

PRESTRESSED CONCRETE STRUCTURES

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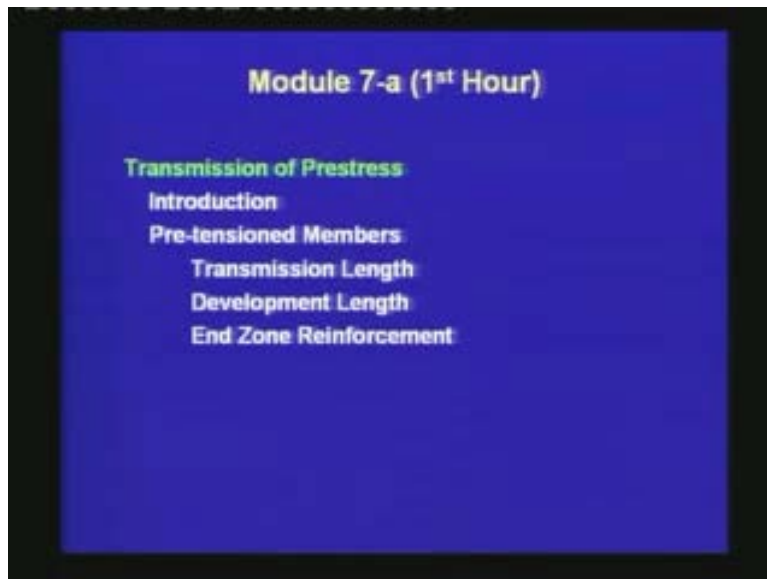
Indian Institute of Technology Madras

Module – 7: Transmission of Prestress

Lecture – 30: Pre-tensioned Members

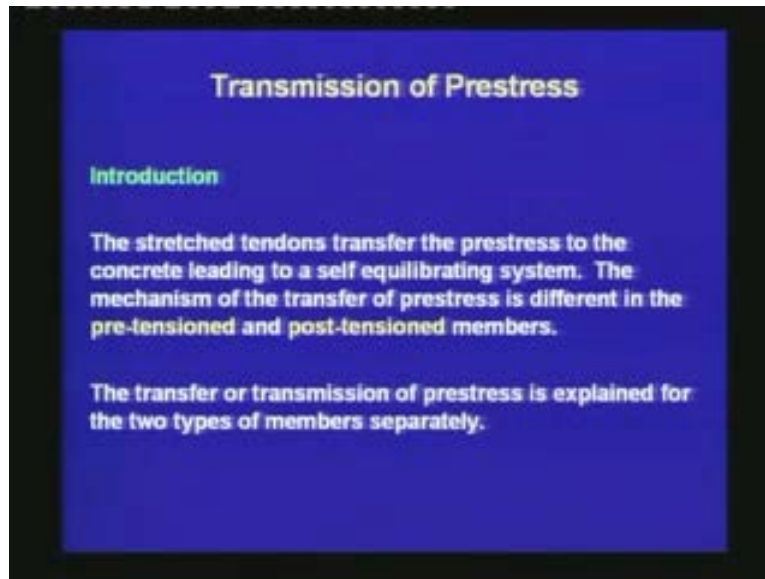
Welcome back to Prestressed Concrete Structures. This is the first lecture of Module 7 on Transmission of Prestress.

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In this lecture after the introduction, first we shall study about pre-tensioned members. For this type of members, we shall learn about the transmission length, the development length and the end zone reinforcement.

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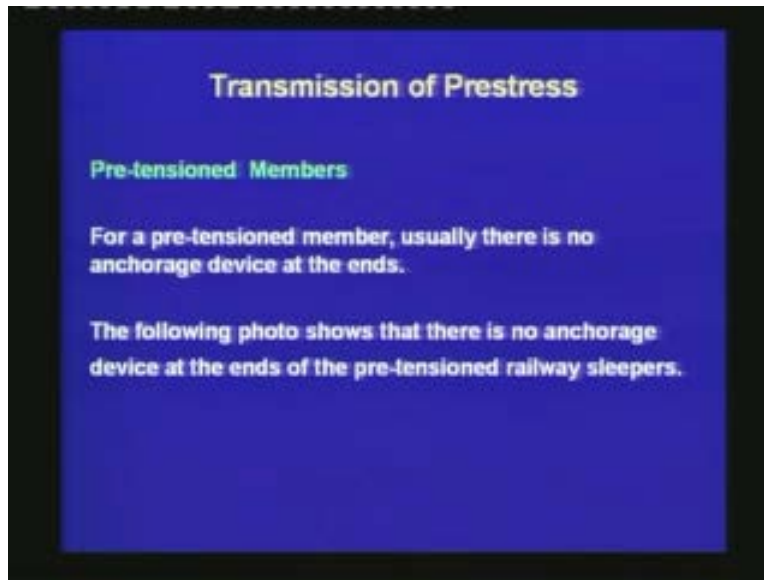


In a prestressed member, the prestress is transferred from the steel tendons to the concrete. The stretched tendons transfer the prestress to the concrete leading to a self-equilibrating system. The mechanism of the transfer of prestress is different in the pre-tensioned and post-tensioned members. The transfer or transmission of prestress is explained for the two types of members separately. First, it is explained for the pre-tensioned members.

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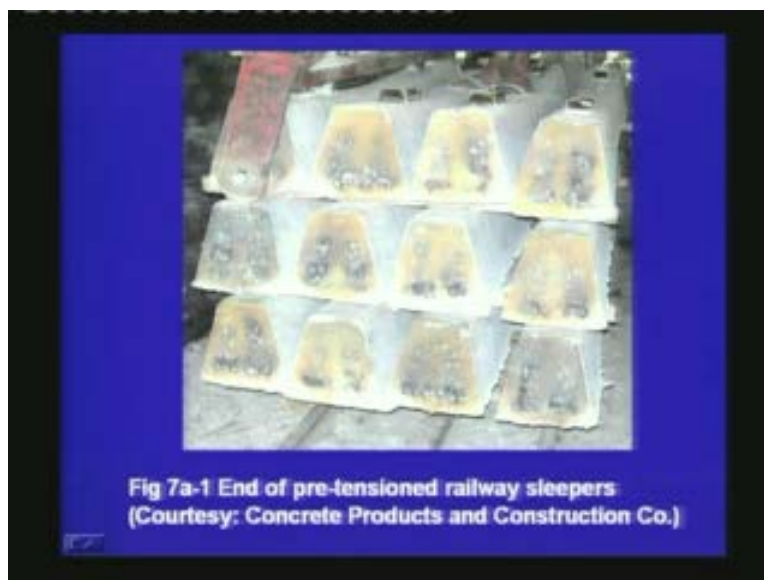


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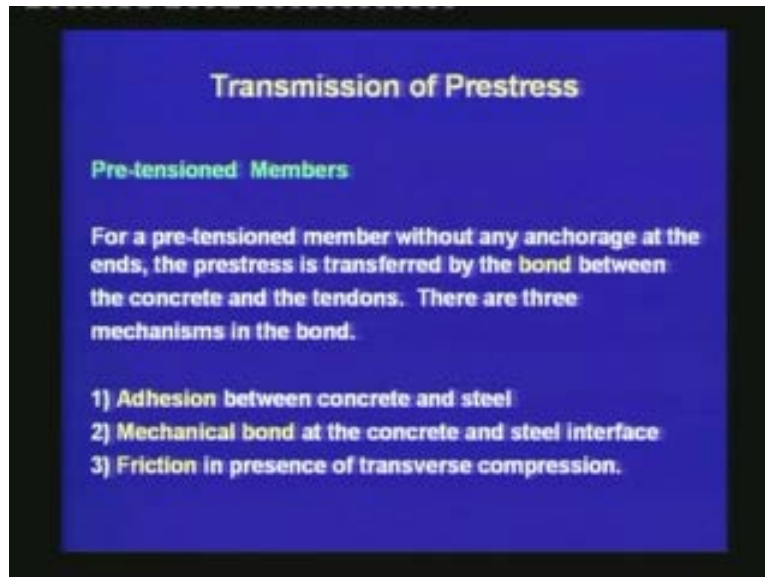
For a pre-tensioned member, usually there is no anchorage device at the ends. The following photo shows that there is no anchorage device at the ends of the pre-tensioned railway sleepers.

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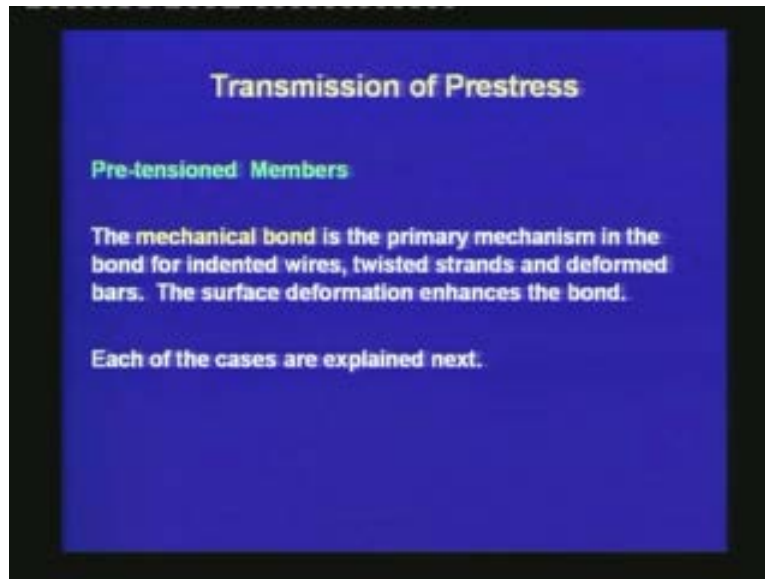
In this photograph, you can observe that the strands have been cut, and there is no anchorage device at the end of these railway sleepers. This is an essential difference of the pre-tensioned members, as compared to the post-tensioned members.

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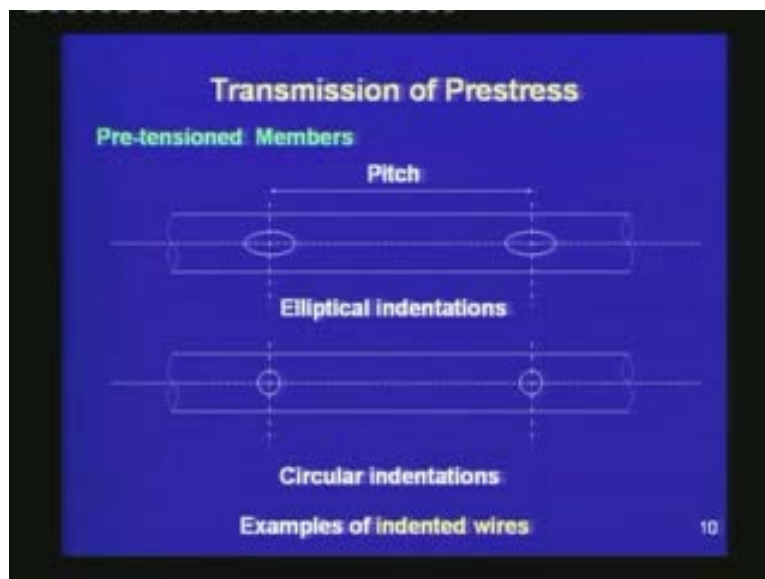
For a pre-tensioned member, without any anchorage at the ends, the prestress is transferred by the bond between the concrete and the tendons. There are three mechanisms in the bond. First is the adhesion between concrete and steel. This occurs due to a chemical reaction at the interface. The second is the mechanical bond at the concrete and steel interface. The third is the friction in presence of transverse compression.

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Out of these three components, the mechanical bond is the primary mechanism in the bond for indented wires, twisted strands and deformed bars. The surface deformation enhances the bond.

