PRESTRESSED CONCRETE STRUCTURES

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Module – 6: Calculation of Deflection and Crack Width

Lecture-29: Calculation of Deflection and Crack Width

Welcome back to prestressed concrete structures. This is the lecture of Module 6 on calculation of deflection and crack width.

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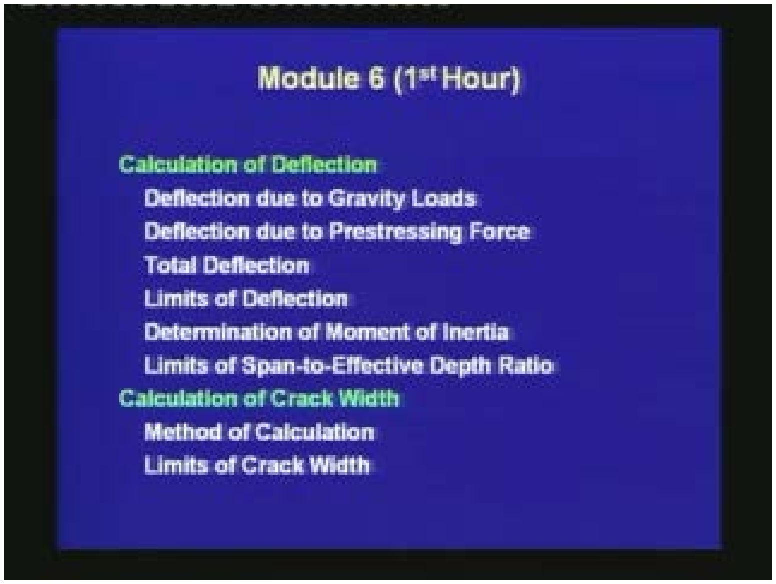

In this lecture, we shall first study the calculation of deflection. Under that we shall learn about the deflection due to gravity loads, deflection due to prestressing force, the calculation of total deflection, the limits of deflection, determination of moment of inertia and limits of span-to-effective depth ratio. Next, we shall study about the calculation of crack width, under which we shall study the method of calculation and the limits of crack width.

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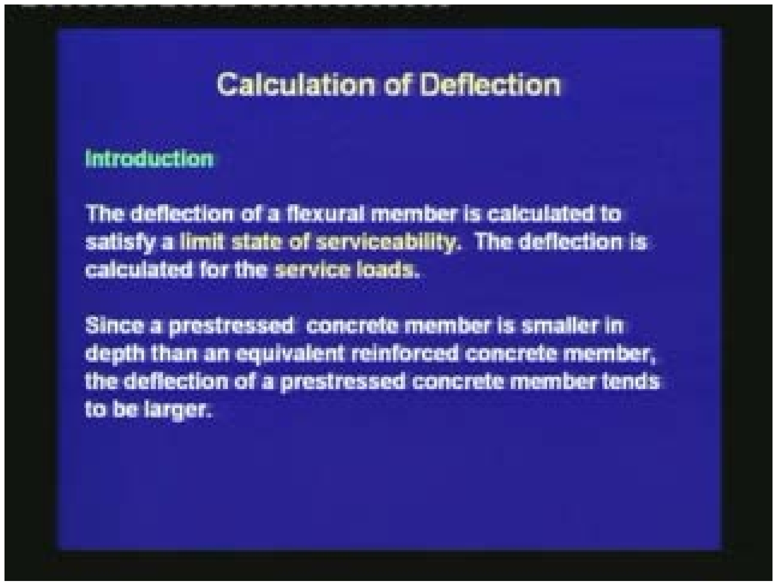

First is the calculation of deflection. The deflection of a flexural member is calculated to satisfy a limit state of serviceability. The deflection is calculated for the service loads. Since a prestressed concrete member is smaller in depth than an equivalent reinforced concrete member, the deflection of a prestressed concrete member tends to be larger. This is one drawback, that if very shallow members are used then there may be deflection problems. Hence the calculation of deflection becomes necessary, when the members are shallow. We have to make sure that the deflection does not cause any problem in the functioning of the member.

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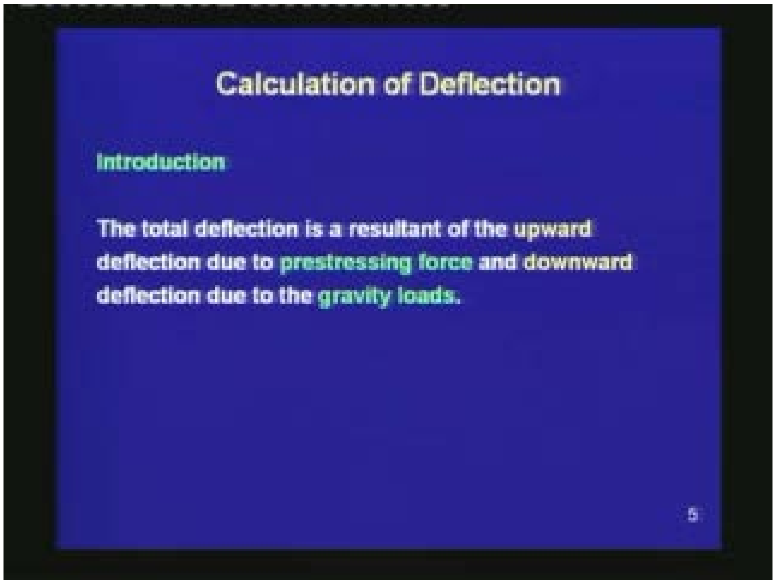

The total deflection in a prestressed concrete member is a resultant of the upward deflection due to prestressing force and downward deflection due to the gravity loads. Thus, unlike reinforced concrete member here we have another component of deflection, which is the deflection due to the prestressing force. For a simply supported beam, the prestressing tendon provides an upward thrust to the member, which results in hogging during the prestressing of the member. The total deflection is a summation of the upward deflection due to the prestressing force and the downward deflection due to the gravity loads.

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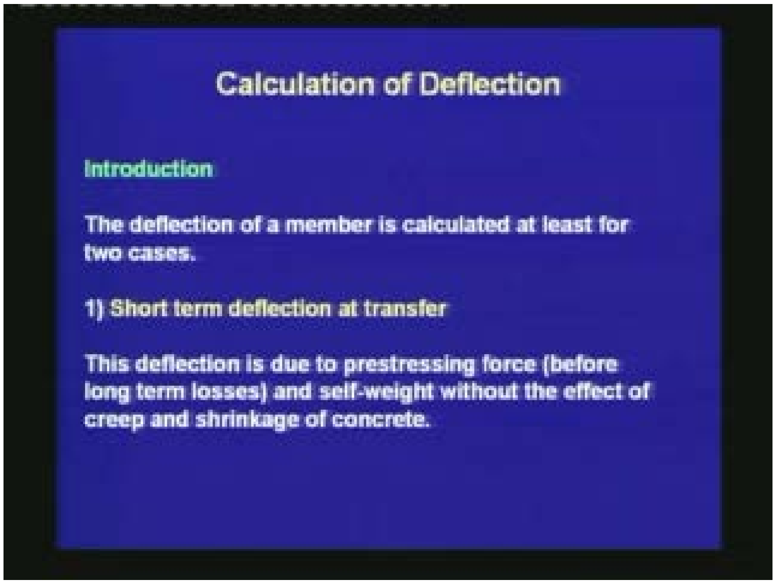

The deflection of a member is calculated at least for two cases. First is the short term deflection at transfer. This deflection is due to prestressing force before long term losses, and self-weight without the effect of creep and shrinkage of concrete. Thus, when the prestress is transferred the member may have a resultant upward deflection (camber) depending on the amount of prestressing force and the self weight of the member. It has to be checked, whether the camber is too much during the prestressing operation or not. If there is some finishing over the member then, it has to be checked whether there will be any cracking on the finishes or not.

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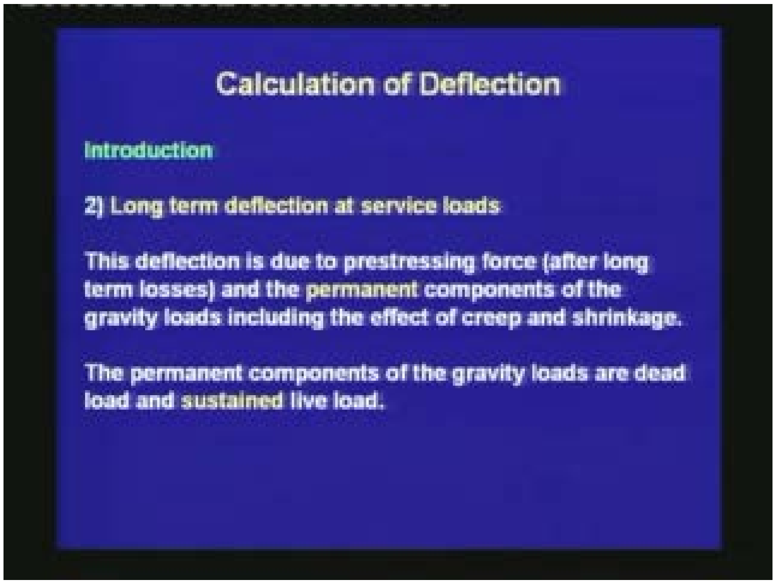

The second case is the long term deflection at service loads. This deflection is due to prestressing force after long term losses, and the permanent components of the gravity loads, including the effects of creep and shrinkage. Thus, at service loads the deflection has two components. One is due to the prestressing force after the long term losses, and next is that due to the gravity loads. For the later, only the permanent components of the gravity loads are considered in the long term deflections. The permanent components of the gravity load include the dead load and sustained component of the live load.

Now, here comes an engineering judgment; that how much live load should be considered to be sustained, that depends upon the analyst. In the permanent load, we are including only the sustained part of the live load and not the total live load. If we are interested in the total deflection, then we may include the deflection due to the additional component of the live load as well. First, we shall see the calculation of deflection due to gravity loads.

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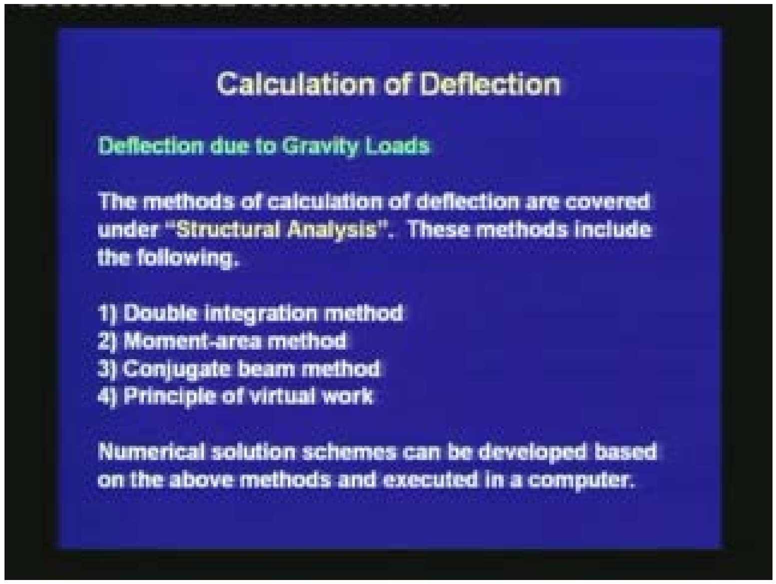

The methods of calculation of deflection are covered under structural analysis. These methods include the following: double integration method, moment–area method, conjugate beam method and principle of virtual work. A student should have studied these methods in a course on structural analysis. In this lecture, we are not going into the details of the evaluation of deflection based on these methods, but we shall consider the end results from these methods. Numerical solutions schemes can be implemented in a computer, based on the above methods.

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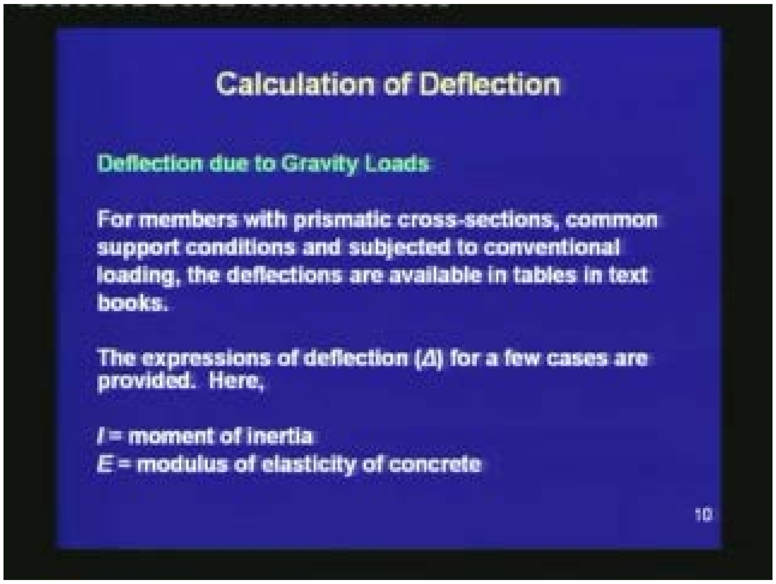

For members with prismatic cross-sections (that is, the cross-section is constant throughout the span of a member), common support conditions and subjected to conventional loading, the deflections are available in tables in text books. The expressions of deflection (which is represented as Δ) for a few cases are provided here. I is the moment of inertia of the section, and E is the modulus of elasticity of concrete. What is the value of the moment of inertia we need to consider, that we shall discuss later.

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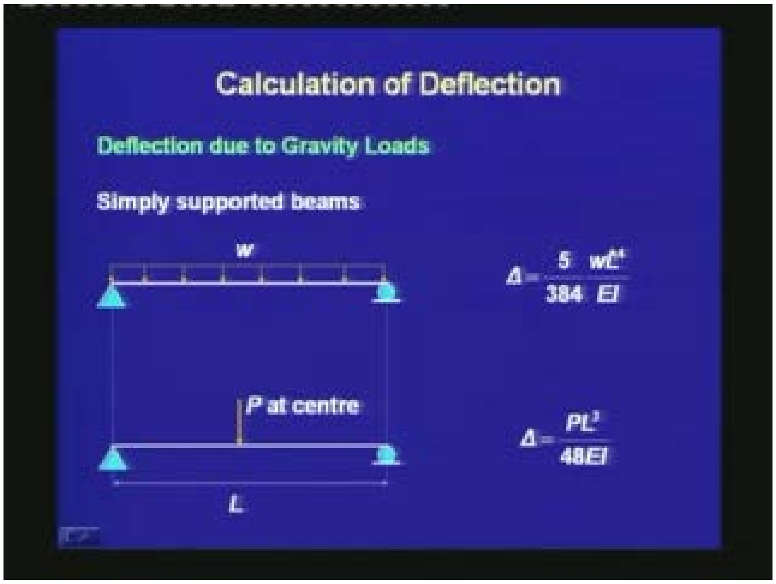

In this figure, we can see two simply supported beams. For the top one, the beam is subjected to a uniformly distributed load and the deflection at the centre is given as $\Delta = (5/384) \times wL^4/EI$. In the figure at the bottom, a beam is subjected to a point load at the centre, and $\Delta = PL^3/48EI$.

