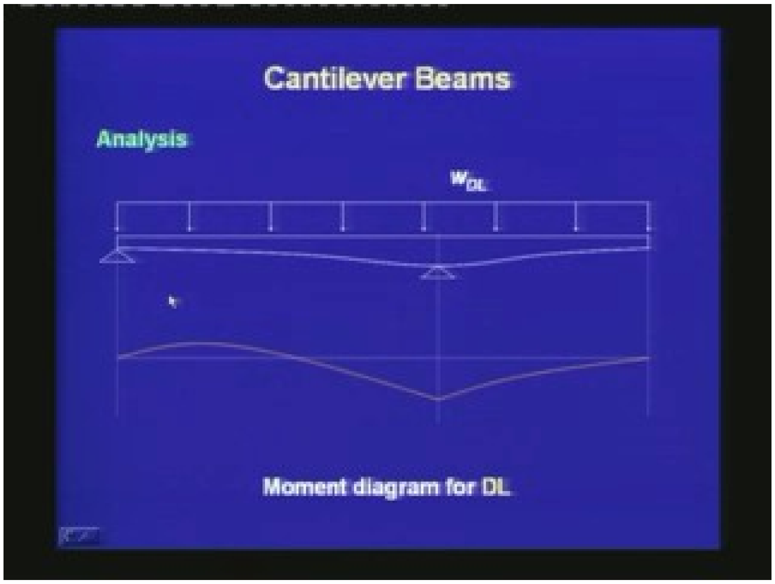

This is the sketch of a cantilever on the right side and has back span on the left side and the dead load is throughout the length of the member; it need not be uniform, if the depth of the member is varying, but for the sake of convenience, right now, we are showing it as a uniform load. Due to the dead load, the moment in the cantilever is negative with 0 at the end and with increasing value towards the support. When we go to the back span then we observe that close to the support we have negative moments and then as we move away from the support and proceed toward the left we can have positive moment in the back span. This is the moment diagram due to the dead load which occurs throughout the length of the beam.

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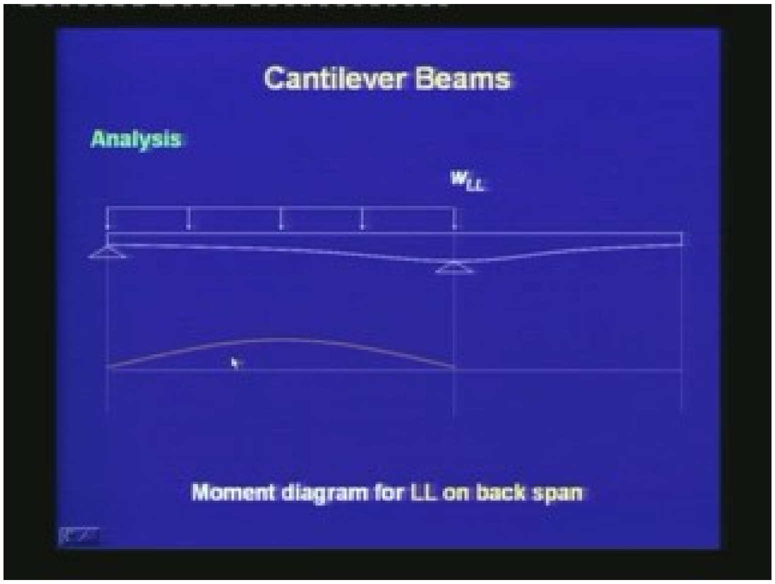

In the second figure, we are having the live load only on the back span. Due to the live load, the back span experiences a positive moment and depending on the magnitude of the live load, the location of the maximum moment can be anywhere in the middle of the back span. Here for convenience we have shown a uniform live load throughout the back span. Observe that throughout the back span the moment is positive.

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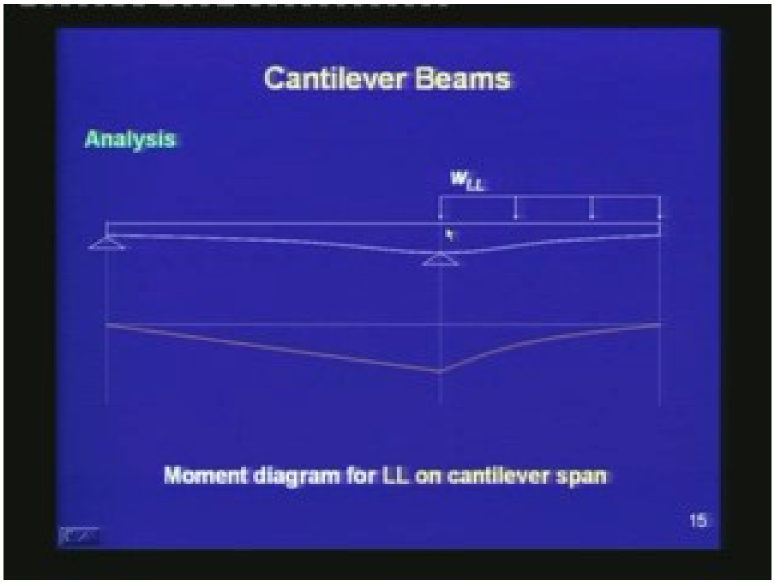

Third, the live load is placed only on the cantilever span. Here we find that the cantilever is subjected to a negative moment which increases from the 0 value at the end towards the support and then in the back span, the moment varies linearly from the support at the right to the support at the left, where the moment drops down to 0.

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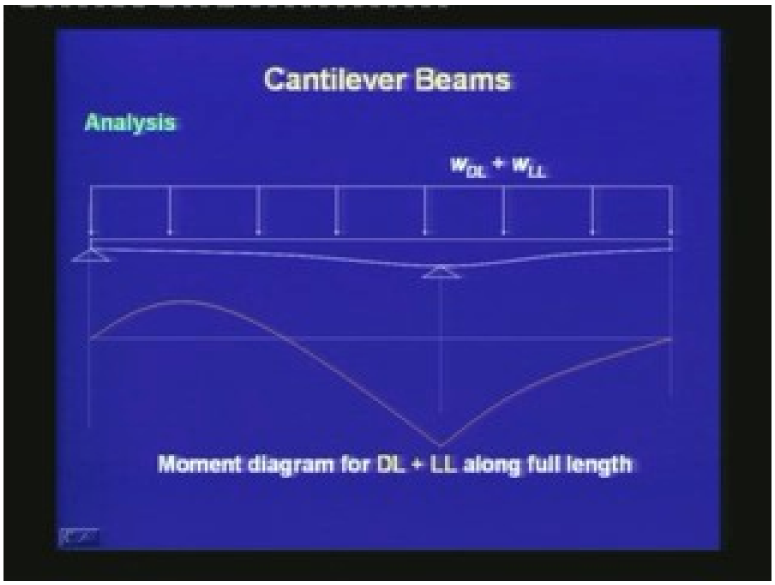

If you have the dead load and the live load acting throughout the length of the beam, then we have a combined moment diagram where the cantilever has a high negative moment. Then, as we enter the back span, we have negative moment near the support and a positive moment towards the left support.

Now from these diagrams, suppose, we have the live load on the back span, or the live load in the cantilever span, or the live load throughout - from these different placement of the live load, we develop the envelop moment diagrams for the total beam.

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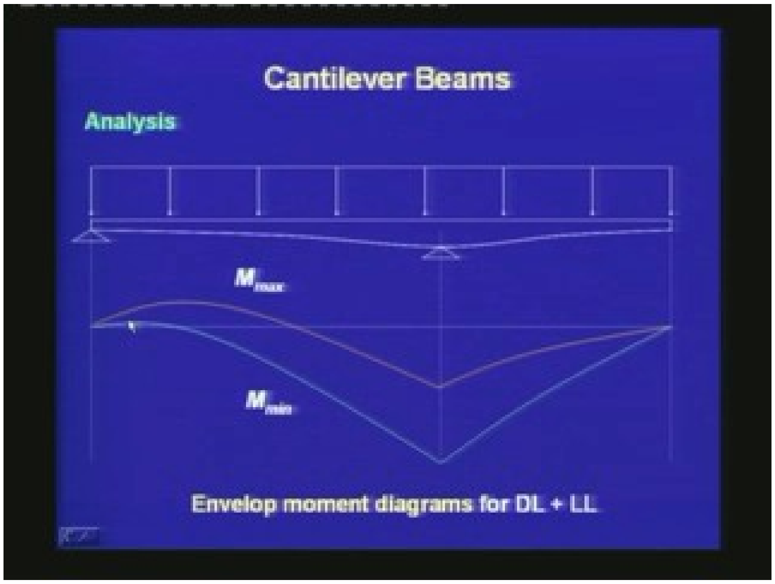

In this figure we are seeing the envelop moment diagrams. The orange line is the maximum moment, in an algebraic sense. We see that in the back span we do have high positive moments towards the left support. Then as we are coming towards the support on the right, we will not observe any positive moment, but the value of the negative moment can be less under certain loading conditions. In the cantilever the M_{max} value is always negative depending on the load distribution on the beam.

If we are taking the M_{min}, which is the minimum value in an algebraic sense, in that case we find that the moment in the cantilever is substantially high close to the support. Then, in the back span as we move from right to left, the moment drops down. There may be a region where there is no negative moment generated which is close to the left support, but otherwise most of the back span has a negative moment which is given by the envelop diagram of M_{min}. Notice that certain portions of the back span can be subjected to both a negative moment and a positive moment depending on the load conditions.

If I pick up a section somewhere at the middle of the back span, we observe that M_{max} is positive and M_{min} is negative. Since the two envelop values have opposite sign that means that particular location will experience both positive moment and negative moment under the service loads.