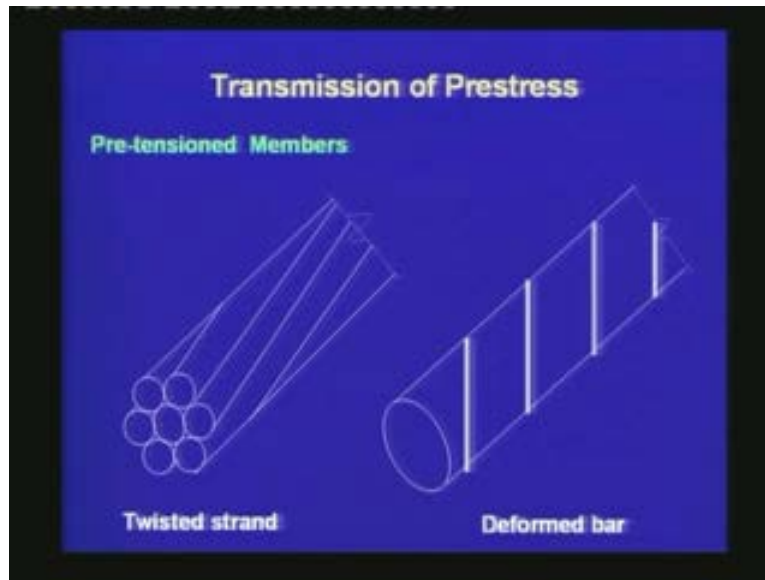
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Let us now see three examples of the surface deformations. In the above sketch, you can observe that the wires have been provided with some indentations. The indentations can

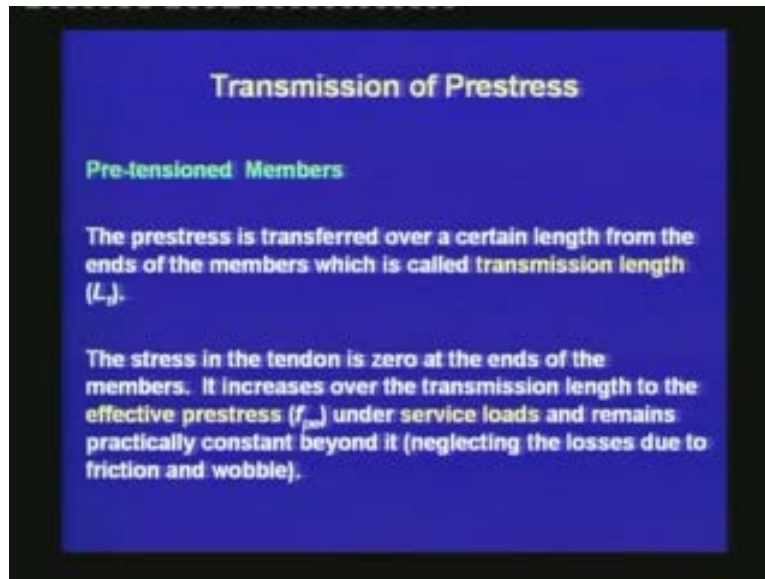
be circular or they can be elliptical. The indentations are provided at a certain pitch. This type of wires is better than the plain wires, regarding the mechanical bond.

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For a twisted strand, say a seven-wire strand, the surface itself is quite deformed, because the individual wires are twisted around a central wire. Hence, the mechanical bond in a twisted strand is much better than a plain wire. If a deformed bar is used for pre-tensioning, the surface deformations on the bar help in generating additional mechanical bond in the concrete and steel interface.

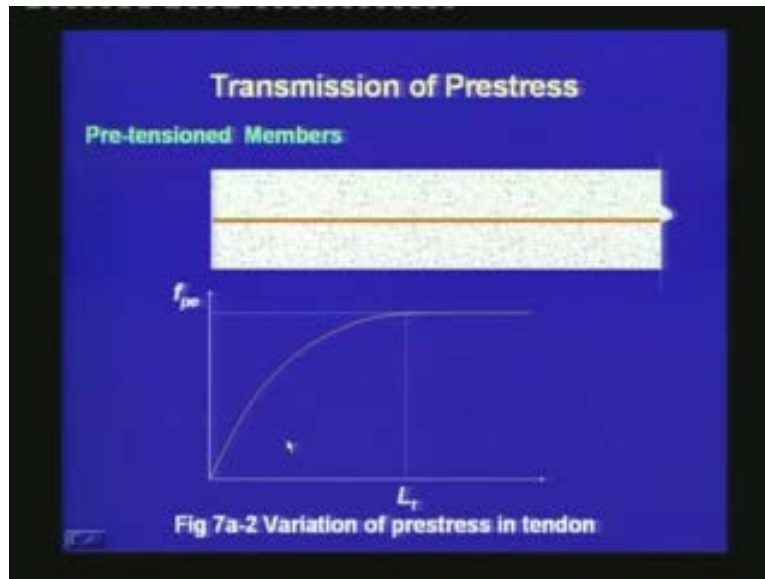
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The prestress is transferred over a certain length from the ends of the members, which is called the transmission length or the transfer length. This will be denoted as L_t . This is a very important concept for a pre-tensioned member, that the prestress is not transferred right at the ends, but it is transferred over a certain length from the ends.

The stress in the tendon is zero at the ends of the members. It increases over the transmission length to the effective prestress which is f_{pe} under service loads, and remains practically constant beyond it. When the external load is applied, the stress may increase from f_{pe} to a higher value at the critical section depending upon the amount of load, but that increase is substantially less compared to the increase over the transmission length.

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This sketch shows the variation of the prestress in the tendon. The prestress varies substantially in the transmission length at the end. Under service loads, it increases up to f_{pe} and then it stays more or less constant.

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The text explains the Hoyer effect, stating that after stretching the tendon, the diameter reduces due to Poisson's effect. When the prestress is transferred after the hardening of concrete, the ends of the tendon sink in concrete, and the prestress at the ends is zero. The diameter of the tendon regains its original value over the transmission length.

Transmission of Prestress

Pre-tensioned Members

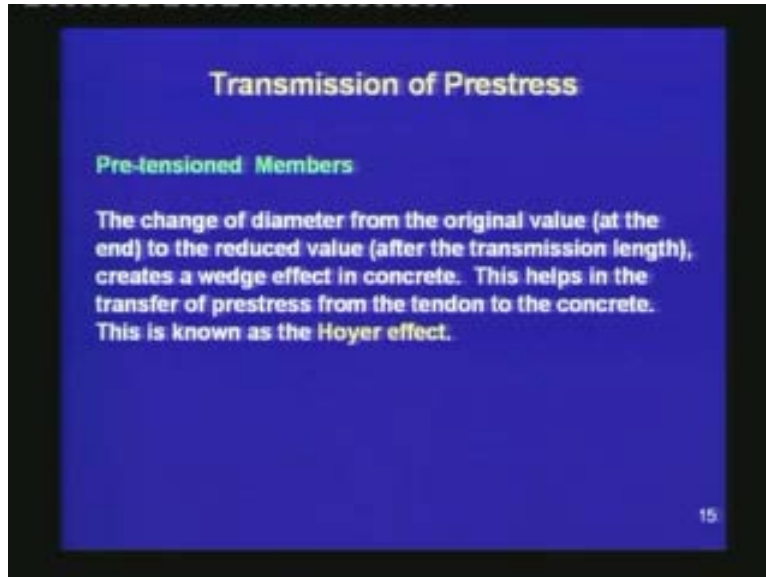
The Hoyer effect that develops in a tendon is explained.

After stretching the tendon, the diameter reduces from the original value due to the Poisson's effect. When the prestress is transferred after the hardening of concrete, the ends of the tendon sink in concrete. The prestress at the ends of the tendon is zero. The diameter of the tendon regains its original value over the transmission length.

Next, we shall talk about the Hoyer effect. After stretching the tendon, the diameter reduces from the original value due to the Poisson's effect. When the prestress is

transferred after the hardening of concrete, the ends of the tendon sink in concrete. The prestress at the ends of the tendon is 0. The diameter of the tendon regains its original value over the transmission length.

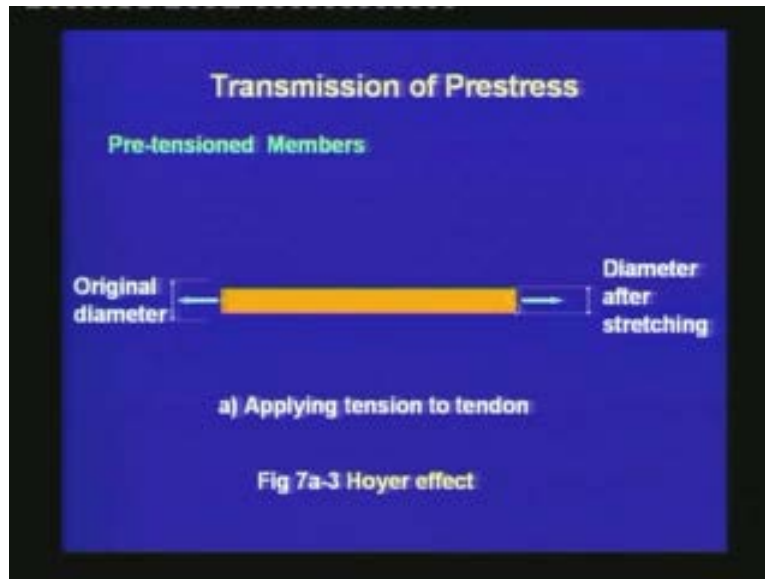
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The change of diameter from the original value at the end to the reduced value after the transmission length, creates a wedge effect in concrete. This helps in the transfer of prestress from the tendon to the concrete. This is known as the Hoyer effect.

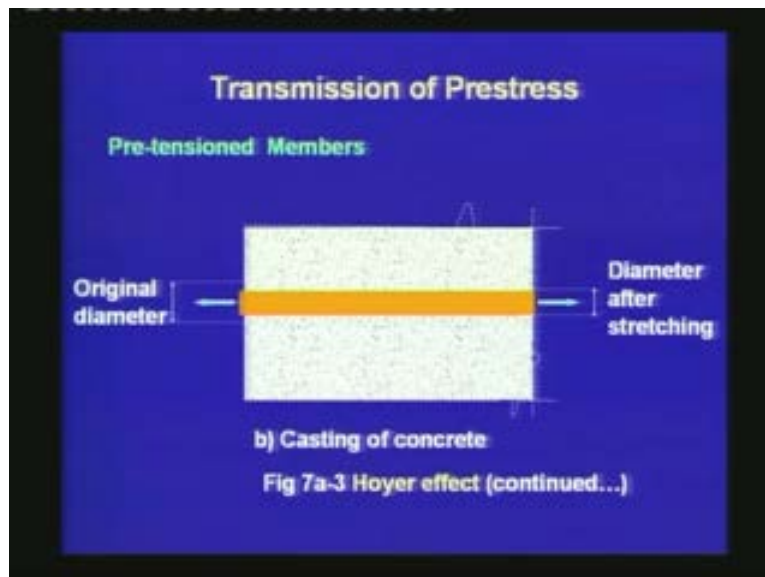
Let us try to understand the Hoyer effect by a few sketches.

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In this sketch, after the tendon has been stretched, its diameter has reduced from the original value due to the Poisson's effect. That means, after stretching, the diameter of the tendon is less than the original diameter.

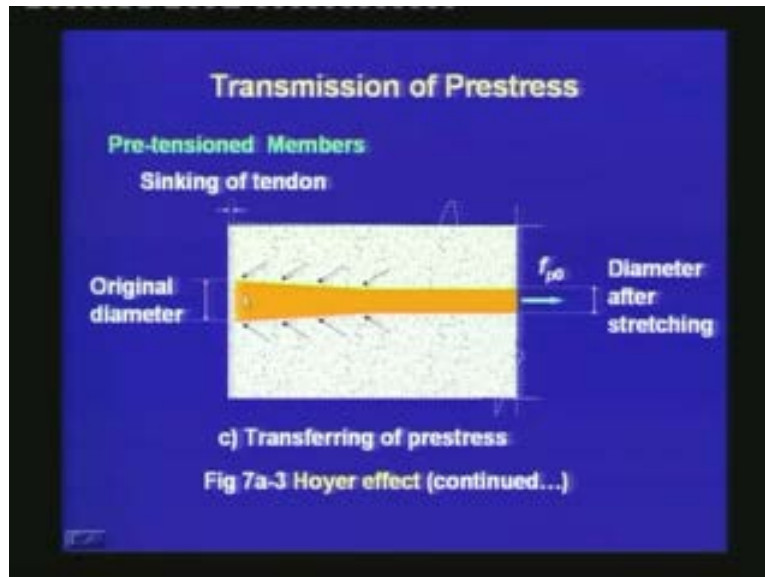
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Next, the concrete is poured and it is allowed to harden. In the above sketch, the concrete has been cast and the tendon is about to be cut. Till this stage, the diameter of the tendon

is constant throughout the length. After the concrete has hardened to the required strength, the tendons are cut and the prestress is transferred to the concrete. In this process, the tendon slightly sinks within the concrete and the stress at the end drops to 0.