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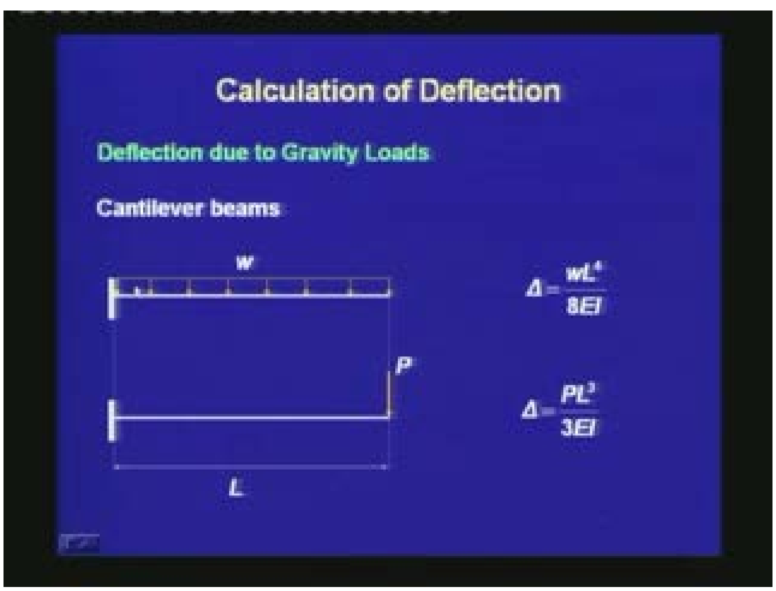

For a cantilever beam under a uniformly distributed load, the deflection at the end is given as $\Delta = wL^4/8EI$. For a cantilever beam with a point load at the end, $\Delta = PL^3/3EI$. In most of the prestressed concrete applications, the beams are simply supported. Hence, we can use the standard expressions for simply supported beams, for calculating the deflections due to the gravity loads. Next, we are calculating the deflections due to prestressing force.

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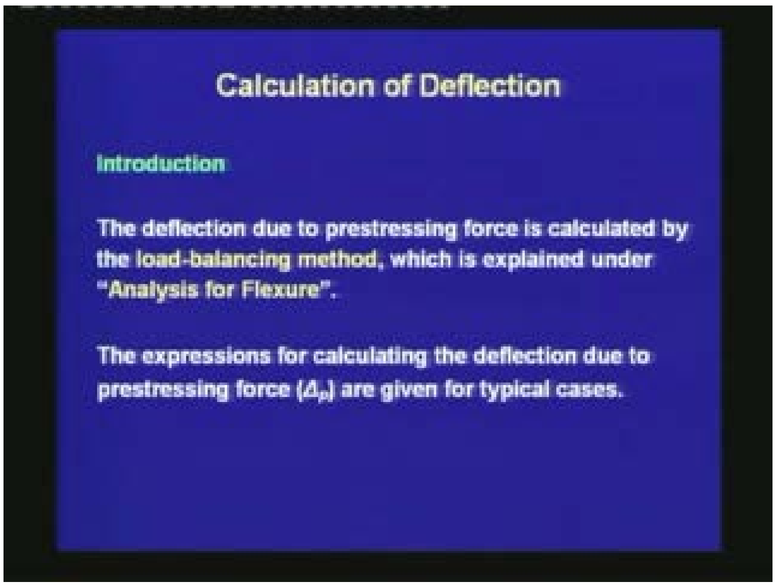

The deflection due to prestressing force is calculated by the load-balancing method, which is explained under "Analysis for Flexure". Earlier, we had seen that there are three methods of analysis of a prestressed member. First is based on the stress concept, the second is based on the force concept and the third is based on the load-balancing method. Now, the third method is used to calculate the deflection due to prestressing force. Here, we shall see the expressions of the deflections due to prestressing force for the standard cases. The deflection due to prestressing force is represented as Δ_p.

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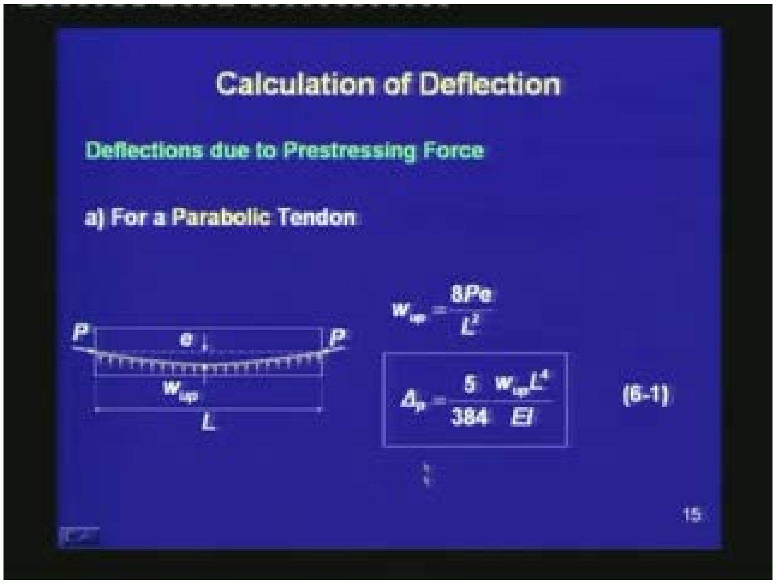

For a parabolic tendon with prestressing force P and eccentricity 'e' at the middle, there is a uniform upward load, which is represented as w_{up}. The span of the beam is equal to L. The upward load is given as $w_{up} = 8Pe/L^2$. Then, we can calculate the upward deflection. $\Delta_p = (5/384)\, w_{up}L^4/EI$. Thus, this is the expression of upward deflection of a beam that is prestressed with a parabolic tendon.

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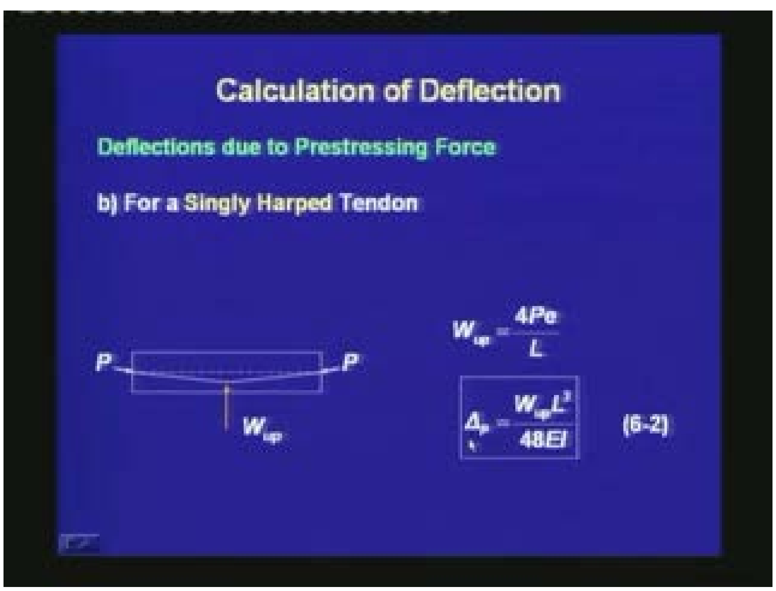

For a singly harped tendon, there is an upward force at the location of the harping, which is denoted as W_{up}. $W_{up} = 4Pe/L$. Then $\Delta_p = W_{up}L^3/48EI$. This is the expression of the deflection of a beam that is prestressed with a singly harped tendon, and the harping point is at the middle of the beam.

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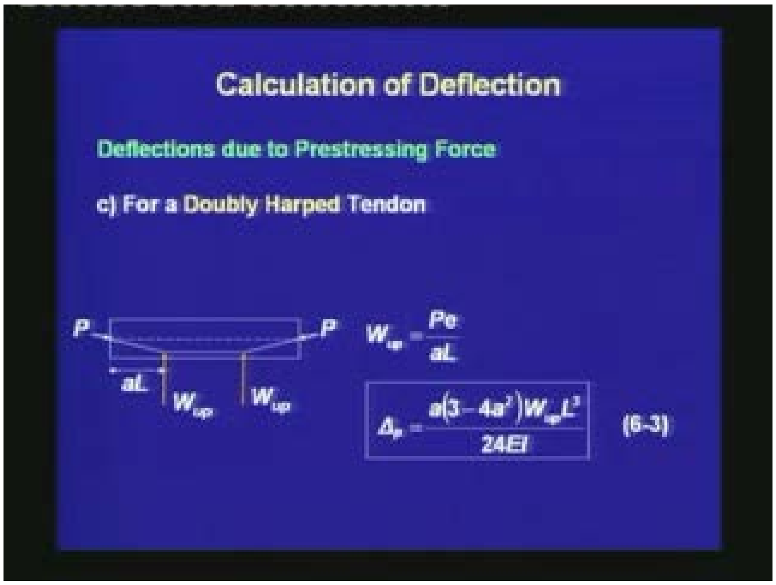

Next, we are going on to the expression for a beam with a doubly harped tendon. Here, the harping points are symmetric, and each harping point is at a distance aL from the support. In this figure, we see that there are two upward forces, corresponding to the two harping points, and each upward force is represented as W_{up}. $W_{up} = Pe/aL$. Then, $\Delta_p = a(3 - 4a^2)W_{up}L^3/24EI$.

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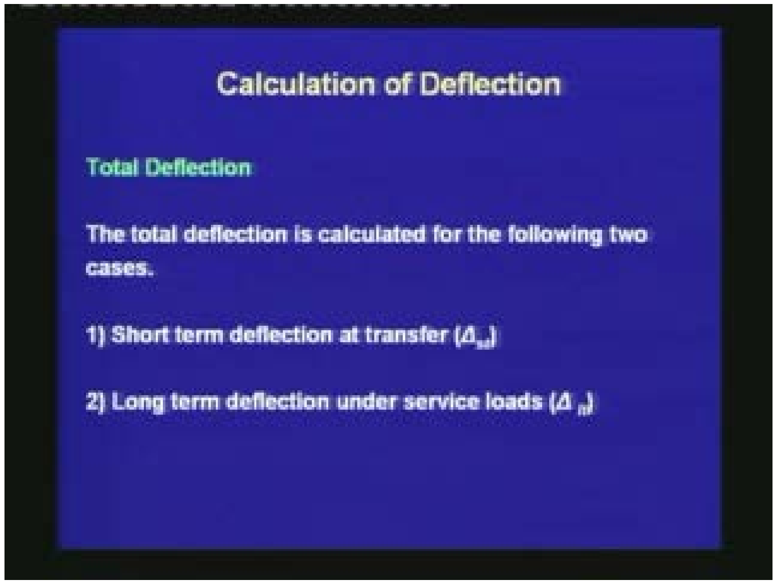

Next, we are calculating the total deflection due to the prestressing force and the gravity loads. As we said before, that the total deflection is calculated for the two cases: first, the short term deflection at transfer, which we shall denote as Δ_{st}, and the second is the long term deflection under service loads, which is denoted as Δ_{lt}.

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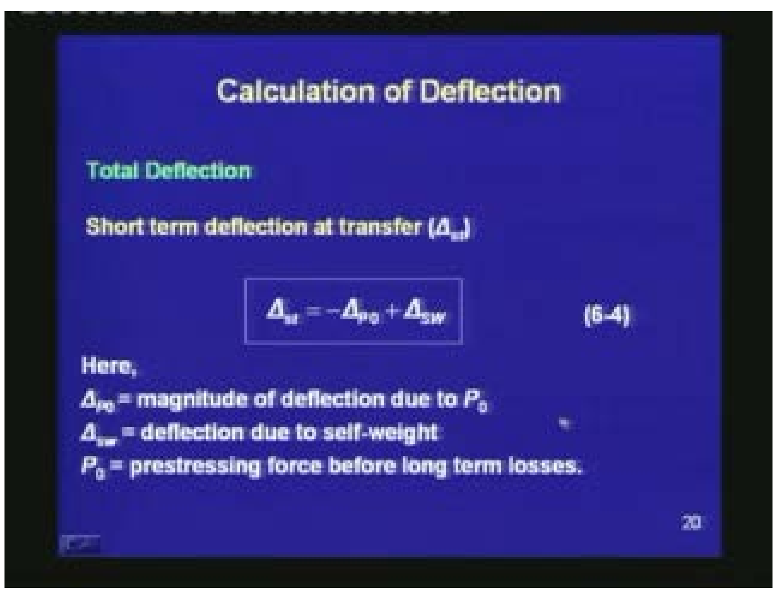

The short term deflection at transfer, $\Delta_{st} = -\Delta_{Po} + \Delta_{sw}$. Here, Δ_{Po} is the magnitude of deflection due to the prestress at transfer (P_o) which is before the long term losses. Δ_{sw} is the deflection due to the self-weight, which is downwards. Note, that the sign of the two deflections are opposite. A negative sign has been placed for the deflection due to the prestressing force, since this is upwards. Thus, in presence of the prestressing force, the total deflection may become negative, if the value of Δ_{sw} is numerically smaller than Δ_{po}. In that case, the beam will have a camber and it will deflect upwards.