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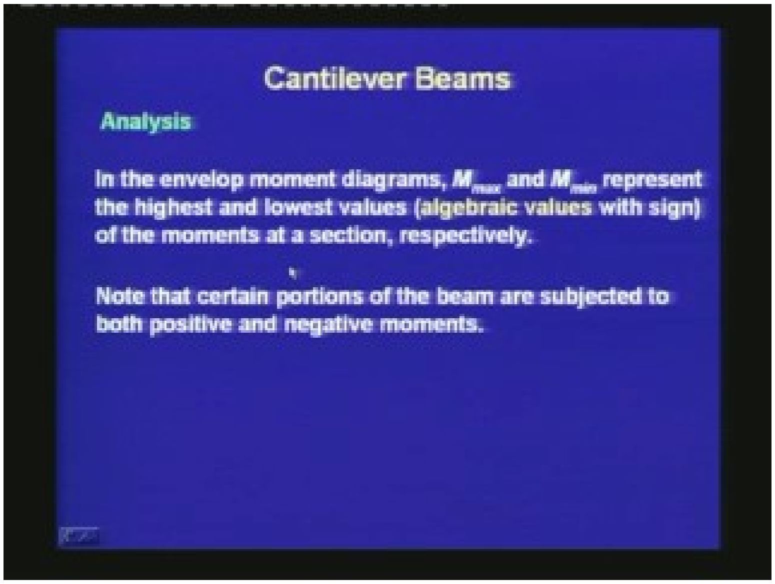

In the envelop moment diagrams, M_{max} and M_{min} represent the highest and lowest values. These are algebraic values with sign of the moments at a section respectively. Thus M_{max} is the maximum value in an algebraic sense; M_{min} is the minimum value in the algebraic sense. Why we are saying it is algebraic is that we are not comparing the numeric value of these two yet; we are considering these two moments with the respective signs.

Note that certain portions of the beam are subjected to both positive and negative moments. This is an important aspect of a cantilever beam with the back span that there are locations in the back span which is subjected to a positive or a negative moment under the service loads.

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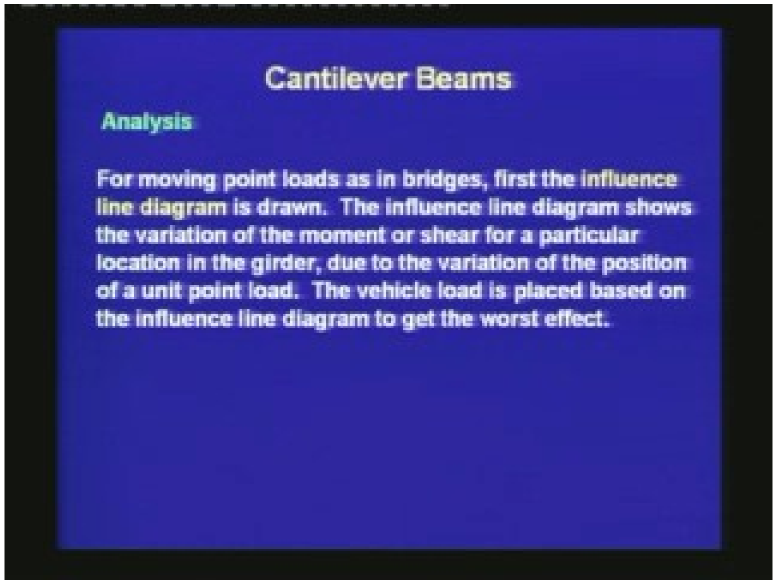

For moving point loads as in bridges, first the influence line diagram is drawn. The influence line diagram shows the variation of the moment or shear for a particular location in the girder, due to the variation of the position of a unit point load. The concept of influence line diagram is covered in the Structural Analysis course. In this particular topic we are not covering influence line diagram, but we are briefly mentioning that the influence line diagram is used if there is a moving point load. The influence line diagram shows the variation of moment or shear at a particular location due to the variation of the position of a unit load along the length of the beam. Once the influence line is first developed then the vehicle load is placed based on the influence line diagram to get the worst effect.

Thus, in bridges the effect of live load is designed placed on influence line diagram where once the influence line diagrams are available, then the vehicle load is placed in such a way so that we get the worst effect - whether it is the positive moment or whether it is a negative moment or whether it is the shear; depending on each of these variables from the corresponding influence line diagrams we can place the load in the worst condition.

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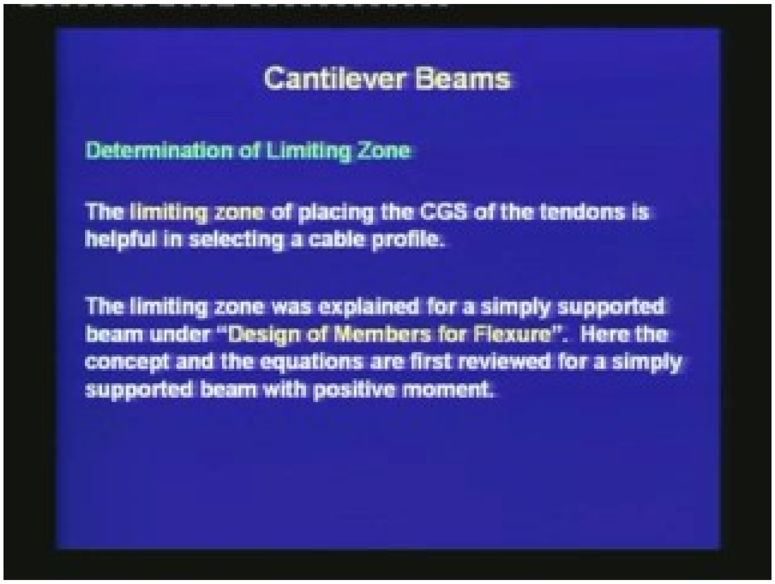

Next, we are moving on to the determination of limiting zone which is done after the analysis of the beam. The limiting zone of placing the CGS of the tendons is helpful in selecting a cable profile. I said earlier that once the analysis of a cantilever is performed, the sectional analysis or design is very similar to the simply supported beam, but the difference is since the moment has different sign and the sign of the moment can change based on the location of the beam, selecting an appropriate cable profile is important in the design of a cantilever beam. To have a proper cable profile, first the limiting zone is determined and here we shall recollect the concept of limiting zone.

The limiting zone was explained for a simply supported beam under design of members for flexure. Here, the concept and the equations are first reviewed for a simply supported beam with positive moment; that means, we shall first check how did we develop the equations for a simply supported beam; the same equations can be used for a cantilever beam provided we are particular about the sign of the moment and then we can determine the limiting zone of placing the CGS of the tendons in the cantilever beam.

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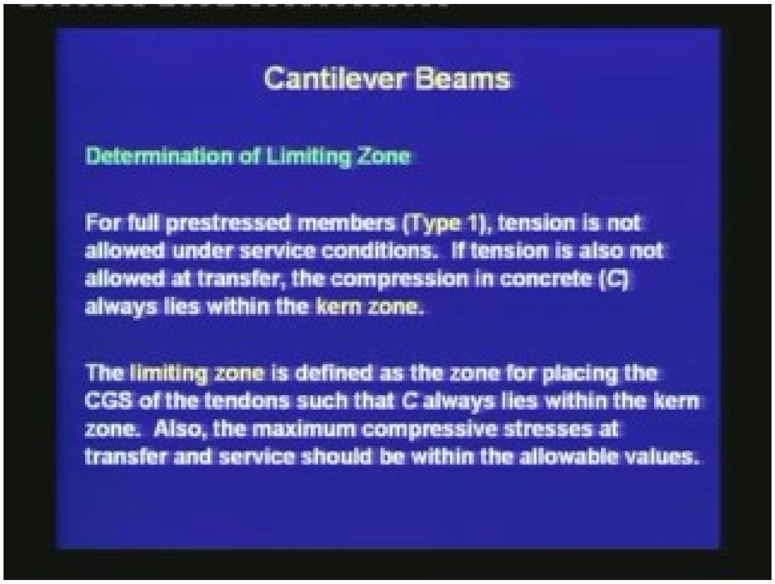

There are three types of prestress members as per our code. For a fully prestressed member which is referred to as Type 1 - tension is not allowed under service conditions. If tension is also not allowed at transfer, the compression in concrete, which will be referred to as C, always lies within the kern zone. The limiting zone is defined as the zone for placing the CGS of the tendons such that C always lies within the kern zone. Also the maximum compressive stresses at transfer and service should be within the allowable values.

That is - how do we determine the limiting zone for a simply supported beam? For a simply supported beam, under positive moment, we find out extreme position of C that is possible without creating any tension for a Type 1 member. The minimum moment is at transfer and at that time C can push down to the lowest kern point and the maximum moment is under service loads where C can be shifted to the upper kern point. This maximum travel of the C is helpful to have an economical section.

Now, since we are not allowing any tension in the section, the C always lies within the kern zone of the section; this determines the limiting zone of a simply supported beam. Let us now check what the equations are to find out the limiting zone.

