PRESTRESSED CONCRETE STRUCTURES

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Module – 5: Analysis and Design for Shear and Torsion

Lecture-28: Design for Torsion (Part - 2)

Welcome back to prestressed concrete structures. This is the sixth lecture of Module 5 on analysis and design for shear and torsion.

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In this lecture, we shall study about design for torsion. First, we shall study the detailing requirements; next, we shall move on to some general comments; and then we shall learn about the design steps.

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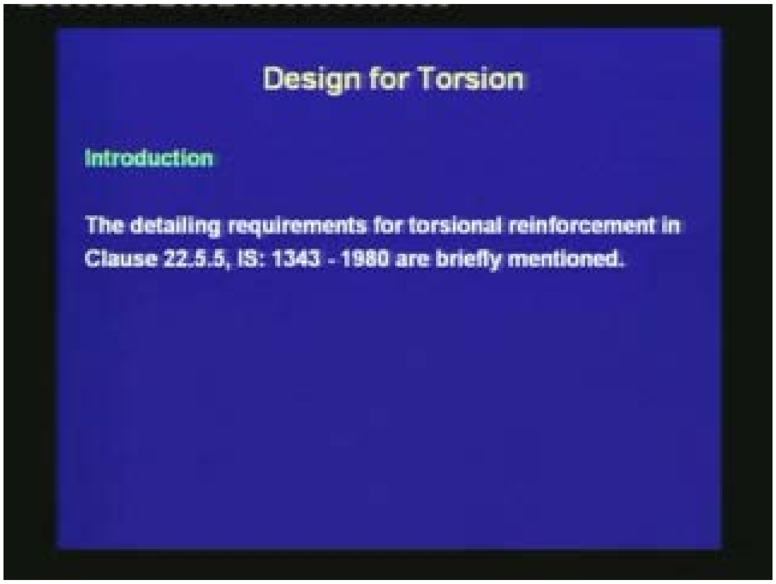

The detailing requirements for torsional reinforcement in Clause 22.5.5, IS: 1343-1980 are briefly mentioned.

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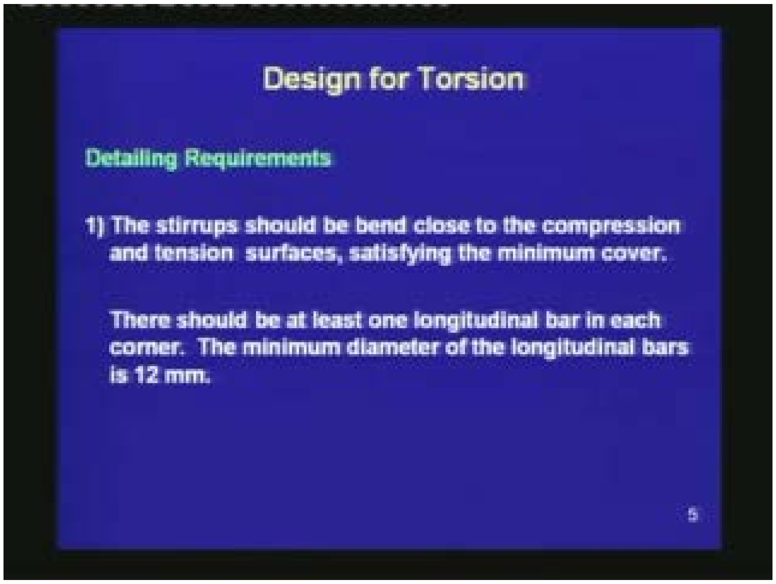

1) The stirrups should be bend close to the compression and tension surfaces, satisfying the minimum cover. There should be at least one longitudinal bar in

each corner. The minimum diameter of the longitudinal bars is 12 mm. Thus, we are trying to have the stirrups as close to the periphery as possible.

We have to satisfy the minimum cover requirements and we have to hold the stirrups by the longitudinal bars. The longitudinal bars have to be stiff enough to keep the stirrups in position. Hence, the minimum diameter of the longitudinal bar is recommended.

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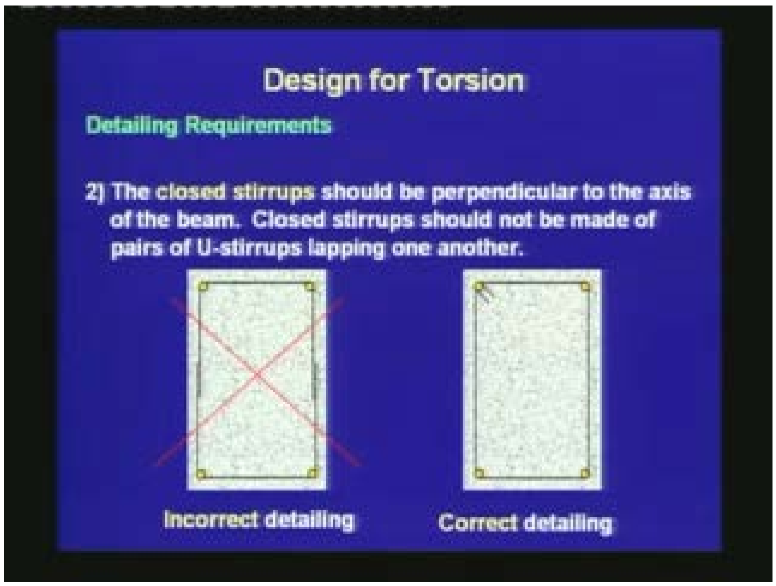

2) The closed stirrups should be perpendicular to the axis of the beam. Closed stirrups should not be made of pairs of U-stirrups lapping one another. As we had known earlier, for torsion we need close stirrups, to resist the circulatory shear.

The closed stirrups should be made of only one piece of steel bar. We cannot make close stirrups by overlapping two U-bars, although that is more convenient in construction. The figure on the left is an incorrect detail because it is showing two U-bars overlapping each other. The correct detailing is to have one piece of stirrup with proper anchorage at the ends, as shown in the figure on the right.

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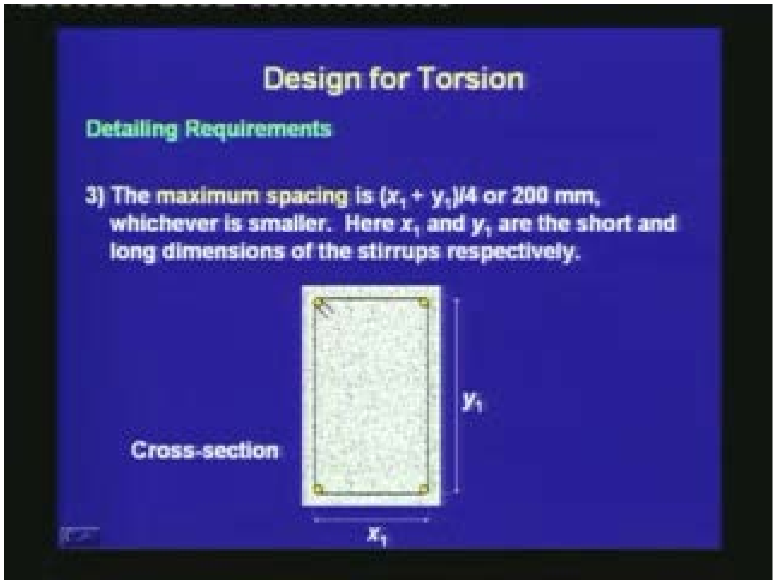

3) The maximum spacing of the stirrups is $(x_1 + y_1)/4$ or 200 mm, whichever is smaller. Here, x_1 and y_1 are the short and long dimensions of the stirrups, respectively.

This requirement has come from the space truss analogy, that the stirrups should traverse all the inclined cracks. Based on that requirement, the maximum spacing is $(x_1 + y_1)/4$. There is in upper limit to the spacing which is 200 mm.

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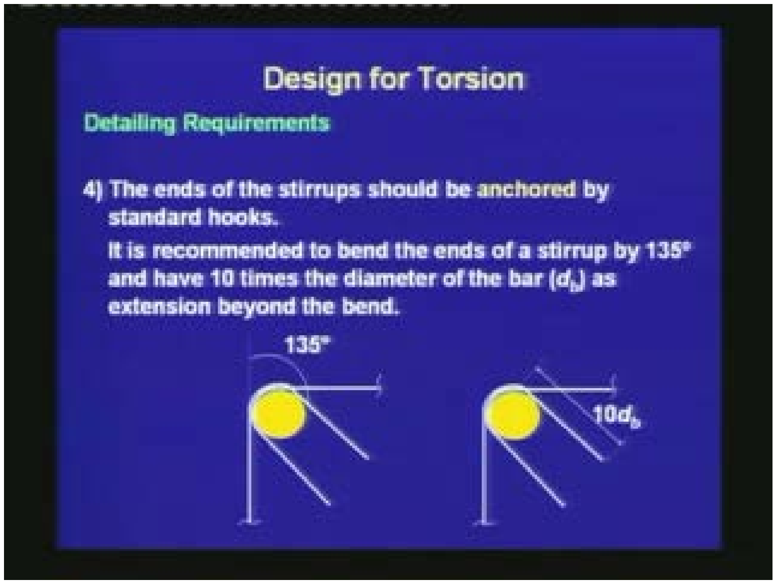

4) The ends of the stirrups should be anchored by standard hooks. Unless the stirrup is properly anchored at the ends, it will not develop its yield stress. Hence, proper anchorage is necessary at the ends of the closed stirrups.

It is recommended to bend the ends of a stirrup by 135° and have 10 times the diameter of the bar, which is represented as d_b, as extension beyond the bend. One end of the bar has been bent by 135° and it has been inserted within the core concrete. The other end of the bar is also bent by 135° and inserted within the core concrete. The extension of the bars beyond the bends should be 10 times the diameter of the bars. This detailing helps to retain the shape of the stirrups, under torsion. The stirrups have to be in the original form when the torsion is acting, and they should not open up in presence of torsion. To maintain this shape, the ends of the bars are provided with 135° hooks, and the extensions beyond the hooks are placed within the core concrete.

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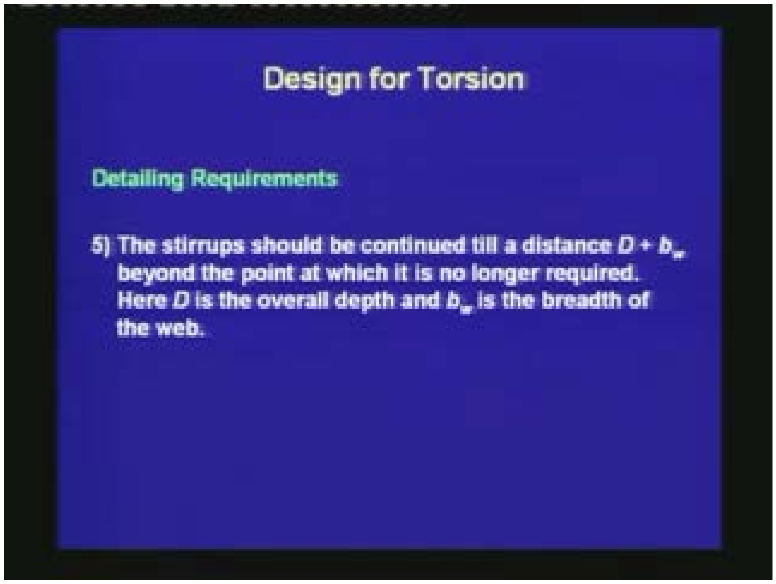

5) The stirrups should be continued till a distance D + b_w beyond the point at which it is no longer required. Here, D is the overall depth and b_w is the breadth of the web.

This requirement has also come from the space truss analogy. After the concrete forms struts and the steel bars form the ties tying the concrete struts, the stresses that we calculate at one particular section is shifted due to the formation of the struts. Based on this, it is recommended that the stirrups have to be continued beyond the point of calculation by a distance D + b_w. This is to maintain the truss action in the resistance for torsion.

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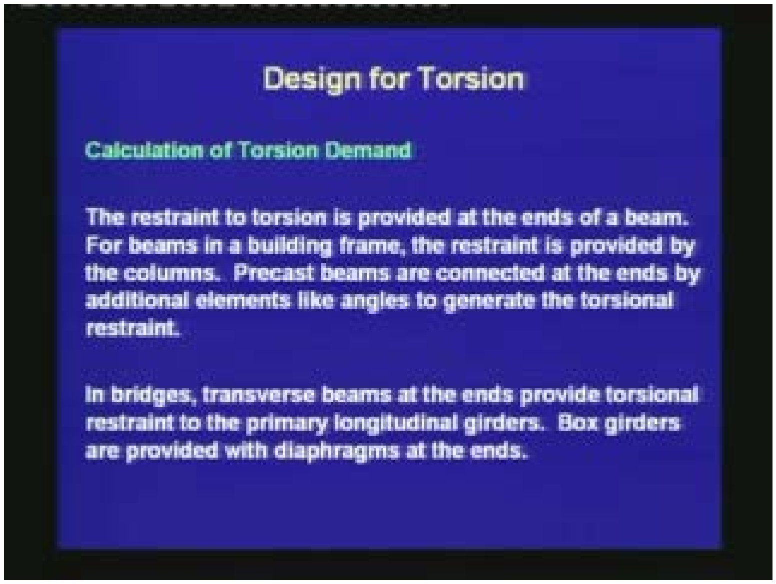

Next, we shall be discussing some general comments in the design for torsion. The restraint to torsion is provided at the ends of a beam. For beams in a building frame, the restraint is provided by the columns. Precast beams are connected at the ends by additional elements like angles to generate the torsional restraint.

In bridges, transverse beams at the ends provide torsional restraint to the primary longitudinal girders. Box girders are provided with diaphragms at the ends.

Thus, the beams that are subjected to torsion need to have some restraint at the ends to resist the torsion. For an individual beam, like precast beams, torsional restraint is provided by some attachment at the ends like angle attachments. In bridges, transverse beams are provided or in a box girder, diaphragms are provided at the ends to provide the torsional restraint. Thus, when there is torque acting along the length of the beam, the restraint comes from the two ends of the beam.