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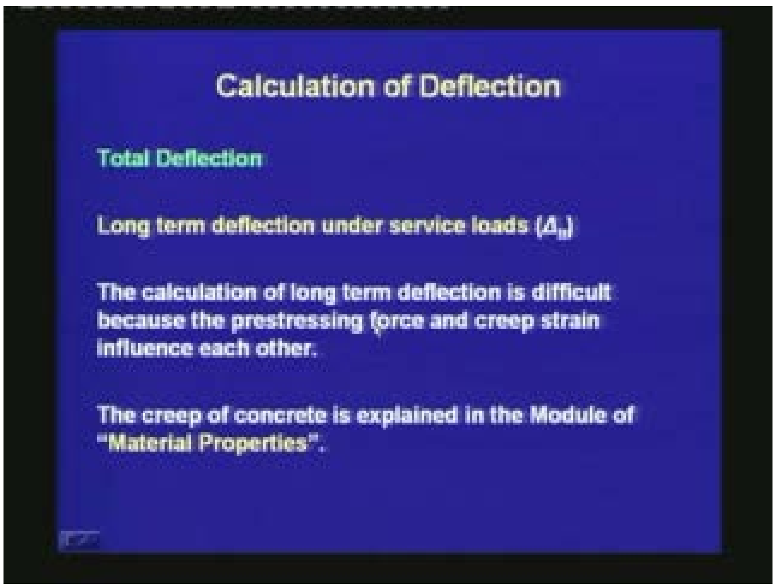

Now, we are calculating the long term deflection under service loads. The calculation of long term deflection is difficult because the prestressing force and creep strain influence each other. The creep of concrete is explained in the module of material properties.

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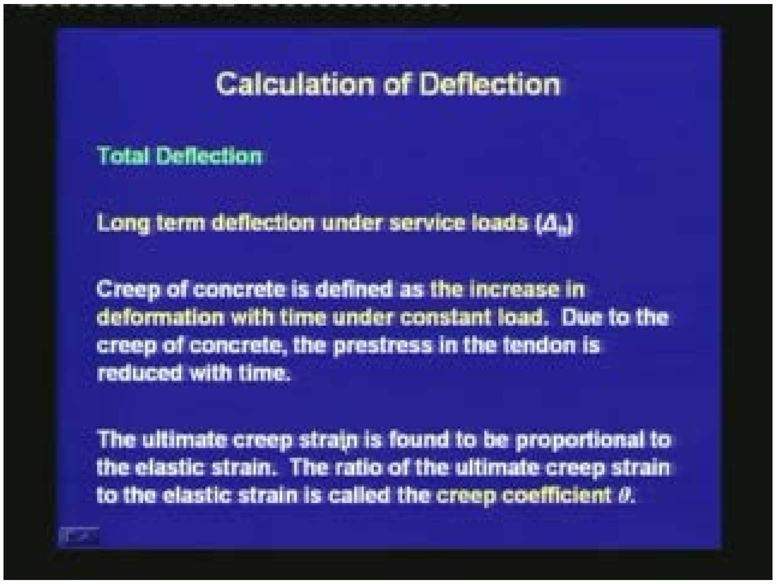

The creep of concrete is defined as the increase in deformation with time under constant load. Due to the creep of concrete, the prestress in the tendon is reduced with time. This is an important aspect in the calculation of long term deflections that the concrete deforms with time due to the permanent load, and due to the deformation of the concrete the prestressing force gets reduced. There is a loss in the prestressing force. Thus, creep and the prestressing force influence each other. Hence, the exact calculation of long term deflection gets difficult.

The creep was discussed in detail in the module of material properties. Here we are having a quick review of how to measure the creep strain in concrete. The ultimate creep strain is found to be proportional to the elastic strain. The ratio of the ultimate creep strain to the elastic strain is called the creep coefficient θ.

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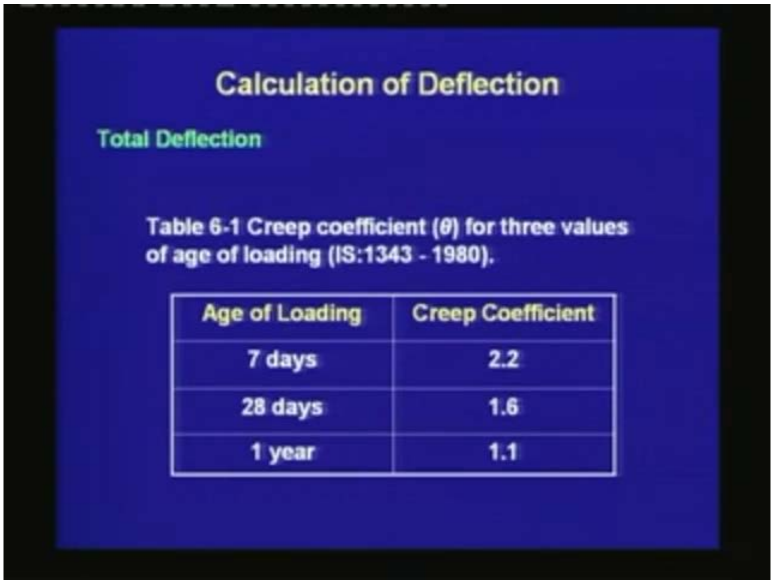

The creep coefficient θ, for three values of age of concrete at prestressing (termed as age of loading, or age of prestressing) as per the code IS: 1343–1980 is given in the table. At seven days of prestressing, the creep coefficient is 2.2; that means if the prestressing force is transferred when the concrete age is seven days then the creep strain is 2.2 times the elastic strain. If the prestress is transferred at twenty eight days, then the creep coefficient is 1.6. Thus, the ultimate creep strain is reduced to 1.6 times the elastic strain. Finally, if the transfer of prestress is at one year, then the creep coefficient is 1.1. That is, the ultimate creep strain is 1.1 times the elastic strain.

In order to reduce the long term deflection, we should delay the application of the prestressing force such that the concrete gains adequate strength, and the effect of creep is reduced. In this table the only factor which has been considered in evaluating the creep strain is the age of loading for the concrete. There are other factors which influence creep. In case if more accurate evaluation of creep is necessary, with the time as a variable, then we need to look into specialised literature.

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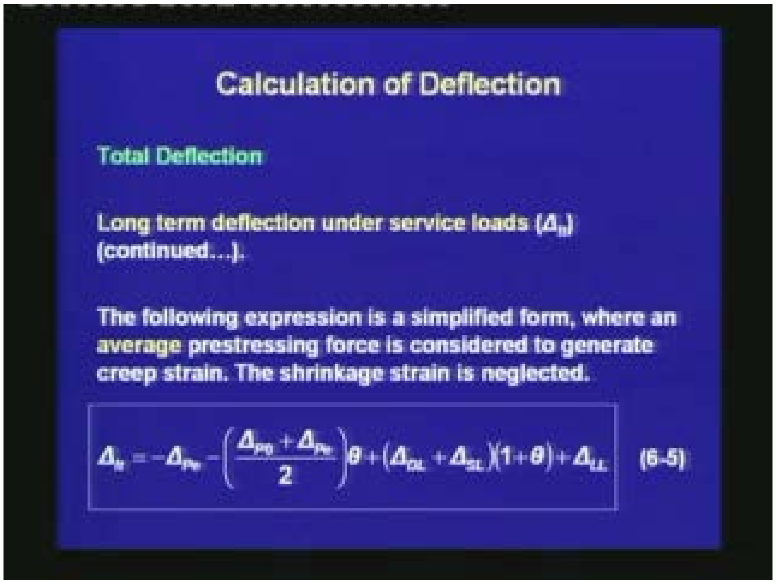

The following expression is a simplified form, where an average prestressing force is considered to generate creep strain. The shrinkage strain is neglected.

$\Delta_{lt} = -\Delta_{Pe} - (\Delta_{Po} + \Delta_{Pe})/2 \times \theta + (\Delta_{DL} + \Delta_{SL})(1 + \theta) + \Delta_{LL}$

This is an expression of the deflection due to long term loads, where we have considered an average value of the prestressing force which causes the creep.

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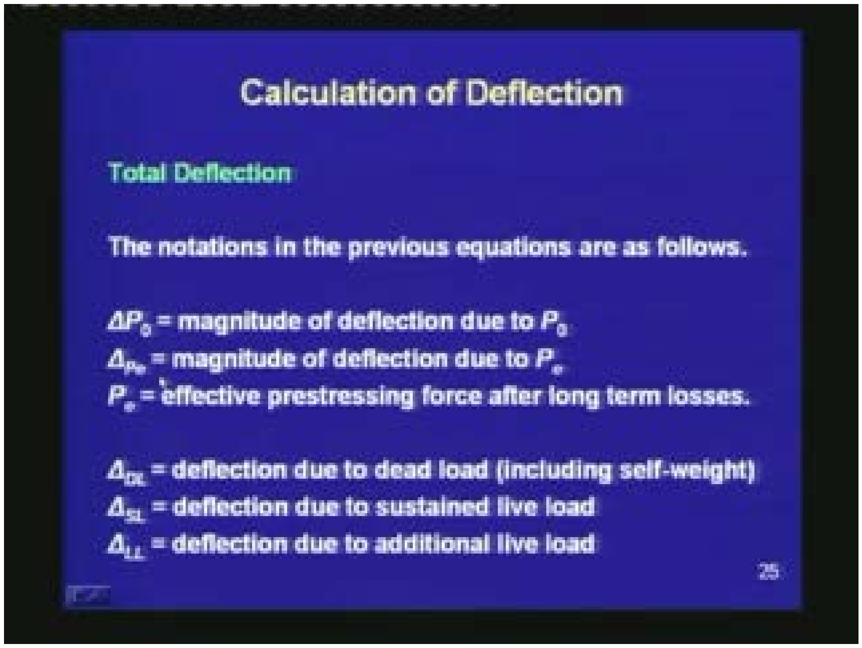

To summarize the notations in the previous expression:

Δ_{Po} = magnitude of deflection due to P_o, the prestress before long term losses;

Δ_{Pe} = magnitude of deflection due to P_e, where P_e is the effective prestress after long term losses;

Δ_{DL} = deflection due to the dead load, including self-weight;

Δ_{SL} = deflection due to sustained live load;

Δ_{LL} = deflection due to additional live load.

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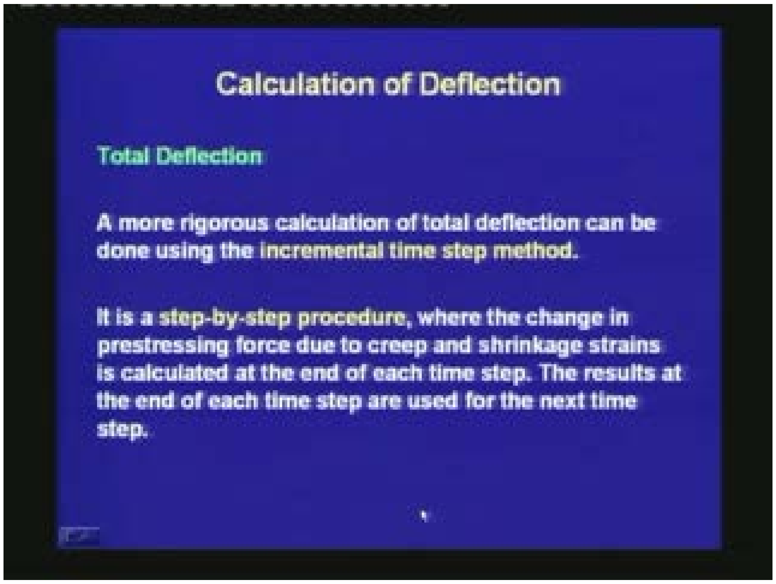

A more rigorous calculation of total deflection can be done using the incremental time step method. It is a step-by-step procedure, where the change in prestressing force due to creep and shrinkage strains is calculated at the end of each time step. The results at the end of each time step are used for the next time step.

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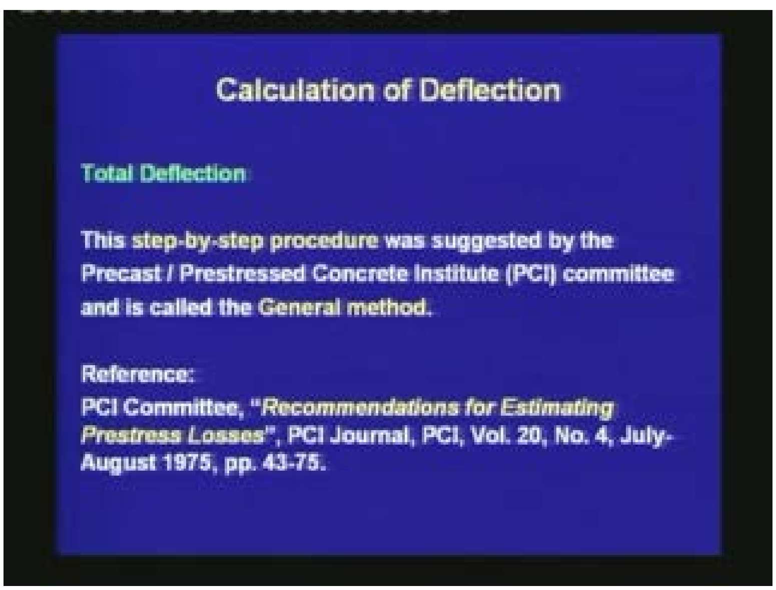

This step-by-step procedure was suggested by the Precast/Prestressed Concrete Institute (PCI) committee. The title of the paper is “Recommendations for Estimating Prestress Losses”. It was published in the PCI Journal, Volume 20, Number 4, and in the month of July to August, 1975. The pages are from 43 to 75. This method is called the General Method of calculating the prestressing force with time.

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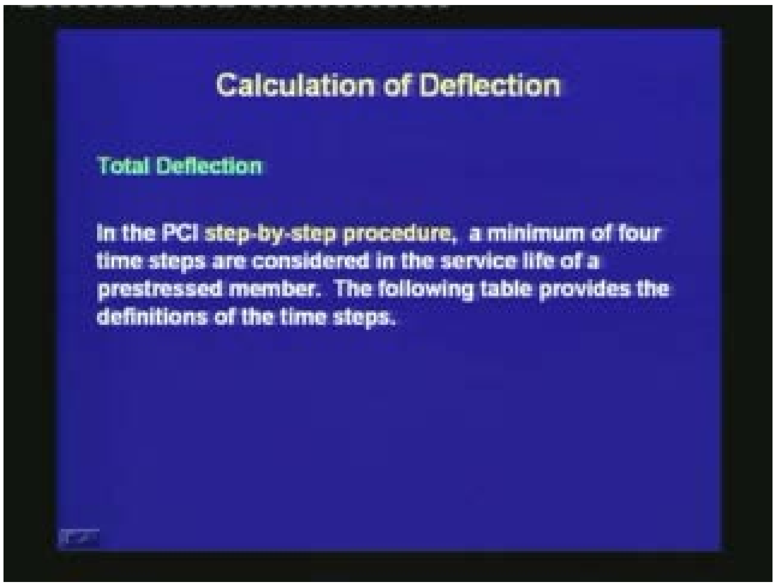

In this method, a minimum of four time steps are considered in the service life of a prestressed member. The following table provides the definitions of the time steps.