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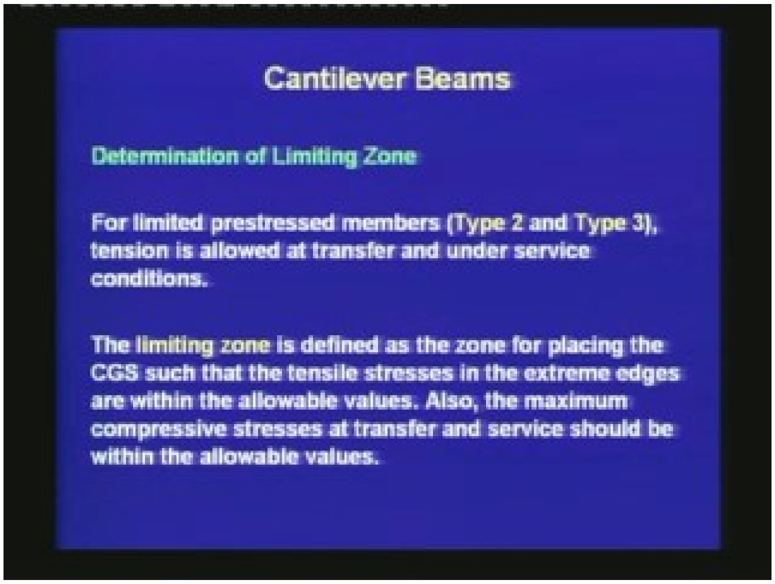

For limited prestressed members like Type 2 and Type 3, tension is allowed at transfer and under service conditions. The limiting zone is defined as the zone for placing the CGS such that the tensile stresses in the extreme edges are within the allowable values. Also the maximum compressive stresses at transfer and service should be within the allowable values. The difference of determination of the limiting zone for Type 2 and Type 3 members as compared to a Type 1 member is that in Type 2 and Type 3 we allow tensile stresses at transfer as well as under service and based on the allowable stresses we find expressions of the limiting zone, where we place the CGS is such a way that C may lie outside the kern zone provided that the tensile stresses in the opposite phase is within the allowable value. Also, the compressive stresses should be within the allowable values for both at transfer and at service.

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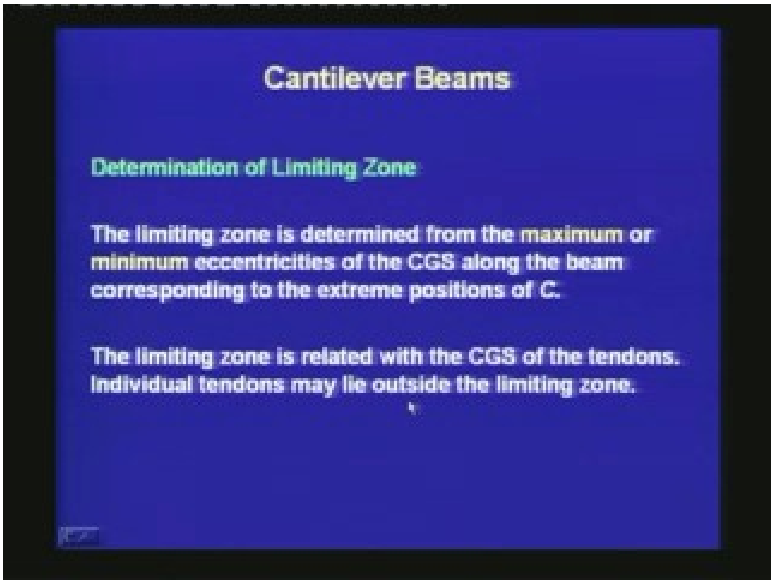

The limiting zone is determined from the maximum or minimum eccentricities of the CGS along the beam corresponding to the extreme positions of C. Thus, once we know the extreme position of C, we can determine the corresponding maximum and minimum eccentricities of the CGS at a particular section. When we draw the low side of this maximum and the minimum positions along the length of the beam, we determine the limiting zone. Remember that the limiting zone is related with the CGS of the tendons. Individual tendons may lie outside the limiting zone; that is, when we are talking of limiting zone, we are talking about the placement of the CGS within the limiting zone. Individual tendons may lie outside the limiting zone provided a CGS is lying within the limiting zone.

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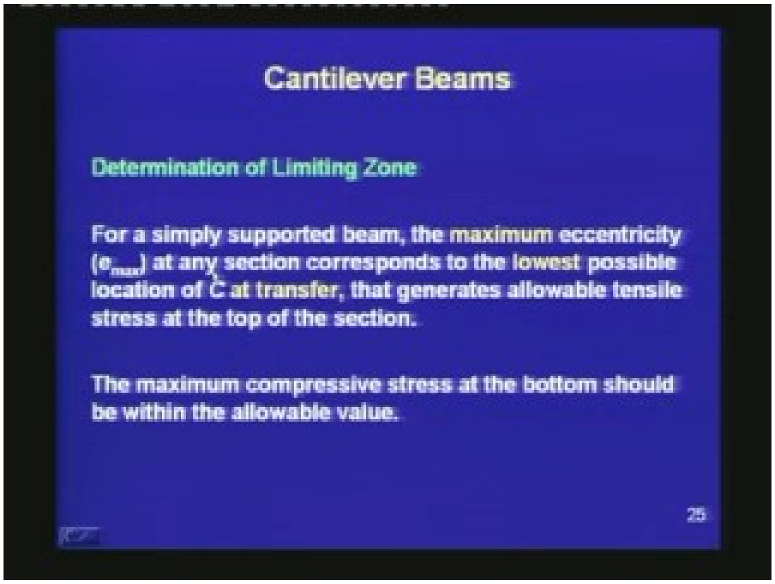

For a simply supported beam, the maximum eccentricity, which will be referred to as e_{max}, at any section corresponds to the lowest possible location of C at transfer. This generates allowable tensile stress at the top of the section. Thus, the maximum eccentricity - e_{max} - is determined corresponding to the lowest position of C at transfer. The maximum compressive stress at the bottom should also be within the allowable value. The minimum eccentricity which is represented as e_{min} at any section corresponds to the highest possible location of C at service that generates allowable tensile stress at the bottom of the section.

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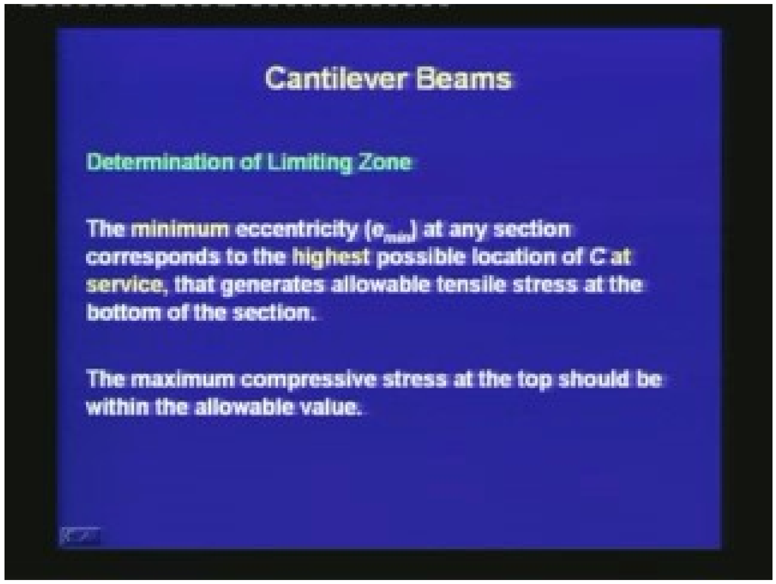

Thus the minimum eccentricity of the CGS is calculated from the highest position of the C under service loads. This will generate allowable tensile stress at the bottom of the section. The maximum compressive stress at the top should also be checked to be within the allowable value.

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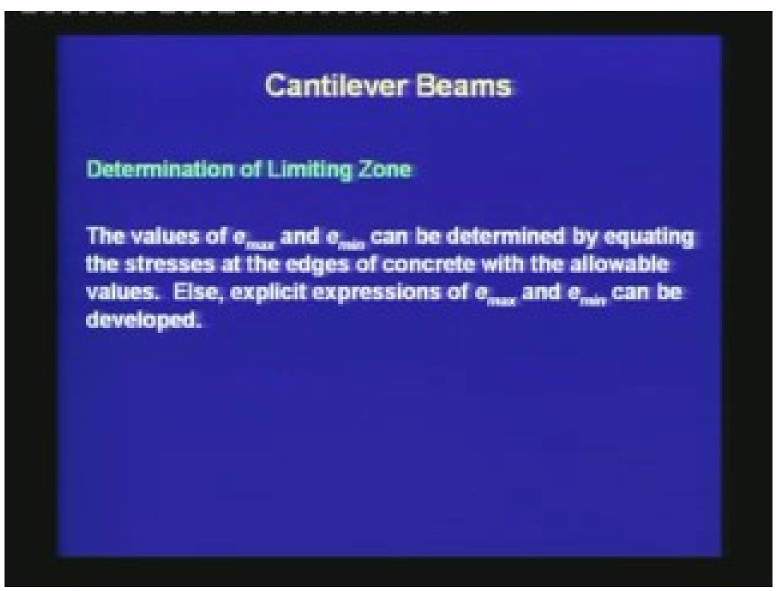

The values of e_{max} and e_{min} can be determined by equating the stresses at the edges of concrete within the allowable values; else, explicit expressions of e_{max} and e_{min} can be developed. Thus, in order to determine the limiting zone we can calculate e_{max} and e_{min} for a particular beam, for the given loading conditions, or else we can try to determine them from some explicit expressions which are discussed here. These expressions help us to determine the maximum and minimum eccentricities at several locations along the length of the beam.

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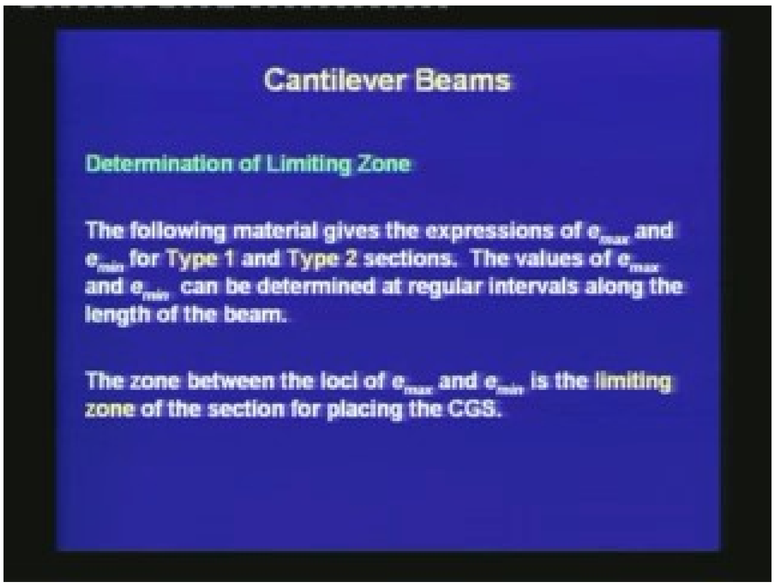

The following material gives the expressions of e_{max} and e_{min} for Type 1 and Type 2 sections. The values of e_{max} and e_{min} can be determined at regular intervals along the length of the beam from which we shall determine the limiting zone.