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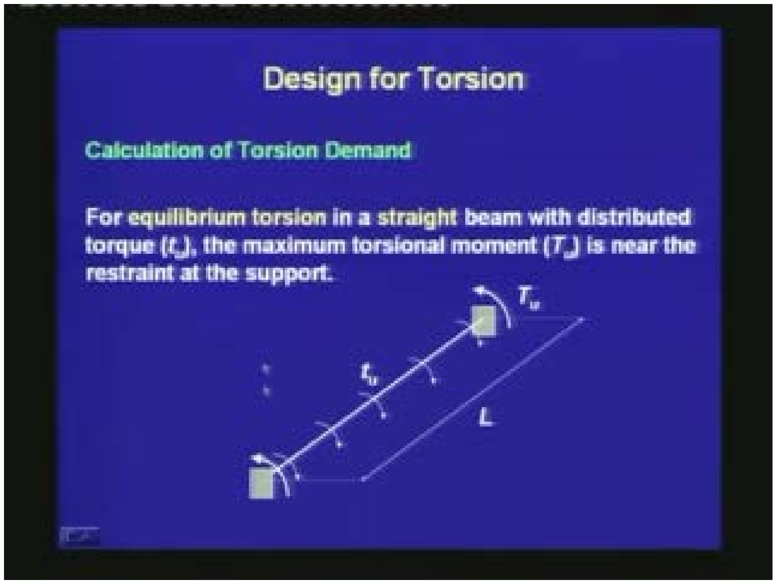

For equilibrium torsion in a straight beam with distributed torque t_u, the maximum torsional moment T_u is near the restraint at the support. This figure shows a beam, which is statically determinate and it is subjected to a distributed torque t_u per unit length. In this beam, the maximum torque comes at the two ends near the restraints at the supports and is denoted as T_u.

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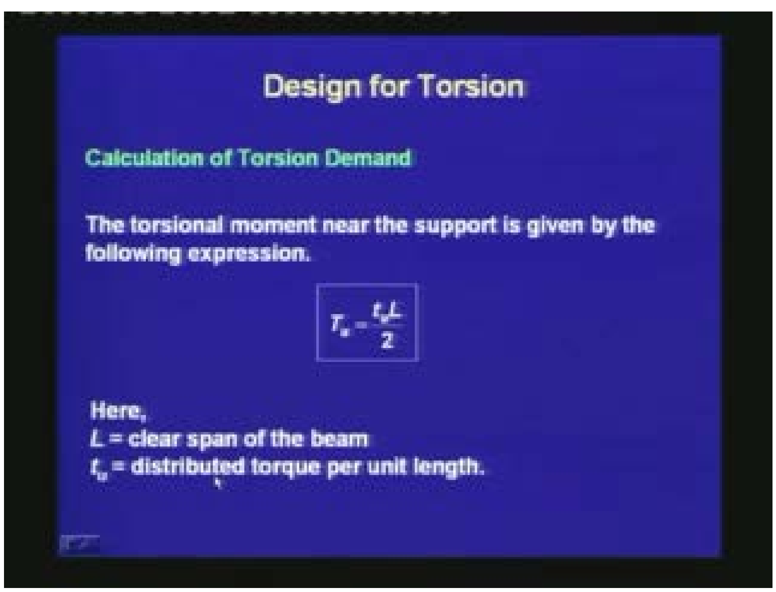

The torsional moment near the support is given by the following expression.

$T_u = t_uL/2$

Here, L is the clear span of the beam and t_u is the distributed torque per unit length of the beam.

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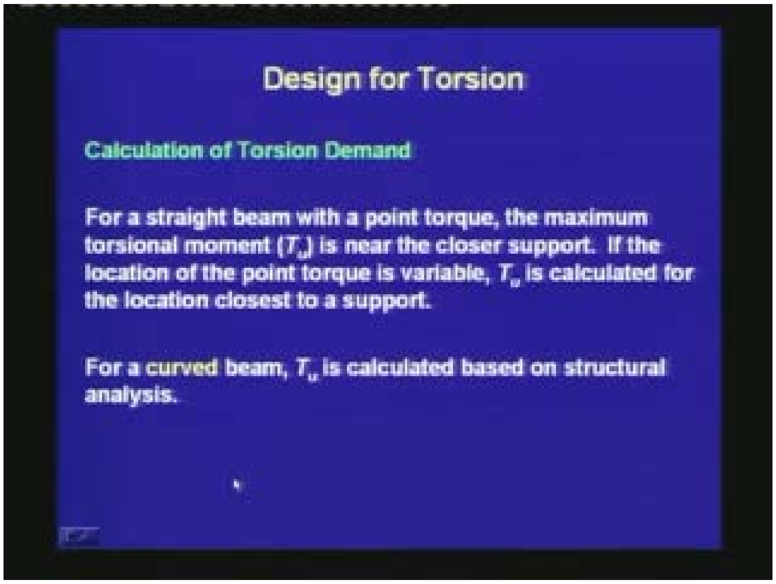

For a straight beam with a point torque, the maximum torsional moment T_u is near the closer support. If the location of the point torque is variable, T_u is calculated for the location closest to a support.

The analogy can be drawn from shear. When there is a point load whose location is variable, the shear is designed for the location of the load which generates the maximum value of the shear. Similarly for torsion, if it is generated due to a point load whose location is variable, such as the vehicle load in a bridge, then the torque is calculated by placing the load at the position which generates the maximum value of the torque. Hence, the load has to be placed closest to the support to generate the maximum torsion in the beam.

In a curved beam, the calculation of T_u is more involved. It is calculated based on structural analysis.

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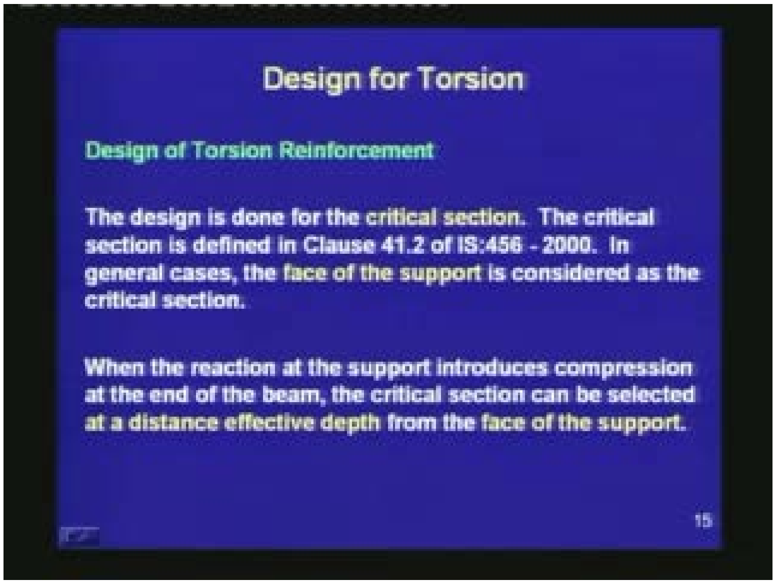

The design is done for the critical section. The critical section is defined in Clause 41.2 of IS: 456-2000. In general cases, the face of the support is considered as the critical section. When the reaction at the support introduces compression at the end of the beam, the critical section can be selected at a distance effective depth from the face of the support.

Thus, although the maximum torque occurs at the face of the support, if the beam near the support is under compression, then we can select a section which is at the distance effective depth from the face of the support as the critical section. The capacity of the concrete in the beam in between the critical section and the face of the support is larger as compared to the rest of the beam. Hence, the critical section can be selected at a distance effective depth from the face of the support.

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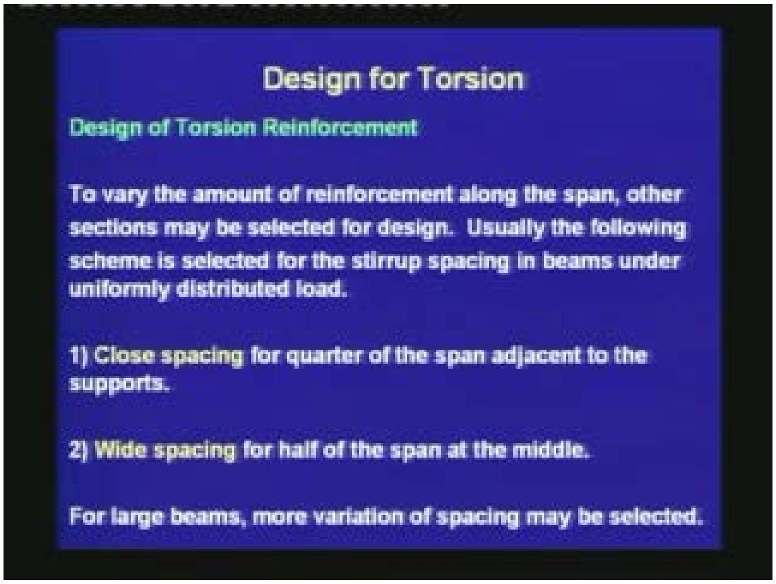

To vary the amount of reinforcement along the span, other sections may be selected for design. Usually, the following scheme is selected for the stirrup spacing in beams under uniformly distributed load: close spacing for quarter of the span adjacent to the supports; wide spacing for half of the span at the middle. For large beams, more variation of spacing may be selected.

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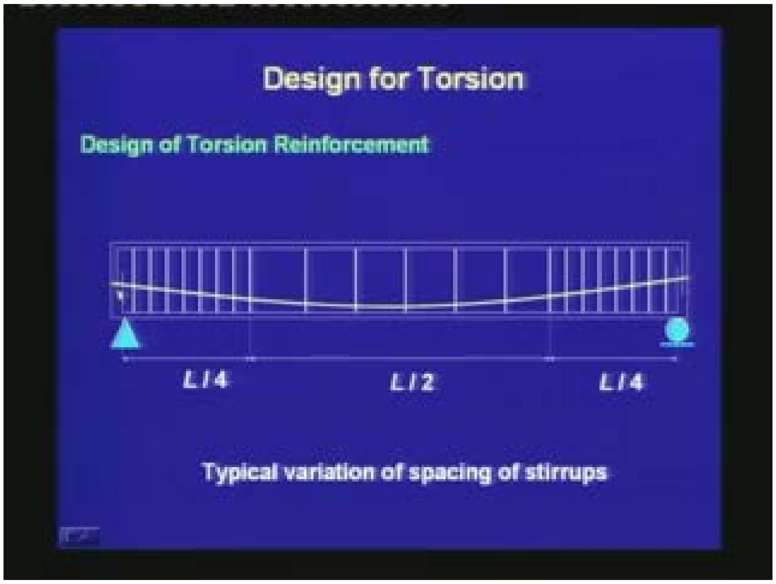

Let us understand this by a sketch. In this figure, you can see that the spacing of the stirrups at the two ends is close; whereas, the spacing at the middle half of the beam is large. Hence, if we want to vary the spacing of the stirrups, then we have to select more than one section for design. In this situation, we need at least two sections: one is the section which is at the effective distance from the face of the support, and another section which is at a distance quarter of the span of the beam.

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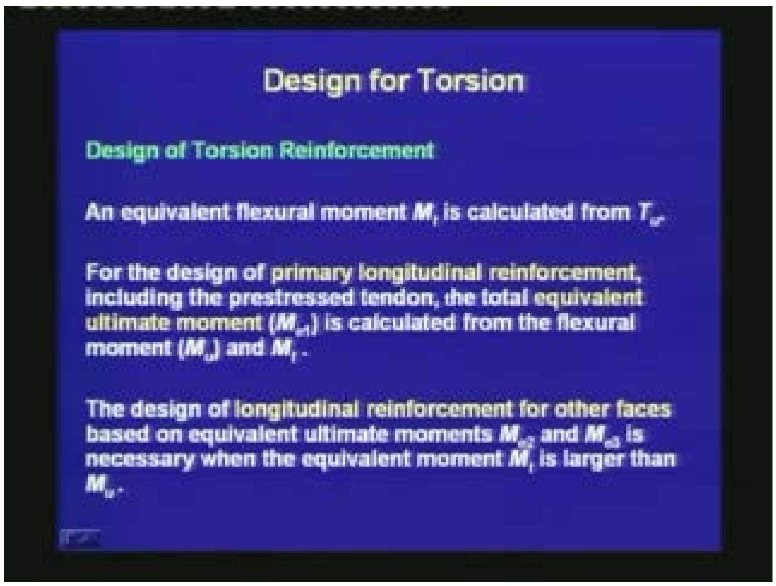

Next, an equivalent flexural moment M_t is calculated from T_u. For the design of primary longitudinal reinforcement, including the prestressed tendon, the total equivalent ultimate moment M_{e1} is calculated from the flexural moment M_u and M_t.

We have the flexural moment M_u and we have the equivalent flexural moment M_t due to the torsion. We add these two to get the total equivalent moment which is M_{e1}, and with that we design the primary longitudinal reinforcement including the prestressed tendon.

The design of the longitudinal reinforcement for other faces based on equivalent ultimate moments M_{e2} and M_{e3} is necessary when the equivalent moment M_t is larger than M_u. Thus, if the torsion is substantially high, then we may need to design for the other two equivalent moments M_{e2} and M_{e3}.

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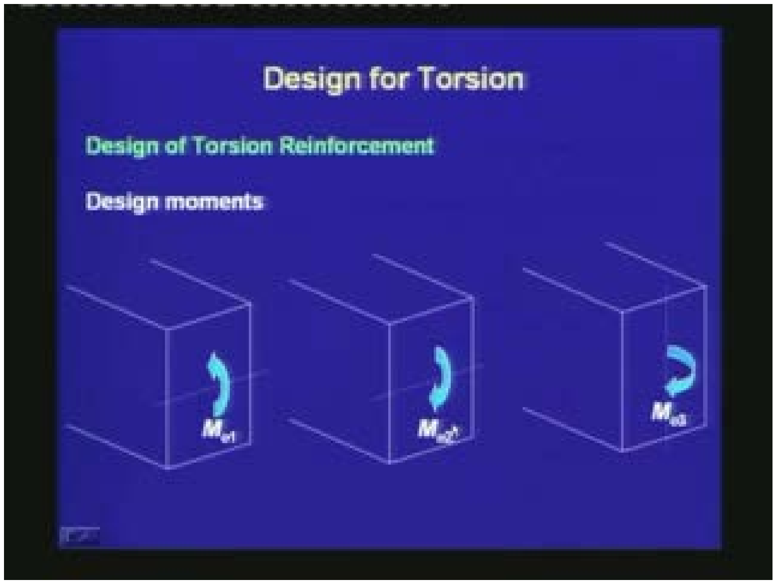

This figure shows the three design moments. The first one is M_{e1}, which is in the same sense as that of the flexural moments. If the flexural moment causes compression at the top, M_{e1} will also cause compression at the top. The value of M_{e1} is larger than the flexural moment due to the effect of torsion. Second, if the torsional moment is substantially high, then the failure may occur by the crushing of concrete at the bottom and yielding of the steel at the top. To check such a case, we design for a moment M_{e2}. We generate compression at the bottom and tension at the top. This is in an opposite sense to that of the flexural moment M_u. Third, which is usually required in a flanged section with thin web, that we may have to design for a moment which occurs about a vertical axis to cause transverse bending. This moment M_{e3} generates compression at one side face and tension in the opposite side face. Thus, the axes about which M_{e3} acts is vertical, unlike that of M_{e1} and M_{e2}. Observe the orientation of the three moments for the design of torsional reinforcement.