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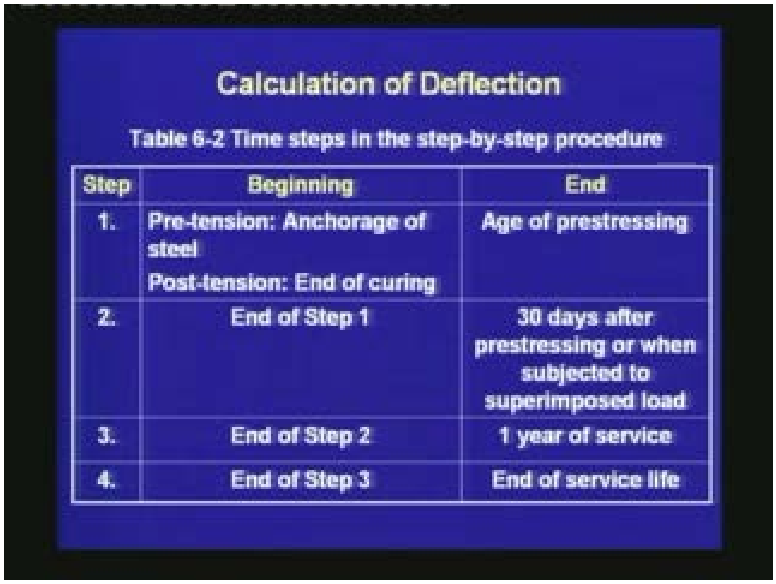

Step	Beginning	End
1.	Pre-tension: Anchorage of steel Post-tension: End of curing	Age of prestressing
2.	End of Step 1	30 days after prestressing or when subjected to superimposed load
3.	End of Step 2	1 year of service
4.	End of Step 3	End of service life

First, the time scale is discretised into four steps, and the discretization is based on the variation of the prestressing force with time. The method suggests that at least a minimum four times steps should be considered and those time steps are as follows. For a pre-tensioned member, the beginning of the time step is the anchoring of the steel. For a post-tensioned member, the beginning is the end of curing. The end of the first time step is the age of prestressing. Thus, within this period for the pre-tensioned member there can be some relaxation losses after the tension has been applied on the steel. For the post-tensioned member, there will be some shrinkage in the concrete which is neglected in the calculation of the loss of prestress. For the second time step, the beginning of the step is the end of the first time step, and the end of the second step is thirty days after prestressing or when subjected to superimposed load. Thus, the first one month after prestressing is important in the variation of the prestressing force, and the creep and shrinkage strains.

For the third time step, the beginning is the end of Step 2 and the end is one year of service. The fourth time step begins at the end of Step 3 and ends at the end of service life. Thus, these are the minimum four time steps that the committee recommended to monitor the prestressing force with time considering the creep and shrinkage strains in the concrete.

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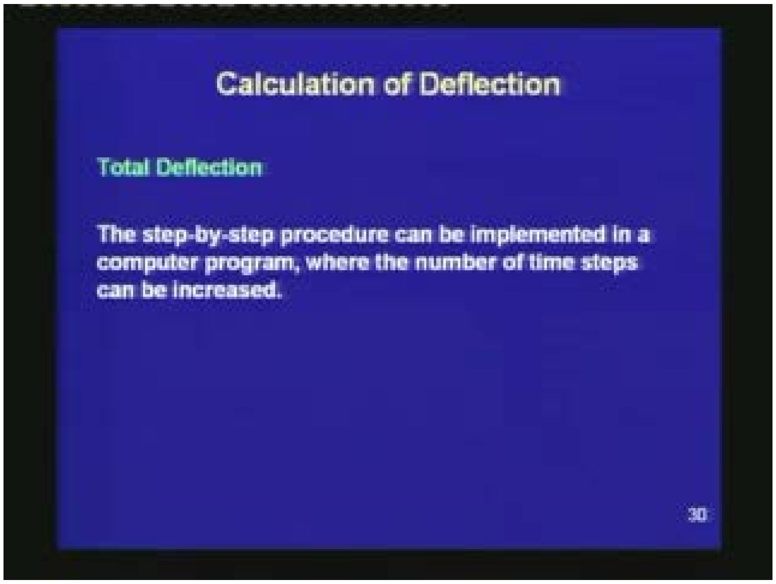

The step-by-step procedure can be implemented in a computer program, where the number of time steps can be increased. Thus, we may not stick to four time steps, we can have even larger number of time steps, which can be implemented in a computer program. In this method, we need more accurate expressions of the creep and shrinkage strains, which are functions of time.

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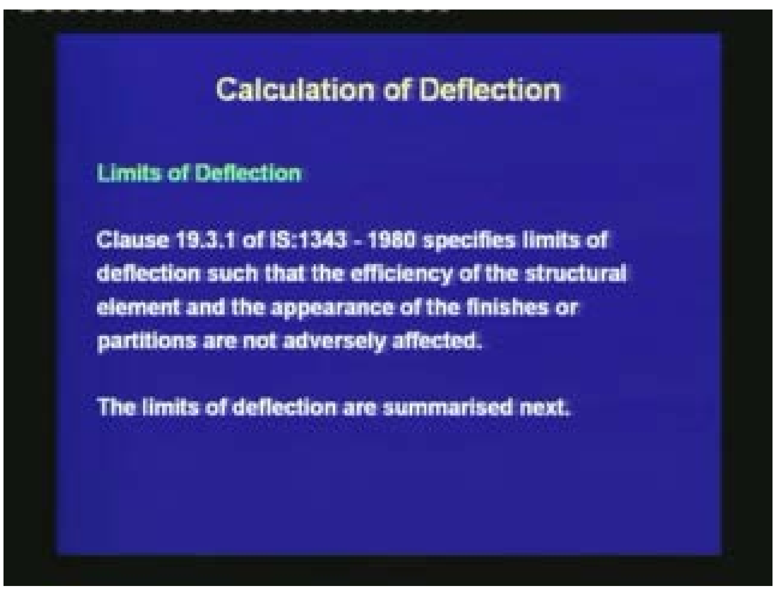

Next, we are calculating the limits of deflection. Clause 19.3.1 of IS: 1343–1980 specifies limits of deflection such that the efficiency of the structural element and the appearance of the finishes or partitions, are not adversely affected. The limits of deflection are summarized next.

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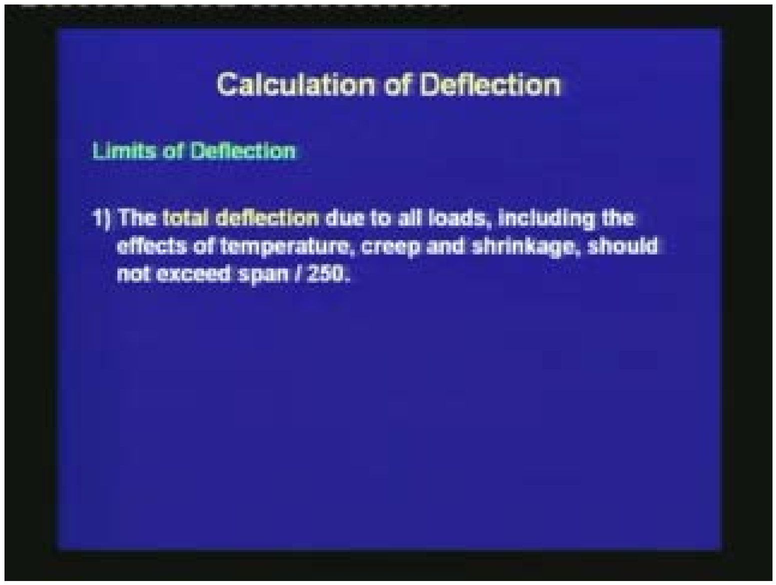

1) The total deflection due to all loads, including the effects of temperature, creep and shrinkage, should not exceed span divided by 250.

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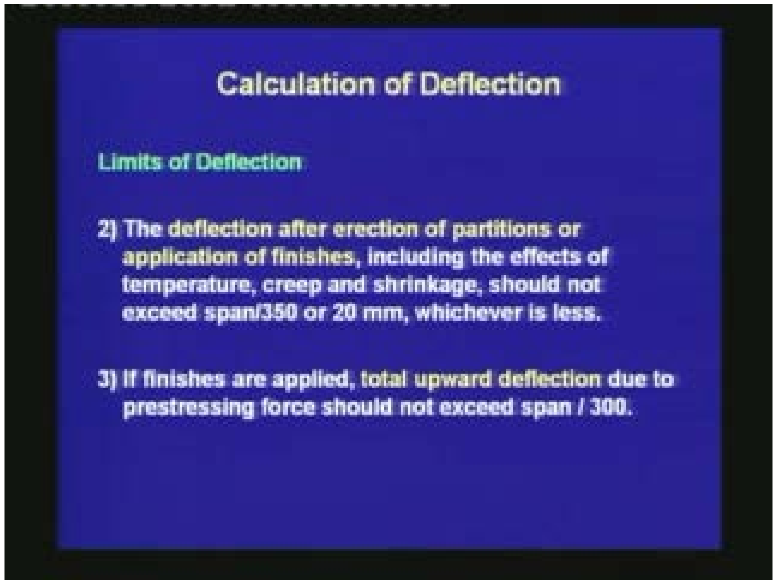

2) The next requirement is that the deflection after erection of partitions or application of finishes, including the effects of temperature, creep and shrinkage, should not exceed span divided by 350 or 20 mm, whichever is less. Thus, if there are partitions or finishes we may need to calculate deflections before the finishes or partitions are applied, because we are calculating the additional deflection after the partitions or finishes are placed.

3) The third limit is that if finishes are applied at the top of a beam, then the total upward deflection due to the prestressing force should not exceed span divided by 300.

These are the limits that the code specifies. For special structures, additional limits may be considered depending upon the situation. Next, we are moving on to the determination of moment of inertia.

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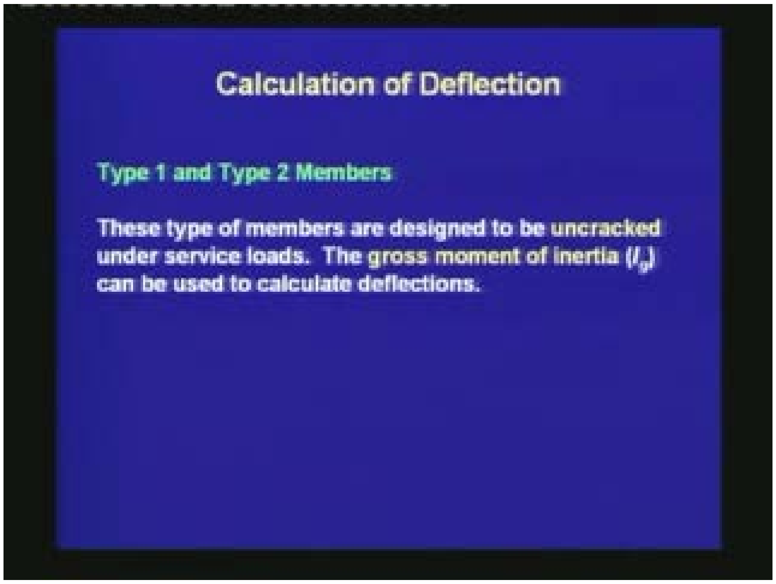

For Type 1 and Type 2 members, since they are designed to be uncracked under service loads, the gross moment of inertia which is represented as I_g can be used to calculate deflections. That means, the moment of inertia can be calculated from the total section and it can be substituted in the expressions of deflection.

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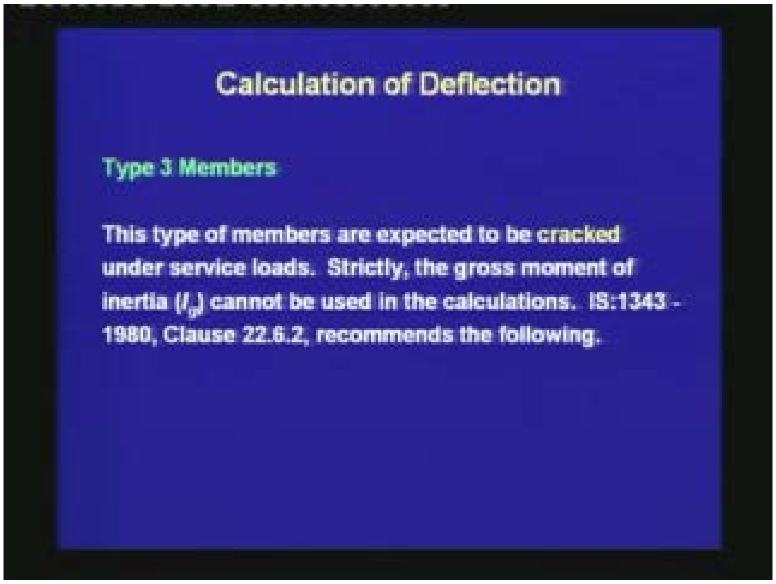

Type 3 members are expected to be cracked under service loads. Strictly, the gross moment of inertia cannot be used in the calculations. IS: 1343 – 1980, Clause 22.6.2, recommends the following:

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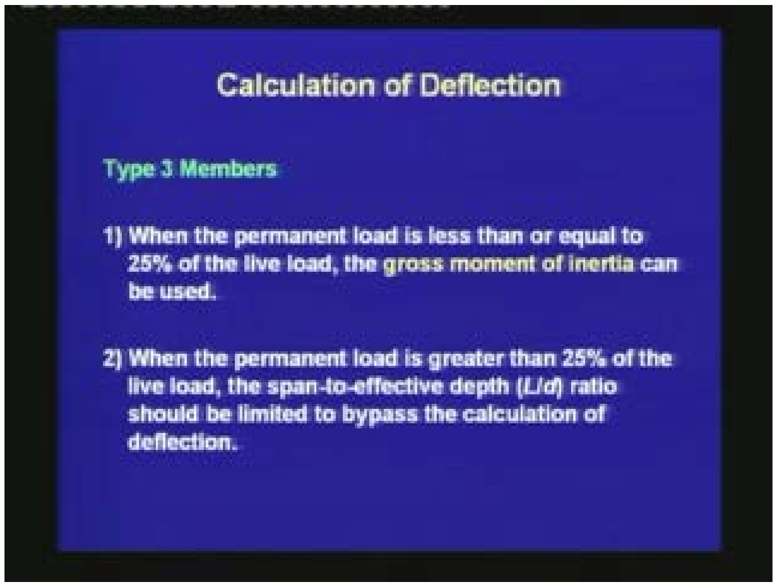

1) When the permanent load is less than or equal to 25% of the live load, the gross moment of inertia can be used. If the permanent component of the live load is very small, then most of the time the section will remain under compression. Hence we can use the gross moment of inertia.