The zone between the loci of e_{max} and e_{min} is the limiting zone of the section for placing the CGS. Here we shall recapitulate the expressions of e_{max} and e_{min} for Type 1 and Type 2 members. For Type 3 members the expressions are similar to the expressions of Type 2 members; the only difference being the value of the allowable tensile stress. Once we have determined e_{max} and e_{min} at regular intervals along the length of the beam, then you have the limiting zone, which is in between the loci of e_{max} and e_{min} along the length of the beam.

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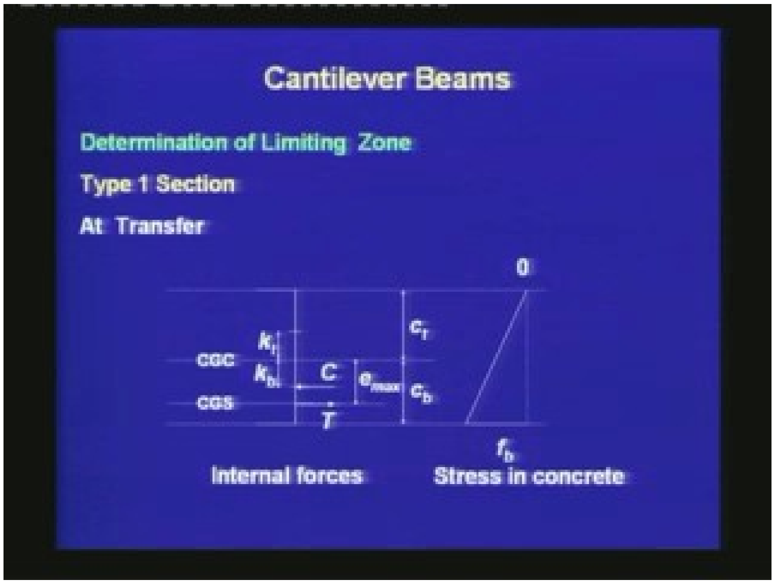

To have an analytical expression you are first seeing a Type 1 section at transfer. The lowest possible location of C is at the bottom kern point; the tension T is at the level of the CGS; the stress at the top is 0 for a Type 1 section at transfer and the stress at the bottom is f_b. K_t and K_b represents the kern distances or the distances of the kern points from the CGC. C_t and C_b represent the distances of the top and the bottom fiber from the CGC. e_{max} is the distance of the CGS from the CGC when C lies at the bottom kern point. We are using this stress diagram to develop the expression of e_{max}.

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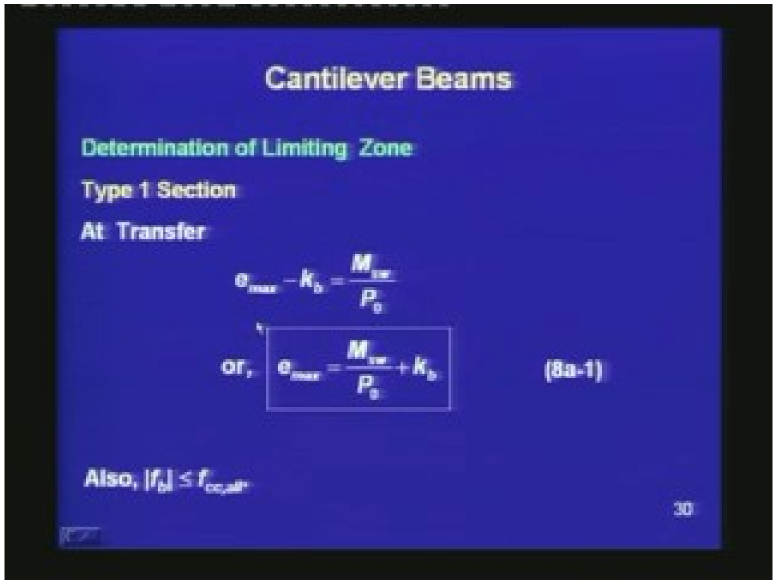

e_{max} minus K_b which is the lever arm by which C has shifted from T, is equal to the self-weight moment that is acting as transfer divided by the prestress at transfer which is denoted as P_0. Transposing K_b towards the right side, we have e_{max} equal to M_{sw} divided by P_0 plus K_b.

Thus this is the expression of e_{max} for a simply supported beam, for the load condition at transfer, where we have the moment due to the self-weight, the prestressing P_0 and the geometric property K_b. Using these expressions we can calculate the value of e_{max} at a particular location. We have to check that the stress at the bottom should be less than the allowable compressive stress at transfer.

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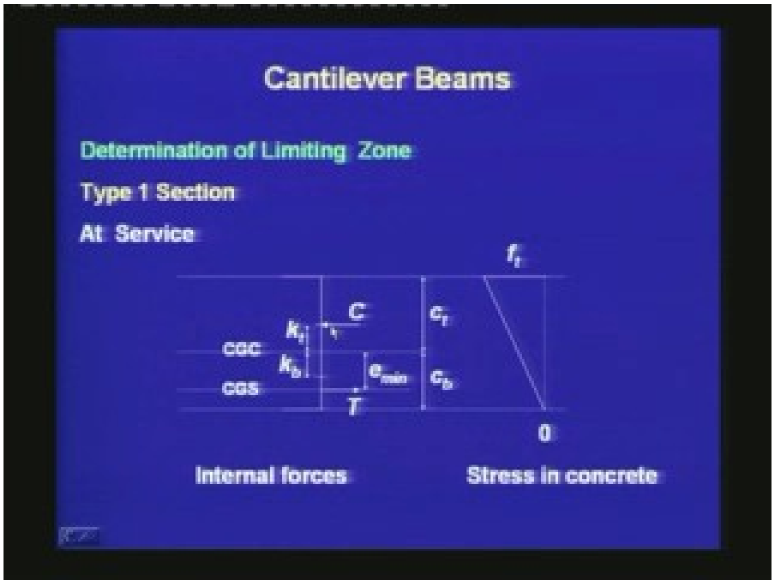

Next, we are determining e_{min} for Type 1 section under service loads. Under service loads we are allowing C to traverse as high as possible, so that it is at the top kern point under service loads, for which the stress at the bottom is 0 and the stress at the top is represented as f_t. The location of the CGS for the upper most location of C is the e_{min} or the minimum possible eccentricity.

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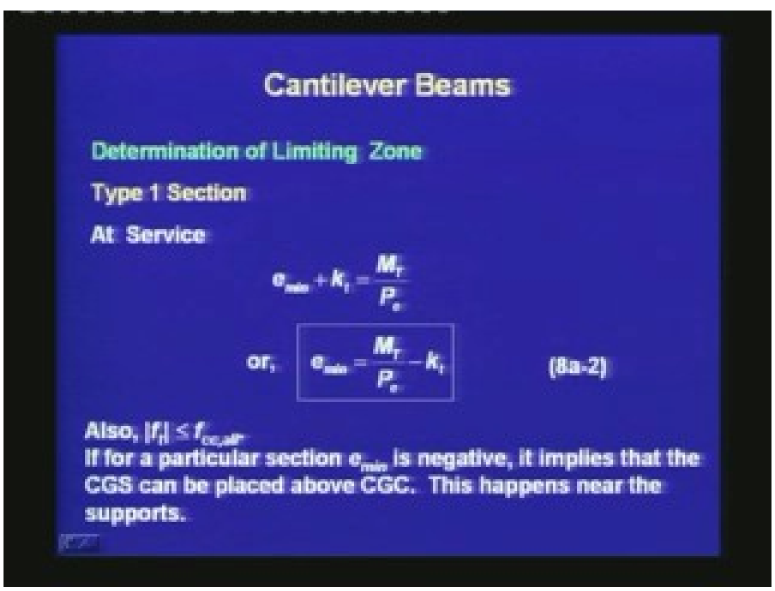

From the stress diagram we can find that the lever arm through which C has shifted which is e_{min} plus K_t, this is equal to the moment under service loads which is represented as M_t divided by the effective prestress at service which is denoted as P_e. Transposing the term K_t on the right side, we have an explicit expression of e_{min} equal to M_t divided by P_e minus K_t. Thus, given the values of M_t, P_e and the geometric variable K_t we can determine e_{min} at a particular location. Of course, we need to check that the stress at the top is less than the allowable compressive stress under service loads. If for a particular section $e_{minimum}$ comes out to be a negative, it implies that the CGS can be placed above CGC. This happens near the supports of a simply supported beam.