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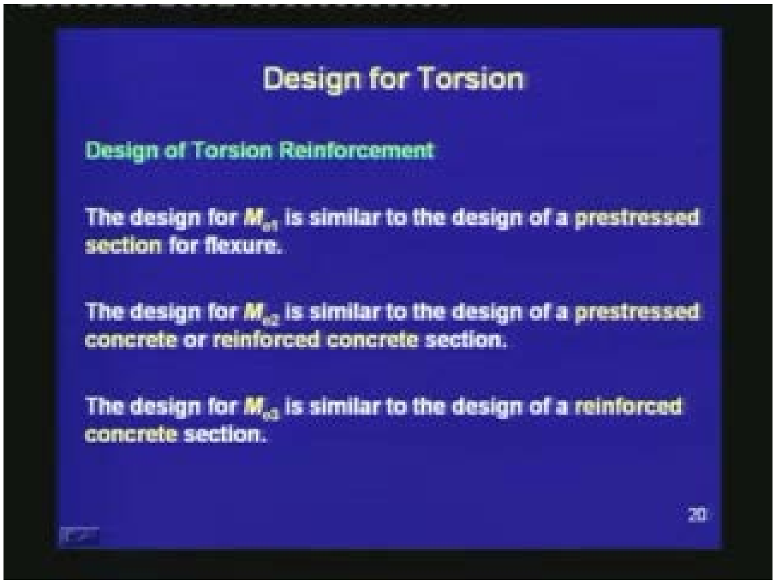

The design for M_{e1} is similar to the design of a prestressed section for flexure. Earlier in the chapter of design of members, we had studied in details, the design of members for flexure. In presence of torsion, once we calculate M_{e1} the design is exactly similar to what you have studied for flexure.

The design for M_{e2} is similar to the design of a prestressed concrete or reinforced concrete section. M_{e2} is the negative bending. We can design the section as a reinforcement concrete section, if there is no prestressing steel at the top, or we can design the section as a prestressed concrete section if there is some prestressing steel at the top.

The design for M_{e3} is similar to the design of a reinforced concrete section. For the transverse bending, we design it like a reinforced concrete section.

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The design of stirrups including torsion is similar to the design of stirrups in absence of torsion. In the design for shear, we had studied how to calculate the steel and provide the spacing. When there is torsion, the design is similar to the procedure that for shear, except for the amount of torsional reinforcement and the spacing.

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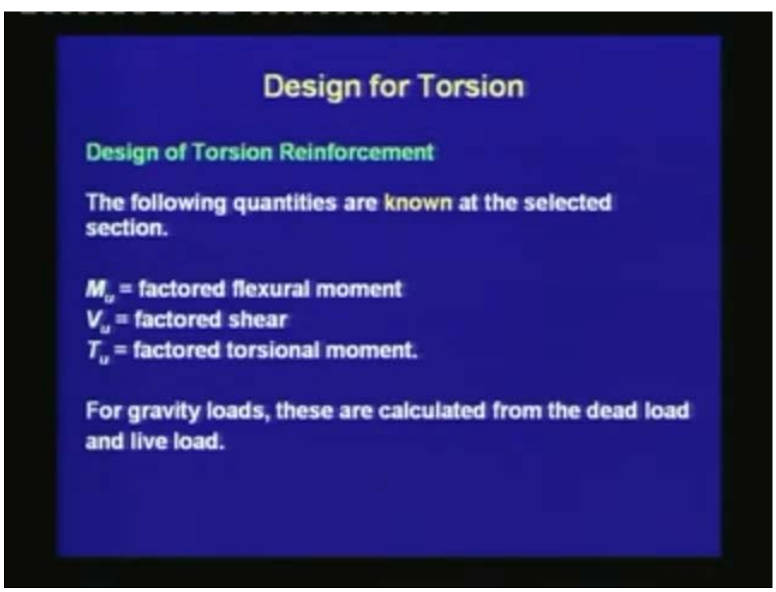

The following quantities are known at the selected section: M_u which is the factored flexural moment; V_u is the factored shear; T_u is the factored torsional moment. For gravity loads, these are calculated from the dead load and live load.

Thus, before we start the design, from structural analysis we have calculated the three actions acting in the particular section: one is the flexural moment M_u, next is the shear V_u and the torsional moment T_u. These are the known quantities before the design of the section.

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The grades of concrete and steel have to be selected before design. As per IS: 1343-1980, the grade of steel for stirrups is limited to Fe 415.

The selection of the material properties depend on what material will be supplied and on the type of structure that is being built. Before we start the design calculations, the material properties should be known to us, so that we can use those properties in the design equations.

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For the design of longitudinal reinforcement, the following quantities are unknown: the member cross-section; M_{e1}, M_{e2}, M_{e3} are the total equivalent flexural moments; A_p is the amount of prestressing steel; P_e is the effective prestress; e the eccentricity of CGS with respect to CGC; A_s is the area of longitudinal reinforcement; $A_s^{/}$ is the area of longitudinal reinforcement in opposite face. Prestressing steel $A_p^{/}$ may be provided in the opposite face.

If M_{e2} is large, then we may provide prestressing steel at the top also, and design the section as a prestressed section. Thus, before the design we know what are the unknown quantities that we need to solve to come to the designed section.

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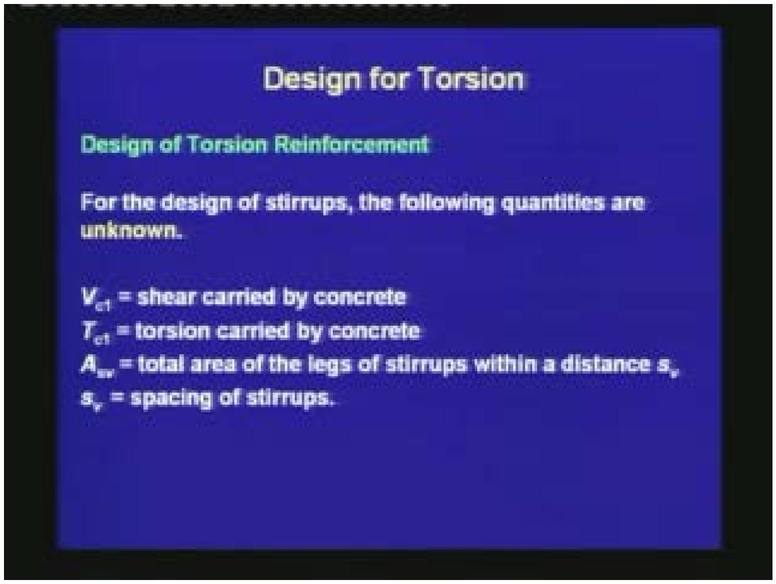

For the design of stirrups, the following quantities are unknown: V_{c1} is the shear carried by concrete; T_{c1} is the torsion carried by concrete; A_{sv} is the total area of the legs of stirrups within a distance s_v; and s_v is the spacing of stirrups. Remember that T_{c1} and V_{c1} are the reduced values from the capacities of concrete under the individual actions.

Next, we shall discuss about the design steps of the torsional reinforcement.

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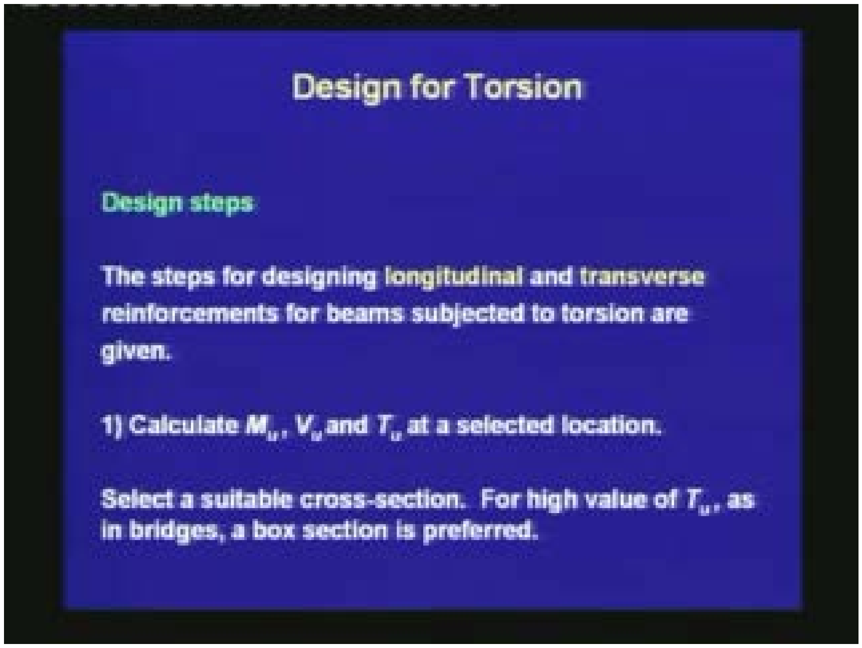

The steps for designing longitudinal and transverse reinforcement for beams subjected to torsion are given. First, calculate $M_{u,}$ V_u and T_u at a selected location. Select the suitable cross-section. For high value of T_u, as in bridges, a box section is preferred.

The selection of sections was discussed when we studied the design of members for flexure. The selection depends on the application and on the relative values of the forces. In buildings, usually we have rectangular section, or we can have flanged section for precast joists. For bridges, we can have rectangular section for small spans; but as the span increases, flanged section is preferred, or if the torsion is high then we select box section. Thus, the selection of a section is important as per the application and the design forces acting on the section.

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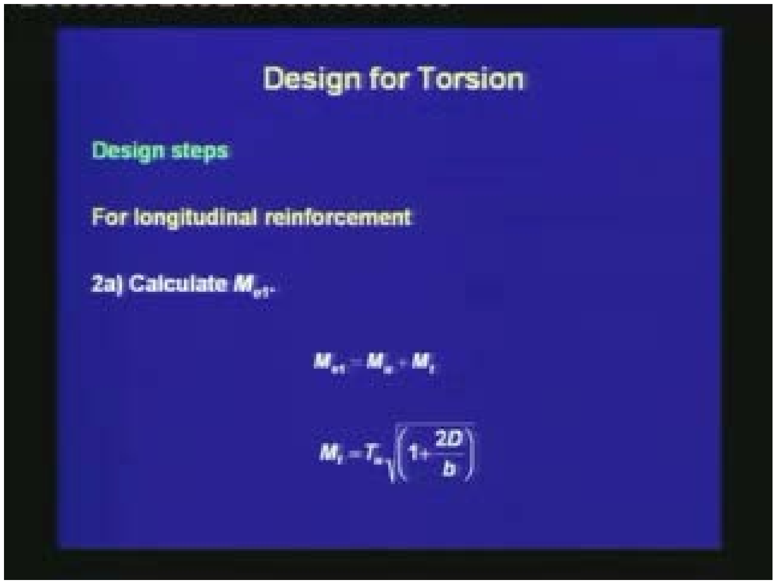

The next step is to calculate the equivalent moment M_{e1}. M_{e1} is equal to $M_u + M_t$, where M_t is calculated from the following expression.

$M_t = T_u \sqrt{(1 + 2D/b)}$

M_t is the equivalent flexural moment corresponding to the torsional moment T_u. We add that to the flexural moment M_u to get the total equivalent moment at ultimate, which is represented as M_{e1}.

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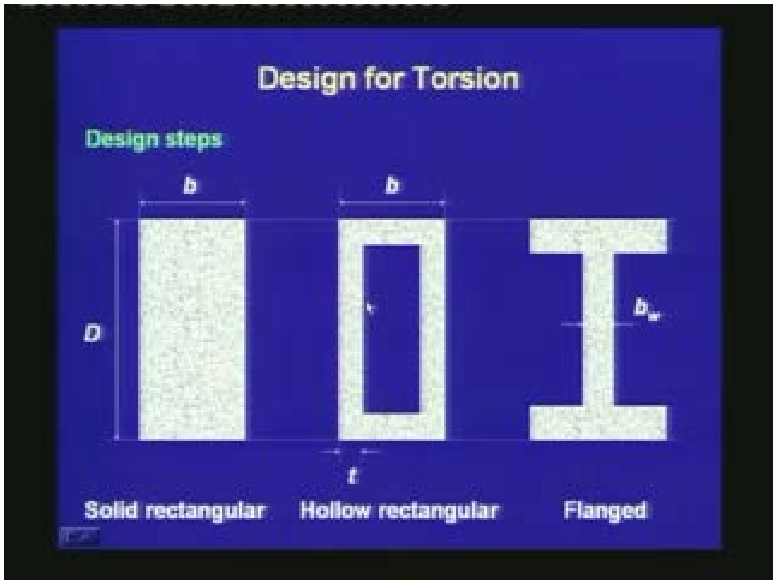

The geometric variables are defined for the different types of section as follows: b is the breadth which is the smaller dimension; D is the depth which is the longer dimension. For a hollow rectangular section, the thickness should be at least quarter of the breadth so that we have adequate shear flow zone. For a flanged section, the breadth of the web b_w substitutes b in the previous expression.