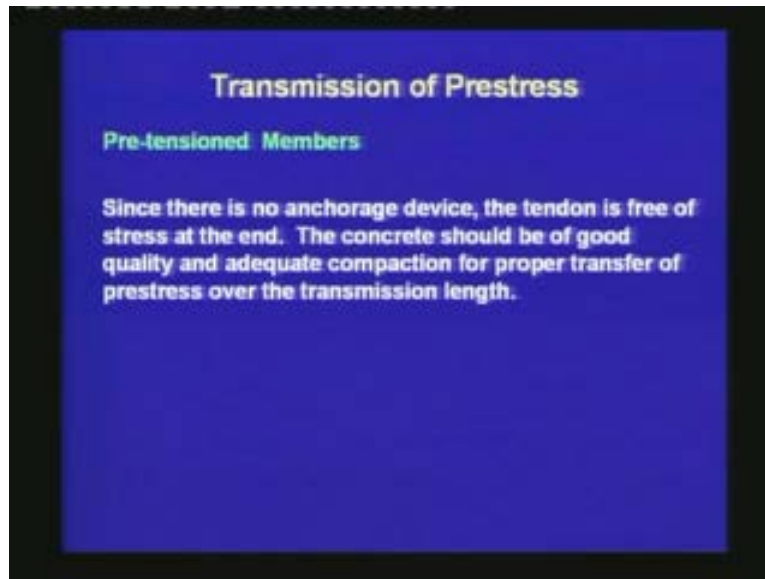
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Since the stress has dropped to 0, the diameter at the end has regained its original value. Over the transmission length, the diameter reduces as the prestress increases. This change in diameter from the original value to the reduced value over the transmission length, creates a wedge effect in the concrete. That means there are forces which are not allowing the tendon to move inside the concrete any further. In reaction to this, the tendon is applying the prestress to the concrete. Thus, the Hoyer effect is beneficial for the transmission of prestress from the tendon to the concrete.

Right after transfer, the prestress beyond the transmission length is f_{p0} . With time after the long term losses, the value of f_{p0} will drop down to the effective prestress f_{pe} .

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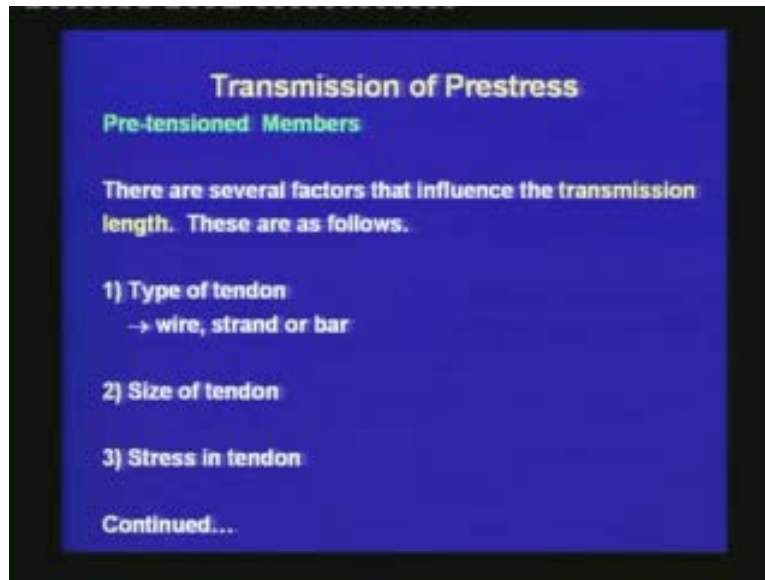
Since there is no anchorage device, the tendon is free of stress at the end. The concrete should be of good quality, and adequately compacted for proper transfer of prestress over the transmission length. Since the transfer of prestress solely depends on the bond between the concrete and the steel, the quality and construction of concrete should be paid attention to.

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Next, we shall discuss the estimate of transmission length. This to make sure that high external load is not placed over the transmission length. There should be adequate provision of the transmission length in a member, such that the prestress is developed when an external moment demand generates in a section.

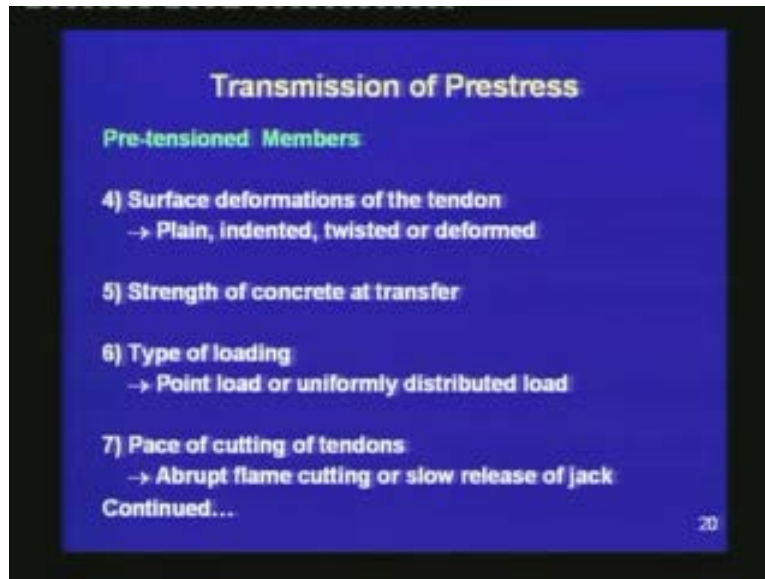
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There are several factors that influence the transmission length. These factors are as follows:

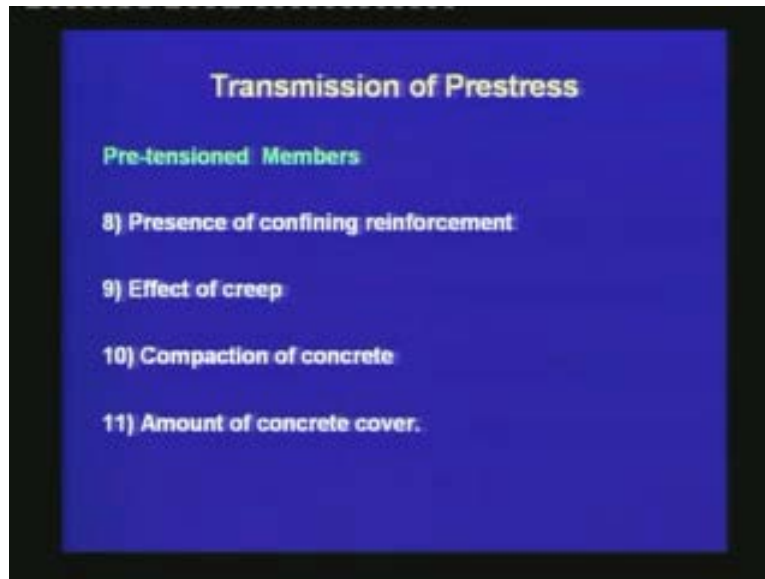
- 1) The type of tendon: whether it is a wire, strand or bar. We have observed earlier that each of this tendon is different. Hence, the transmission lengths are different.
- 2) The second factor is the size of the tendon. For smaller size tendon, the perimeter is large and hence, the bond developed is better.
- 3) The third factor is the stress in the tendon. If the applied prestress is high, then the transmission length required will also be high.

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- 4) The fourth factor is the surface deformations of the tendon. The mechanical bond depends on the surface deformations. It is small for a plain wire. For an indented wire or twisted strand or deformed bar, the surface deformations are more and the mechanical bond is also more.
- 5) The fifth factor is the strength of concrete at transfer. If the concrete is not of adequate strength, then the transmission length will be large.
- 6) The sixth factor is the type of loading. If it is a point load or whether it is a uniformly distributed load, depending on that the transmission length can vary.
- 7) The seventh factor is the pace of cutting of the tendons. Are we abruptly cutting the tendons by a flame cutter or are we slowly releasing the jack to transfer the prestress? It is found that the transmission length is smaller if we are slowly releasing the jack.

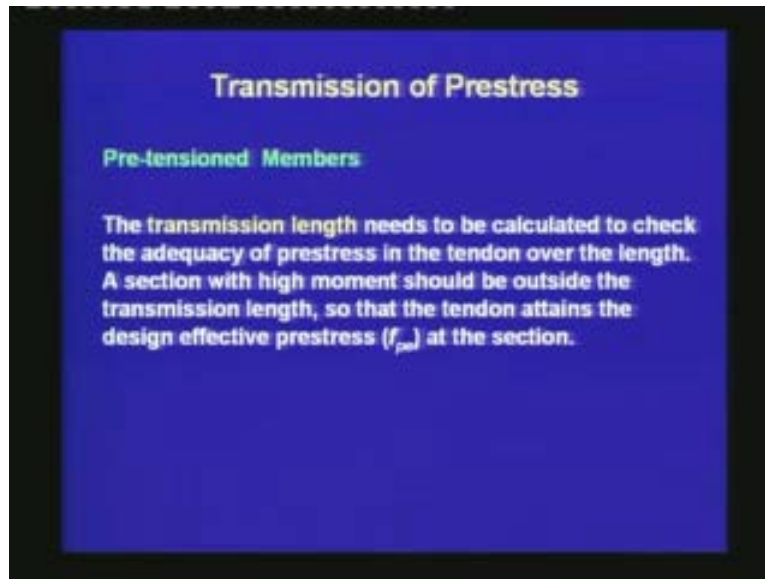
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- 8) If there is some confining reinforcement to check the development of cracks in concrete, then the transmission is better.
- 9) The effect of creep in concrete can also influence the transmission.
- 10) The compaction of the concrete is very important. Well compacted concrete will have a lesser transmission length and the transfer of prestress will be better.
- 11) The transmission length also depends on the amount of concrete cover around the tendons. If there is more concrete surrounding the tendons, then the transfer of prestress will be better.

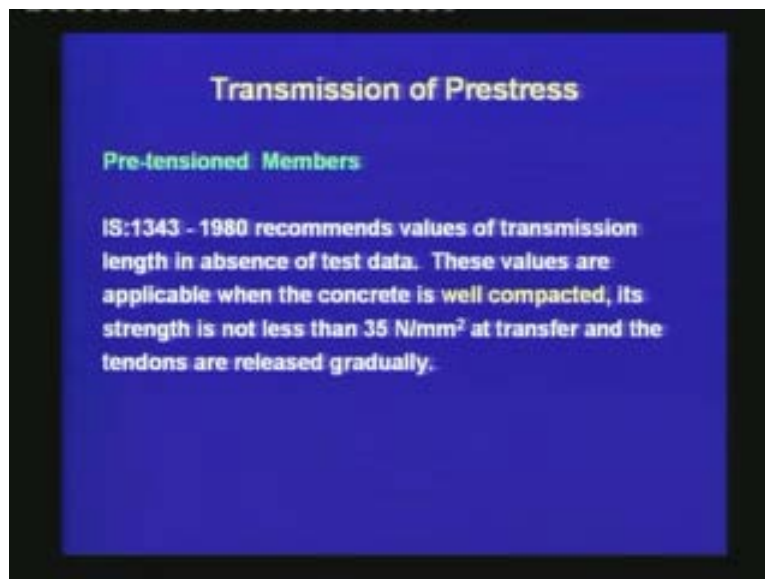
Thus, the transmission length depends on several factors.