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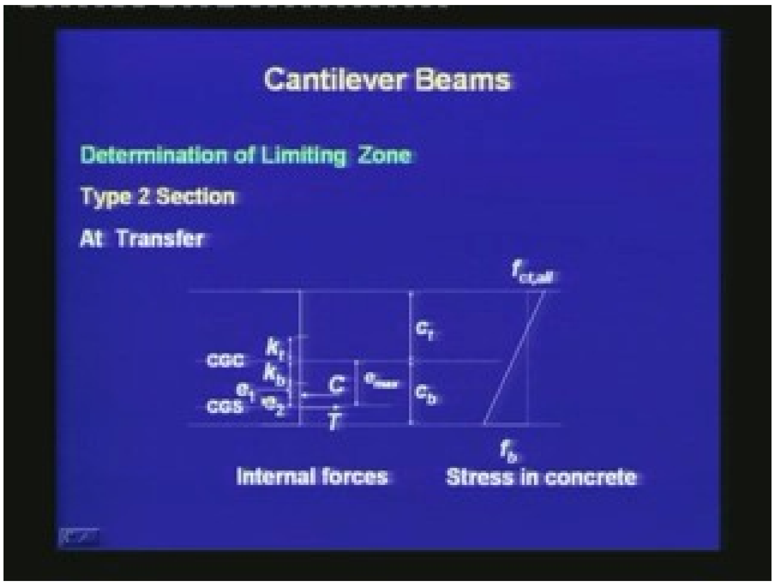

Next, we are recollecting the expressions of e_{max} and e_{min} for Type 2 sections. At transfer there is allowable tensile stress at the top; the position of C is outside the kern region by a distance e_1 and the lever arm between C and T is represented as e_2. e_{max} is the position of the CGS, such that C is in the bottom most location which creates allowable tensile stress at the top during transfer.

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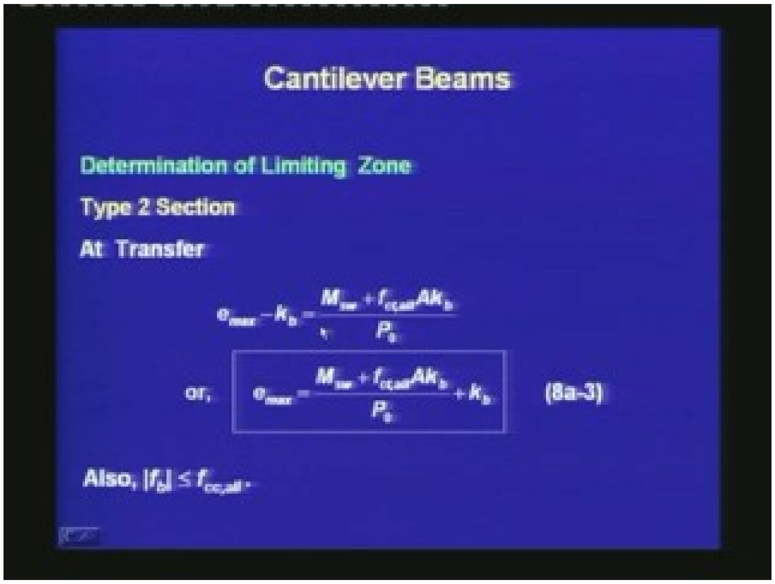

The lever arm by which the C traverses is e_{max} minus K_b which is equal to M_{sw} plus $f_{ct,\ allowable}$ times A times K_b divided by P_0. This expression is a general form of the expression that we have seen for the Type 1 members. Transposing the term of K_b on the right side we have e_{max} equal to M_{sw} which is the moment due to self-weight plus $f_{ct,\ allowable}$ which is the allowable tensile stress in the concrete at transfer times the area of the section times K_b whole divided by P_0 plus K_b. Thus, this expression is applicable for both Type 2 and Type 3 members with the appropriate value of $f_{ct,\ allowable}$. Note, that if $f_{ct,\ allowable}$ is made 0, then this expression becomes same as that for a Type 1 member. Thus, this expression is more generic as compared to the expression for a Type 1 member. Also, we have to check the stress at the bottom, f_b, is less than the allowable compressive stress at transfer.

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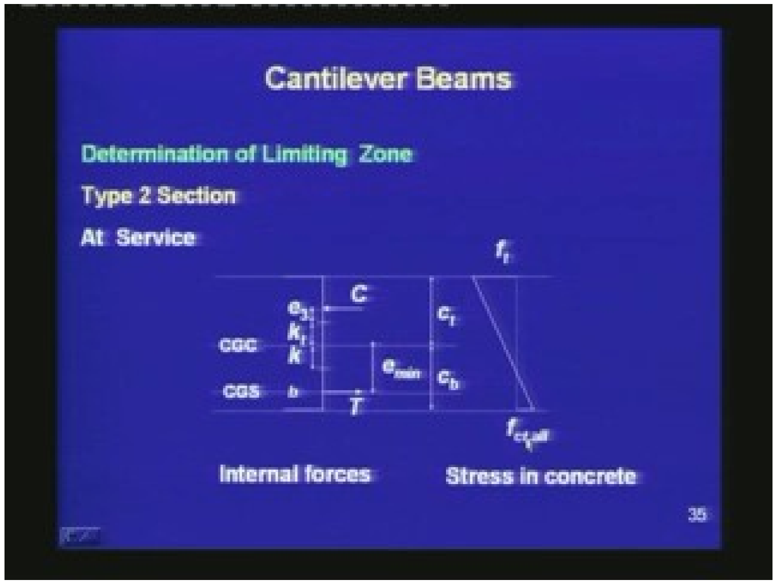

For service conditions, C is at maximum level which can be beyond the kern point. Here the distance of C from the upper kern point is denoted as e_3. The corresponding location of CGS is e_{min}. The stress at the top is represented as f_t and the stress at the bottom is the allowable tensile stress in the concrete under service conditions. With this stress block, we can write the expressions of $e_{minimum}$ plus K_t which is equal to M_t minus $f_{ct, allowable}$ times A times K_t divided by P_e. Transposing the term of K_t on the right side we have e_{min} is equal to M_t minus $f_{ct, allowable}$ times A times K_t divided by P_e minus K_t.

Thus, this is an explicit expression of $e_{minimum}$ corresponding to a Type 2 section under service load conditions. The same expression can be used for a Type 3 member if you have the appropriate value of $f_{ct, allowable}$. Note, that this expression is more generic form of the expression for a Type 1 section, because, once we substitute $f_{ct, allowable}$ equal to 0, we find that this expression become same as expression for a Type 1 section. Also, we need to check that the stress at the top should be less than the allowable compressive stress under service conditions.

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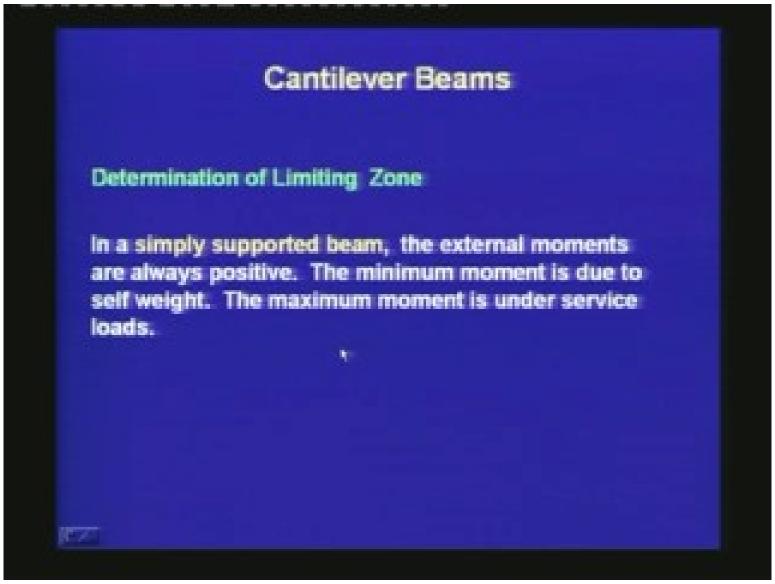

With this recapitulation of the determination of limiting zone for a simply supported beam, we are moving on to the determination of limiting zone for a cantilever beam. In a simply supported beam, the external moments are always positive. The minimum moment is due to self-weight; the maximum moment is under service loads.

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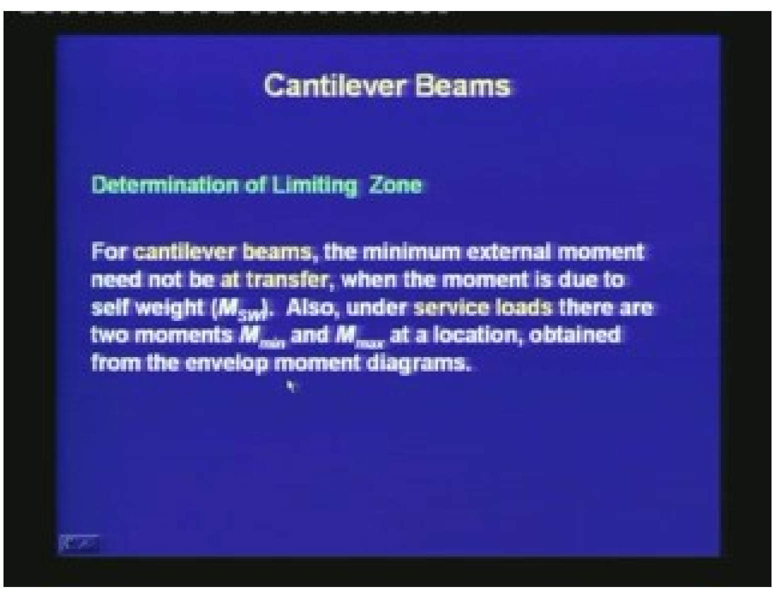

For cantilever beams, the minimum external moment need not be at transfer, when the moment is due to self-weight; that means, when we are trying to determine the limiting zone for a cantilever, we have to be careful that the minimum moment may not be at transfer; the moment at transfer is due to self-weight only.

We have to also check the moment conditions under service loads. Under service loads, there are two moments M_{min} and M_{max} at a location which obtained from the envelop moment diagrams. Unlike a simply supported beam where we have only one value of the moment, under service conditions in a cantilever beam, you can have two moments M_{max} and M_{min} if there are of opposite sign then we have to consider both the values. This is the essential difference between the analysis of a cantilever beam with respect to that of a simply supported beam.