2) If the permanent component of the live load exceeds 25%, then the code recommends that the span-to-effective depth ratio, which is denoted as L/d, should be limited to bypass the calculation of deflection.

Thus, if the span-to-depth ratio is limited to a certain value, which we shall learn next, then we can bypass the calculation of deflection, because it is considered that the deflection will not be of any problem.

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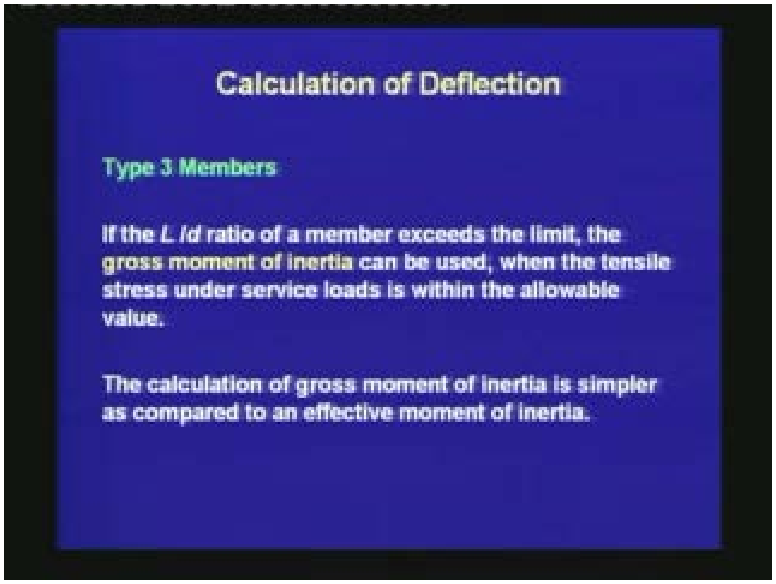

If the L/d ratio exceeds the limit, then the gross moment of inertia can be used, when the tensile stress under service loads is within the allowable value. The calculation of gross moment of inertia is simpler as compared to an effective moment of inertia.

In reinforced concrete, we use an effective moment of inertia to consider the variation of moment of inertia along the span. For prestressed concrete, even for a Type 3 member if the tensile stress is limited to the allowable value, then we may use the gross moment of inertia.

Next, we are learning about the limits of span-to-effective depth ratio.

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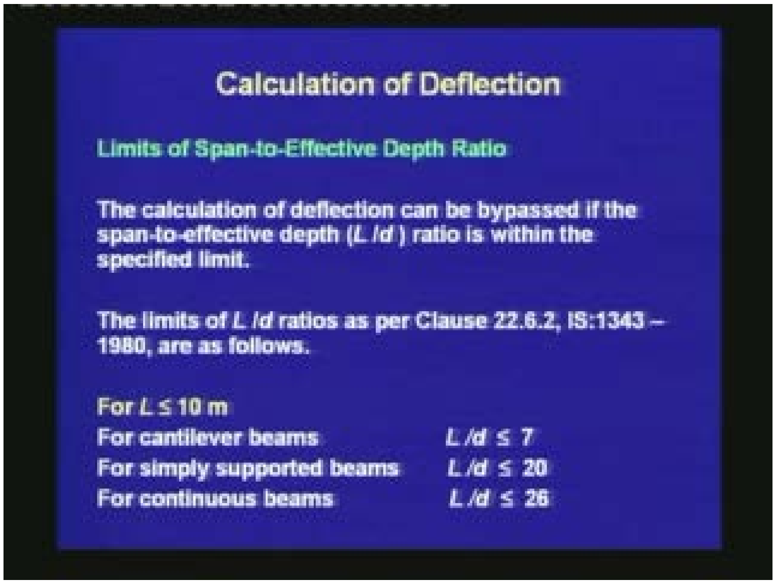

The calculation of deflection can be bypassed if the span-to-effective depth ratio, which is represented as L/d, is within the specified limit. The limits of L/d ratio, as per Clause 22.6.2 of IS: 1343 – 1980 are as follows. For span (L) less than 10 m, for cantilever beams L/d should be less than 7; for simply supported beams the ratio should be less than 20; for continuous beams the ratio should be less than 26.

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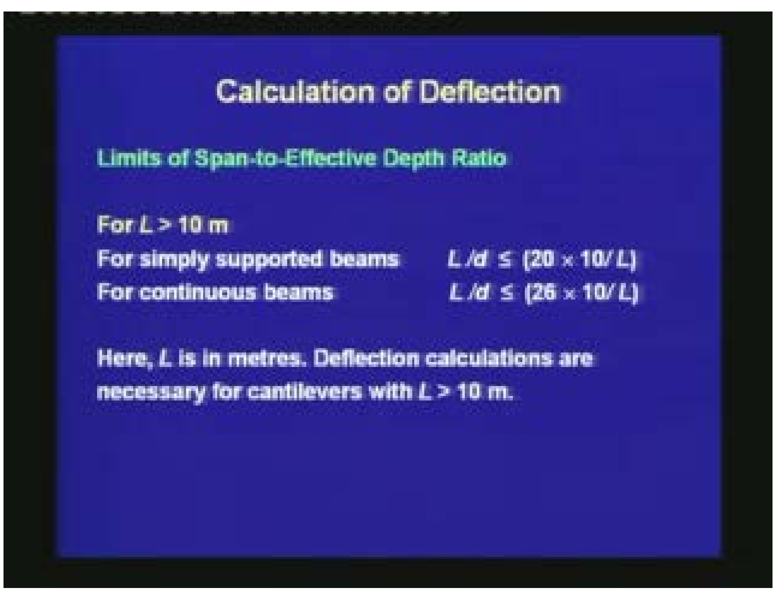

If the span exceeds 10 m, then we have to modify these limits as follows. For simply supported beams, L/d should be less than 20 × 10/L. For continuous beams, L/d should be less than 26 × 10/L. Here, L is in meters. Deflection calculations are necessary for cantilevers with L greater than 10 m.

Next, we are moving on to the second serviceability check, which is the calculation of crack width.

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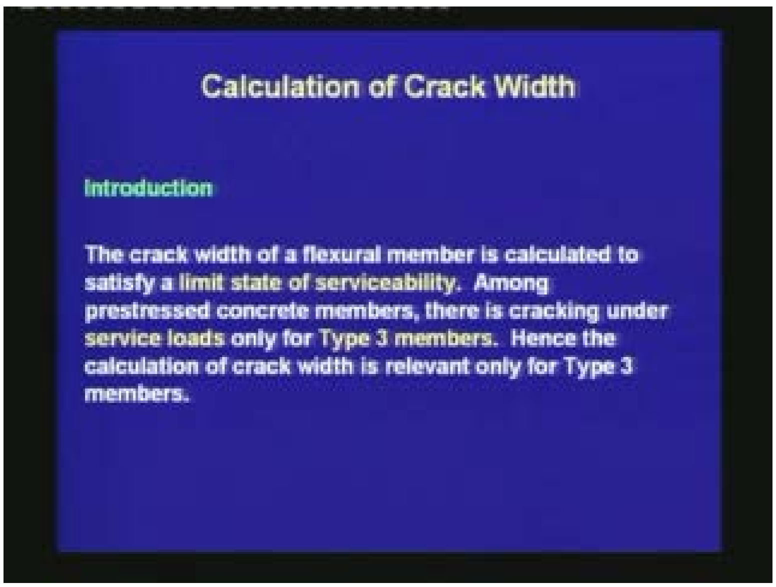

The crack width of a flexural member is calculated to satisfy a limit state of serviceability. Among prestressed concrete members, there is cracking under service loads only for Type 3 members. Hence the cracking and the calculation of crack width is relevant only for Type 3 members.

We have learnt earlier that Type 1 member is designed such that, there is no tensile stress in the member under service loads. Type 2 member is designed such that, there can be tensile stress in the member but the tensile stress is less than the cracking stress at service loads. For Type 3 member cracking is allowed, but it is limited by limiting the crack width. Thus, the calculation of crack width is relevant only for Type 3 members and the crack widths are calculated for the service loads.

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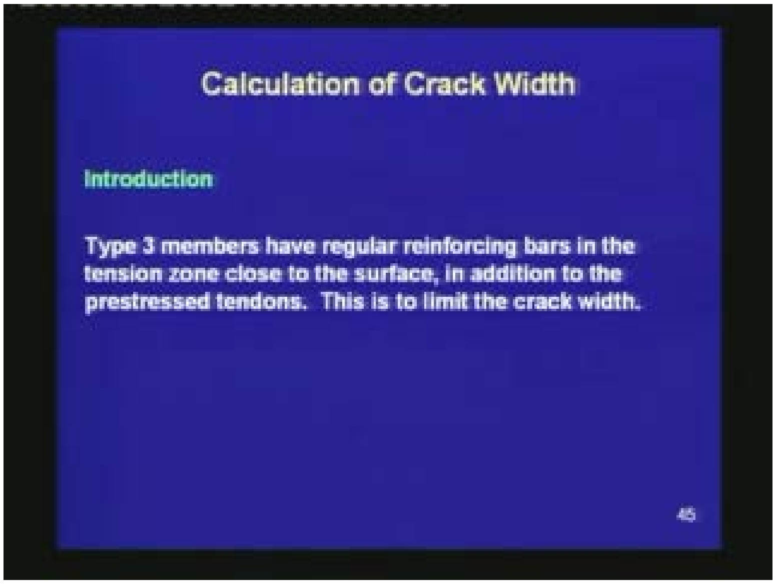

Type 3 members have regular reinforcing bars which are non-prestressed, in the tension zone close to the surface, in addition to the prestressed tendons. This is to limit the crack width and to distribute the cracking.

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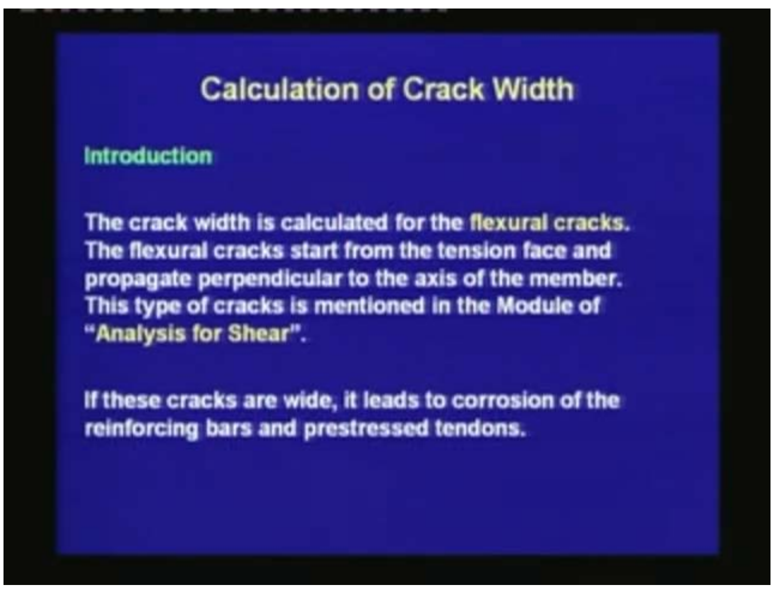

The crack width is calculated for the flexural cracks. The flexural cracks start from the tension face and propagate perpendicular to the axis of the member. This type of cracks is mentioned in the module of “Analysis for Shear”. If these cracks are wide, it leads to corrosion of the reinforcing bars and prestressed tendons. Also the appearance becomes bad. The crack width calculation is related to the width of the flexural cracks.

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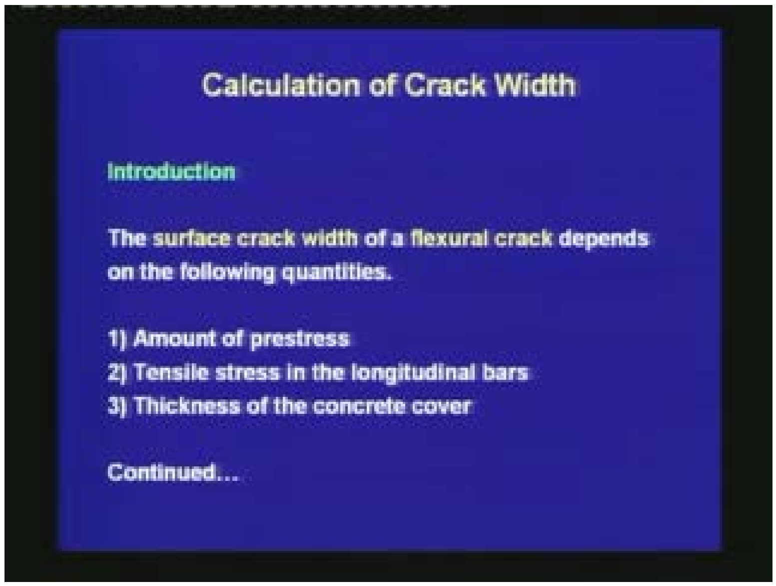

The surface crack width of a flexural crack depends on the following quantities:

1) Amount of prestress,
2) Tensile strength in the longitudinal bars,
3) Thickness of the concrete cover,
4) Diameter and spacing of longitudinal bars,
5) Depth of member and location of neutral axis,
6) Bond strength and
7) Tensile strength of concrete.

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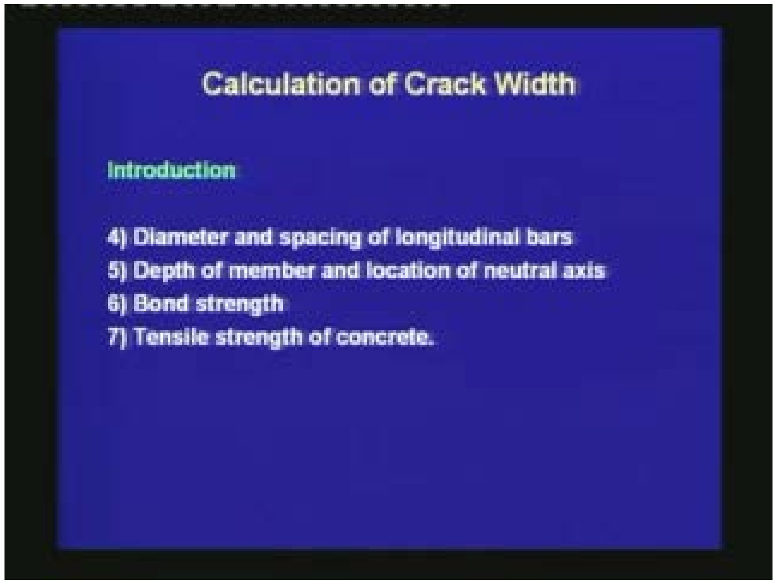

The crack width calculation is a difficult process. There is a fracture mechanics based approach to calculate the crack width. But the recommendations in reinforced concrete design are simpler for our day-to-day use in design checks. We have to appreciate that the crack width depends on several factors, and the expression provides only an estimate of the crack width. When we are experimentally investigating the crack width, we may find variations in the observed crack width.