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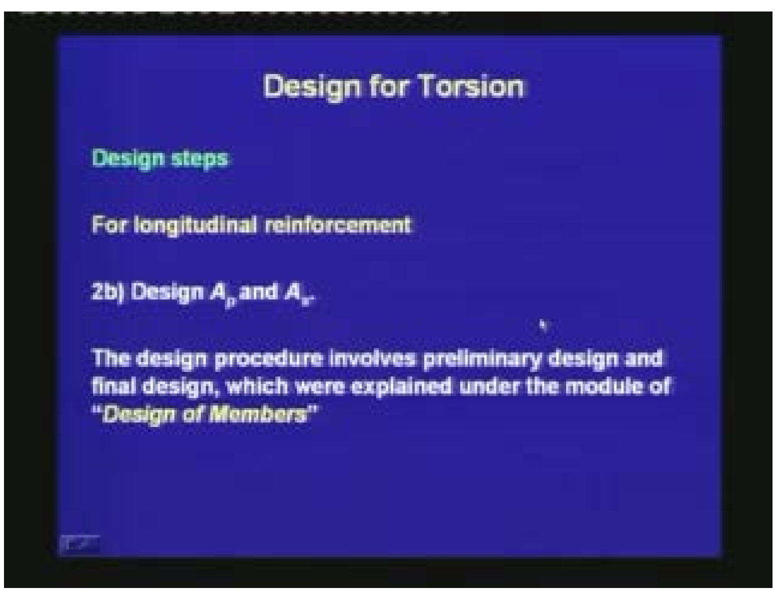

In the next step, we have to design A_p and A_s. A_p is the amount of prestressing steel and A_s is the amount of longitudinal steel. The design procedure involves preliminary design and final design, which were explained under the module of “Design of Members”.

The flexural design is a systematic procedure to select the maximum eccentricity of the prestressing steel so as to have an economical amount of the steel. We do this by having the maximum possible lever arm at service loads. The design steps are divided into two stages: first, is the preliminary stage; second, is the final stage. Thus, when the member has torsion, the design of the primary longitudinal reinforcement is similar to the design of members for flexure. The only difference is that we are substituting the total equivalent moment M_{e1} in place of the flexural moment M_u.

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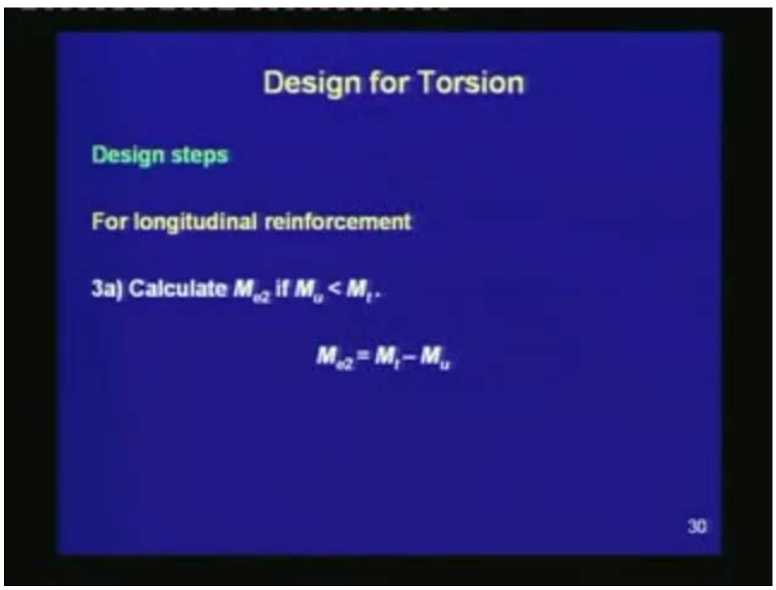

The third step is to calculate the other equivalent moment M_{e2}, if M_u is less that M_t.

$M_{e2} = M_t – M_u$

Thus, if the torsion is substantially high, there is a chance of the negative bending failure. In that situation, we need to calculate M_{e2} and design for M_{e2}.

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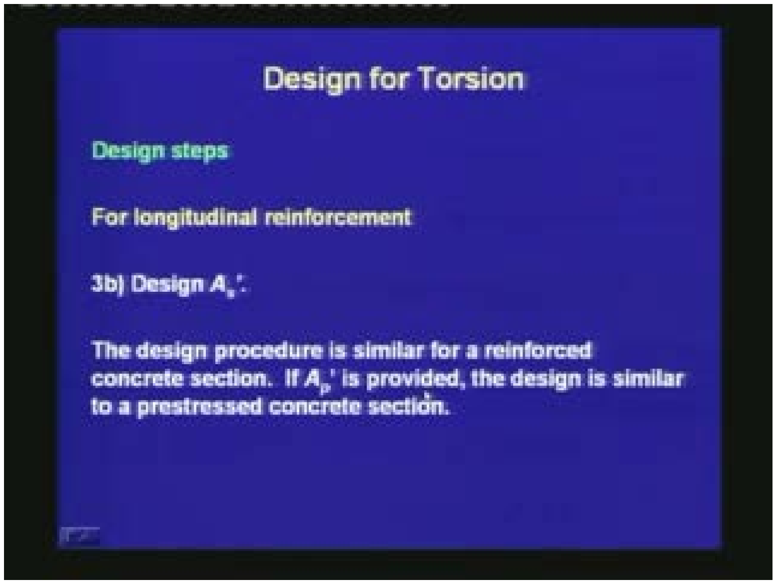

Design $A_s^{/}$ for M_{e2}. The design procedure is similar for a reinforced concrete section. If $A_p^{/}$ is provided, the design is similar to a prestressed concrete section. Once we have M_{e2}, we can design the section as a reinforced concrete section where we do not provide any prestressing steel at the top near the tension zone. If we provide prestressing steel at the tension zone, then we have to design the section as a prestressed section. The design procedure is similar to the design for flexure.

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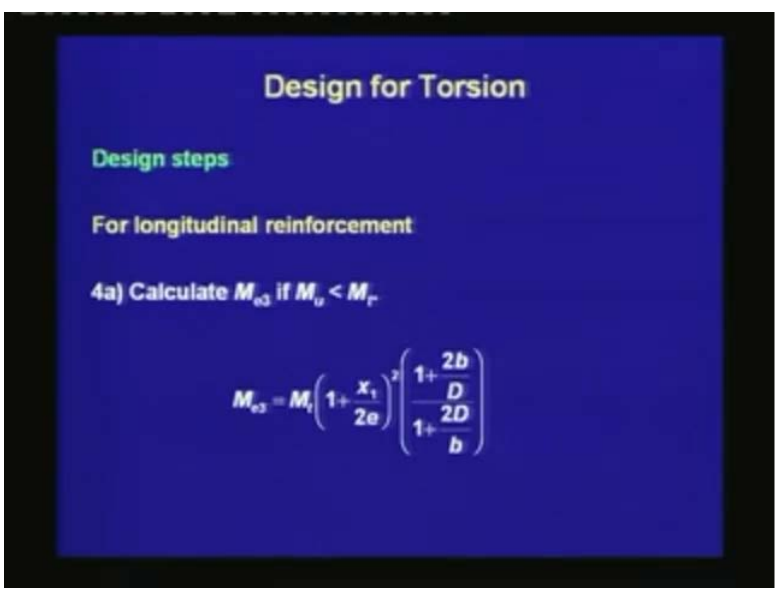

The fourth step is to calculate the third equivalent moment M_{e3} for transverse bending, if M_u is less than M_t. The expression of M_{e3} is given as follows.

$M_{e3} = M_t (1 + (x_1/2e))^2 ((1 + 2b/D)/(1 + 2D/b))$

The moment acts about the vertical axis causing transverse bending. We need to design for the side face reinforcement.

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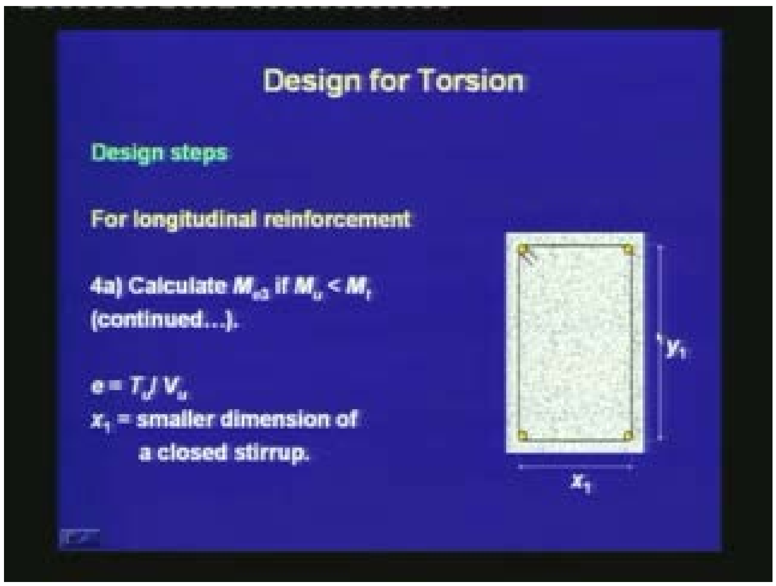

In the previous expression, $e = T_u/V_u$. Note that, this e is not the eccentricity of the prestressing steel. Here, e is the ratio of the ultimate torsional moment to the ultimate shear. For a closed stirrup, we are representing x_1 as the smaller dimension and y_1 as the larger dimension.

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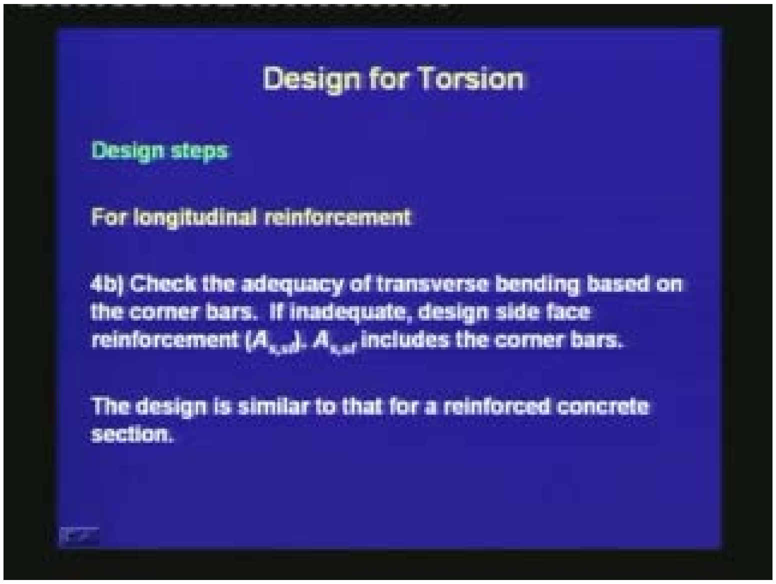

Once we have calculated M_{e3}, we need to check the adequacy of transverse bending based on the corner bars. If inadequate, design the side face reinforcement $A_{s,sf}$. $A_{s,sf}$ includes the corner bars. The design is similar to that for a reinforced concrete section.

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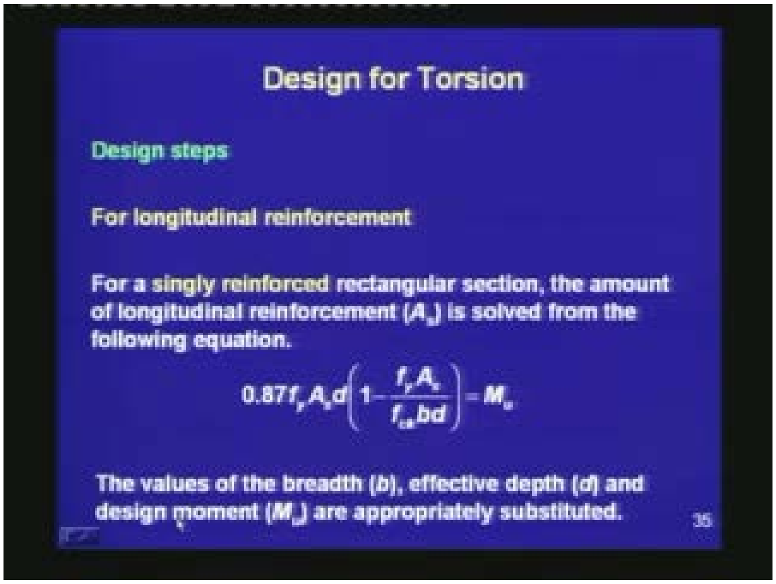

For a singly reinforced rectangular section, the amount of longitudinal reinforcement A_s is solved from the following equation.

$0.87 f_y A_s d (1 – f_y A_s/f_{ck}bd) = M_u$

The values of the breadth b, effective depth d and design moment M_u are appropriately substituted. Here, b will be the vertical dimension of the beam and d will be the horizontal effective depth of the reinforcement. Substituting the variables, we can find out the value of A_s.

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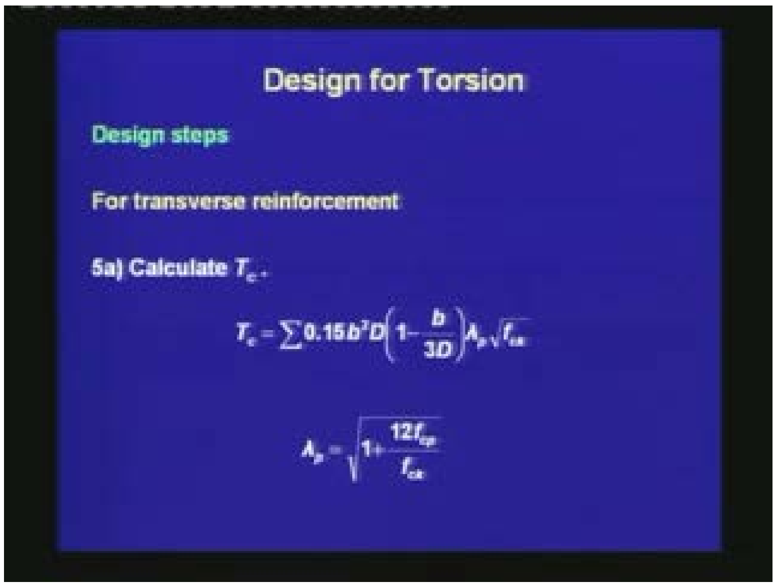

Next, we are moving on to the design of transverse reinforcement. The first step is to calculate the capacity of concrete to resist torsion.