Thus, we have three moment values at a particular location: one is due to the self-weight and another is from the envelop moment diagrams under service loads which give a min and a max.

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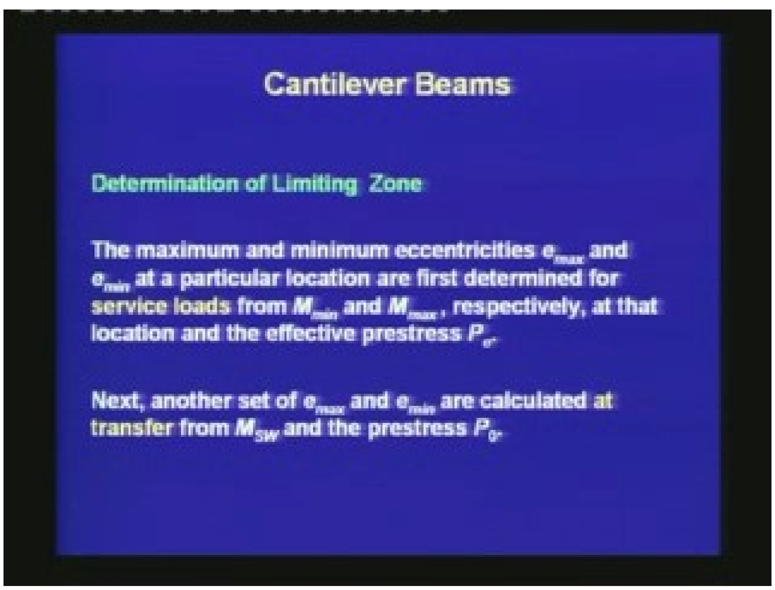

The maximum and minimum eccentricities - e_{max} and e_{min} - at a particular location are first determined for service loads from M_{min} and M_{max} respectively at that location and the effective prestress P_e. That means first e_{max} is calculated at a particular location from M_{min} which is obtained from the envelop moment diagram; e_{min} is calculated from M_{max} at that particular location; both e_{max} and e_{min} we are calculating first for the service loads and then we are checking for the transfer condition. We calculate another set of e_{max} and e_{min} for the loads at transfer from the self-weight moment M_{sw} and the prestress of transfer that is P_0.

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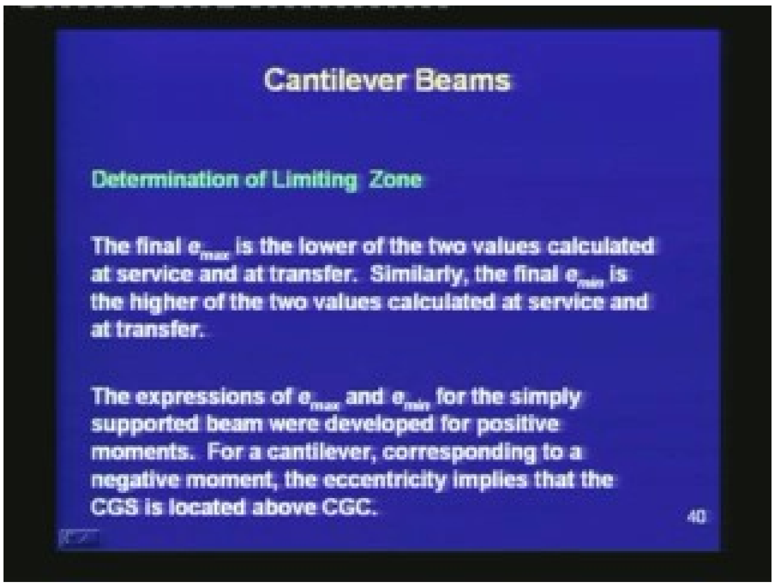

The final e_{max} is the lower of the two values calculated at service and at transfer. Similarly, the final e_{min} is the higher of the two values calculated at service and at transfer. Thus, we have two sets of calculations - one for the service loads and another for the transfer. The values are selected judiciously that for e_{max} we have two values - one for service and one for transfer - whichever is lower will satisfy both of them and that is selected as the final e_{max}.

Similarly, we have two values of e_{min} - one for service and one for transfer - which ever is higher of these two, that location will satisfy the strength condition both at transfer and at service and we select that as the e_{min}. The expressions of e_{max} and e_{min} for simply supported beam were developed for positive moments. For a cantilever corresponding to a negative moment, the eccentricity implies that the CGS is located above CGC. Thus, when we are using the expressions from the simply supported beam, we have to be careful that the sign of the moment may be opposite; in that case, eccentricity refers to the location of CGS which is above CGC.

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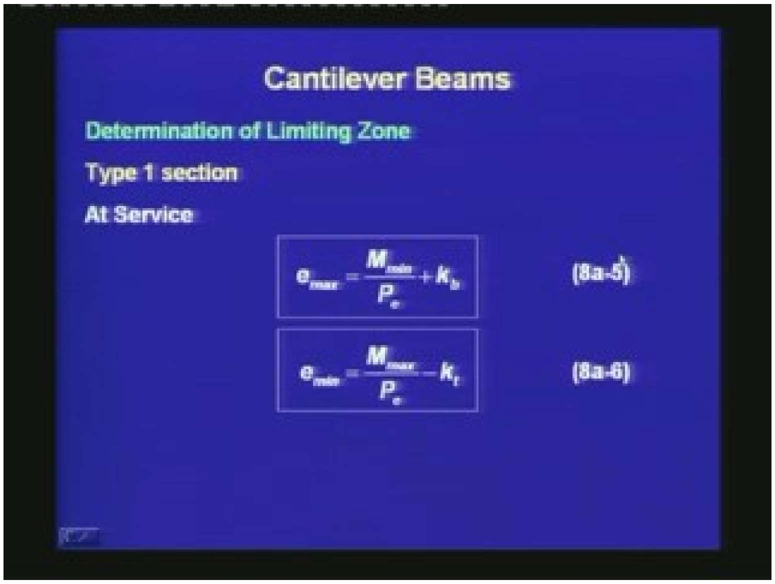

To recapitulate the equations: for a Type 1 section at service we have e_{max} equal to $M_{minimum}$ divided by P_e plus K_b; this is the expression we have got from the simply supported beam where we have substituted M_{min} in place of M_{sw}; e_{min} is equal to M_{max} divided by P_e minus K_t. Both e_{max} and e_{min} are being calculated at service and hence we have P_e - the effective prestress - for both the expressions.

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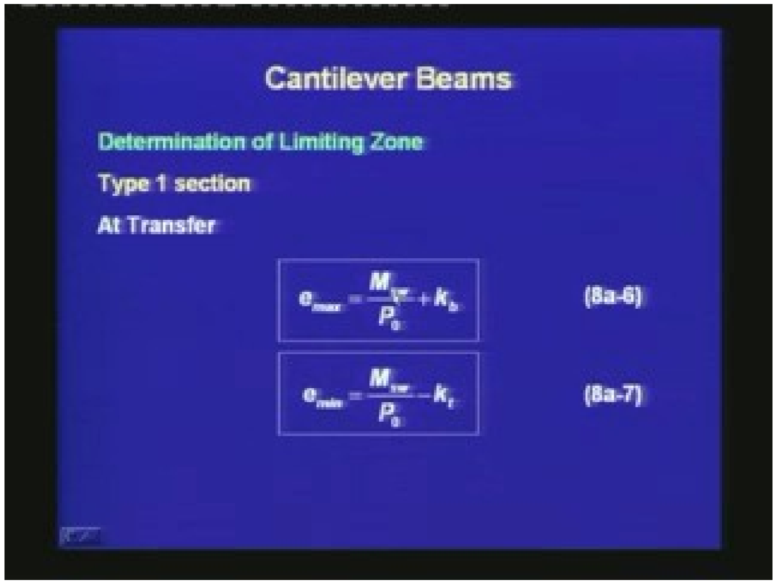

At transfer, we have another set of equations where they are given based on the self-eight moment M_{sw} and the prestress at transfer to P_0. e_{max} is equal to M_{sw} divided by P_0 plus K_b; e_{min} is equal to M_{sw} divided by P_0 minus K_t. Again, this is an expression which is similar to that of a simply supported beam, but we are applying both the expressions for transfer and that is why we have M_{sw} and P_0 in both the expressions. Thus we have two expressions of e_{min} - one for service and one for transfer; similarly, we have two expressions for e_{max} - one for service and one for transfer and from this you finally calculate the e_{max} and e_{min} at a particular location of a cantilever beam. This is the way we determine the limiting zone for a cantilever beam.

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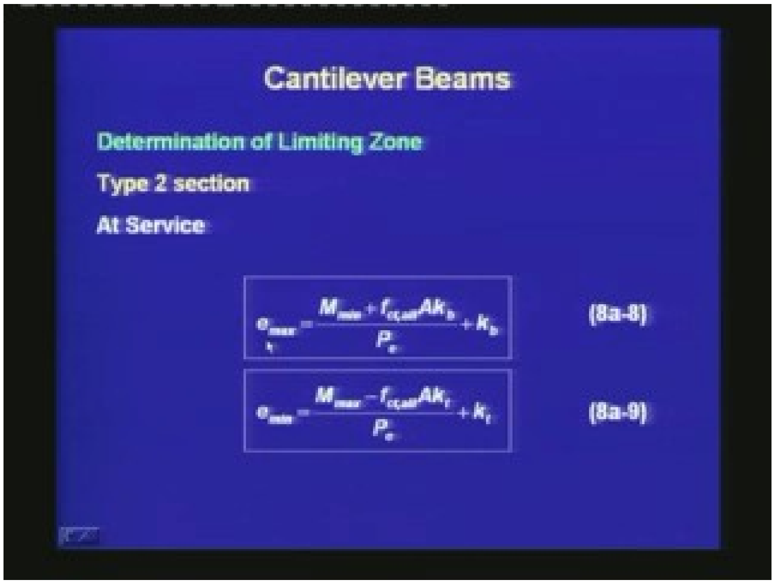

For a Type 2 section we have e_{max} equal to $M_{minimum}$ at service plus $f_{ct,\ allowable}$ times A times K_b whole divided by P_e plus K_b; e_{min} is equal to M_{max} minus $f_{ct,\ allowable}$ times A times K_t divided by P_e plus K_t. Again, note that both e_{max} and e_{min} are calculated at transfer corresponding to the minimum moment and the maximum moment respectively and using the effective prestress P_e.