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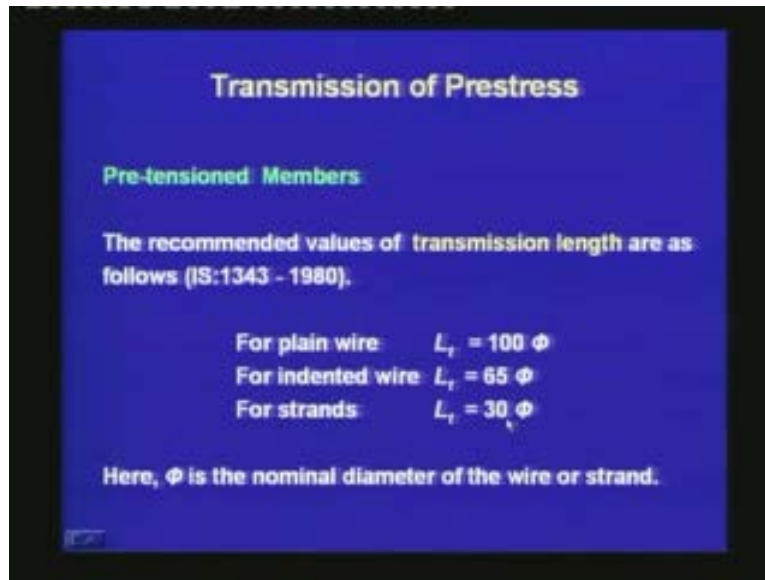
The transmission length needs to be calculated to check the adequacy of prestress in the tendon over the length. A section with high moment should be outside the transmission length so that the tendon attains the design effective prestress (f_{pe}) at the section. In case if there is a point load very close to the end of the member and within the transmission length, the capacity should be reduced at that location.

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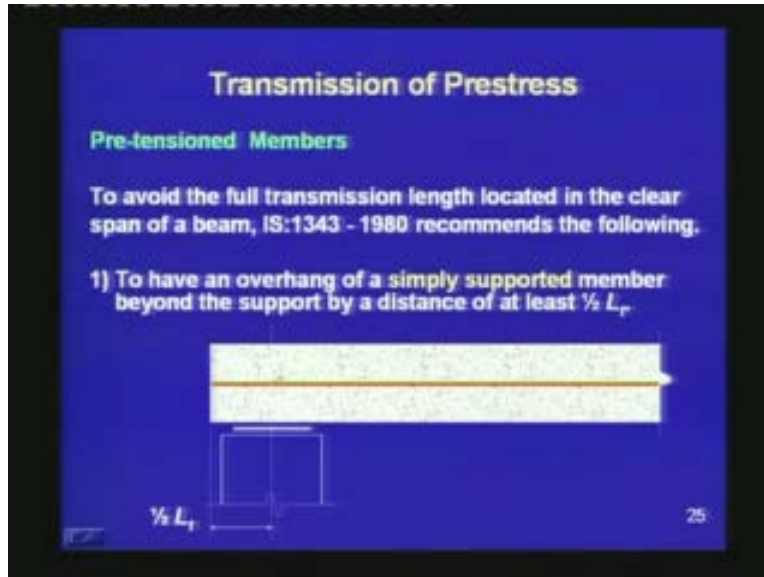
The code IS: 1343 - 1980 recommends values of transmission length in absence of test data. These values are applicable when the concrete is well compacted, its strength is not less than 35 N/mm^2 at transfer, and the tendons are released gradually. The recommendations of the code are simple. They do not involve the different factors that were mentioned.

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As per the recommendation of the code, for plain wire the transmission length should be 100 times the diameter of the wire. For indented wire, the transmission length is smaller, which is 65 times the diameter. For twisted strands, the transmission length is even smaller, which is 30 times the nominal diameter of the strands. Thus, we can see that the only factor which is being considered in the transmission length is the size of the tendon. There are more refined estimates of the transmission length which takes account of some more factors.

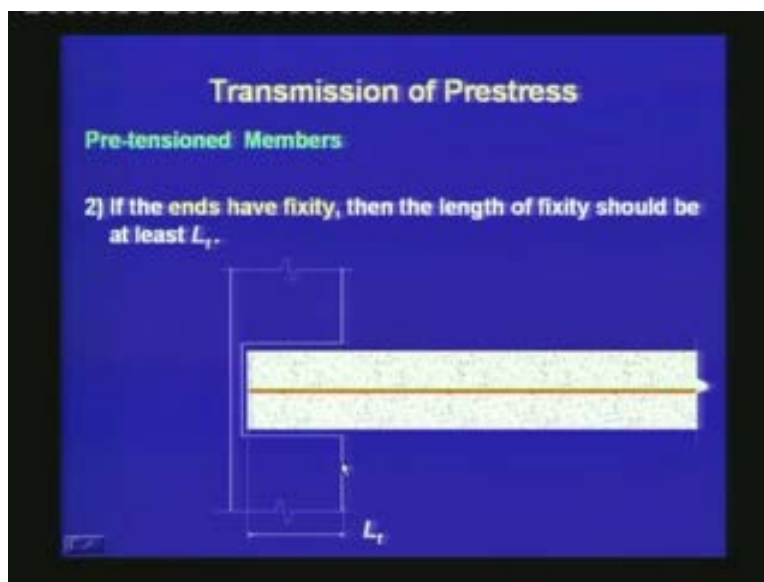
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To avoid the full transmission length located in the clear span of a beam, IS: 1343 – 1980 recommends the following:

- 1) To have an overhang of a simply supported member beyond the central line of the support by a distance of at least half of the transmission length (L_t). Thus, at the inside face of the support the prestress is totally transferred to the concrete.

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- 2) If the ends have fixity, then the length of fixity should be at least L_t . If the end of a pre-tensioned member is in a socket, there is some fixity at the end. Then the length of the fixity should be at least equal to the transmission length. Beyond the face of the support, the prestress is totally transferred to the concrete.

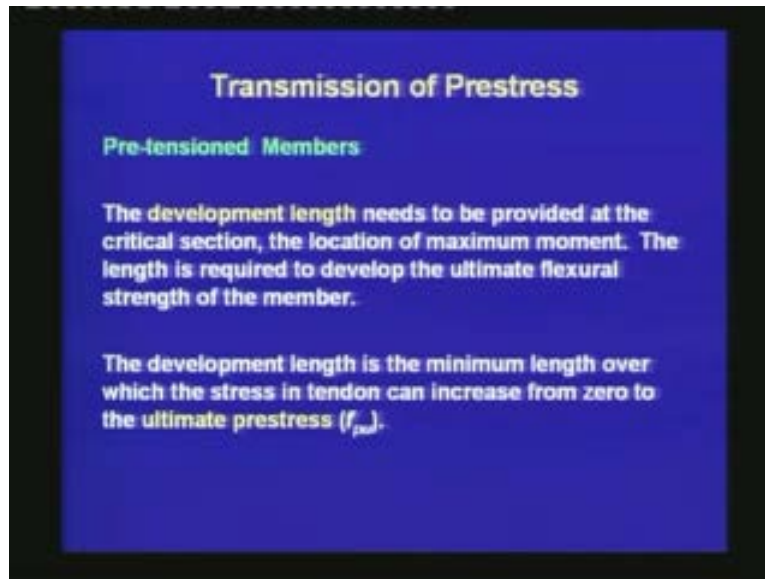
These recommendations are to ensure that the point of maximum moment is outside the transmission length from the ends.

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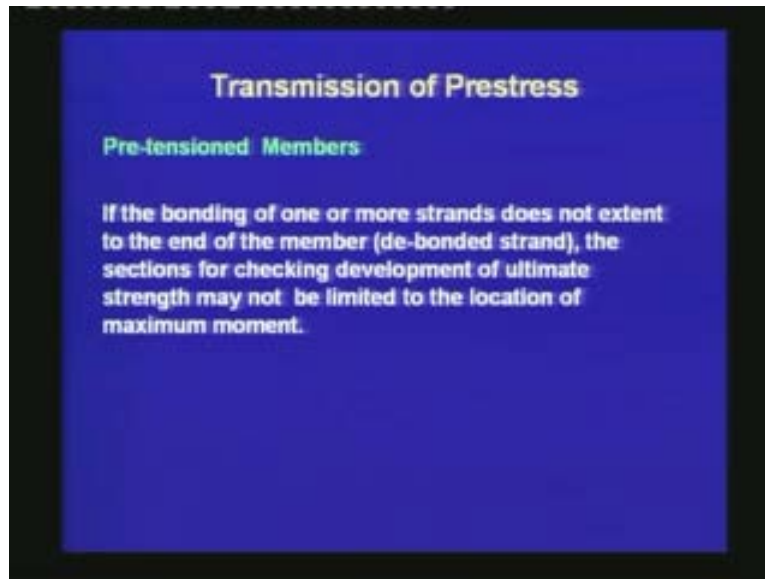
Next, we are moving on to another concept which is the development length.

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The development length needs to be provided at the critical section, the location of maximum moment. The length is required to develop the ultimate flexural strength of the member. The development length is the minimum length over which the stress in tendon can increase from zero to the ultimate prestress (f_{pu}). The difference between the transmission length and the development length is that, in the transmission length the stress in the tendon develops from zero to the effective prestress (f_{pe}). For the development length, the stress is expected to develop from zero to the ultimate prestress (f_{pu}). The member should attain the maximum flexural capacity under ultimate loads at the end of the development length. Thus, the transmission length is required to achieve its service capacity, whereas the development length is required to achieve its ultimate capacity.

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If the bonding of one or more strands does not extend to the end of the member, which are called de-bonded strands, then the sections for checking development of ultimate strength may not be limited to the location of maximum moment.

Sometimes the strands are de-bonded towards the end to reduce the effect of the stress concentration at the ends. In such a situation, the critical section to check development length may not be just the location of maximum moment, but we may have to check some other locations depending on the length of de-bonding of the strands. In this lecture, we shall consider that all the strands are having bond through out the length of the member.

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Transmission of Prestress

Pre-tensioned Members

The development length (L_d) is the sum of the transmission length (L_t) and the bond length (L_b).

$$L_d = L_t + L_b \quad (7a-1)$$

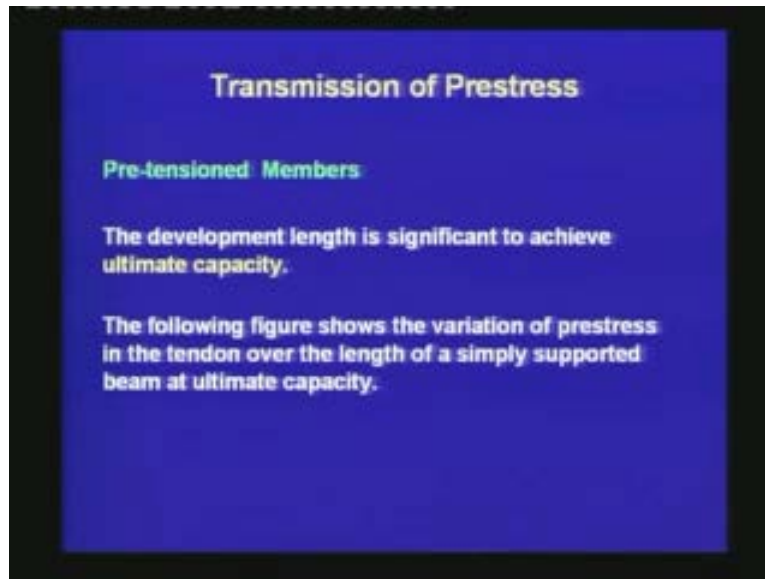
The bond length is the minimum length over which, the stress in the tendon can increase from the effective prestress (f_{pe}) to the ultimate prestress (f_{pu}) at the critical location.

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The development length (L_d) is the sum of the transmission length (L_t) and the bond length (L_b). Thus, $L_d = L_t + L_b$. The bond length is the minimum length over which, the stress in the tendon can increase from the effective prestress (f_{pe}) to the ultimate prestress (f_{pu}) at the critical location.

Thus, L_d is the length over which the stress in the tendon is developing from 0 to f_{pu} . This consists of two components: one is L_t , where the stress develops from 0 to f_{pe} and another component is L_b , where the stress develops from f_{pe} to f_{pu} . The second component is called the bond length.