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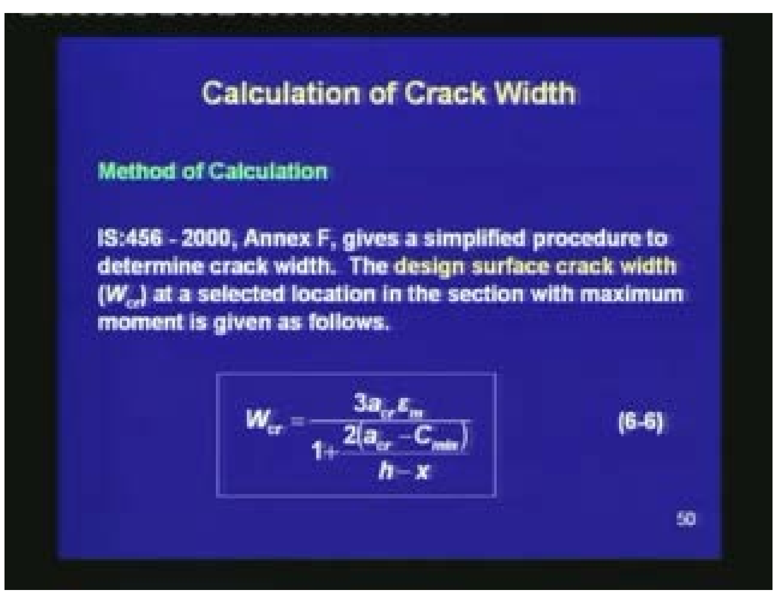

IS: 456 – 2000, Annex F, gives a simplified procedure to determine crack width. The design surface crack width (which will be represented as W_{cr}) at a selected location in the section with maximum moment, is given as follows. For that particular section, we can select any location along the periphery in the tensile region.

$W_{cr} = 3a_{cr}\ \varepsilon_m / (1 + 2(a_{cr} - C_{min})/(h - x)$

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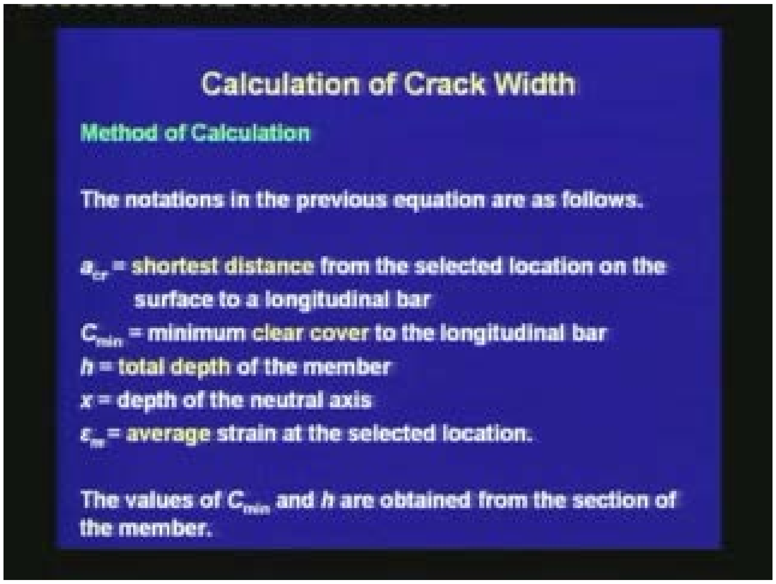

To summarize the notations:

a_{cr} = shortest distance from the selected location on the surface to a longitudinal bar,

C_{min} = minimum clear cover to the longitudinal bar,

h = total depth of the member,

x = depth of the neutral axis,

ε_m = average strain at the selected location.

The zone below the neutral axis is the zone under tension. Thus, h – x is the depth of the zone of concrete under tension.

We shall discuss later what is meant by an average. The values of C_{min} and h are obtained from the section of the member. Next, we need to calculate a_{cr} and then we need to calculate x and ε_m.

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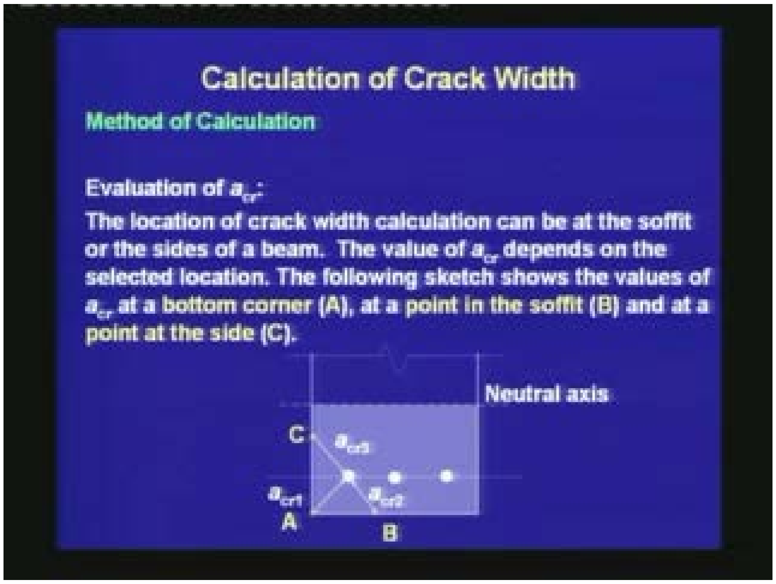

Evaluation of a_{cr}:

The location of the crack width calculation can be at the soffit or the sides of the beam. The value of a_{cr} depends on the selected location. The following sketch shows the values of a_{cr} at a bottom corner (A), at a point in the soffit of the beam (B) and at a point at the side (C). For these three points, the distance to the nearest longitudinal bar has been represented by a_{cr}. Thus, the value of a_{cr} can be found out from the design section, based on the location of calculation of crack width.

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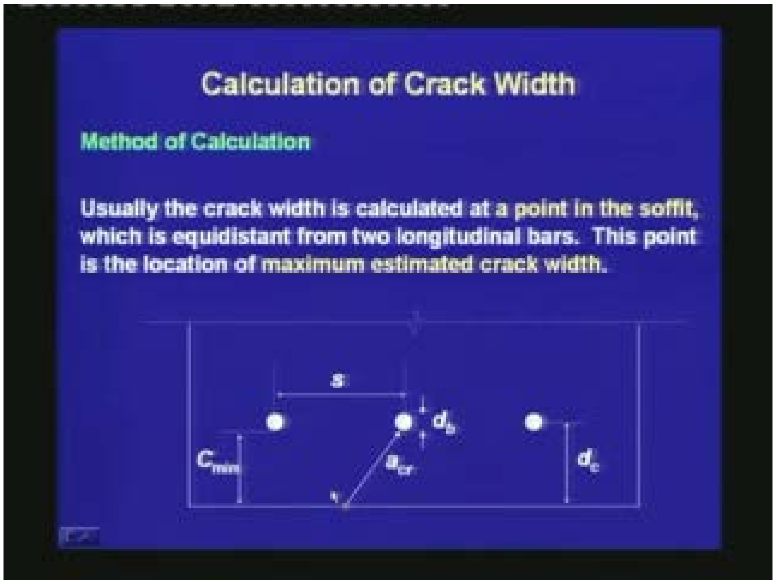

Usually, the crack width is calculated at a point in the soffit, which is equidistant from two longitudinal bars. This point is the location of maximum estimated crack width. Hence, first we calculate the crack width at the soffit, which is in between two longitudinal bars.

In this sketch of the cross section of the beam, C_{min} is the clear cover, s is the spacing of the longitudinal bars, d_b is the diameter of the longitudinal bars, a_{cr} is the distance from the point of investigation to the nearest longitudinal bar and d_c is the effective cover to the reinforcing bars.

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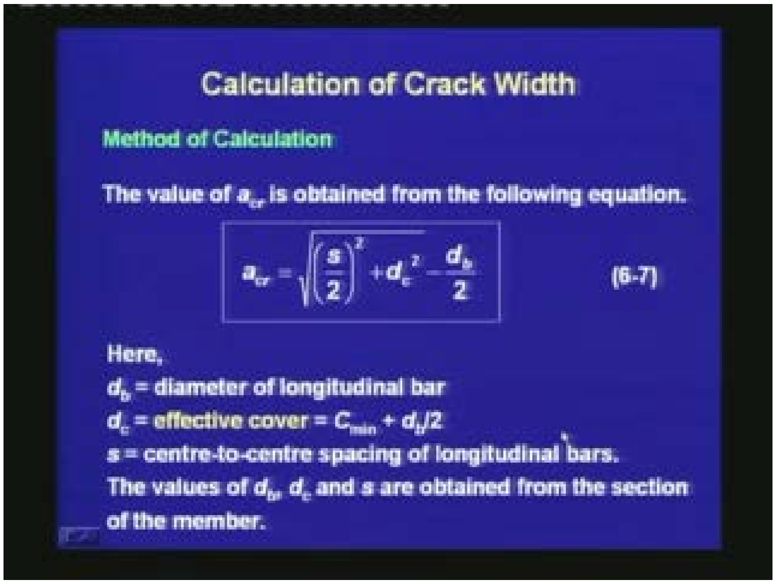

The value of a_{cr} is obtained from the following equation based on the Pythagorean theorem. It is the radial distance from the point of investigation to the center of the nearest bar.

$$a_{cr} = \sqrt{[(s/2)^2 + d_c^2]} - (d_b/2)$$

In this expression,

d_b = diameter of a longitudinal bar

d_c = effective cover = C_{min} + $d_b/2$

s = center-to-centre spacing of longitudinal bars.

The values of d_b, d_c and s are obtained from the section of the member. Thus, once the member has been designed for flexure, these variables are available and we can calculate the distance a_{cr}.

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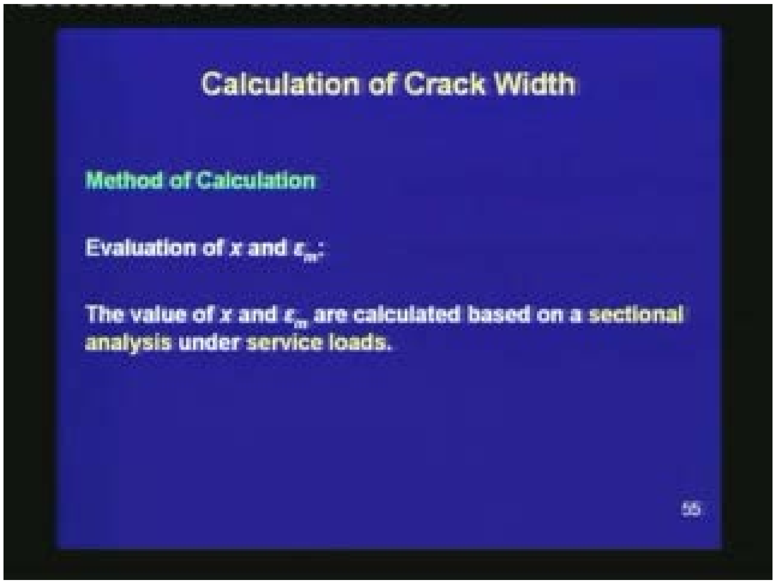

Next, we are evaluating the depth of the neutral axis and the average strain at the level of investigation of crack width. The values of x and ε_m are calculated based on a sectional analysis under service loads.

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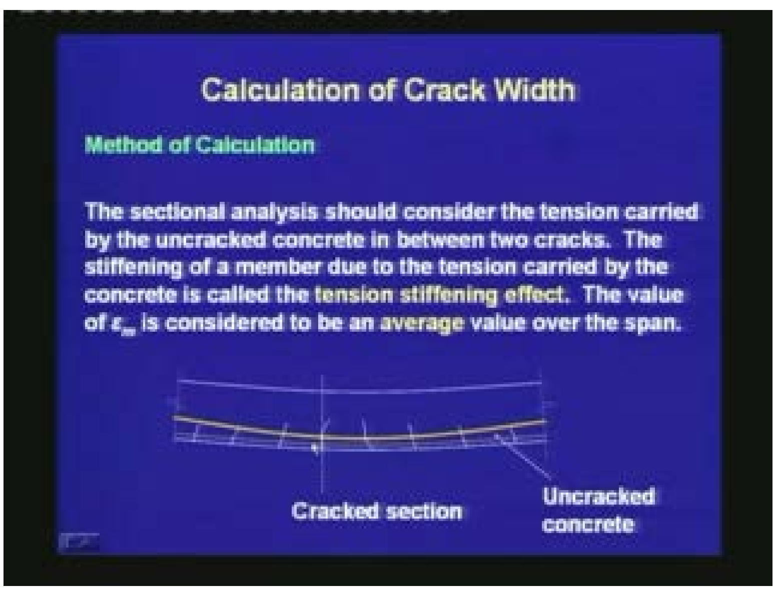

The sectional analysis should consider the tension carried by the uncracked concrete in between two cracks. The stiffening of a member due to the tension carried by the concrete is called the tension stiffening effect. The value of ε_m is considered to be an average value over the span. This is a new concept which we are observing. For flexure, usually we do a sectional analysis at the critical section, which is a cracked section. But when we are trying to find out the crack width, if you do a cracked section analysis, then the crack width is over estimated. The reason behind this is that, if we just do a cracked section analysis then we are neglecting the effect of concrete, which is in between the cracks. The concrete in between the cracks has some tensile strength, and that tensile strength reduces the crack width if we just calculate it based on a cracked section. The effect of tension in the concrete in between two cracks is called the tension stiffening effect. It reduces the crack width, and it also reduces the deflection from the value calculated based only on cracked section.

In this figure you can see that the cracked section is at the location of a crack, whereas the uncracked concrete is in between two cracks, which helps the beam to reduce deflection. The contribution of uncracked concrete is called the tension stiffening effect. When we are calculating ε_m at the soffit of the beam, since ε_m various along the length of the span, we are calculating an average value, which should include the tension stiffening effect.