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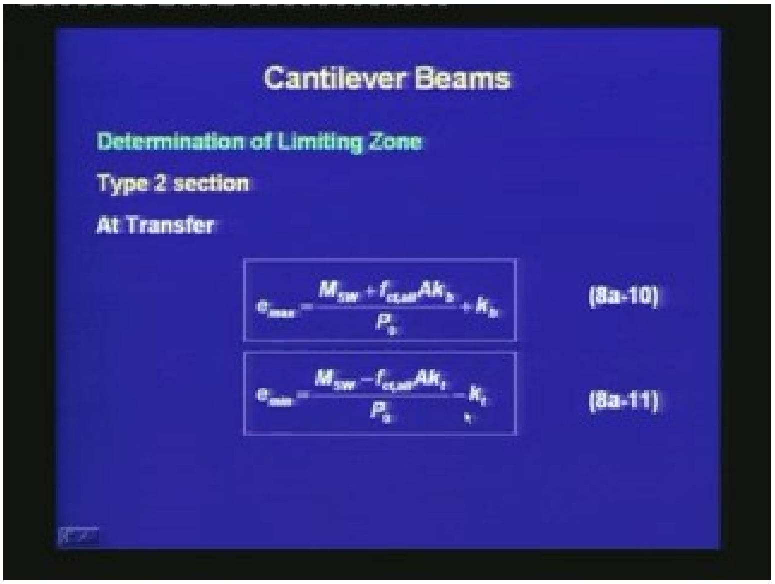

Similarly, at transfer we have two expressions of e_{max} and e_{min} which uses the self-weight moment M_{sw} and the prestress at transfer $P_{0.}$ e_{max} is equal to M_{sw} plus $f_{ct,\ allowable}$ times A times K_b whole divided by P_0 plus K_b. e_{min} is equal to M_{sw} minus $f_{ct,\ allowable}$ times A times K_t whole divided by P_0 minus K_t. Once we have these expressions of e_{max} and e_{min} we can determine the limiting zone for a Type 2 section. For a Type 3 sections the expressions are same; the only difference is that you have different values of the allowable tensile stress at transfer and at service.

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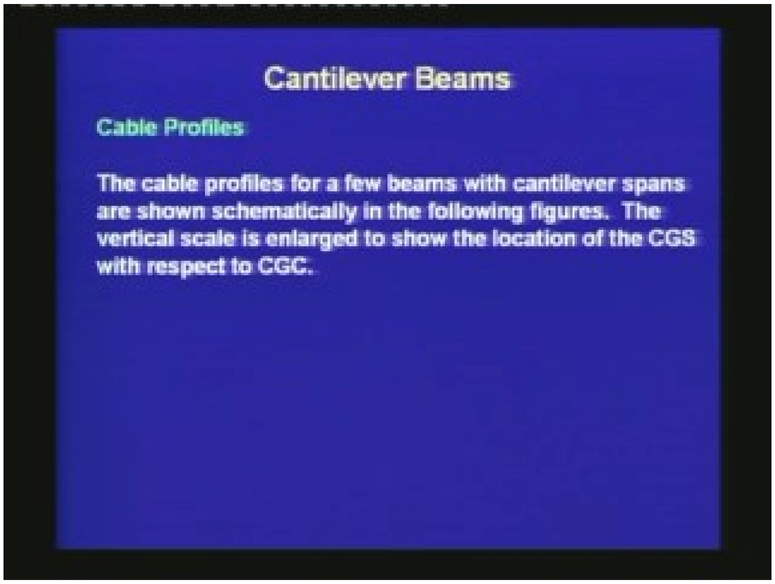

Next, we are moving on to have an idea of the cable profiles in a cantilever beam. The cable profiles for a few beams with cantilever spans are shown schematically in the following figures. The vertical scale is enlarged to show the location of the CGS with respect to the CGC.

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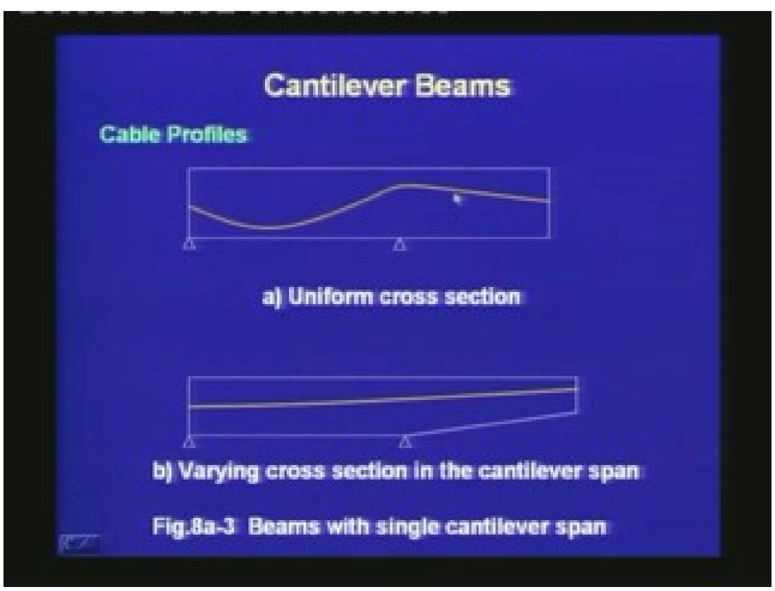

In this figures, we have enlarged the vertical scale to have a feel of the cable profiles. That top one is a beam with a single cantilever and you observe that in the back span region since it is subjected to a positive moment, we have a cable profile which is going down the CGC. Near the support, we always have a negative moment and hence the CGS is going above CGC and in the cantilever part the moment is always negative; hence, the CGS is always above CGC. Note that the cable profile which can be selected from the limiting zone has a variation which is similar to the moment diagram. The difference is near the support region, since, in a cable we cannot have a sharp kink we are providing and intermediate curve at the support region, without the kink.

If you have a varying cross section in the cantilever span, we can change the location of the CGS within the limiting zone such that we have minimum bend along the length of the beam; this helps us in placing the tendon easily and it reduces the friction losses in a post tensioned beam.

Thus, the straight profile of the tendon can be selected from the limiting zone. The straight profile has less friction losses and it is easy to place before the post tensioning operation.

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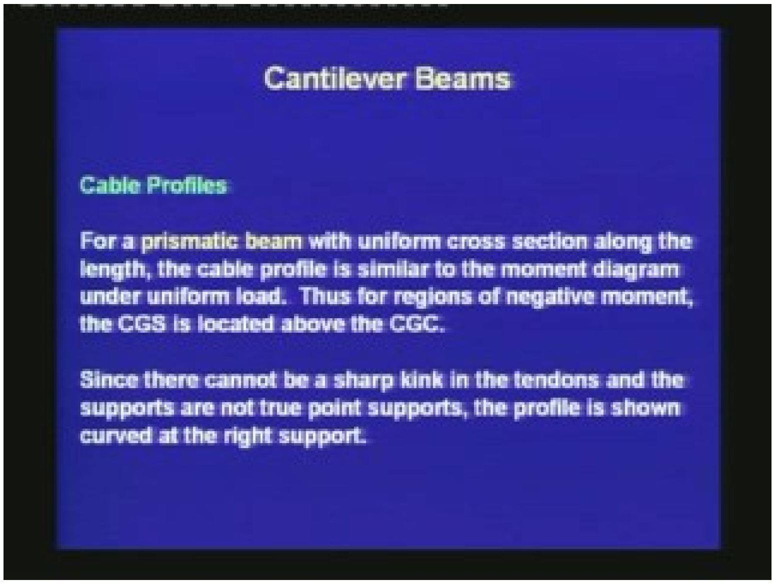

For a prismatic beam with uniform cross section along the length, the cable profile is similar to the moment diagram under uniform load. Thus, for regions of negative moment the CGS is located above the CGC since there cannot be a sharp kink in the tendons and the supports are not true point supports, the profile is shown curved at the right support.

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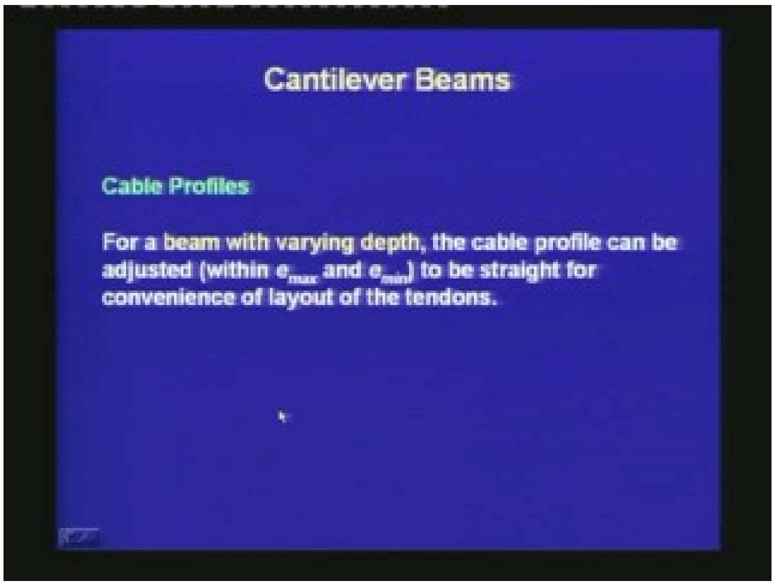

For a beam with varying depth, the cable profile can be adjusted within e_{max} and e_{min} to be straight for convenience of layout of the tendons.