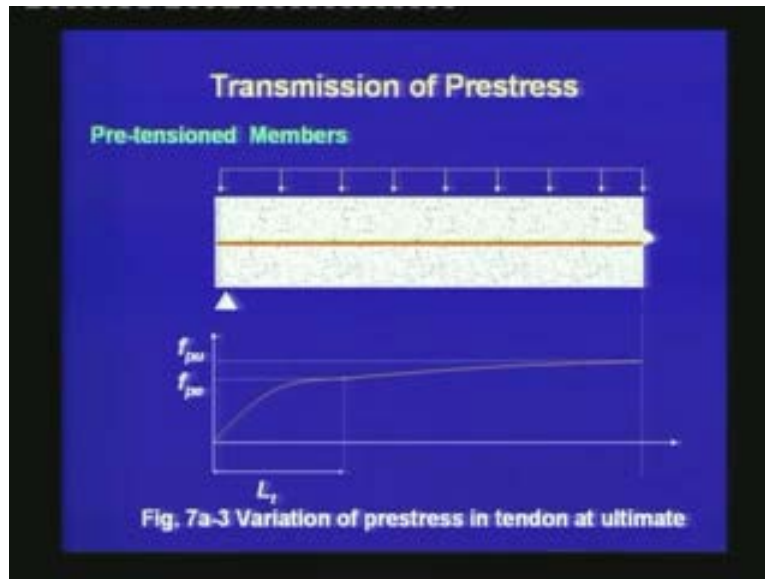
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The development length is significant to achieve ultimate capacity. That means, whenever we are checking the flexural capacity at a particular critical location, we need to make sure that adequate development length has been provided from the end to that particular location, so that the stress in the tendon can reach the ultimate stress f_{pu} .

The following figure shows the variation of prestress in the tendon over the length of a simply supported beam at ultimate capacity.

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Here, the simply supported beam is subjected to a uniformly distributed load. At the mid-span, it is expected to have the ultimate capacity with the prestress equal to f_{pu} . From the end, the distance L_t is required to develop the stress from 0 to f_{pe} . After that, there has to be a minimum length to develop the stress from f_{pe} to f_{pu} . The stress needs to be developed at the critical section where the moment is maximum.

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Transmission of Prestress

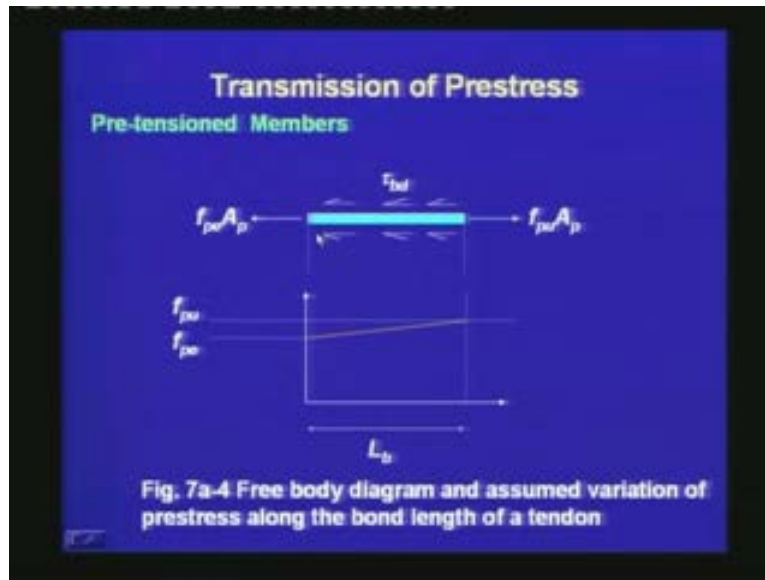
Pre-tensioned Members

The calculation of the bond length is based on an average design bond stress (τ_{bd}). A linear variation of the prestress in the tendon along the bond length is assumed.

The following sketch shows a free body diagram of a tendon along the bond length.

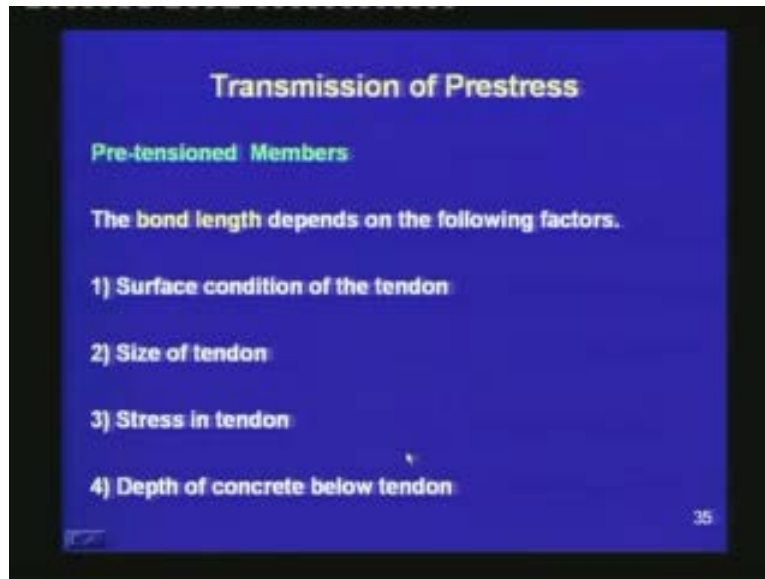
The calculation of the bond length is based on an average design bond stress, which is denoted as τ_{bd} . A linear variation of the prestress in the tendon along the bond length is assumed. The following sketch shows a free body diagram of a tendon along the bond length.

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On the left side of the tendon, the force is the effective prestress times the area of the steel A_p , whereas on the right side which has developed the ultimate capacity, the force is f_{pu} times the area of the steel. This is under equilibrium along with the tractive force due to the bond stress τ_{bd} , which is assumed to be constant over the length. If τ_{bd} is constant, then the increase in stress from f_{pe} to f_{pu} is linear over the bond length L_b .

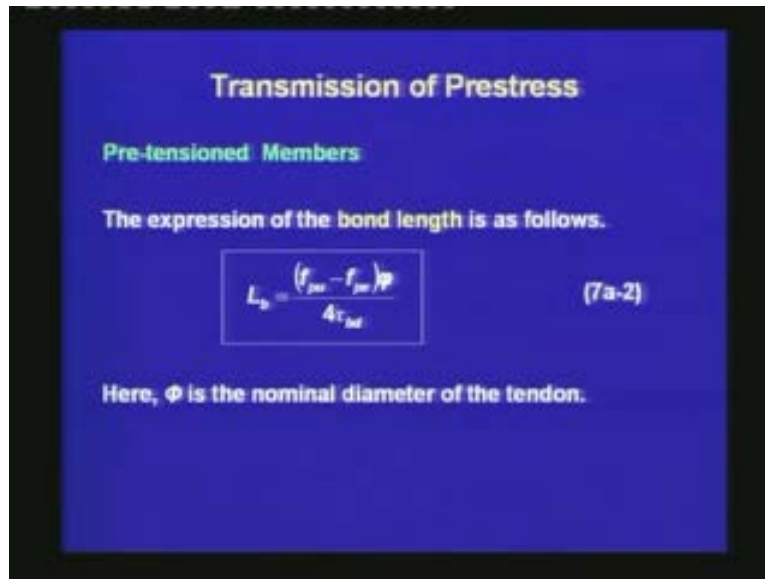
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The bond length depends on several factors.

- 1) First is the surface condition of the tendon. Similar to that for the transmission length, the surface condition influences the bond length.
- 2) The second factor is the size of the tendon. For a larger tendon the surface area per unit length is small, and the bond length is larger.
- 3) Stress in the tendon: the values of f_{pe} and f_{pu} influence the bond length.
- 4) Depth of concrete below the tendon: for more concrete, the development of the stress is better.

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Transmission of Prestress

Pre-tensioned Members

The expression of the bond length is as follows.

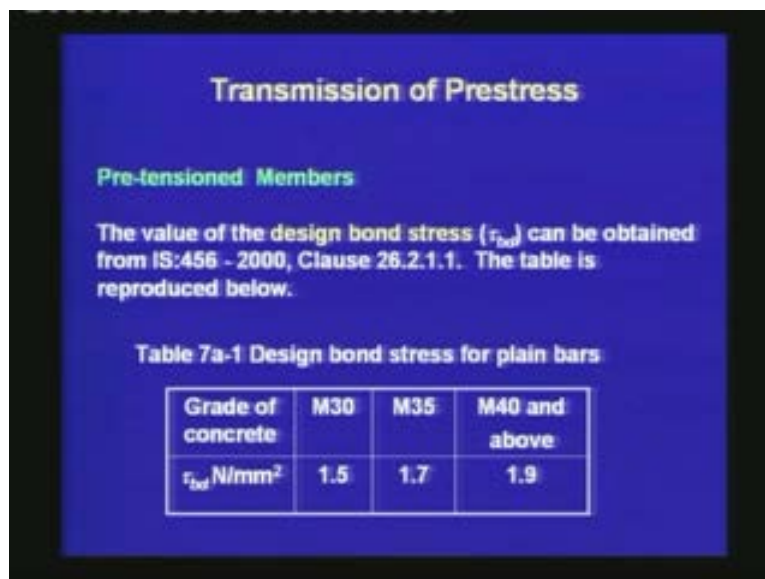
$$L_b = \frac{(f_{pu} - f_{pe})\phi}{4\tau_{bd}} \quad (7a-2)$$

Here, ϕ is the nominal diameter of the tendon.

Based on the free body diagram, an expression of the bond length is written in terms of f_{pu} , f_{pe} , the nominal diameter of the tendon (ϕ) and τ_{bd} . Thus, the expression is:

$$L_b = (f_{pu} - f_{pe})\phi/4\tau_{bd}.$$

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Transmission of Prestress

Pre-tensioned Members

The value of the design bond stress (τ_{bd}) can be obtained from IS:456 - 2000, Clause 26.2.1.1. The table is reproduced below.

Table 7a-1 Design bond stress for plain bars

Grade of concrete	M30	M35	M40 and above
τ_{bd} N/mm ²	1.5	1.7	1.9

The value of the design bond stress τ_{bd} can be obtained from IS: 456 – 2000, Clause 26.2.1.1. The table is reproduced here. The design bond stress for plain bars is: for M30

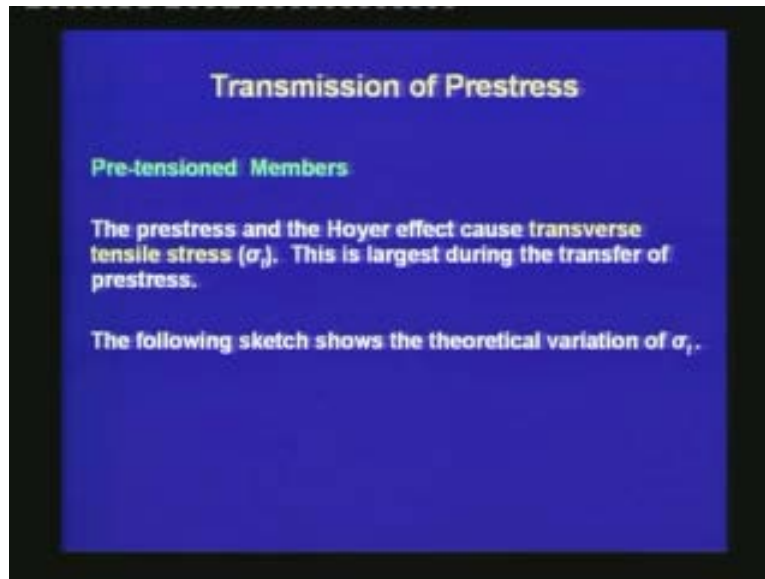
concrete $\tau_{bd} = 1.5 \text{ N/mm}^2$, for M35 concrete $\tau_{bd} = 1.7 \text{ N/mm}^2$, for M40 concrete and above $\tau_{bd} = 1.9 \text{ N/mm}^2$. Thus, for a particular pre-tensioned member, once we know f_{pu} and f_{pe} and the size of the tendon, we can calculate the bond length for that particular application. We add the transmission length to the bond length to get the development length.

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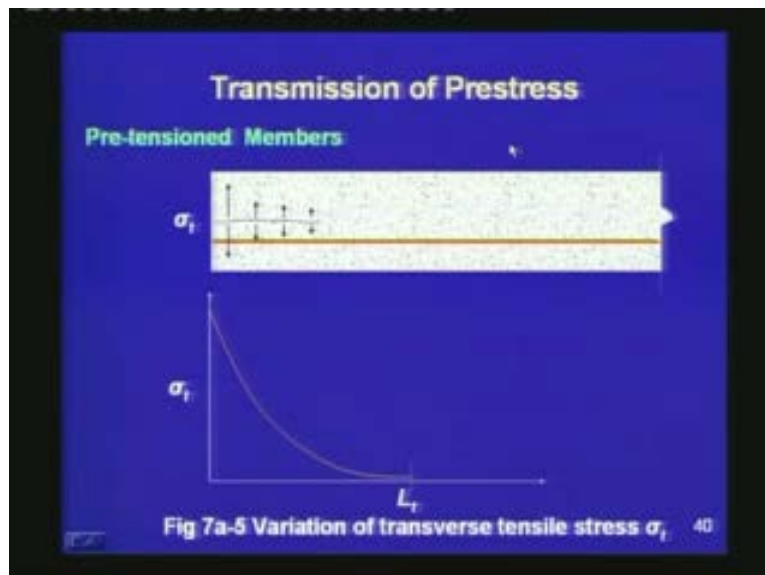
Next, we are moving on to the design of the end zone reinforcement.