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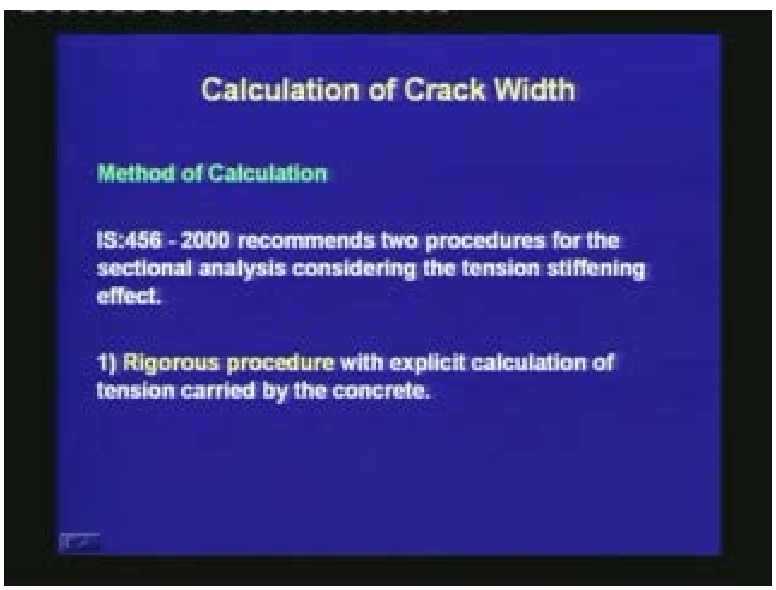

IS: 456 - 2000 recommends two procedures for the sectional analysis, considering the tension stiffening effect. The first one is a rigorous procedure with explicit calculation of tension carried by the concrete.

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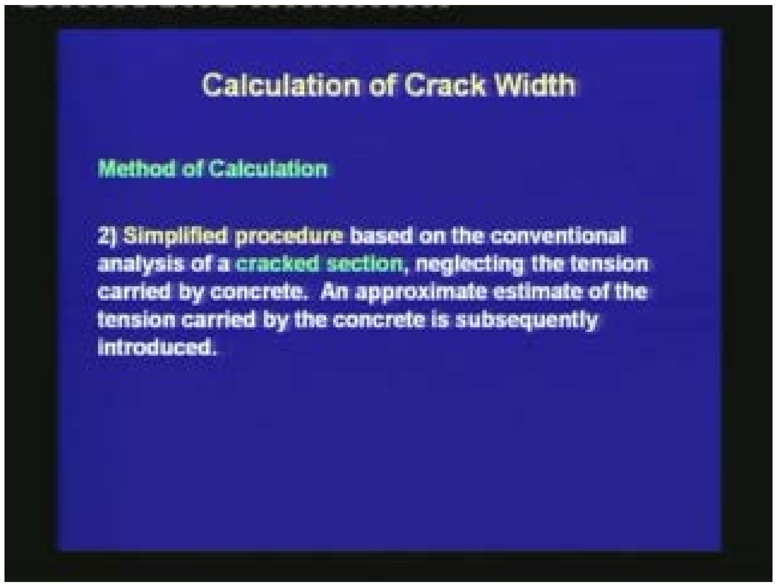

The second one is a simplified procedure based on the conventional analysis of a cracked section, neglecting the tension carried by concrete. An approximate estimate of the tension carried by the concrete is subsequently introduced.

Thus, IS: 456 gives us two procedures to do the sectional analysis to calculate x and ε_m. The first one is a rigorous procedure, where we consider a section with tension in the concrete below the neutral axis. The second procedure is the conventional cracked section analysis, where we neglect any tension in the concrete below the neutral axis. But then the strain is modified to take account of the tension in the concrete. In this lecture, we shall explain the simpler procedure which is based on a conventional cracked section analysis.

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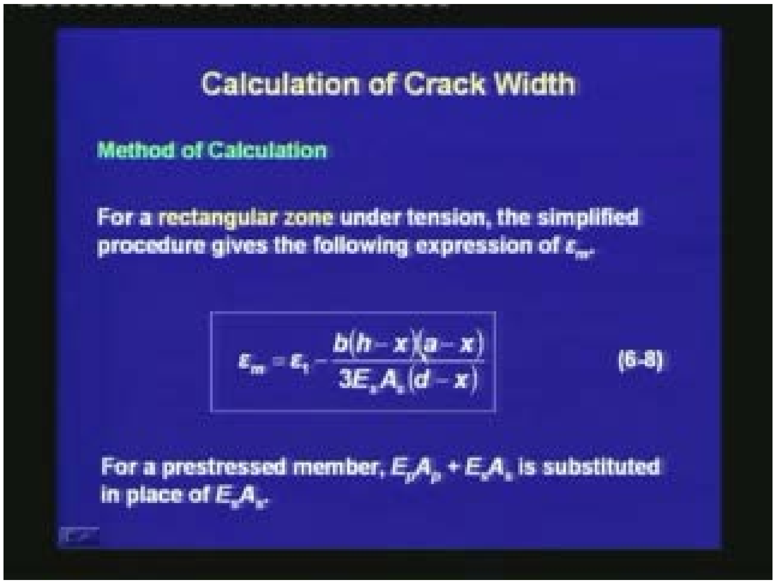

For a rectangular zone under tension,

$\varepsilon_m = \varepsilon_1 - [b (h - x) (a - x)]/[3 E_s A_s (d - x)]$

For a prestressed member, $E_pA_p + E_sA_s$ is substituted in place of E_sA_s. We shall understand the notation of each of these terms in the next slide.

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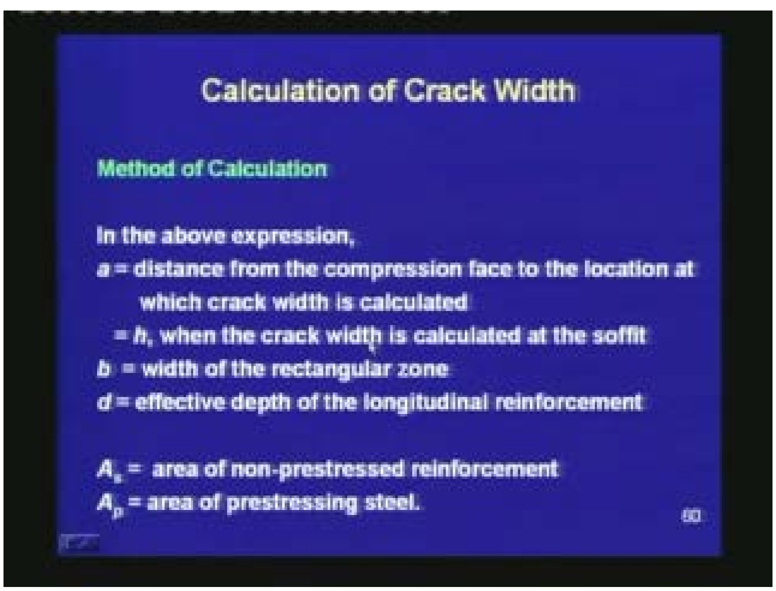

In the above expression,

a = distance from the compression face to the location at which crack width is calculated, which is same as h when the crack width is calculated at the soffit

b = width of the rectangular zone. In most of our applications we will have a rectangular zone at the bottom and hence this formula will be applicable, where b is the width of the rectangular zone.

d = effective depth of the longitudinal reinforcement, that means it is the effective depth of the non-prestressed steel

A_s = area of non-prestressed reinforcement

A_p = area of prestressing steel.

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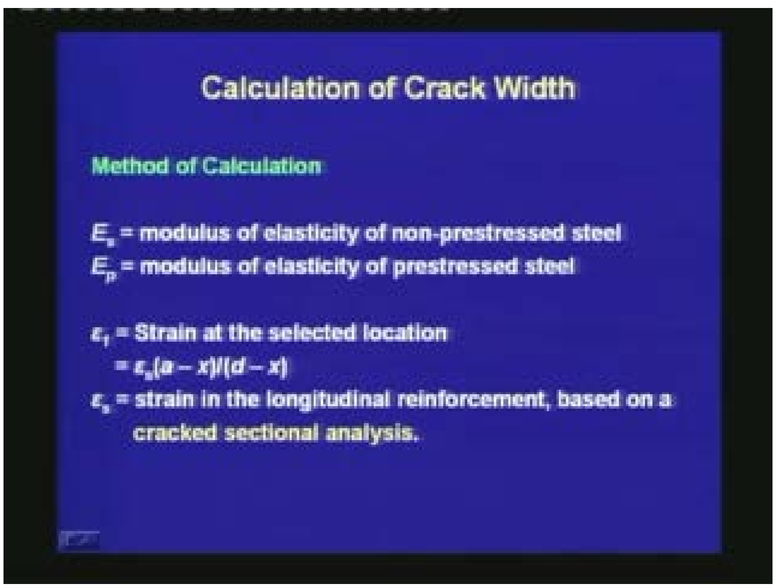

E_s = modulus of elasticity of non-prestressed steel,

E_p = modulus of elasticity of prestressed steel.

All these variables are available from the section and the material properties of the beam. The two variables ε_1 and ε_s which are used in ε_m need to be calculated.

ε_1 = strain at the selected location. For the soffit, it is the strain at the soffit. By similarity of triangles, $\varepsilon_1 = \varepsilon_s(a - x)/(d - x)$.

ε_s = strain in the longitudinal reinforcement, based on the cracked section analysis.

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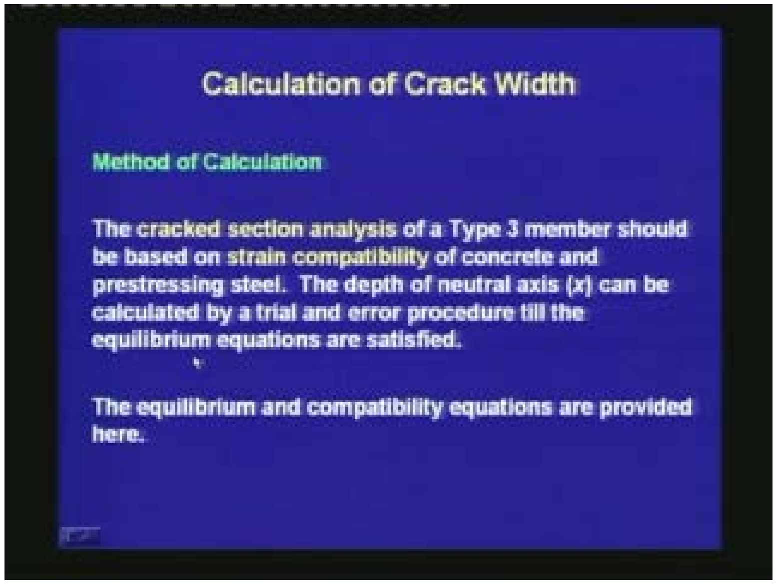

The cracked section analysis of a Type 3 member should be based on strain compatibility of concrete and prestressing steel. The depth of neutral axis (x) can be calculated by a trial and error procedure, till the equilibrium equations are satisfied. The equilibrium and compatibility equations are provided here.

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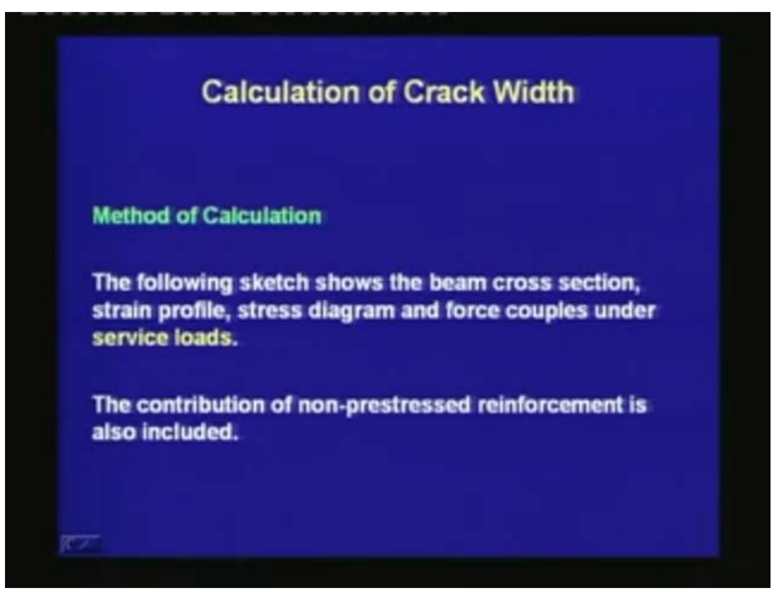

The following sketch shows the beam cross section, strain profile, stress diagram and force couples under service loads. The contribution of non-prestressed reinforcement is also included.

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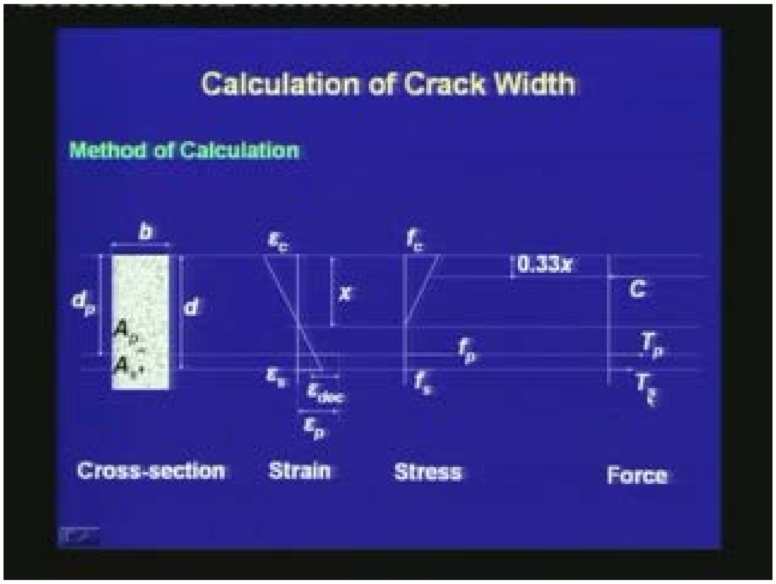

In this figure, we see that for a rectangular section b is the breadth, d_p is the depth of the prestressing steel, d is the depth of the non-prestressed reinforcement. In the strain diagram ε_c is the strain in the concrete at the top, ε_s is the strain in the non-prestressed steel and ε_{dec} is the strain at decompression. The total strain in the prestressing steel ε_p is the strain in the concrete at the level of the prestressing steel plus ε_{dec}. The strain diagram considers the strain compatibility of the concrete and the prestressing steel, at the level of the prestressing steel. The stress diagram in concrete is linear. The maximum stress in concrete is f_c, and the stress in the prestressing steel and the non-prestressed steel are f_p and f_s, respectively. The resultant compression C occurs at one-third the depth of the neutral axis, and the tension are represented as T_p for the prestressing steel and T_s for the non-prestressed steel.