$T_c = \Sigma 0.15b^2D (1 – b/3D) \lambda_p\sqrt{f_{ck}}$

$\lambda_p = \sqrt{(1 + 12 f_{cp}/f_{ck})}$

In the last lecture, we saw that the capacity of concrete is based on the generation of a torsional crack at the mid-depth of the longer side. If it is a flanged section, then the expression considers the flanged section as a compound section made up of several rectangles, and we use the summation symbol. The effect of prestressing is considered by the factor λ_p, which shows that by increasing the prestress the strength of concrete will increase. This expression of T_c helps us to calculate the capacity of concrete to resist pure torsion.

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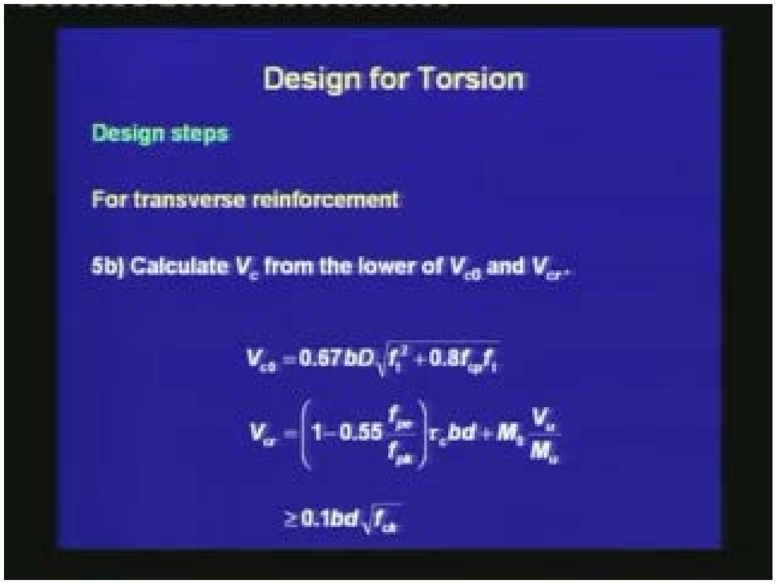

Calculate V_c, the capacity of concrete for shear in absence of torsion, from the lower of V_{c0} and V_{cr}. To recollect, V_{c0} is the capacity of concrete for an uncracked section. V_{c0} corresponds to the shear corresponding to web shear crack. V_{cr} is the capacity of concrete for a cracked section. It is equal to the shear that causes a flexure–shear crack. The expressions of these two quantities V_{c0} and V_{cr} are as follows.

$V_{c0} = 0.67bD\ \sqrt{(f_t^2 + 0.8\ f_{cp}\ f_t)}$

In this expression, f_t is the tensile strength of concrete and f_{cp} is the average prestress at the level of the CGC.

$V_{cr} = (1 - 0.55\ f_{pe}/f_{pk})\tau_c\ bd + M_0\ (V/M)$

The expression of V_{cr} contains two terms: the first term, is the amount of shear that extends the flexural crack to a flexure–shear crack. This term depends on τ_c which is the capacity of concrete to resist shear. The second term is the shear corresponding to flexural crack. The value of V_{cr} need not to be smaller than $0.1bd\sqrt{f_{ck}}$. Thus, here we have two expressions of the capacity of concrete to resist the shear in absence of torsion, and whichever gives the smaller value, that value is assigned to V_c.

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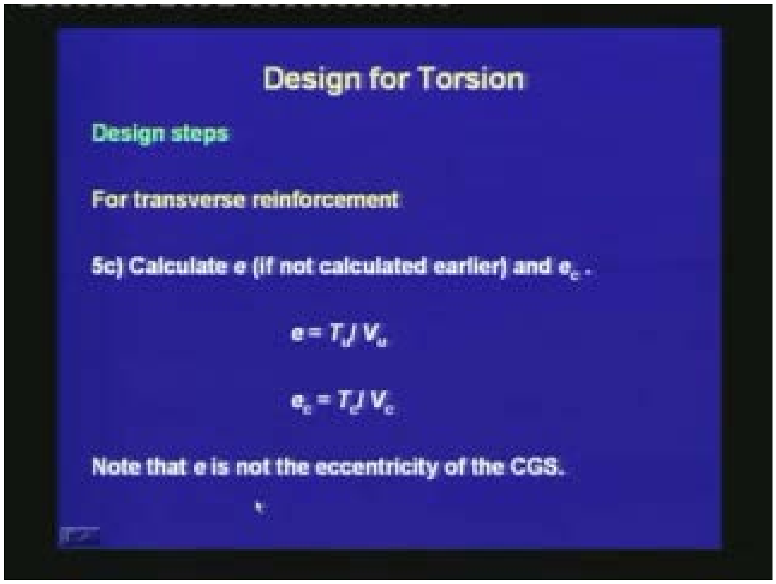

Next, we are calculating the parameters e and e_c, which are the ratios of the torsion and shear. e is the ratio of the ultimate torsion to the ultimate shear and e_c is the ratio of the capacity of concrete to resist torsion to the capacity of concrete to resist shear. Note that e is not the eccentricity of the CGS.

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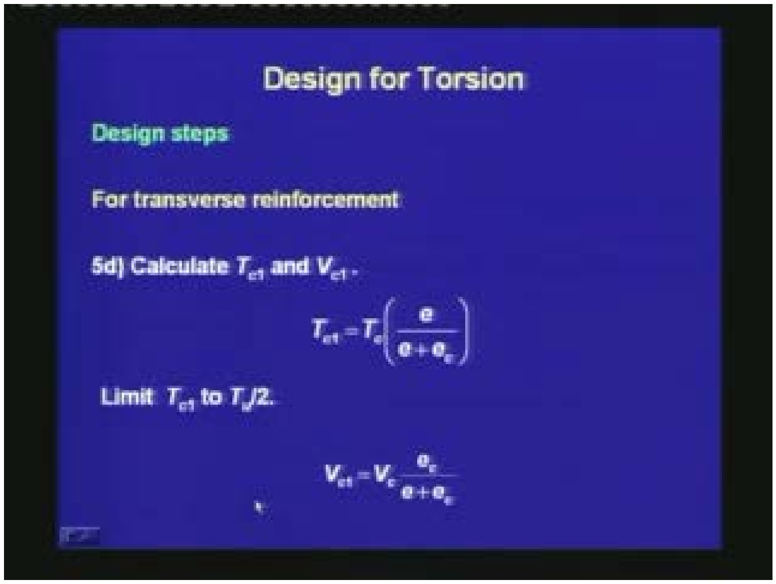

The parameters e and e_c are used in the interaction equation of the torsion and shear, from which we calculate the capacity of concrete to resist torsion and shear when both of them are acting together. In this step, we are calculating T_{c1} and V_{c1}.

$T_{c1} = T_c\, e/(e + e_c)$

Here, T_c is the capacity in absence of shear. T_{c1} is limited to half of the ultimate torque.

$V_{c1} = V_c\, e_c/(e + e_c)$

Now, we have got two capacities of concrete, one to resist torsion and other to resist shear, when both of them are acting together.

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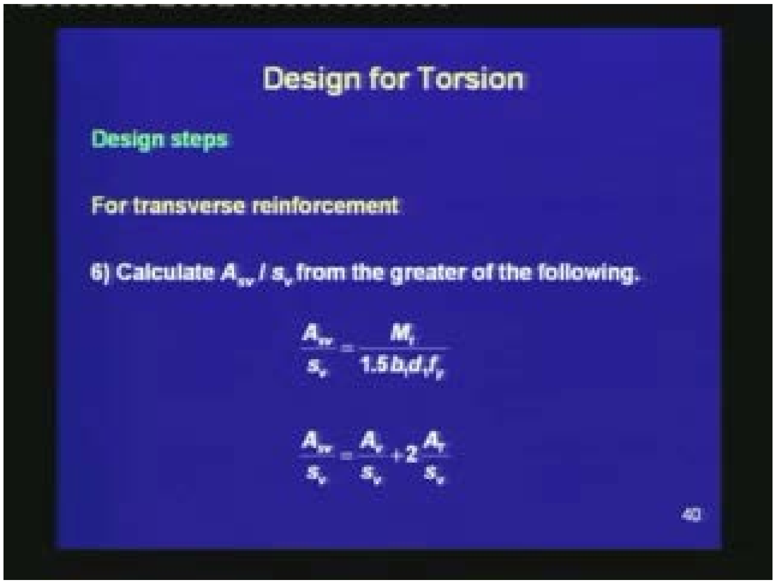

Now we are calculating the amount of transverse reinforcement from the greater of the two expressions of A_{sv}/s_v. The first expression is based on the skew bending theory, where M_t is the equivalent flexural moment.

$A_{sv}/s_v = M_t/(1.5b_1d_1f_y)$

The second expression of A_{sv}/s_v is from the requirement of total shear.

$A_{sv}/s_v = A_v/s_v + 2A_T/s_v$

The first term is the requirement based on flexural shear. The second term is the requirement based on torsional shear. They are considered to be additive in both the vertical sides of the shear flow zone.

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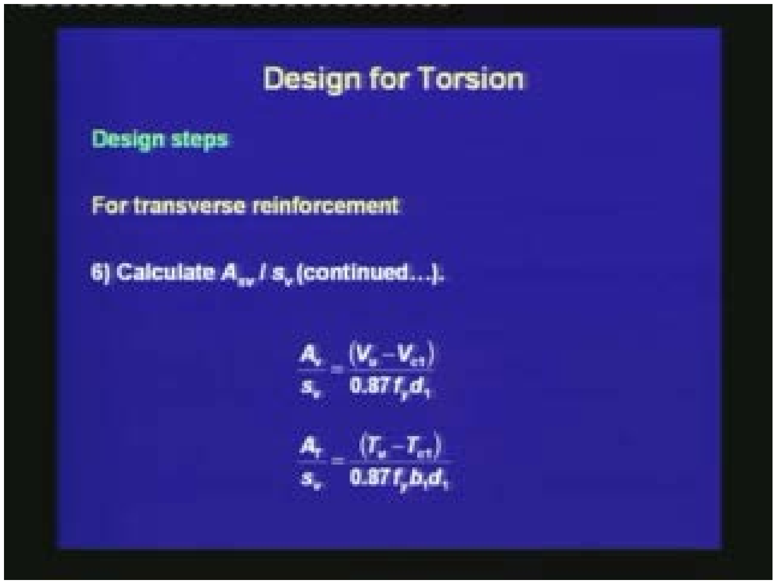

In the previous expression A_v/s_v is calculated from the shear to be carried by the steel which is $(V_u - V_{c1})$. A_T/s_v corresponds to the torsional shear that is to be carried by the steel which is $(T_u - T_{c1})$.

$A_v/s_v = (V_u - V_{c1})/0.87f_yd_1$

$A_T/s_v = (T_u - T_{c1})/0.87f_yb_1d_1$

The two amounts are combined to get the value of A_{sv}/s_v.

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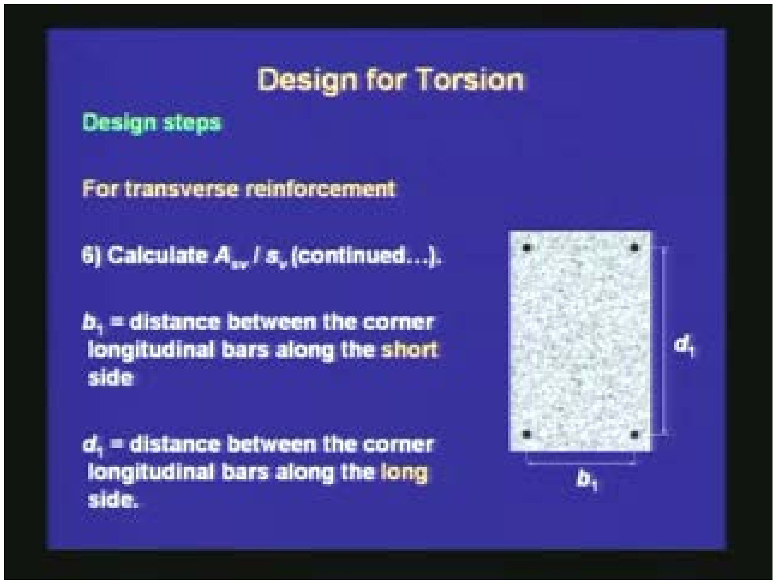

In the previous expression, b_1 is the distance between the corner longitudinal bars along the short side, and d_1 is the distance between the corner longitudinal bars along the long side. At the beginning, these values are not available exactly, but are estimated based on the cover requirement and estimated size of the transverse reinforcement.

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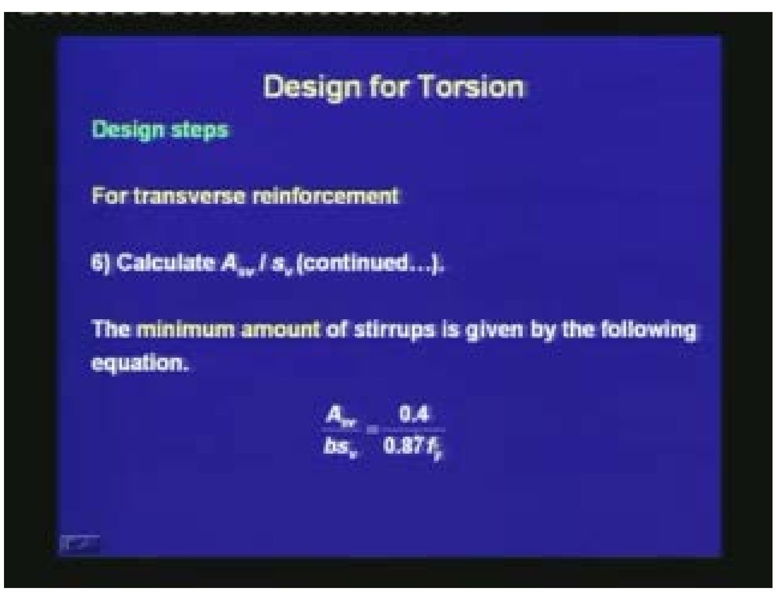

We have to satisfy the minimum amount of transverse reinforcement. The minimum amount of stirrups is given by the following equation.