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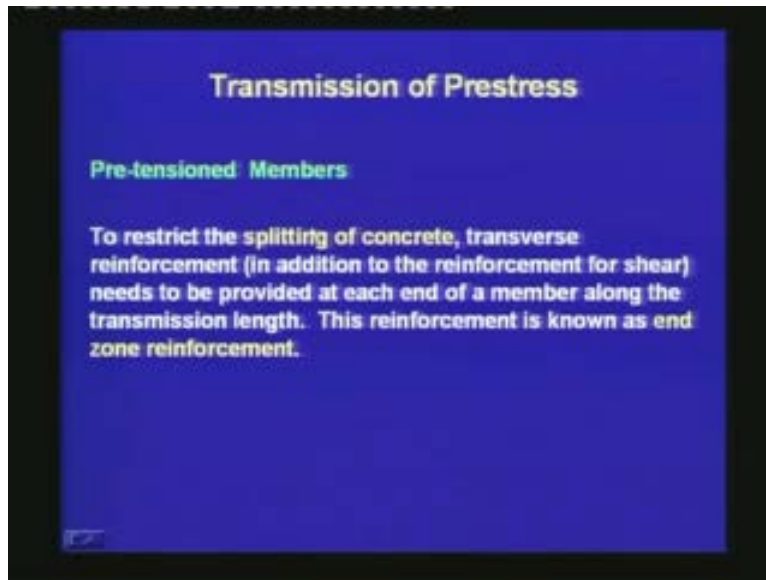
The prestress and the Hoyer effect cause transverse tensile stress, which is denoted as σ_t . This is largest during the transfer of prestress. Since there is a wedge effect, which is known as the Hoyer effect, tensile stress develops in the perpendicular (transverse) direction to the axis of the beam. The tensile stress can cause cracking in the member. We may have to provide reinforcement depending on the amount of stress developed, to check the cracking. The following sketch shows the theoretical variation of σ_t .

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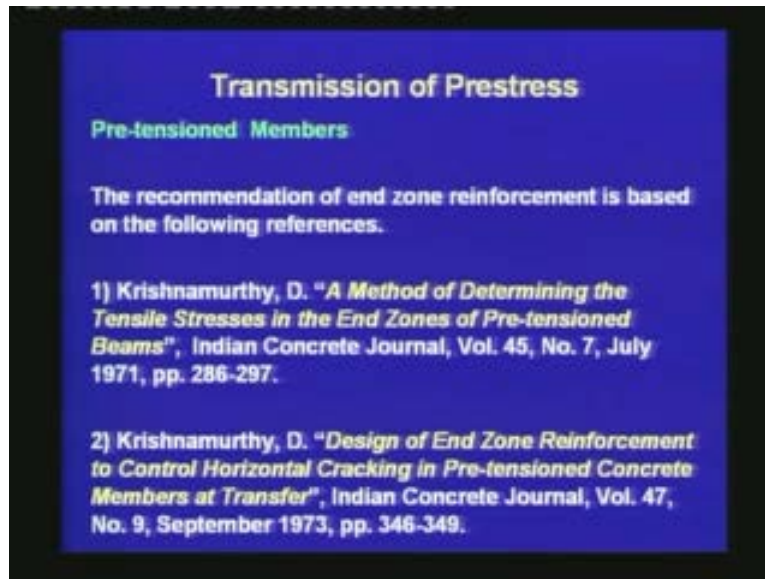
When the prestress is transferred to the concrete, transverse tensile stress develops which is maximum at the end of the member, and it drops down over the transmission length. Beyond the transmission length, the stress is practically zero. Unless reinforcement is provided, there is a chance that this will lead to cracking at the end of the member.

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To restrict the splitting of concrete, transverse reinforcement (in addition to the reinforcement for shear) needs to be provided at each and of a member along the transmission length. This reinforcement is known as end zone reinforcement.

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The recommendation of the end zone reinforcement is based on the following two references. Both the references are written by Professor D. Krishnamurthy. The title of the first reference is "A Method of Determining the Tensile Stresses in the End Zones of Pre-tensioned Beams". This was published in the Indian Concrete Journal, Volume 45, Number 7, July 1971, and the pages are from 286 to 297. The title of the second reference is "Design of End Zone Reinforcement to Control Horizontal Cracking in Pre-tensioned Concrete Members at Transfer". This was also published in the Indian Concrete Journal, Volume 47, Number 9, September 1973, and the pages are from 346 to 349.

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Transmission of Prestress

Pre-tensioned Members

The generation of the transverse tensile stress can be explained by the free body diagram of the following zone below crack. Tension (T), compression (C) and shear (V) are generated due to the moment acting on the horizontal plane at the level of the crack.



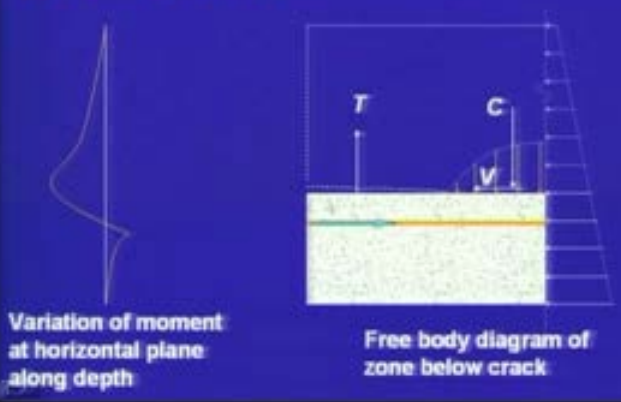
The diagram shows a horizontal beam with a crack. A yellow shaded rectangular zone is highlighted below the crack, representing the transmission length. A dimension line below this zone is labeled L_t .

The generation of the transverse tensile stress can be explained by the free body diagram of the zone below a crack. The tension, compression and shear are generated due to the moment acting on the horizontal plane at the level of the crack. The length of the zone is the transmission length.

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Transmission of Prestress

Pre-tensioned Members



The left diagram shows a parabolic curve representing the variation of moment along the depth of the beam. The right diagram is a free body diagram of a zone below a crack, showing tension (T), compression (C), and shear (V) forces.

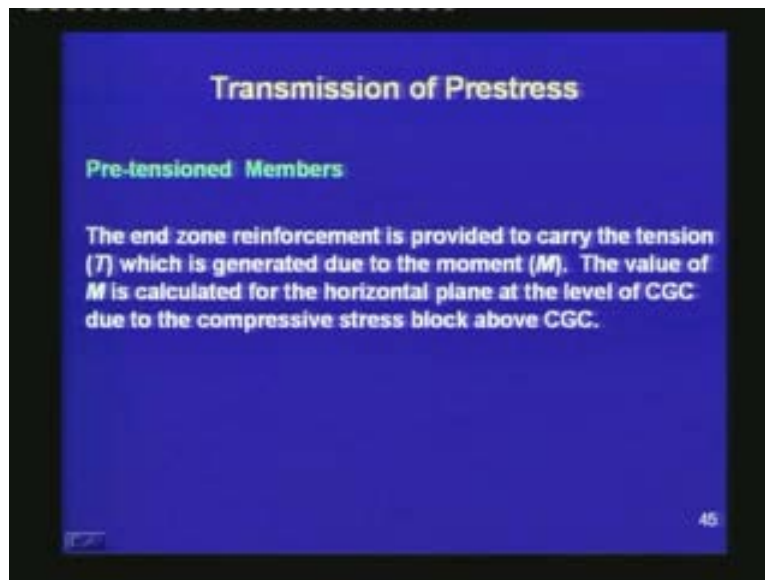
Variation of moment at horizontal plane along depth

Free body diagram of zone below crack

The moment acting in the horizontal plane generates a force couple. There is tension near the edge, which leads to the cracking, and there is compression away from the edge. There is also a shear force along the horizontal plane.

The tension leads to splitting crack in the concrete. With this free body diagram, based on the moment that acts in the horizontal plane, an expression of the amount of reinforcement has been suggested.

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The end zone reinforcement is provided to carry the tension T . The value of the moment M is calculated for the horizontal plane at the level of CGC due to the compressive stress block above CGC. Thus, the level of the CGC is selected as the level of probable crack, and at that level the moment is calculated.

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Transmission of Prestress

Pre-tensioned Members

The minimum amount of end zone reinforcement (A_{st}) is given in terms of the moment (M) as follows.

$$A_{st} = \frac{2.5M}{f_s h} \quad (7a-3)$$

The allowable stress in the transverse reinforcement (f_s) is selected based on a maximum strain.

The minimum amount of end zone reinforcement (A_{st}) is given in terms of the moment as follows. $A_{st} = 2.5M/f_s h$. Here, f_s is the allowable stress in the transverse reinforcement, and it is selected based on a maximum strain. That means, if we allow a strain up to say 0.001, then $f_s = 0.001E_s$ where E_s is the modulus of the transverse reinforcement.

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Transmission of Prestress

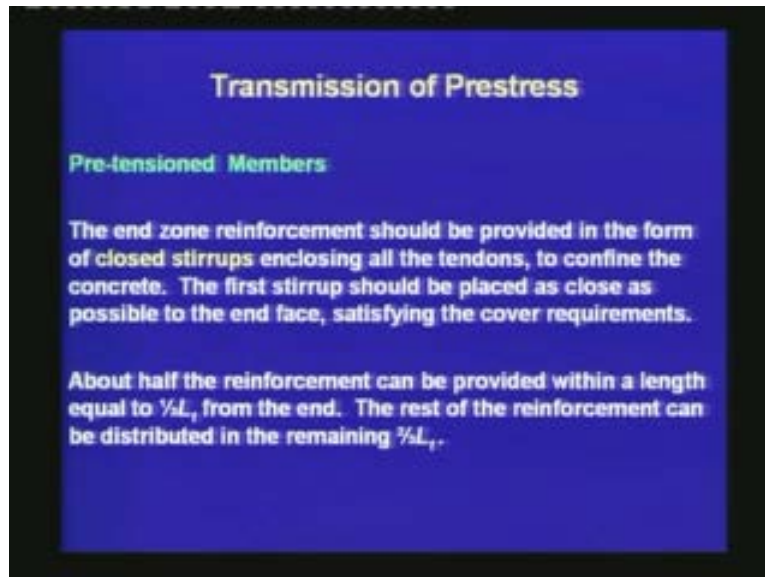
Pre-tensioned Members

In the previous equation,

h = total depth of the section
 M = moment at the horizontal plane at the level of CGC due to the compressive stress block above CGC
 f_s = allowable stress in end zone reinforcement.

In the previous equation, h is the total depth of the section; M is the moment at the horizontal plane at the level of CGC due to the compressive stress block above CGC; and f_s is the allowable stress in end zone reinforcement.

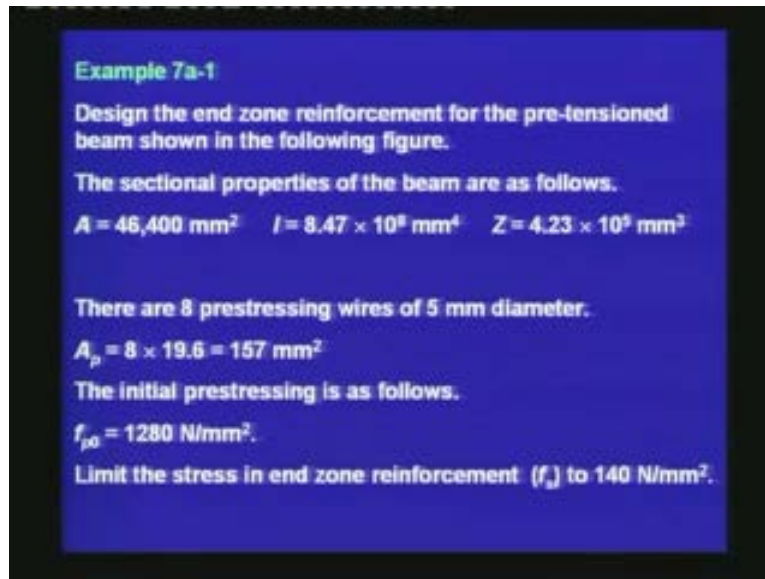
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The end zone reinforcement should be provided in the form of closed stirrups enclosing all the tendons, to confine the concrete. Earlier it was observed that the transverse stress is maximum at the end. Hence, the first stirrup should be placed as close as possible to the end, satisfying the cover requirements.