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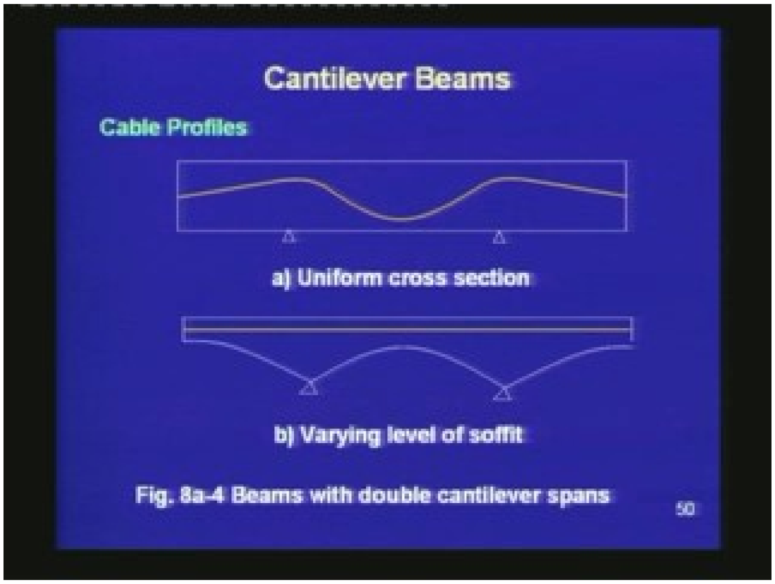

For beams with double cantilever spans, again for a uniform cross-section, you observe that the cable profile mimics the moment diagram where the CGS is below the CGC in the positive moment regions of the middle span. As we go close to the supports, the CGS is above CGC in the regions where the moment is negative. In the cantilever spans the CGS is always above the CGC, because, the cantilevers are always subjected to a negative moment. If the depth of the beam is varied, then we can select a cable profile within the limiting zone which is convenient to place and has less loss due to friction. In this beam with double cantilevers, a straight tendon can be selected; in fact this straight tendon will be located above the CGC near the supports and it will come beneath the CGC near the middle of the mid span. The selection of a straight cable is more for convenience and to reduce the friction losses in the post tensioning operation.

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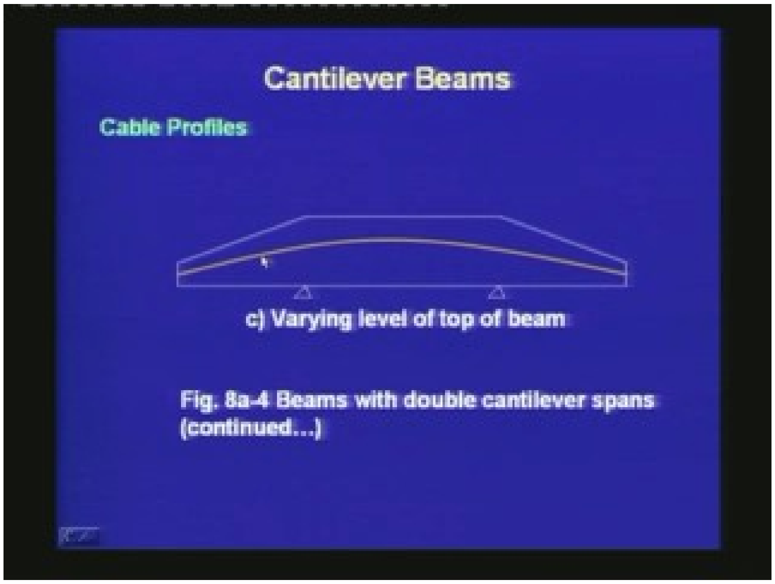

Sometimes we may observe that the top of the beam is varying in the elevations; for a beam with varying level of top of the beam, the cable profile can be selected somewhat like this, where the CGS is above the CGC in the cantilever portions and in the central region the CGS is located based on the limiting zone. We are trying to avoid sharp variations in curvature so as to minimize the losses due to friction.

Thus the selection of a cable profile is dependent on the limiting zone that we have determined and also on the placement of the tendons and friction losses. A cable profile is selected such that the tendon can be laid conveniently and there will be minimal friction losses during the post tensioned operations.

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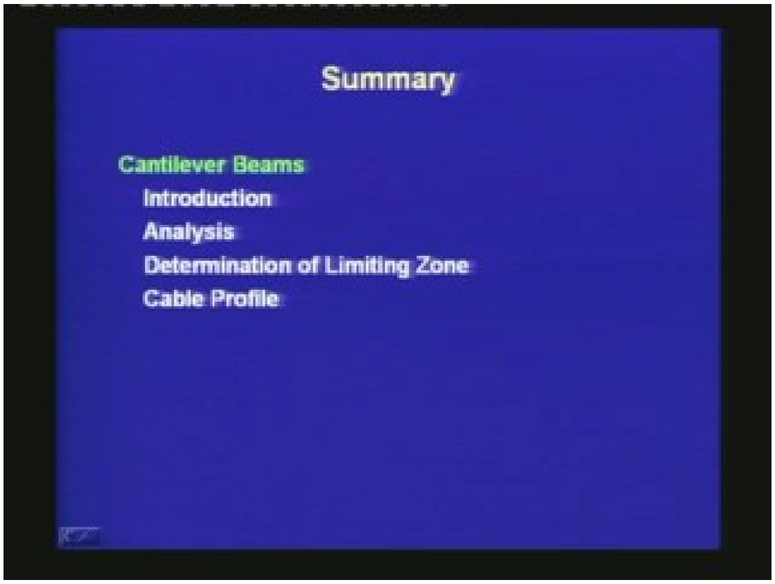

In today's lecture, we covered the analysis and design of cantilever beams. First, we observed the different types of cantilevers that can occur. In a building the cantilever can be a part of continuous beam, but the cantilever extends outside the last support. If there is not a continuous beam, usually a cantilever is provided with the back span to reduce the torsion in the supporting column. In bridges, there can be cantilevers in a balanced cantilever type of construction that means, over a pier, two cantilevers are progressively projecting out during the construction. There can be cantilever spans with an intermediate span in between. The analysis of a cantilever beam has to be done carefully by a proper placement of the live load. As compared to a simply supported beam, the main difference of a cantilever beam is that under different positioning of the live load, one particular section can have either a positive moment or a negative moment. If we place the live load in the cantilever span we get a certain moment diagram, if you place the live load in the back span we get another moment diagram. If you have a live load through out then it is a third type of moment diagram. From these conditions, we determine the envelop moment diagrams, which keeps as a maximum value M_{max} and a minimum value M_{min}; these values are algebraic, in the sense, the M_{max} can have a positive value or it can have the least negative value and M_{min} can have a least positive value, but it has the highest negative value.

Once we have done the analysis properly with different positioning of the live load we have the design moments - M_{max} M_{min}- for the design of a section of the cantilever.

For the placement of the tendons we use the concept of limiting zone. Now, first we revised the expressions of the limiting zone for a simply supported beam. We have found that the maximum eccentricity corresponds to the minimum moment at transfer and the minimum eccentricity corresponds to the maximum moment under service loads; these are for a simply supported beam.

For a cantilever beam the minimum moment need not be at transfer. We have extra moment conditions; we have one condition at transfer where the moment is due to the self-weight and then under service we have two values of moments M_{max} and M_{min} for a particular section which is available from the envelop moment diagrams. Thus to determine the limiting zone we first use the M_{max} and M_{min} values at service from which we determine one set of e_{max} and e_{min}. Then, we again calculate e_{max} and e_{min} for transfer and then we pick up the values of e_{max} and e_{min} which satisfies the stress conditions both for transfer and for service. The equations that we have written are for Type 1 member and Type 2 members. The equations are similar; the equations for Type 2 members are more generic, because, if we substitute the allowable tensile stress to be 0, then we get back the equations for Type 1 member. For Type 3 member the expressions are same as that for Type 2 member, provided we substitute the appropriate value of the allowable tensile stress.

After we revised the expressions for a simply supported beam, we got the expressions for a cantilever which is the same expressions, but now for service loads we are calculating e_{max} and e_{min} corresponding to $M_{minimum}$ and $M_{maximum}$ respectively. Then at transfer you are calculating another set of e_{max} and e_{min} corresponding to M_{sw}.

Thus, once we have these two values of both e_{max} and e_{min} we finally come to the values of e_{max} and e_{min} which satisfies the stress condition both at transfer and at service. Once the limiting zone has been determined, we select the cable profile. The cable profile can be different type depending on the situation. If we have small beams with uniform cross section, then the cable profile mimics the moment diagram, where the CGS lies below the

CGC in the positive moment regions and the CGS lies above CGC in the negative moment regions. Near the supports the cable profile deviates from the moment diagram, because, we cannot provide a sharp kink near the supports and also the supports are not true point supports.

If the depth of the beam is varying for large constructions, then the profile is adjusted within the limiting zone such that it is convenient to place the tendons and you have reduced friction losses during the post tensioning operation. We can try to have a straight cable profile within the limiting zone, which will be convenient to place the tendon and will have minimal friction effects. We have seen the cable profiles for beams with a single cantilever and also we have seen the cable profiles for beams with two cantilevers on the two sides.

In our next class, we shall move on to the discussion of a continuous beam, which is the extension of the concept of a cantilever beam, but there the number of spans is more than 2 or 3. We shall observe that the cable profile is similar to what you have seen for cantilever beams.

Thank you.