$A_{sv}/bs_v = 0.4/0.87f_y$

Minimum transverse reinforcement is required to check a sudden failure due to shear.

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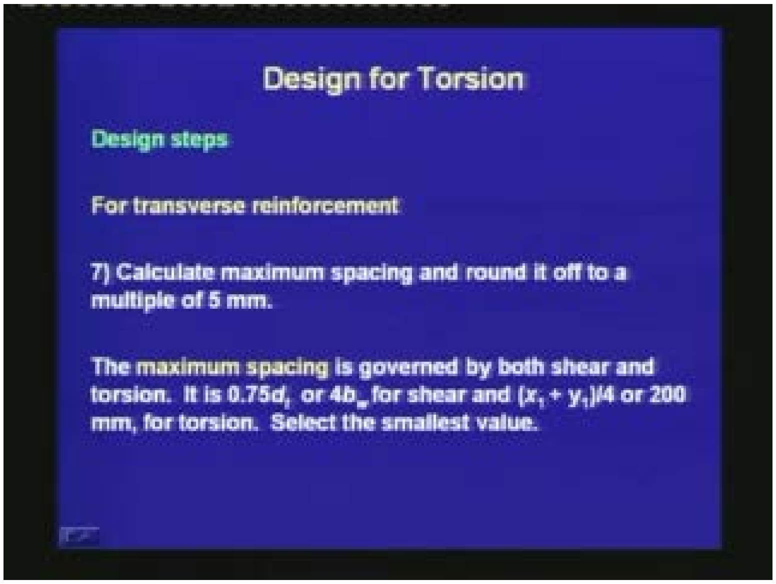

The seventh step is to calculate the maximum spacing of the stirrups, and round it off to a multiple of 5 mm. The maximum spacing is governed by both shear and torsion. It is $0.75d_t$ or $4b_w$ for shear, and $(x_1 + y_1)/4$ or 200 mm, for torsion. Select the smallest value. Thus, there are two requirements of maximum spacing. The first one is based on the plane truss analogy for shear; there is a maximum limit of $4b_w$ to that. The second requirement is based on the space truss analogy for torsion; there the maximum limit is 200 mm. Once we calculate these four values, the smallest value gives the maximum spacing for the transverse reinforcement.

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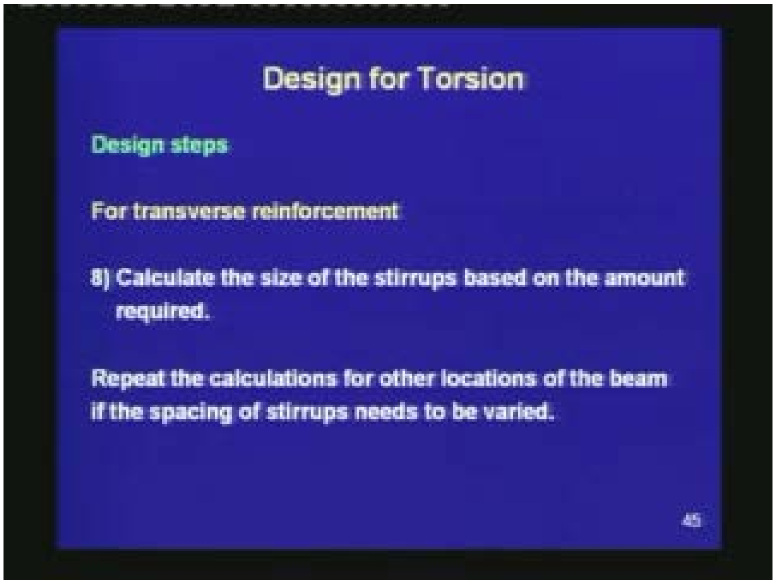

The eighth step is to calculate the size of the stirrups based on the amount required. Once we know s_v, we can calculate A_{sv} from the value of A_{sv}/s_v. From there, we calculate the size of the stirrups and the number of legs that will satisfy the value of A_{sv}.

This whole procedure can be done for other sections, if you need to vary the spacing of the stirrups. It depends upon the designer, that how many sections we are designing for, in a particular beam.

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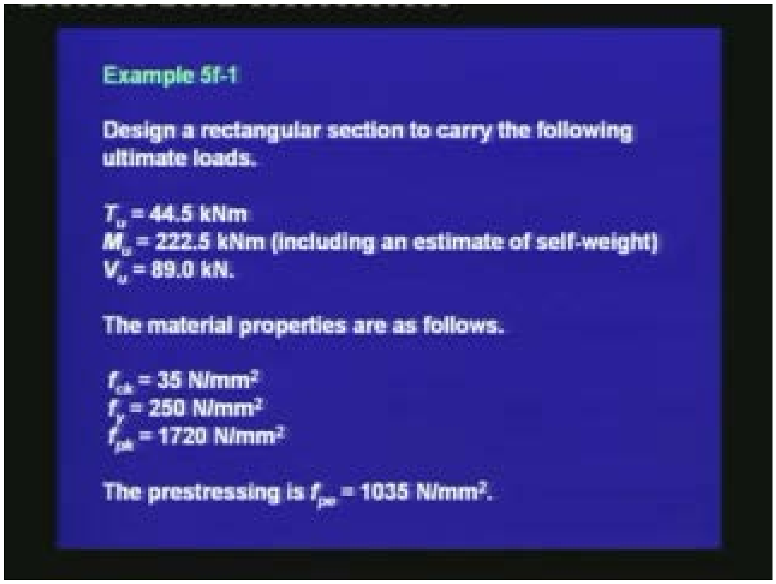

Let us understand the design procedure by the help of an example. Design a rectangular section to carry the following ultimate loads: T_u = 44.5 kNm, M_u = 222.5 kNm including an estimate of self-weight, V_u = 89 kN. Note that here the value of T_u is substantially small than M_u. The material properties are as follows: f_{ck} = 35 N/mm^2; f_y = 250 N/mm^2; f_{pk} = 1720 N/mm^2. The effective prestress after the losses is f_{pe} = 1035 N/mm^2.

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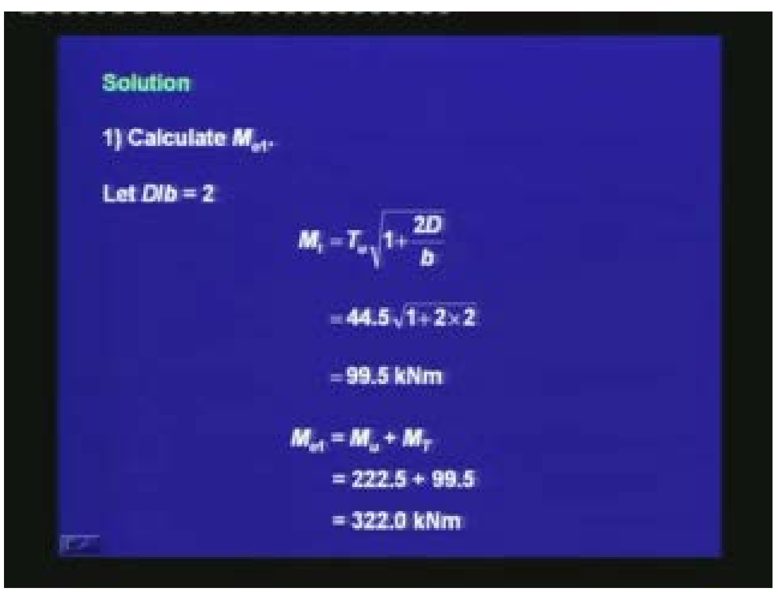

The first step is to calculate the equivalent moment M_{e1}. For the rectangular section, let the depth D divided by the breadth b is equal to 2; this gives an economical section. With this value of D/b we can calculate M_t from T_u. M_t = 99.5 kNm. Thus, $M_{e1} = M_u + M_t$ = 222.5 + 99.5 = 322 kNm.

Thus, in presence of torsion, we are designing for a higher flexural moment as compared to the original flexural moment. The original flexural moment was 222.5 kNm, but in presence of the torsion we are designing for a moment which is 322.0 kNm.

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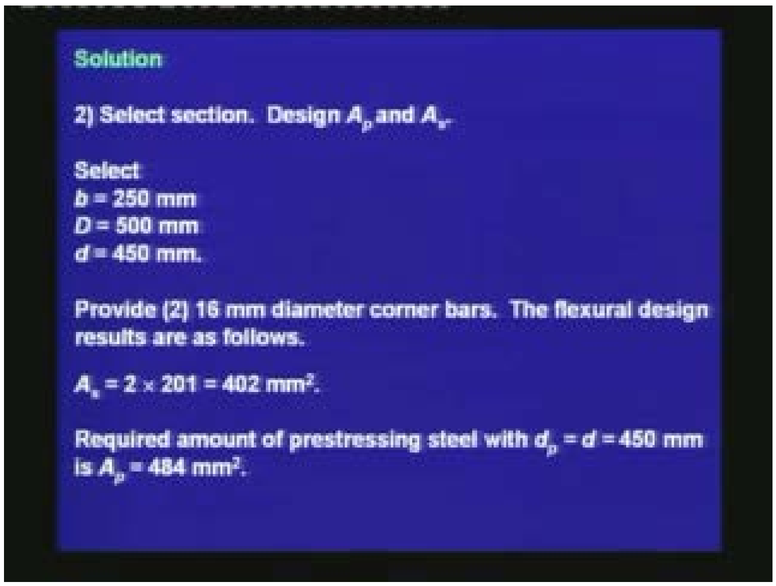

The second step is to select the section, and design A_p and A_s. For the rectangular section, let b = 250 mm, D = 500 mm, the effective depth d = 450 mm. Based on the design steps, provide two numbers of 16 mm diameter corner bars. The flexural design results are as follows: $A_s = 2 \times 201 = 402$ mm^2. The required amount of prestressing steel with $d_p = d$ = 450 mm is A_p = 484 mm^2. The design has been done based on the steps that we do for a conventional flexural design. This is a partially prestressed section, where we are taking advantage of the longitudinal bars to provide flexural capacity.

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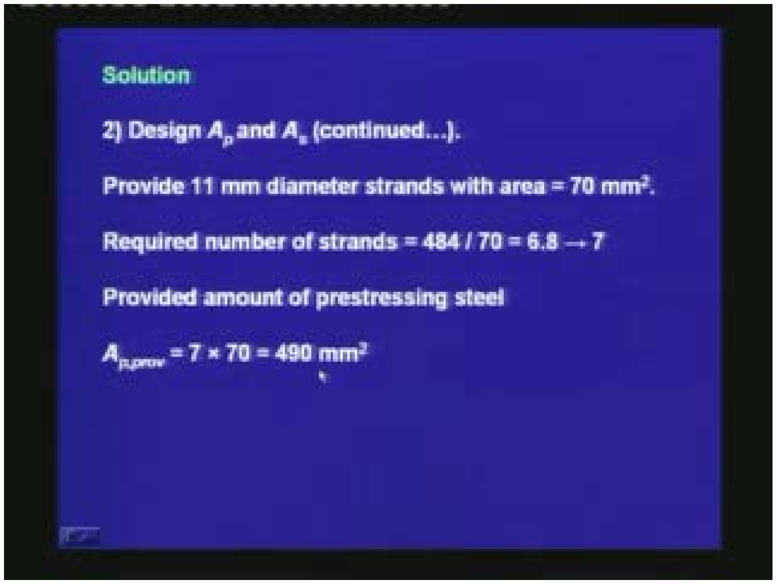

Provide 11 mm diameter strands with area 70 mm^2. At this stage, we are selecting what type of strands we shall use for prestressing. The required number of strands is equal to 484 ÷ 70 = 6.8, which is rounded off to 7. Provided amount of prestressing steel is $A_{p,prov}$ = 7 × 70 = 490 mm^2. Thus, the amount of prestressing steel for the primary longitudinal reinforcement has been designed at the stage, and it is 7 of 11 mm diameter strands.

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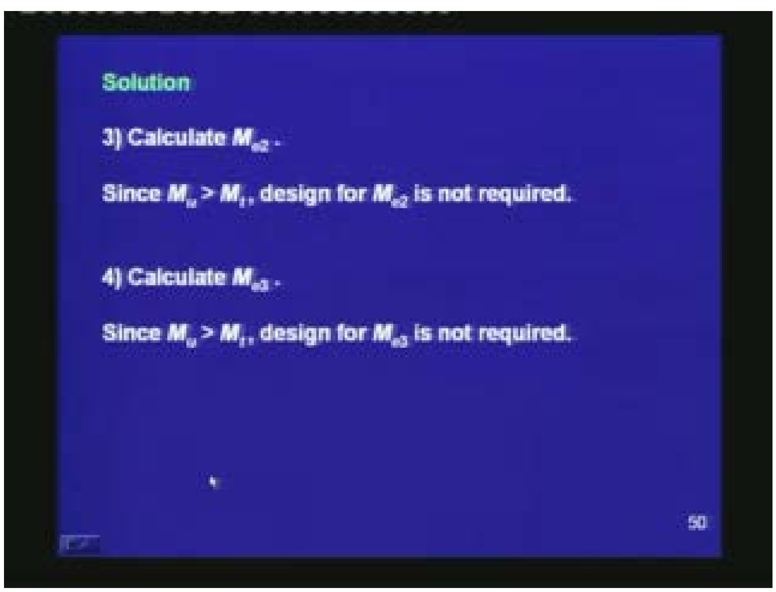

The third step is to calculate the next equivalent moment, M_{e2}. In this example, M_u is larger than M_t and hence design for M_{e2} is not required. The fourth step is to calculate M_{e3}. Here, M_u is larger than M_t and hence design for M_{e3} is not required.

In most of the situations, we may not have to design for M_{e2} and M_{e3}, because the flexural moment (M_u) will be substantially high compared to the equivalent moment due to torsion (M_t). But there may be cases where the equivalent moment is high and in that situation, we have to design for M_{e2} and M_{e3} as well.