The stirrup should be closed which confines the concrete, and helps to check the formation of splitting cracks. About half of the reinforcement can be provided within a length equal to $\frac{1}{3}L_t$ from the end. The rest of the reinforcement can be distributed in the remaining $\frac{2}{3}L_t$. Thus, since the stress varies quite rapidly at the end, half of the calculated end zone reinforcement is provided in the first $\frac{1}{3}L_t$, and the other half in the remaining $\frac{2}{3}L_t$.

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Example 7a-1

Design the end zone reinforcement for the pre-tensioned beam shown in the following figure.

The sectional properties of the beam are as follows.

$A = 46,400 \text{ mm}^2$ $I = 8.47 \times 10^8 \text{ mm}^4$ $Z = 4.23 \times 10^5 \text{ mm}^3$

There are 8 prestressing wires of 5 mm diameter.

$A_p = 8 \times 19.6 = 157 \text{ mm}^2$

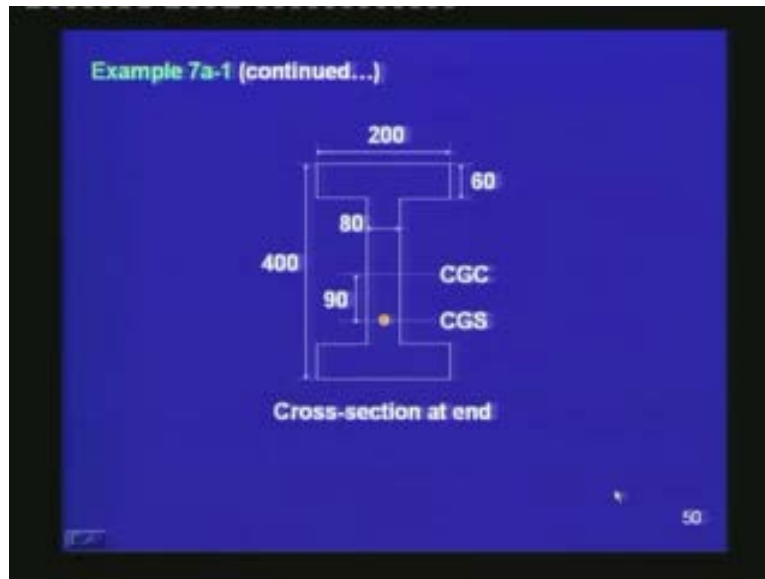
The initial prestressing is as follows.

$f_{p0} = 1280 \text{ N/mm}^2$.

Limit the stress in end zone reinforcement (f_s) to 140 N/mm^2 .

Let us understand the design of the end zone reinforcement by the help of an example. Design the end zone reinforcement for the pre-tensioned beam shown in the following figure. The sectional properties of the beam are as follows. Area $A = 46,400 \text{ mm}^2$, moment of inertia $I = 8.47 \times 10^8 \text{ mm}^4$, section modulus $Z = 4.23 \times 10^5 \text{ mm}^3$. There are eight prestressing wires of 5 mm diameter. Thus, $A_p = 8 \times 19.6 = 157 \text{ mm}^2$. The initial prestressing is as follows: $f_{p0} = 1280 \text{ N/mm}^2$. Limit the stress in end zone reinforcement (f_s) to 140 N/mm^2 .

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The end zone reinforcement is being designed for the prestress at transfer. The section is symmetric about the horizontal axis and the CGC is located at half the depth. The flange width is 200 mm, the flange depth is 60 mm, the breadth of the web is 80 mm, the total depth h is equal to 400 mm, and the eccentricity of the CGS at the end is 90 mm. Given the amount of prestress at transfer, the end zone reinforcement needs to be designed for the section.

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Solution

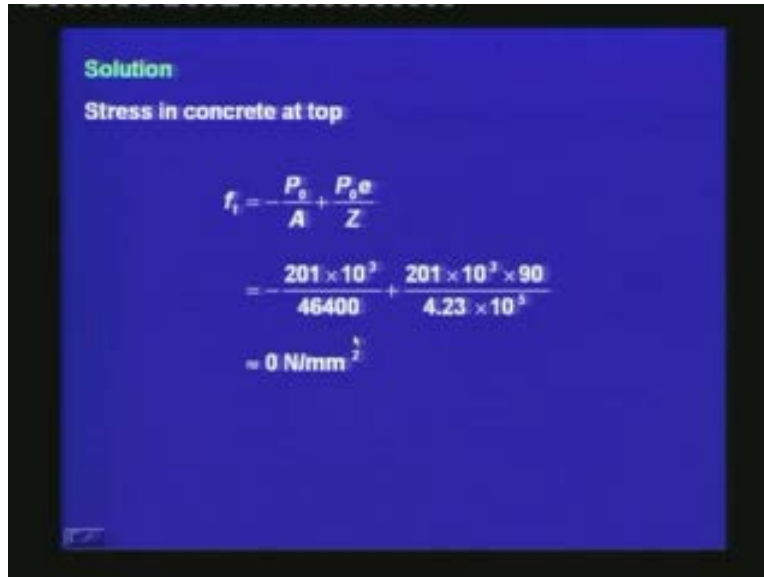
1) Determination of stress block above CGC

Initial prestressing force:

$$P_0 = A_p f_{po}$$
$$= 157 \times 1280 \text{ N}$$
$$= 201 \text{ kN}$$

The first step is to determine the stress block above CGC. The moment in the horizontal plane depends on the part of the stress block which comes above the level of CGC. The initial prestressing force is $P_0 = A_p f_{p0} = 157 \times 1280 \text{ N} = 201 \text{ kN}$.

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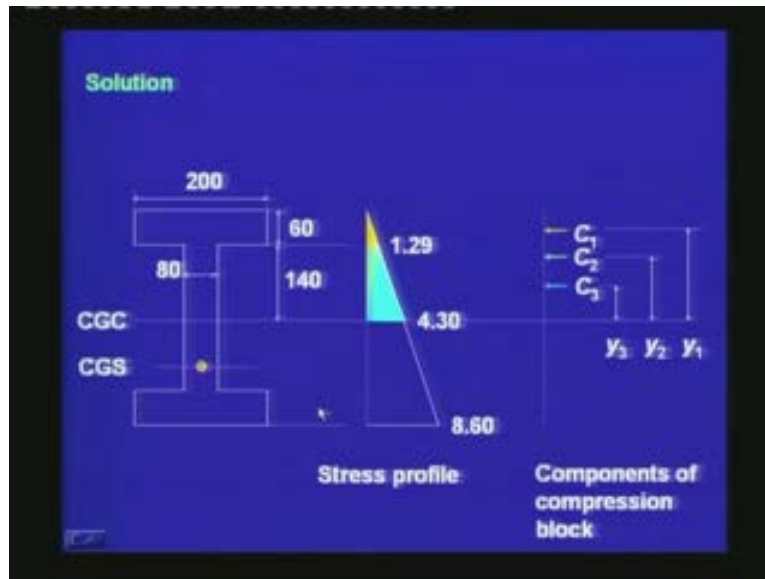
Solution

Stress in concrete at top:

$$f_t = -\frac{P_0}{A} + \frac{P_0 e}{Z}$$
$$= -\frac{201 \times 10^3}{46400} + \frac{201 \times 10^3 \times 90}{4.23 \times 10^3}$$
$$\approx 0 \text{ N/mm}^2$$

The stress in concrete at the top is given by the expression $f_t = -P_0/A + P_0e/Z$. The first term is negative because it is uniform compression due to the prestressing force. The second term is positive for the top, because the moment due to the prestressing force causes tensile stress at the top. Substituting the values of P_0 , e , A and Z , the stress at the top is practically 0 N/mm^2 .

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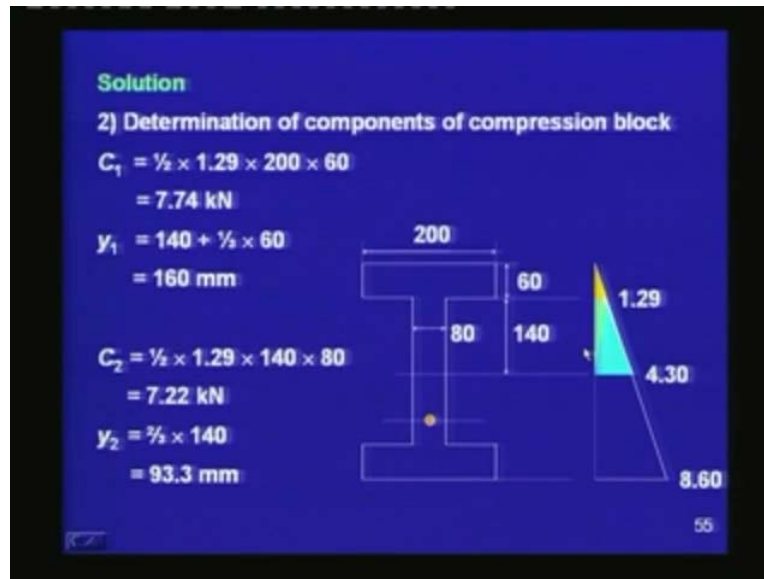


The stress in the bottom is also calculated by a similar expression. The second term is now negative because the moment due to the prestressing force is creating a compression at the bottom. Substituting the values of P_0 , e , A and Z , the stress at the bottom is 8.60 N/mm^2 .

Next, the moment in the horizontal plane at the level of CGC, is calculated due to the part of the compressive stress block which is above CGC. For this we are calculating two key values of the stress profile. One is at the junction of the flange and the web. From the triangular profile, the value of the stress is 1.29 N/mm^2 . The other is at the level of the CGC, where the value of the stress is 4.30 N/mm^2 .

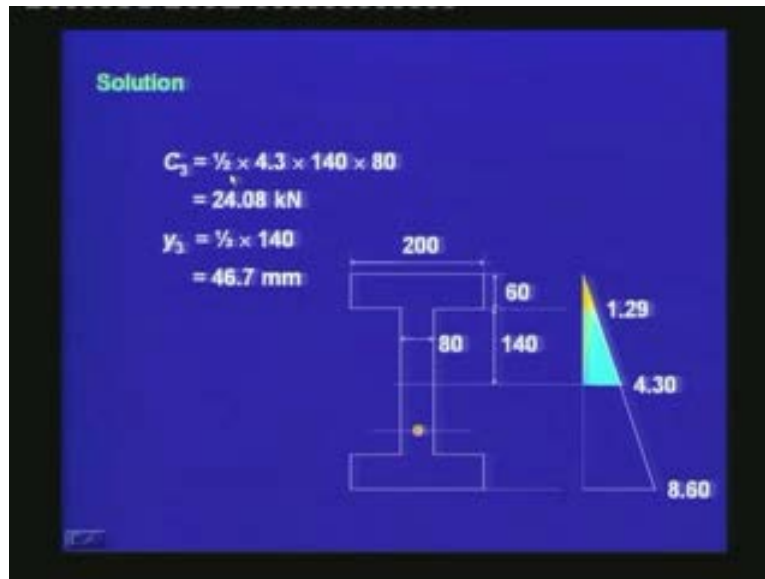
In order to calculate the moment, we are dividing the stress block into three parts: the top triangle shaded in orange is in the flange. For the part in the web, there are two triangles, one in green and the other in blue. For each triangle we can find out the resultant compressive force: C_1 corresponds to the resultant of the top triangle, C_2 corresponds to the resultant of the green triangle, and C_3 is the resultant of the blue triangle. We are also calculating the distances of the locations of C_1 , C_2 , C_3 from the level of CGC. These distances are denoted as y_1 , y_2 and y_3 . These distances are required to calculate the moment due to the stress block.