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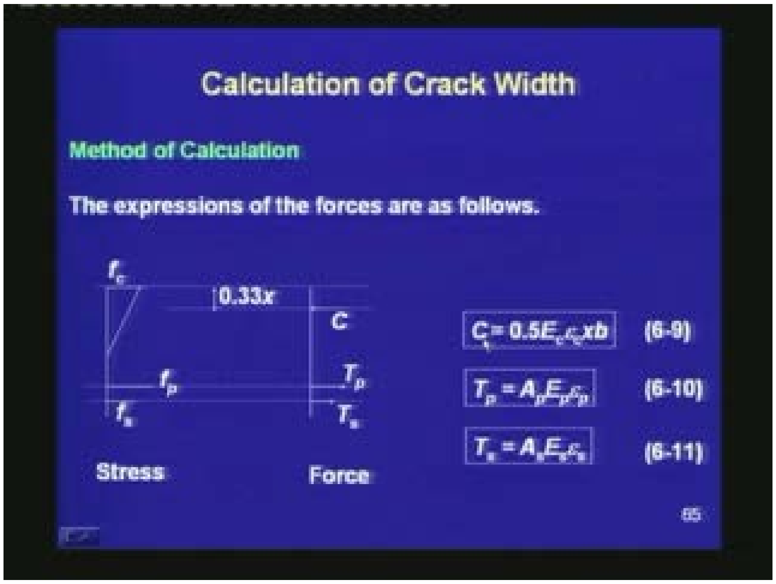

From the stress diagram and the force couples, we can write the expressions of the forces.

$C = 0.5E_c\varepsilon_c xb$, which is the area of the stress triangle.

$T_p = A_pE_p\varepsilon_p$

$T_s = A_sE_s\varepsilon_s$.

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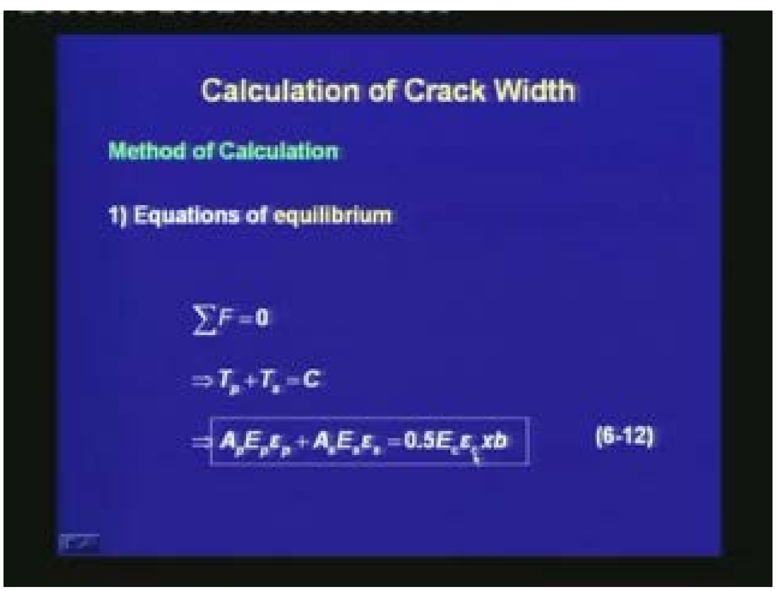

The first equilibrium equation is the equilibrium of the axial forces.

$\Sigma F = 0 \Rightarrow T_p + T_s = C$

Here, we write the expressions of T_p, T_s and C. The value of x should be such that this equation is satisfied.

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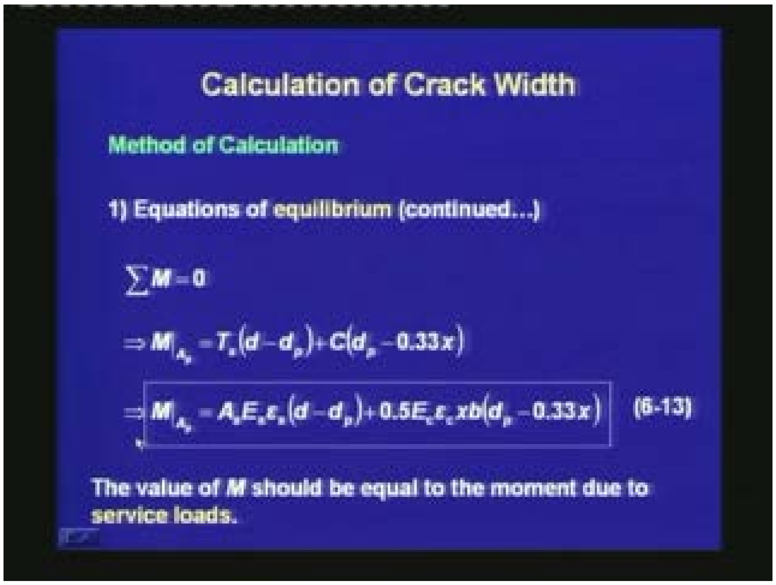

The second equilibrium equation is the moment equation.

$\Sigma M = 0$

Taking the moment about the prestressing steel, we have M is equal to T_s times the distance between the prestressing and the non-prestressed steel, plus C times the distance between the prestressing steel and the location of C. When we substitute the expressions of T_s and C, we get an expression of the moment. The value of the moment should be equal to the moment due to service loads. Thus, x should be such that we need to satisfy both this equilibrium equations.

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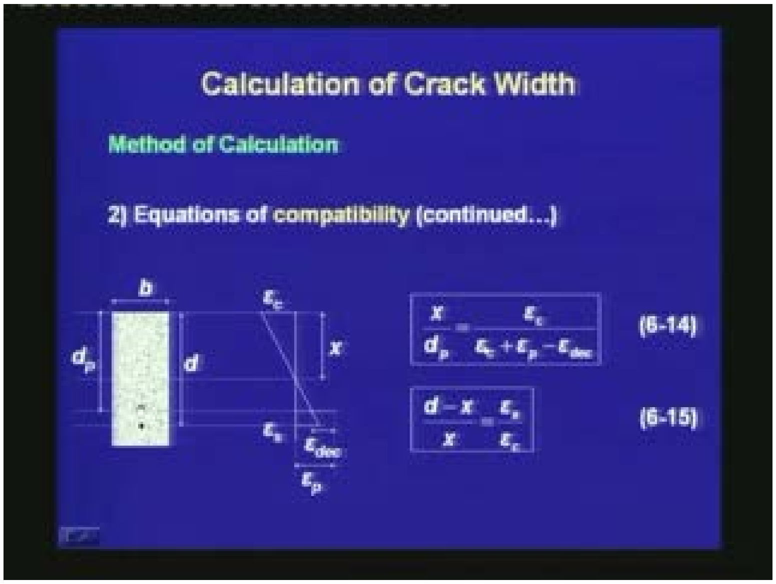

Among the compatibility equations, the first equation relates the compatibility between the prestressing steel and the concrete.

$x/d_p = \varepsilon_c/(\varepsilon_c + \varepsilon_p - \varepsilon_{dec})$

The second equation is for the non-prestressed steel.

$(d - x)/x = \varepsilon_s/\varepsilon_c$.

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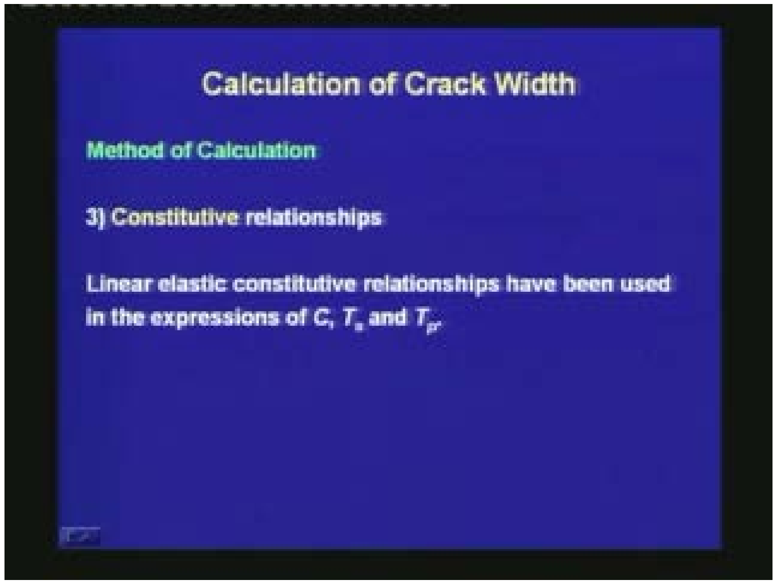

The constitutive relationships have been considered in the expressions of C, T_s and T_p and we have used the elastic relationships to calculate C, T_s and T_p from the respective strains.

The equations have to be solved to evaluate x, which should be substituted in the expressions of ε_m and W_{cr}, to calculate the crack width for a Type 3 member.

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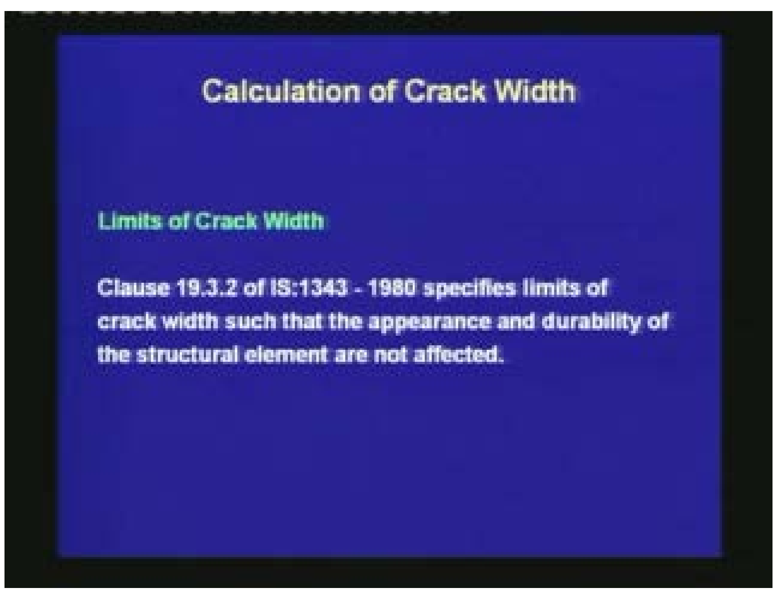

The limits of crack width are as follows. Clause 19.3.2 of the code specifies the limits such that the appearance and durability of the structure elements are not affected.

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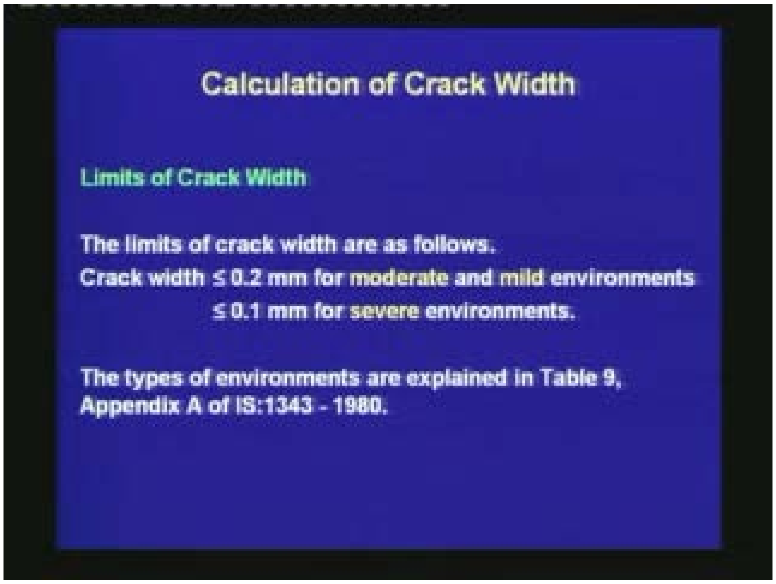

The crack width should be less than 0.2 mm for moderate and mild environments, and 0.1 mm for severe environment. The types of environment are explained in Table 9, Appendix A of IS: 1343 - 1980. Once we calculate the crack width, we should make sure that the crack width is within the limit, depending on the environment the structure is in.

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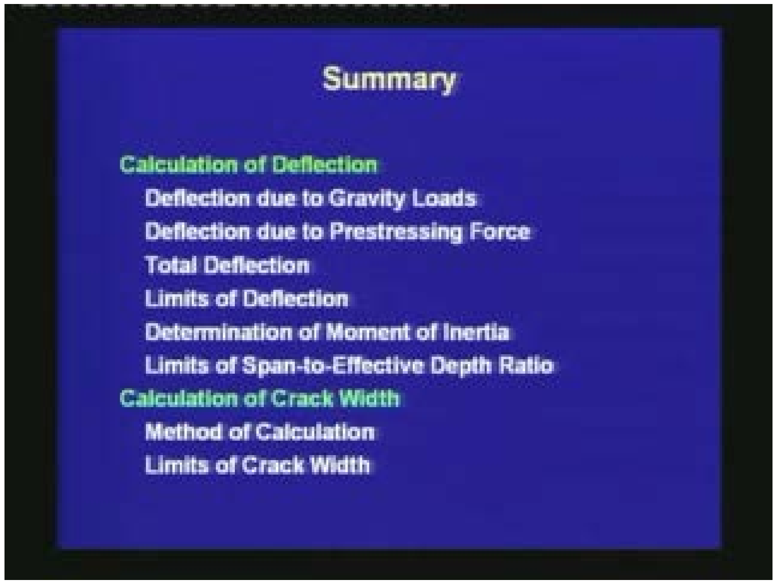

Thus, in today's lecture, we first went through the calculation of deflection. We knew the deflections due to gravity loads; we learned about the deflections due to prestressing force. Then, we studied how to calculate the total deflection, and the limits of deflection the member needs to satisfy. We learned about the determination of moment of inertia. If we satisfy the limit of span-to-effective depth ratio, then we can bypass the deflection calculations.

Next, we studied the calculation of crack width. First we studied the method of calculation and next, we found out the limits of crack width. The calculations of deflection and the crack width help us to satisfy the limits state of serviceability.

Thank you.