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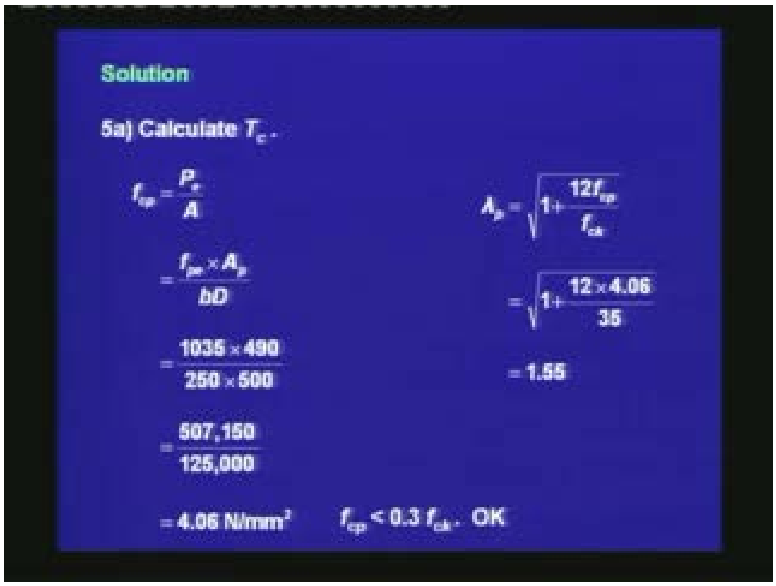

Now, we are moving on to the design of the transverse reinforcement. Calculate T_c. First we are calculating f_{cp} which is the average prestress in the section. The average prestress is calculated as $P_e/A = f_{pe} \times A_p/bD$. Substituting $f_{pe} = 1035$ N/mm^2, $A_p = 490$ mm^2, b and D equal to 250 mm and 500 mm, respectively, $P_e = 507{,}150$ N and $A = 125{,}000$ mm^2. Thus, the average prestress in the concrete is equal to 4.06 N/mm^2. Since, we are interested in the numeric value we are not placing the sign in this expression. We have found that f_{cp} is less than 30% of the characteristic strength. The characteristic strength is 35 and one-third of it is about 12 N/mm^2.

With the value of f_{cp} we are calculating the value of λ_p, the multiplying factor to consider the effect of prestressing force. $\lambda_p = \sqrt{(1 + 12f_{cp}/f_{ck})} = \sqrt{(1 + 12 \times 4.06/35)} = 1.55$. Thus, the effect of prestressing force is considered by a multiplying factor which is 1.55. That means the strength T_c has increased by 55 % due to the presence of the prestressing force.

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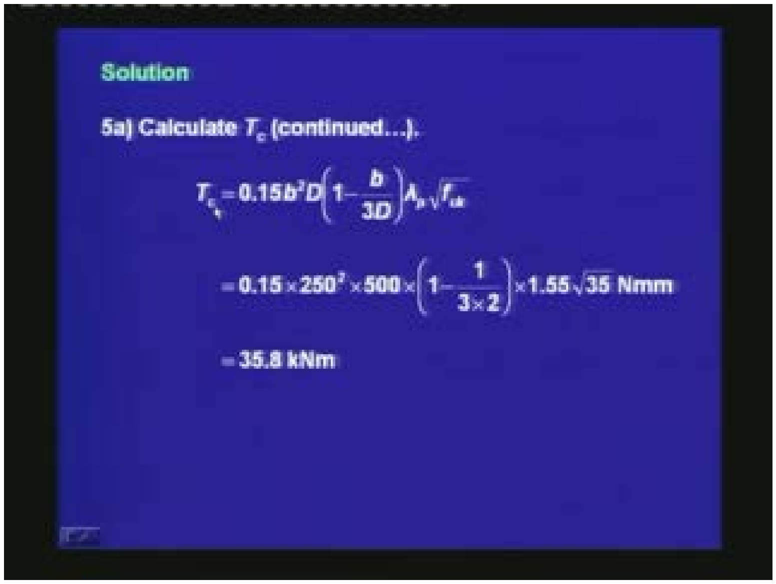

$T_c = 0.15b^2D\ (1 - b/3D)\ \lambda_p\sqrt{f_{ck}} = 0.15 \times 250^2 \times 500 \times (1 - 1/3\times2) \times 1.55 \times \sqrt{35}$ Nmm = 35.8 kNm. That is the capacity of concrete to resist torsion in absence of any shear force.

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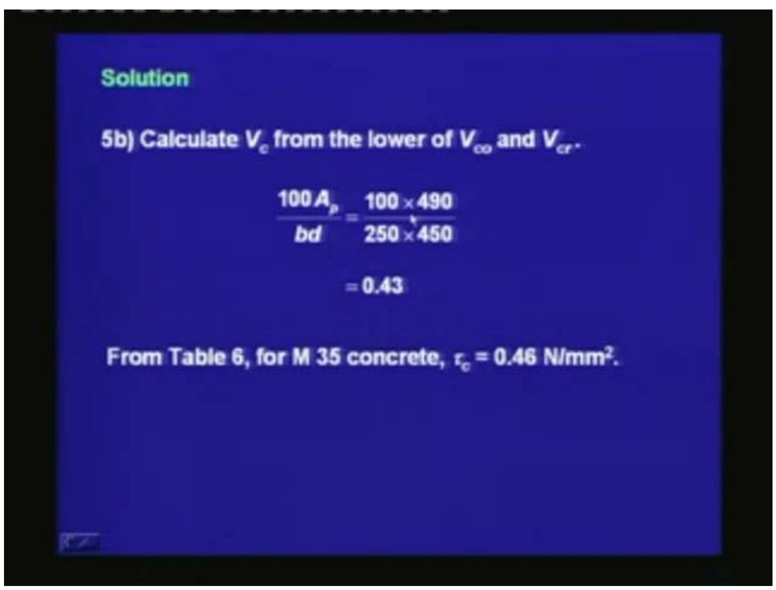

To calculate V_c, we are first calculating the value of V_{cr}. For that we need the amount of prestressing steel as a percentage. $100A_p/bd = 100\times490/250\times450 = 0.43\%$ of prestressing steel. From Table 6, for M35 concrete, $\tau_c = 0.46$ N/mm^2.

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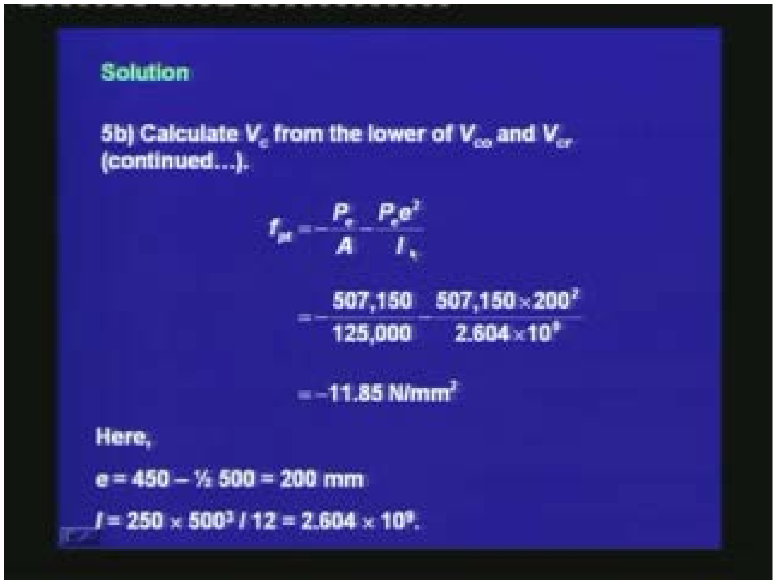

The value of the prestress at the level of the CGS $f_{pt} = -P_e/A - P_e e^2/I$. This expression is available from the stress concept, and once we substitute the value of P_e, A, e and I, we get the value of $f_{pt} = -11.85$ N/mm^2. Here, e has been estimated as the depth of the prestressing steel minus the depth of the CGC, e = 450 – ½ 500 = 200 mm. $I = bd^3/12 = 2.604 \times 10^9$ mm^4.

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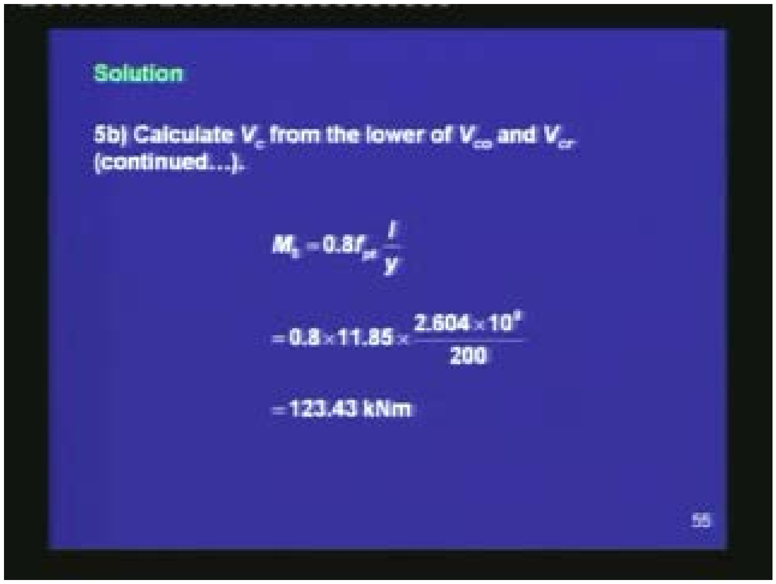

$M_0 = 0.8 f_{pt} I/y$. Once we substitute the values of f_{pt}, I and y, we get $M_0 = 123.43$ kNm.

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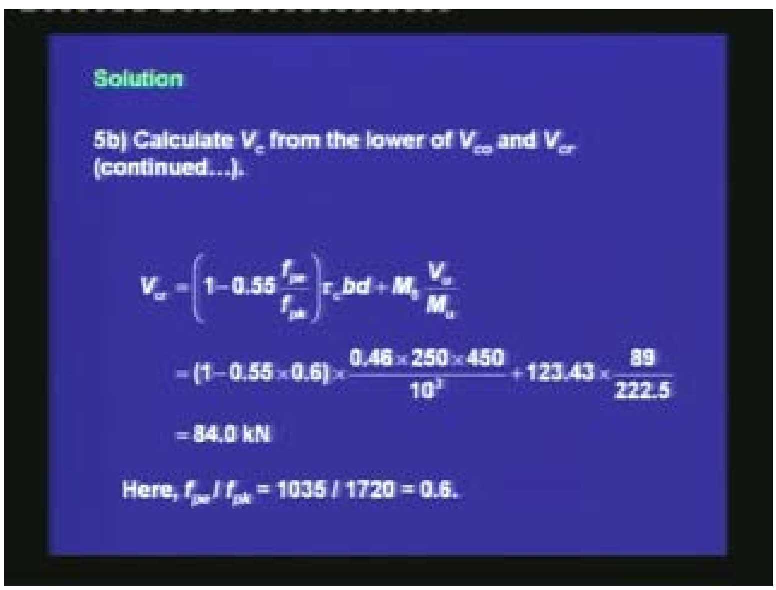

Thus, substituting the values of τ_c, M_0, V_u and M_u we get $V_{cr} = 84.0$ kN. Thus, this is the amount of shear that will generate flexure–shear crack.

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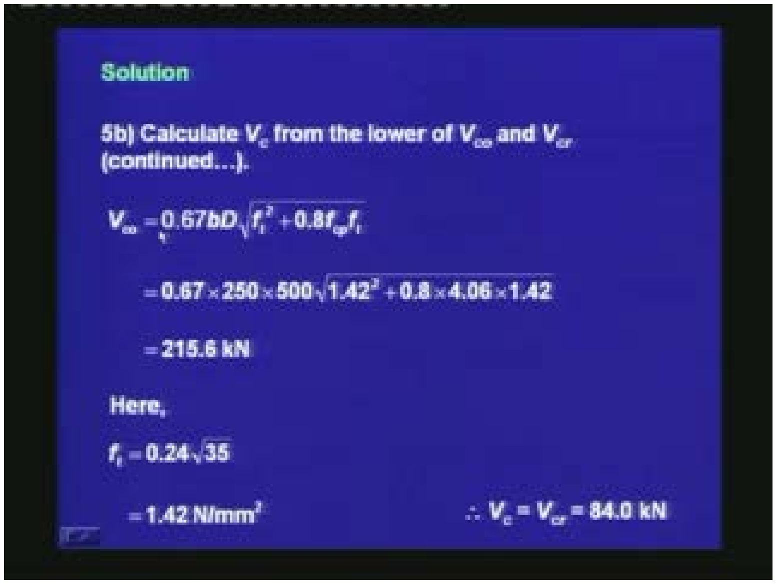

$V_{c0} = 0.67bD\sqrt{(f_t^2 + 0.8f_{cp}f_t)} = 0.67 \times 250 \times 500 \times \sqrt{(1.42^2 + 0.8\times4.06\times1.42)} = 215.6$ kN. V_{c0} is much higher than V_{cr} and hence, we are assigning the value of V_{cr} to V_c. Thus, V_c = 84.0 kN.

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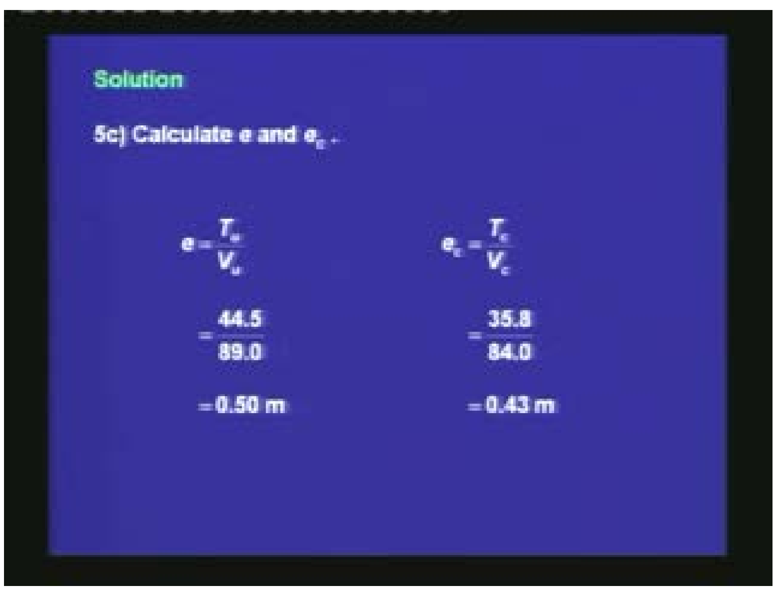

We are calculating the parameters e and e_c. $e = T_u/V_u = 0.5$ m. $e_c = T_c/V_c = 0.43$ m.