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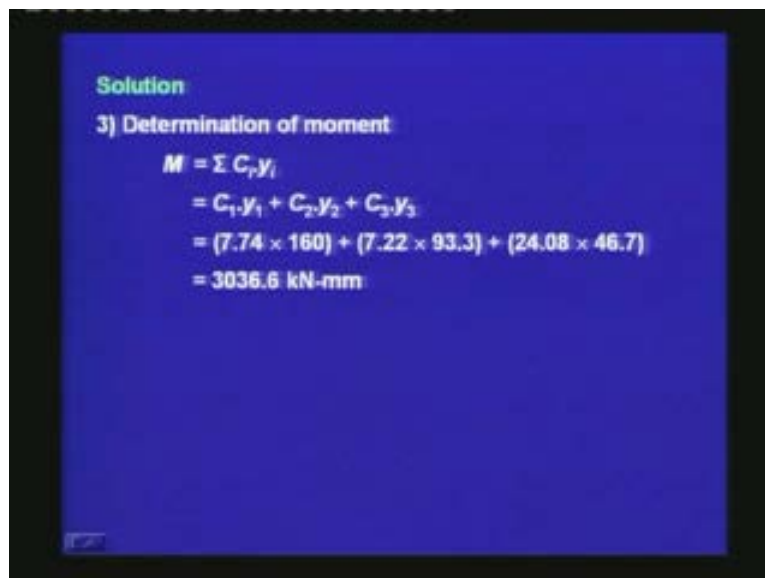
Our second step is the determination of components of the compression block. C_1 is equal to $1/2$ of the stress at the junction of the flange and web which is 1.29, times the area of the flange which is 200×60 . Thus, we get $C_1 = 7.74 \text{ kN}$. The distance of the location of C_1 from the CGC, y_1 is equal to $140 + \frac{1}{3} \times 60 = 160 \text{ mm}$. C_2 which is the area of the green triangle is equal to $\frac{1}{2} \times 1.29$ which is the base, times the altitude which is 140, times the breadth of the web which is 80. This gives $C_2 = 7.22 \text{ kN}$. The location of the centroid of the triangle is $\frac{2}{3} \times 140 = 93.3 \text{ mm}$ from the CGC.

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C_3 the area of the blue triangle is equal to $\frac{1}{2} \times 4.3$ times the area of the web which is equal to 140×80 . This gives $C_3 = 24.08 \text{ kN}$. The location of the centroid of the triangle $y_3 = \frac{1}{3} \times 140 = 46.7 \text{ mm}$.

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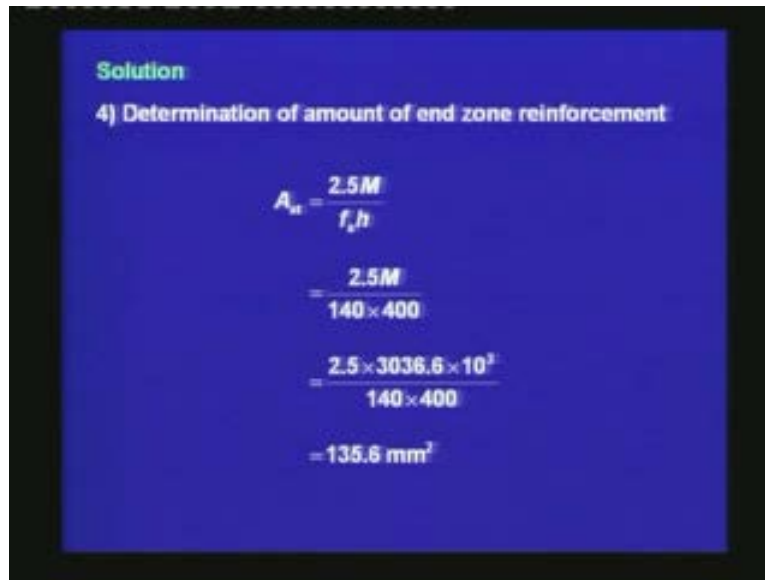


Our third step is to calculate the moment. The moment generated at the horizontal plane at the level of CGC due to the compressive stress block above CGC is given as a

summation of $C_i y_i$. Thus, $M = \Sigma C_i y_i = C_1 y_1 + C_2 y_2 + C_3 y_3$. Thus, we are finding out the moment due to the individual compressive forces and adding them up, to get the total moment above the horizontal plane.

Once we substitute the values of C_1 , C_2 , C_3 and y_1 , y_2 , y_3 we get $M = 3036.6 \text{ kNm}$. Thus, we found the moment which is generated due to the compressive stress block above the level of CGC.

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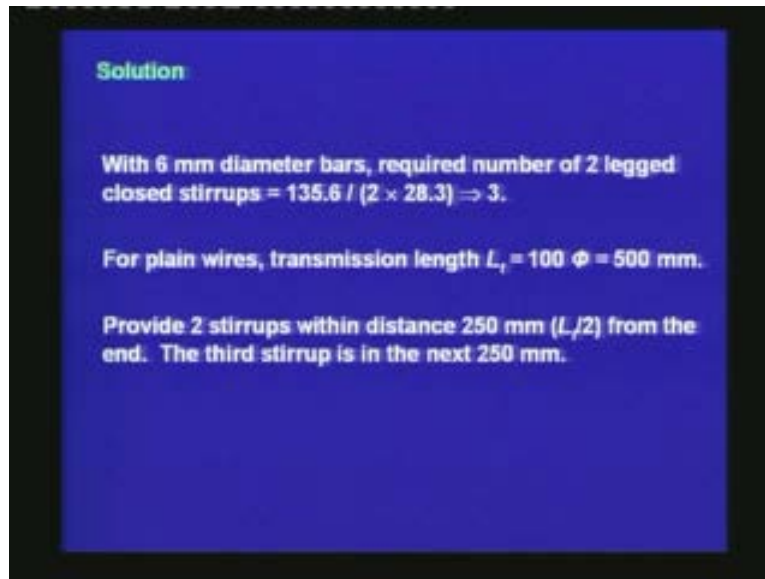
Solution

4) Determination of amount of end zone reinforcement

$$A_{st} = \frac{2.5M}{f_s h}$$
$$= \frac{2.5M}{140 \times 400}$$
$$= \frac{2.5 \times 3036.6 \times 10^3}{140 \times 400}$$
$$= 135.6 \text{ mm}^2$$

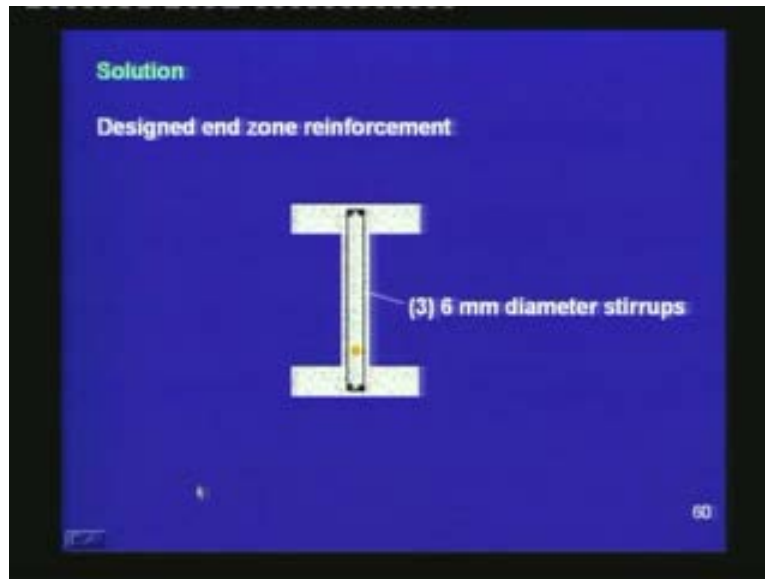
Our fourth step is to determine the amount of end zone reinforcement. A_{st} is given as $2.5M/f_s h$. The value of f_s has been selected to be 140 N/mm^2 and h is the total depth, which is 400 mm . M , we have just calculated, is $3036.6 \times 10^3 \text{ N}$. Thus, once we substitute the values, we get $A_{st} = 135.6 \text{ mm}^2$. Thus, this is the amount of end zone reinforcement that needs to be provided for this particular beam.

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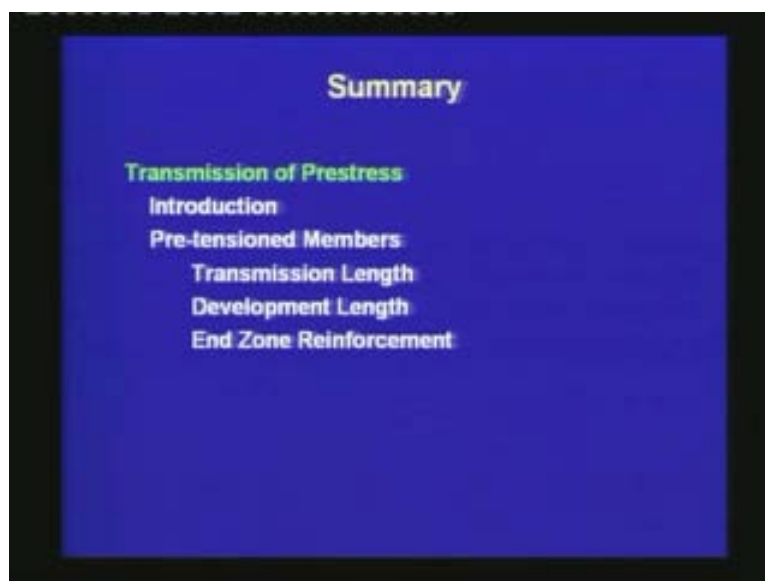
With 6 mm diameter bars, required number of 2 legged closed stirrups is equal to $135.6 / (2 \times 28.3)$, where 28.3 mm^2 is the area of 6 mm diameter bars, and when this is rounded off, we get 3 number of stirrups. This needs to be provided within the transmission length. For plain wires, the transmission length L_t is given as 100 times the diameter. Since we are using 5 mm wires, the transmission length is 500 mm. Provide 2 stirrups within distance $\frac{1}{2} L_t = 250$ mm from the end. The third stirrup is in the next 250 mm. To distribute the end zone reinforcement, we are providing more stirrups towards the end and less stirrup in the second half of the transmission length.

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The designed end zone reinforcement looks like this. We are providing 3 additional 6 mm diameter stirrups, which are closed and enclosing the wires. Out of the three, the first two are provided in the first half of the transmission length and the third one is provided in the second half. The first stirrup should be as close as possible to the end face.

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Thus in today's lecture, we studied the transmission of prestress and found that the transmission of prestress from the tendon to the concrete is different for pre-tensioned and post-tensioned members. In today's lecture, we covered the transmission of prestress for a pre-tensioned member. For such a member, since there is no anchorage device at the end. The prestress is transferred solely by the bond at the transmission length at each end of the member.

We first studied the mechanism of the bond that develops between the steel and the concrete. The tendons which have surface deformations have better bond. For those types of tendons, the transmission length is smaller. The transmission length depends on several factors: like the surface deformations, the size of the tendons, the stress to be developed and the compaction of the concrete. The code gives us a simple estimate of the transmission length, which depends only on the size of the tendons.

The transmission length is the length over which the prestress is transferred to the concrete, and the steel gains the stress from 0 to the effective prestress f_{pe} . The second concept that we studied for the development length, is required at the critical section of maximum moment, where at ultimate state the stress in the prestressing steel is f_{pu} . The development length consists of two parts, one is the transmission length and the second is the bond length. Along the bond length, the stress changes from the effective prestress f_{pe} to the ultimate prestress f_{pu} . The code gives us expressions for the bond length, which is in terms of f_{pe} , f_{pu} , the size of the tendons and a design bond stress. We need to make sure that the development length is provided at the location of the critical section.

The third concept we studied is the end zone reinforcement. There is a chance of cracking due to the transverse tensile stress. To check the cracking we may need to provide stirrups which are in addition to the shear reinforcement. We saw a simple expression to calculate the amount of end zone reinforcement. We studied the application of the expression through an example. In our next class, we shall move on to the other type of prestressed member which is the post-tensioned member.

Thank you