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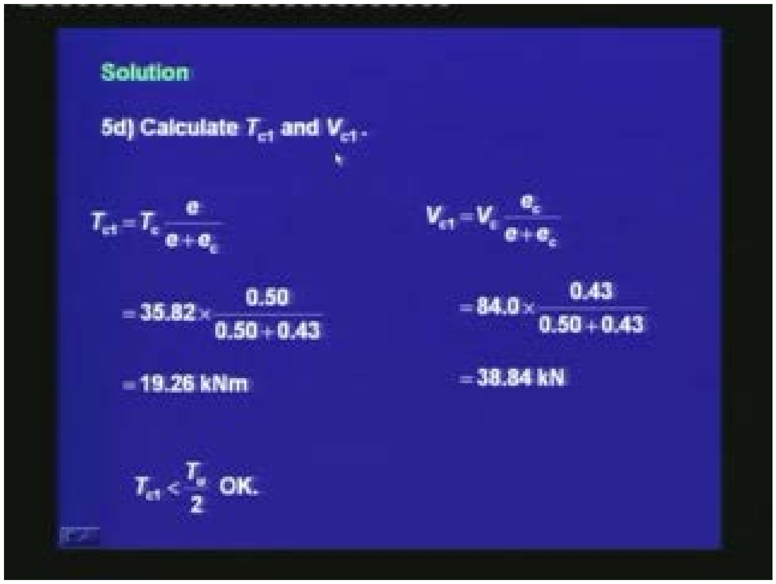

From these we are able to calculate T_{c1} and V_{c1}. T_{c1} is the reduced value from T_c. T_{c1} = 19.26 kNm. Similarly, we get a reduced value of V_{c1} from V_c. V_{c1} = 38.84 kN. Here, T_{c1} is less than $T_u/2$, and hence it is ok.

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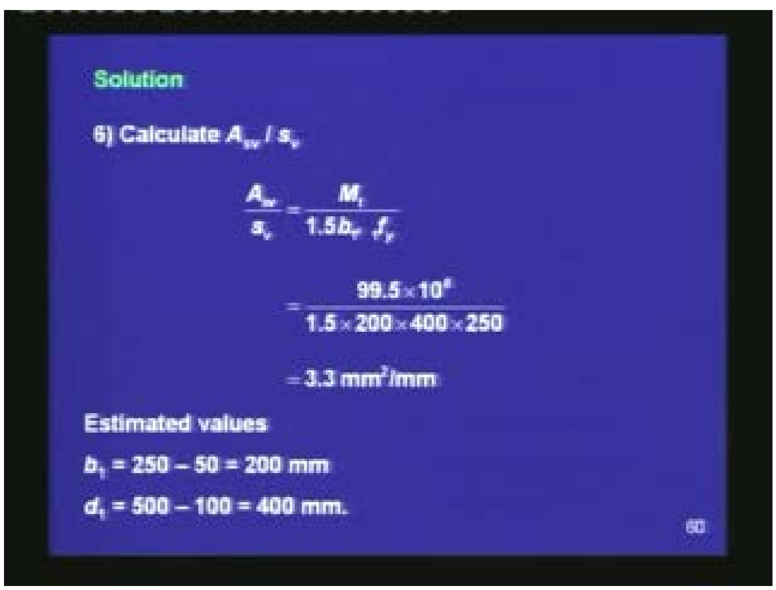

We are calculating A_{sv}/s_v from the equivalent moment M_t. A_{sv}/s_v = 3.3 mm^2/mm. Here, b_1 is estimated as 250 – 50 = 200 mm, and d_1 is estimated as 500 – 100 = 400 mm.

The first term in the second expression of A_{sv}/s_v is based on the shear to be carried by the stirrups, and once we substitute the value of V_u and V_{c1}, we get $A_v/s_v = 0.58\ mm^2/mm$.

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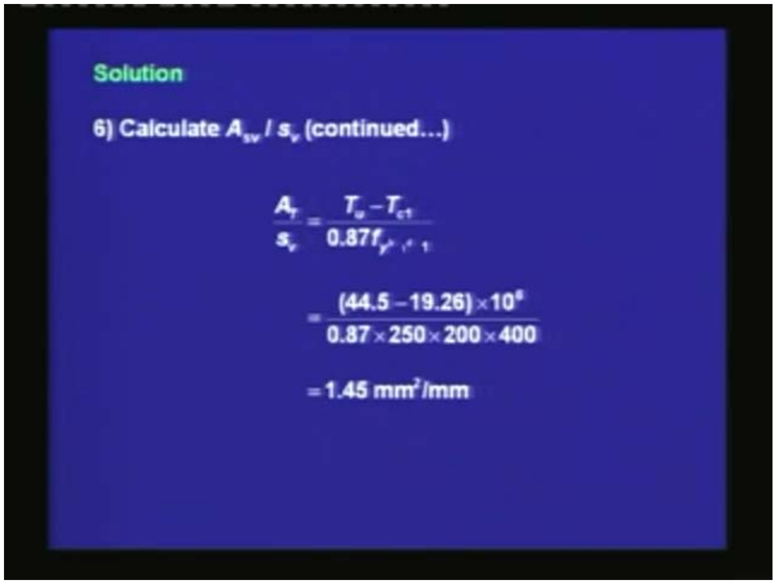

The second term in the second expression of A_{sv}/s_v is based on the torsion to be carried by the stirrups, and once we substitute the value of T_u and T_{c1}, we get $A_v/s_v = 1.45\ mm^2/mm$.

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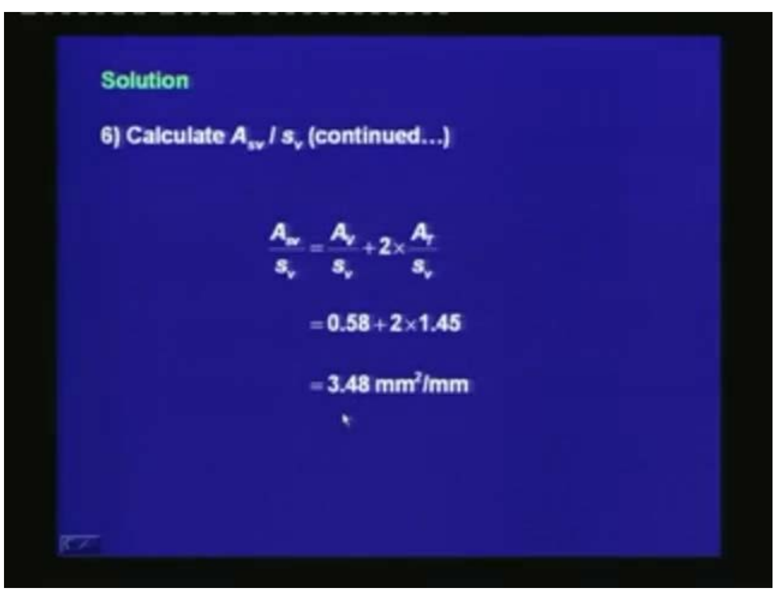

Once we substitute the values in the expression of A_{sv}/s_v, we get a requirement of 3.48 mm^2/mm. This is higher than the previous requirement, and hence we are selecting this value.

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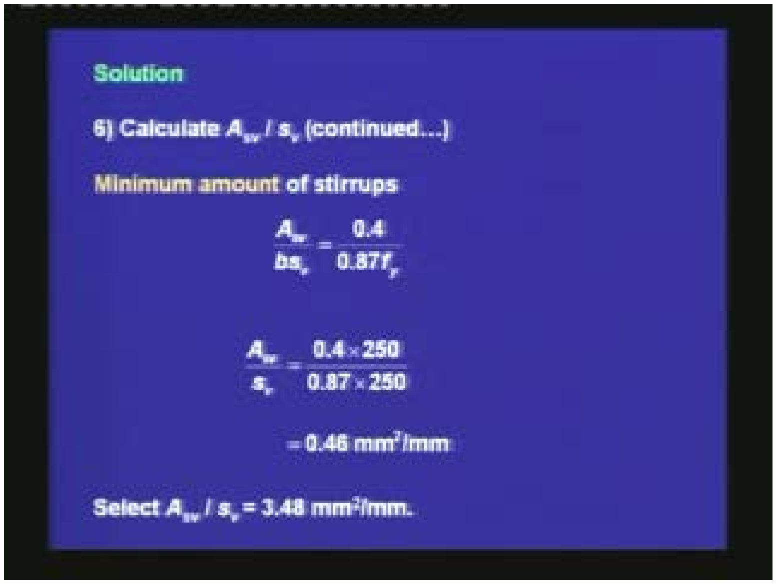

We are checking the minimum amount of stirrups, which is given as $A_{sv}/s_v = 0.4\times250 / 0.87\times250 = 0.46\ mm^2/mm$. The governing value of $A_{sv}/s_v = 3.48\ mm^2/mm$. This satisfies the minimum requirement.

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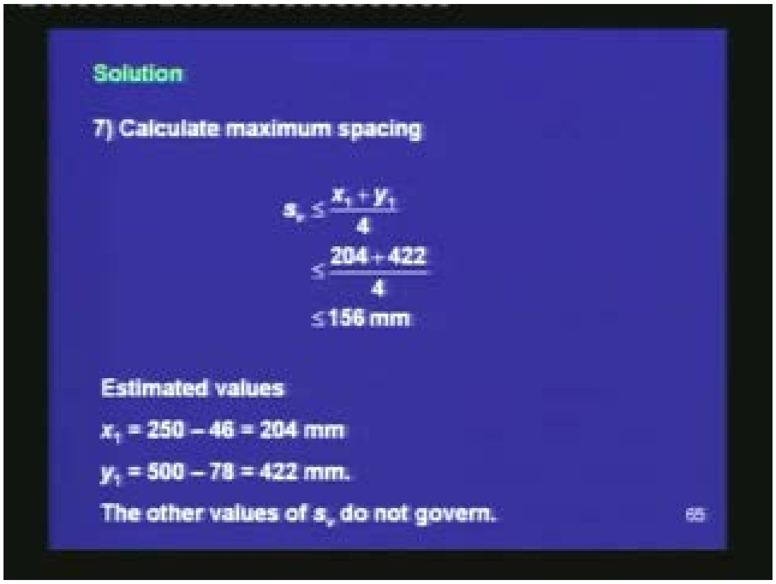

We are calculating the maximum spacing s_v. In this case, by estimating x_1 and y_1 we get maximum s_v = 156 mm. The other values of s_v do not govern.

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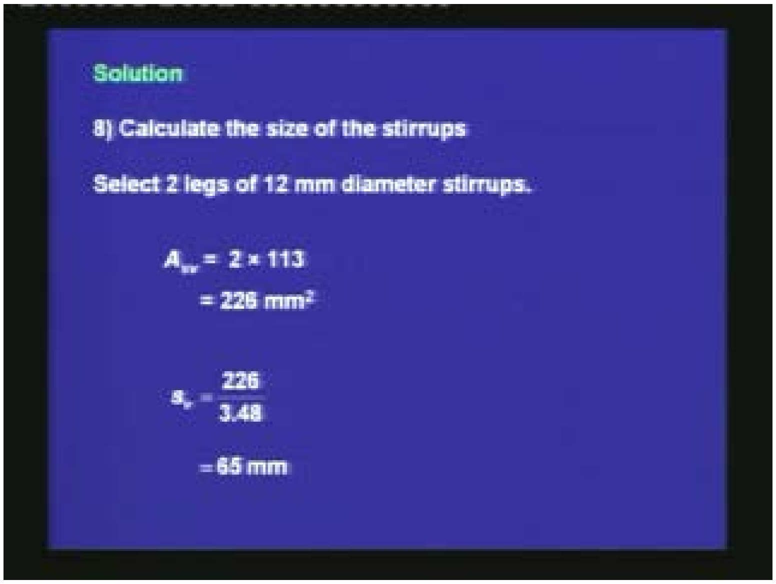

We are selecting the size as two legs of 12 mm diameter stirrups. $A_{sv} = 2 \times 113 = 226$ mm^2, from which we get s_v = 65 mm.

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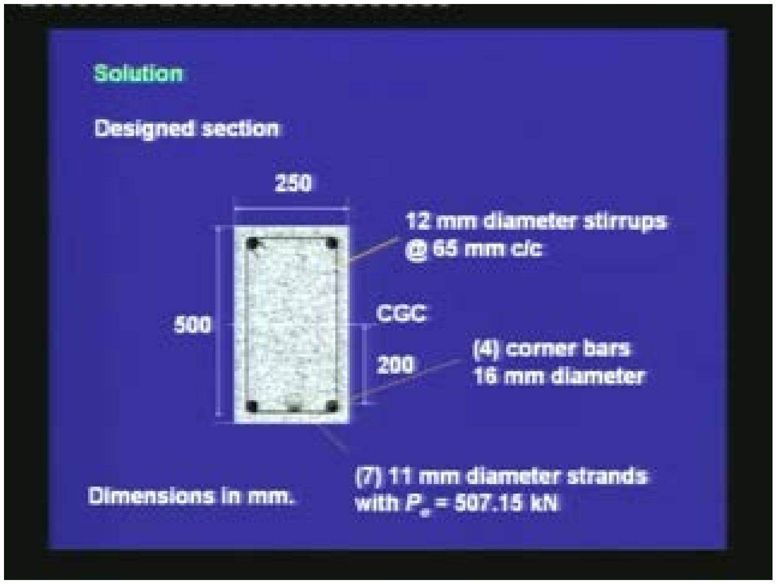

Thus, the designed section is shown here with 12 mm diameter stirrups at 65 mm on center. We are providing four corner bars of 16 mm diameter, and seven number of 11 mm diameter strands with P_e = 507.15 kN.

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Thus, today we covered the detailing requirements for torsional reinforcement. We studied the general comments on the torsional reinforcement. We also studied the design steps for torsional reinforcement. The design for the longitudinal reinforcement is similar to the flexural design for a prestressed section. The design for the transverse reinforcement is similar to the stirrup design that we had seen earlier.

With this, we are ending the design for torsion. In our next module, we shall look into the serviceability requirements for prestressed concrete sections.

Thank you.